Free-Riders and Underdogs: Participation in Corporate Voting

Finance Working Paper N°649/2020
April 2020

Konstantinos Zachariadis
Queen Mary University of London and ECGI

Dragana Cvijanovic
University of Warwick

Moqi Groen-Xu
Queen Mary University of London

ECGI Working Paper Series in Finance

Free-Riders and Underdogs: Participation in Corporate Voting

Working Paper N° 649/2020

April 2020

Konstantinos Zachariadis
Dragana Cvijanovic
Moqi Groen-Xu

We thank Mike Burkart, Cecilia Bustamante, Andras Danis, Amil Dasgupta, Alex Edmans, Daniel Ferreira, Nickolay Gantchev, Emmanuel Guerre, Sarolta Laczo, Erica Li, Claire Lim, Aniol Llorente-Saguer, Michelle Lowry, Nadya Malenko, Ernst Maug, Vikas Mehrota, Clemens Otto, Adam Reed, Jacob Sagi, Christian Strenger, Jason Sturgess, Andrea Tamoni, Christoph Van der Elst, Michela Verardo, Lingling Zheng, the seminar participants at the UNPRI, University of Amsterdam, Birkbeck College, Frankfurt School of Finance and Management, University of Gothenburg, LSE, University of Mannheim, Queen Mary University of London, Swansea, AFA, EFA, ESCP Workshop on Corporate Finance, Finance Theory Group Conference (evening session), European Summer Symposium in Financial Markets (Corporate Finance, evening session), and ABFER, CEPR and CUHK Annual Symposium for their comments. We thank Maya Jalloul for her research assistance and Pedro Saffi for sharing his data with us. Groen-Xu and Zachariadis express their gratitude to STICERD at the LSE for financial support. K.Z. and M.G. wrote the manuscript and analysed the data. K.Z. developed the model and the estimation methods. D.C. performed the sample splits and contributed substantially towards the analysis and writing of earlier drafts.

Abstract

Voting outcomes can differ from underlying preferences due to selection into voting. One source of such selection is lower participation of shareholders with popular preferences (free-rider effect) relative to that of those with unpopular preferences (underdog effect). We illustrate these strategic effects in a rational choice model in which the voting participation decision depends on the probability of being pivotal and the costs and benefits of voting. Based on the model, we structurally estimate unobservable shareholder preferences in US data. We show that strategic selection into voting is relevant: 13% of voting outcomes in shareholder governance proposals represent the minority.

Keywords: voting participation, corporate governance, shareholder proposals, shareholder democracy, structural estimation

JEL Classifications: D72, G23, G34, G38

Konstantinos Zachariadis*
Associate Professor
Queen Mary University of London, School of Economics and Finance
Mile End Road
London E1 4NS, United Kingdom
phone: +44 207 882 8698
e-mail: k.e.zachariadis@qmul.ac.uk

Dragana Cvijanovic
Associate Professor of Finance
University of Warwick, Warwick Business School
Gibbet Hill Rd
Coventry, CV4 7AL, United Kingdom
e-mail: Dragana.Cvijanovic@wbs.ac.uk

Moqi Groen-Xu
Senior Lecturer
Queen Mary University of London, School of Economics and Finance
Mile End Road
London E1 4NS, United Kingdom
phone: +44 77 66 22 75 43
e-mail: moqi.xu@qmul.ac.uk

*Corresponding Author

Free-riders and Underdogs: Participation in Corporate Voting*

Konstantinos E. Zachariadis§ Dragana Cvijanović¶ Moqi Groen-Xu‖

April 2020

Abstract

Voting outcomes can differ from underlying preferences due to selection into voting. One source of such selection is lower participation of shareholders with popular preferences (free-rider effect) relative to that of those with unpopular preferences (underdog effect). We illustrate these strategic effects in a rational choice model in which the voting participation decision depends on the probability of being pivotal and the costs and benefits of voting. Based on the model, we structurally estimate unobservable shareholder preferences in US data. We show that strategic selection into voting is relevant: 13% of voting outcomes in shareholder governance proposals represent the minority.

Keywords: voting participation; corporate governance; shareholder proposals; shareholder democracy; structural estimation

*We thank Mike Burkart, Cecilia Bustamante, Andras Danis, Amil Dasgupta, Alex Edmans, Daniel Ferreira, Nickolay Gantchev, Emmanuel Guerre, Sarolta Laczo, Erica Li, Claire Lim, Aniol Llorente-Saguer, Michelle Lowry, Nadya Malenko, Ernst Maug, Vikas Mehrota, Clemens Otto, Adam Reed, Jacob Sagi, Christian Strenger, Jason Sturgess, Andrea Tamoni, Christoph Van der Elst, Michela Verardo, Lingling Zheng, the seminar participants at the UNPRI, University of Amsterdam, Birkbeck College, Frankfurt School of Finance and Management, University of Gothenburg, LSE, University of Mannheim, Queen Mary University of London, Swansea, AFA, EFA, ESCP Workshop on Corporate Finance, Finance Theory Group Conference (evening session), European Summer Symposium in Financial Markets (Corporate Finance, evening session), and ABFER, CEPR and CUHK Annual Symposium for their comments. We thank Maya Jalloul for her research assistance and Pedro Saffi for sharing his data with us. Groen-Xu and Zachariadis express their gratitude to STICERD at the LSE for financial support. K.Z. and M.G. wrote the manuscript and analysed the data. K.Z. developed the model and the estimation methods. D.C. performed the sample splits and contributed substantially towards the analysis and writing of earlier drafts.

§School of Economics and Finance, Queen Mary University of London. *e-mail*: k.e.zachariadis@qmul.ac.uk (*corresponding author*).

¶Warwick Business School, University of Warwick. *e-mail*: dragana.cvijanovic@wbs.ac.uk.

‖School of Economics and Finance, Queen Mary University of London. *e-mail*: moqi.xu@qmul.ac.uk.

1 Introduction

Shareholders have heterogeneous preferences over how their firms should be managed (Bolton, Li, Ravina, and Rosenthal (2018); Li, Maug, and Schwartz-Ziv (2019)). Corporate voting mechanisms aim to align managerial actions with the majority of shareholders by aggregating shareholders heterogeneous preferences on corporate proposals. However, proponents of shareholder voting often overlook that certain shareholders have discretion over their voting participation. This omission is non-trivial, given that participation is on average 72%. A potential explanation for the observed participation rates is the lower participation of shareholders with popular preferences (*free-rider effect*) relative to the participation of shareholders with unpopular preferences (*underdog effect*). Such strategic selection effects can result in voting outcomes that represent the preferences of a minority while reducing the welfare of the majority of all shareholders (i.e., including non-voters). In this paper, we examine how heterogeneity in shareholder preferences over proposals and strategic selection into voting affect corporate voting outcomes.

Interpreting corporate voting outcomes with unobservable shareholder preferences and selection effects is challenging for at least four reasons. First, direct measures of preferences on proposals and of participation decisions are not available for individual shareholders. Second, although voting outcomes aggregate the preferences of voters over proposals, they ignore selection on participation and do not represent the preferences on proposals of non-voters. Third, share price reactions might incorporate preferences of non-voters but potentially overlook non-shareholder value elements of shareholder welfare (Hart and Zingales (2017)). Fourth, proxy advisory recommendations consider shareholder welfare but can be biased and do not cater to all shareholders (Iliev and Lowry (2014)).

We overcome these challenges by examining a structural estimation of corporate voting and shareholder preferences. The core of our estimation is an extension of Myatt's (2015) political elections pivotal voter model. As in Myatt (2015), we consider costly voting between two options when voters have heterogeneous preferences. We extend Myatt (2015) to capture a key feature of the corporate setup, differences in ownership structure, which give rise to different regimes of voting participation. Specifically, we split shareholders into regular and discretionary voters. Regular voters are investment funds, which are legally required to vote, or blockholders such as family voting trusts that commit to voting. Discretionary voters are dispersed shareholders, such as hedge funds or private wealth managers, who choose whether or not to vote. There is uncertainty over the preferences on proposals of discretionary voters, while those of regular voters are known.

We solve the model with the aim of estimating its key parameters using data on US corporate governance shareholder proposals in 2003–2011. In the model, the following parameters determine a discretionary voter's participation choice: the ownership fraction of regular voters; the known popularity of against vs. for amongst them; the benefit-to-cost ratio, associated with voting, of discretionary voters; and the mean and standard deviation of the popularity of against vs. for amongst them. Differences in ownership structure, regular vs. discretionary, result in a rich set of plausible equilibria with full or no discretionary participation, in addition to the standard equilibrium in political elections with partial participation from both sides. Participation rates per equilibrium and equilibria regions are available in closed form, thus, allowing for better estimates. Our estimation algorithm chooses the preference parameters (i.e., mean and standard deviation), benefit-to-cost ratio per discretionary voter, and equilibrium with the smallest resulting distance to the data.[1] Our structural model predicts voting outcomes out-of-sample significantly better than a linear ordinary least squares (OLS) model, based on the previous literature, even when the latter includes a parsimonious set of fixed effects and explanatory variables.

Our results are as follows. First, in a key departure from the political elections setup, we illustrate how participation and voting outcomes depend on the ownership structure. Consider discretionary voters who agree with the majority of the regular voters. With a stronger majority, their side is more likely to win, and therefore, any individual discretionary voter is less likely to be pivotal. In equilibrium, agreeing discretionary voters will then free-ride on the regular voters and participate less in voting; an effect comparable to free-riding in the context of takeover bids (Grossman and Hart (1980)). In contrast, disagreeing discretionary voters turn out more with weaker support by regular voters, an underdog effect.[2] In the data, free-riding and underdog effects increase the voting support of the minority party by 23% from its base popularity within the entire shareholder population. The over-representation of the minority has an average probability of 13% of swinging the vote towards the minority preference. This probability of a non-representative voting outcome varies across proposal types, with the highest likelihood at 47%, in proposals on

[1] In the US, investment funds (representing 25% of shares, on average) have a fiduciary duty to vote and report their vote on the N-PX form. We use these shareholders as an empirical approximation of the model's regular voters.

[2] In the political science literature, where there is a single group of only discretionary voters, the underdog effect refers to the higher participation rate of supporters of the option that is (ex-ante) less popular among the electorate.

the ability to call special meetings.

Second, we show that the prevalent equilibrium in the corporate context features full participation by disagreeing voters and partial participation (i.e., a mixed strategy) by agreeing voters. This is in contrast to the standard equilibrium in political elections with partial participation on both sides. There, disagreeing voters turn out more to overcome their ex-ante disadvantageous position, to the extent that the average equilibrium outcome is a tie, the so-called full underdog effect. In contrast, the most popular equilibrium in our data exhibits a partial underdog effect: the outcome is, on average, still the favorite of the regular voters, and the underdogs participate over-proportionally despite only a small probability of swinging the vote in their favour. Because of the very different implied outcomes, identifying the equilibrium is important for predictions and counterfactuals—for example, for campaigns to reduce ex-ante uncertainty about discretionary voters' preferences, as in Lee and Souther (2018).

Third, the model exhibits how the cost and benefits of voting can affect participation and, hence, the voting outcome. This is of particular policy relevance: recent major regulatory changes such as the EU Shareholder Rights Directive have aimed to reduce the cost of voting (Pinto (2009)). Counterfactuals allow us to draw the shape of selection effects. Equilibrium outcomes vary with voting costs within the range between the full participation benchmark and the benchmark with no discretionary participation. We locate the full participation benchmark (where the population's favorite option always wins) at a counterfactual voting cost of one quarter of the US level. In contrast, maximum free-riding starts at 30 times the US level. Between these two extremes, the probability of a minority win takes an inverted-U shape in the cost of voting, with a peak at 41% at a voting cost of three times of the US level.

Fourth, a positive relationship between voting participation and the expected likelihood of a close result ("closeness") is a distinctive prediction of pivotal voter models. After all, these models relate participation directly to how likely it is for a voter to affect the outcome (i.e., to be pivotal). In contrast, such a relationship is incompatible with other models of voting participation, for example, private information-driven voting (Feddersen and Pesendorfer (1996)), which the corporate literature has mostly focused on thus far; or even civic duty/ethical reasons for voting (Feddersen and Sandroni (2006)). We show that in our data, total discretionary voter participation does not vary with realized closeness. This would seem to be proof against the pivotal motive of voting, if it were not for the two competing forces that the model highlights: free-riders vs. underdogs. Indeed, participation on the underdog side increases significantly with closeness, while participation on the free-riding side decreases significantly with it. These effects cancel each other out when added to form total participation.

A natural concern is whether non-representative voting outcomes have real effects. Any disparity between the preferences of the majority of the shareholder base and the resulting voting outcome is a distortion to shareholder democracy. As such it may dissuade investors from equity markets. This is akin to weak shareholder rights hindering investment in primary markets (LaPorta, de Silanes, Shleifer, and Vishny (1997, 1998)), and may also result in less liquid and thus less efficient secondary markets. Recently, regulators have taken action to encourage voting participation (SEC, 2015; EU, 2017) suggesting that under-participation has real effects.

Our structural estimation has advantages and disadvantages relative to the literature. The main advantage is that the model allows us to interpret corporate voting outcomes. It accomplishes this by unraveling what affects the strategic participation decision. In particular, the model formalizes the distinction between how popular a proposal is (preferences) from how important voters think winning is (benefit). Hence, for example, our theoretical setup allows for a very important (high benefit) proposal to have low voting participation when most shareholders agree; and vice versa for low importance, but high participation proposals due to disagreement. Therefore, using this distinction offered by the model, we can estimate both unobservable preferences and the benefit-to-cost ratio per voter, which allow us to shed light on observed voting outcomes.

The main disadvantage of our estimation is its reliance on a particular theoretical model. Like any model, it makes assumptions, and two are of particular importance. First, we assume that voting participation is driven by heterogeneity in preferences (private values) rather than heterogeneity in information (private information). In the latter case, the voting mechanism aggregates voters' dispersed private information (e.g., Feddersen and Pesendorfer (1996)). Arguably, information aggregation is particularly important for binding proposals such as those on Mergers & Acquisitions and Financing. In contrast, we focus on non-binding shareholder proposals on corporate governance, where shareholders are less likely to have dispersed private information about the merits of the proposal, but are more likely to disagree on principle, that is, have heterogeneous preferences. Nonetheless, we show that our results are qualitatively similar for subsamples in which information aggregation could be more relevant. Second, we assume that all discretionary voters have the same homogeneous positive cost of voting. To show that this assumption does not drive our results, we study a variant of the model with idiosyncratic costs that can range from zero to an unobserved upper bound. This instance of the model has no closed-form solution, and so the estimation is prone to common issues with numerical solutions. This numerical issue notwithstanding, using the idiosyncratic costs model, the estimation results are qualitatively very similar to our baseline case along several dimensions: the non-representative outcome probability, the participation rates, preference parameters, and benefit-to-cost ratio. Ultimately, it speaks for our baseline model that it predicts notably better than both the variant with idiosyncratic costs and the aforementioned reduced-form OLS model based on the previous literature.

Related Literature. First, we contribute to the theoretical corporate voting literature. Most of the current literature focuses on the aggregation of dispersed private information in a framework with a common value (i.e., heterogeneous information and homogeneous preferences) and no differences in the ownership structure (Maug and Yilmaz (2002); Maug and Rydqvist (2009); Bond and Eraslan (2010); Levit and Malenko (2011)). The aforementioned papers do not allow for voters to abstain and hence do not cater for a study of voting participation.

Two theoretical papers are most related to us. As the aforementioned papers, Bar-Isaac and Shapiro (2017) assume heterogeneous information and homogeneous preferences, but also consider differences in the ownership structure (i.e., blockholders and dispersed shareholders) and allow for abstention. They show that blockholders may not vote all their shares to assist the aggregation of information in the voting process. Hence, their paper has normative implications on current US regulation, which requires blockholders (i.e., regular voters in our terminology) to vote all their shares. In a departure from the previous literature, Levit, Malenko, and Maug (2019) use a setup with heterogeneous preferences to study the interplay between trading and voting. They show that when shareholders have such preferences the stock price of the firm may not reflect shareholders' welfare. However, their model does not feature abstention in the voting game and there are no differences in the ownership structure. We are the first to present a model where the corporate voting participation decision is driven by both differences in the ownership structure and heterogeneous preferences.

Our theoretical work builds on and extends the extensive literature in political elections (for overviews on participation in elections see Feddersen (2004); Geys (2006)). Our work is closer to recent pivotal voter models, which introduce aggregate uncertainty, see Krishna and Morgan (2012); Evren (2012); Myatt (2015). We contribute to this literature by introducing differences in the ownership structure that result in a richer set of equilibria to emerge in the corporate context.

Second, we contribute to a nascent literature on corporate voting participation. Van der Elst (2011) shows empirically that block ownership predicts corporate voting participation. Brav, Cain, and Zytnick (2019) document that retail voters vote more when the firm is smaller, when their ownership stake in the portfolio firm is higher and, consistent with informed choice, when the shareholder receives more information from the firm about the agenda. We contribute to this literature by building and estimating a structural model. Hence, we microfound the parameters that affect the participation decision, and estimate their effect on voting outcomes.[3]

Third, we also contribute to the literature that estimates the benefit of voting. This literature has used: the stock market reaction to the passing of proposals (Cuñat, Gine, and Guadalupe (2012, 2019); Bach and Metzger (2015); Gantchev and Giannetti (2019)); the market for votes in the equity loan market (Christoffersen, Geczy, Musto, and Reed (2007); Aggarwal, Saffi, and Sturgess (2015)); the discrepancy between stock and option prices around voting (Kalay, Karakaş, and Pant (2014)); and trading patterns following voting outcomes (Li, Maug, and Schwartz-Ziv (2019)). We develop an alternative method to estimate the benefit that relies on the voting participation decision.

Fourth, our study is close to papers that structurally estimate voting parameters in the corporate setting. Matvos and Ostrovsky (2010), in the context of director elections, show that (in the terminology of our paper) regular voters vote according to peer effects and heterogeneity in their management friendliness. Bolton, Li, Ravina, and Rosenthal (2018) use a spatial model of voting to show that regular voters vote according to their position in the ideological spectrum, that is, they have heterogeneous preferences.[4] We contribute to this literature by focusing on the participation decision and preferences of discretionary voters.

[3]The empirical literature on corporate voting shows that voting outcomes affect the decision making of firms (Thomas and Cotter (2007); Guercio, Seery, and Woidtke (2008); Cai, Garner, and Walkling (2009); Levit and Malenko (2011); Becht, Polo, and Rossi (2016)).

[4]Bubb and Catan (2018) use model-based cluster analysis and also show that mutual funds (i.e., regular voters) have heterogeneous preferences, in that they vote according to certain 'party' lines.

2 Voting in US data

We begin our analysis with some simple facts about voting participation in US firms.

2.1 Data

We use aggregate voting data from the ISS Voting Results database. For Russell 3000 firms from 2003–2011, the data provides: the voting direction (total votes for, votes against, and empty votes cast, or abstentions); the voting outcome (Pass/Fail); the appropriate base for calculating the voting outcome (for plus against, for plus against plus abstain, or outstanding), the majority rule (simple or super-majority); the recommendation of management and the ISS.

Following the literature on shareholder voting (e.g., Gordon and Pound (1993); Cuñat, Gine, and Guadalupe (2012); Bach and Metzger (2015)), we focus on shareholder-sponsored proposals to change corporate governance. Hence, we exclude other shareholder proposals, most notably director elections because they fall under different voting standards, where abstentions have a different interpretation as a "No" vote (Matvos and Ostrovsky (2010); Cai, Garner, and Walkling (2013)).[5] Moreover, we do not use management-sponsored proposals because most information sensitive proposals (e.g., on mergers, spin-offs, bonds, equity issuance, etc.) fall under this category. Our model is not suitable for cases where aggregation of dispersed private information is the main goal of voting. Finally, in tune with our theoretical analysis, we only focus on simple-majority elections —this is not very restrictive as super-majority voting contests are only 2% of our sample.

We combine the aggregate voting results with the ISS Mutual Fund Voting database, which provides the number of votes per voting direction (for, against, and abstentions) of individual investment funds for each proposal. The source for this database is the mandatory N-PX filing. We aggregate fund-level voting information at the corresponding fund-family level.

We obtain data on institutional ownership from the quarterly 13F filings collected by Thomson Reuters. Institutions that report 13F filings include investment funds, which also disclose their votes on the N-PX forms, as well as hedge funds and other asset managers. We complement this data with the ownership fraction of significant owners, by type (institutional or private), which we

[5]For our baseline analysis we also exclude shareholder proposals that are unrelated to corporate governance, e.g., on environmental and social issues (He, Kahraman, and Lowry (2019)) that are very heterogeneous and hence harder to cluster for our estimation. However, we include them in robustness checks in Section 5.4.

hand-collect from the proxy statements. In the proxy filing, which contains the voting invitation, the firms must report the ownership of blocks greater than 5%.

2.2 Summary Statistics

Table 2 presents the summary statistics. Panel A provides the number of observations per year. Our sample includes 1,460 meetings, for which shareholders submitted corporate-governance proposals. There are, on average, 1.6 such proposals per meeting and 2,307 proposals in total. Panel B presents the characteristics of the firms in our sample. Our firms are comparable to those in the samples used by other papers on shareholder meetings (e.g., Cvijanović, Dasgupta, and Zachariadis (2016)), with an average book asset value of $76 billion, leverage of 26%, and a market to book ratio of 1.7.

Panel C of Table 2 presents the summary statistics for share ownership at the meeting level. The sample firms had, on average, 873 million outstanding shares, of which, on average, institutional investors owned 72%. Among these, 25% reported their votes (N-PX shares). The blocks over 5% account for 17% of the shares. Most of these blocks belonged to institutional shareholders, amounting to a total of 16% of the shares. Private shareholders with blocks over 5% accounted for 0.8% of all shares. Finally, directors owned, on average, 2.3% of the shares.

[Insert Table 2 about here]

2.3 Mandatory voting

In the US, during our sample period 2003–2011, certain shareholders must vote their shares, while others can choose whether or not to vote. In particular, investment funds have a fiduciary duty to vote on behalf of their clients (SEC Final Rule IA-2106). This duty is enforceable for mutual funds and other registered investment management companies, which must disclose their votes on the N-PX forms. As Table 2, Panel C shows, these shareholders hold a significant fraction, 25%, of shares but not typically the majority.

To represent the participation decision accurately, we calculate discretionary participation rates excluding the N-PX shareholders' votes. To that end, we calculate the ownership fraction of the regular voters (henceforth γ) as the fraction of N-PX voters from the 13F filings. To calculate the number of votes by discretionary voters, we subtract the votes of regular (N-PX) voters from the aggregates in each category (for, against, and abstentions). These "NonN-PX" votes can come from other institutional investors such as hedge funds and pension funds as well as from individuals (such as insiders, directors, and dispersed shareholders). We then calculate discretionary participation as the number of these NonN-PX votes out of the total number of NonN-PX shares. Total participation is the number of all votes cast as a percentage of the shares outstanding, or the sum of the discretionary and regular voting participation (100% by definition).

Regular voters can also formally cast abstention votes. The number of these abstentions votes is very small: 1.1% of all N-PX votes. In our results going forward, we include the official abstention votes to the sum of total votes cast.

2.4 Voting Participation: Stylized Facts

To set the stage for our analysis, we show basic summary statistics for voting support (as a fraction of the valid base) and participation in Table 3. The base can be either the number of shares outstanding or the number of voting shares, and depends on state laws and the company charters (Bach and Metzger (2015)).

[Insert Table 3 about here]

Panel A of Table 3 shows that voting participation is non-trivial but also not full, on average. Total participation averages 72% of shares outstanding, and discretionary participation averages 66%. These percentages are substantially higher than participation in political elections, such as the 55% participation in the 2016 US presidential election. [6]

Panel B of Table 3 reports the voting direction and participation by proposal type (see Appendix B for the corresponding definitions). The proposals with the greatest support are on takeover defense (49%). Proposals on executive compensation receive the least support (27%). Discretionary voting participation ranges from 65% in compensation proposals to 67% in defense proposals.

Panel C of Table 3 shows the voting direction and participation by the type of sponsor. Proxy advisors receive the highest support (37%), and proposals by firms and coalitions the highest

[6] Broker non-votes —cases where shareholders did not give voting instructions to the broker holding the shares "in street name"— usually count towards quorum. All of our proposals achieve the simple majority quorum accounting for broker non-votes, and all except for 27 would achieve it even if broker non-votes did not count towards quorum. Broker non-votes are not counted towards the outcome of non-routine proposals such as the ones in our sample.

discretionary participation (72% and 68%, respectively). Next, we present the theoretical model on which we base our structural estimation.

3 Model

In this section, we present a rational-choice model of voting participation. In this model, which is an extension of Myatt (2015), the important variation is that we allow for differences in the ownership structure, which give rise to different regimes of voting participation. We solve the model with the intent of unraveling new effects that are unique to corporate voting and of linking these effects to certain parameters, which we then estimate using US voting data.

Setup. Consider a corporate proposal in which shareholders choose between two options, R and L. Shareholders have heterogeneous preferences over the proposal. This means that if they vote they do so according to a pre-determined preference type, R or L, regardless of others' preferences and voting participation decision. Furthermore, shareholders own in total $n+1$ voting shares, which are split between two different groups.[7] A fraction $\gamma \in [0,1)$ of the n shares belongs to regular voters, while a fraction $1-\gamma$ of n belongs to discretionary voters with a single voting share and, thus, a single vote each —where the last voting share also belongs to a discretionary voter.

Regular voters always vote. We capture their voting preference by a constant $q \in (1/2, 1)$, which is the fraction of this group who vote for R. Thus, we can think of regular voters as: either i) two subgroups (blockholders) with sizes q and $1-q$ supporting R and L, respectively; or, alternatively, ii) coalitions of dispersed shareholders who always participate and vote in proportion q and $1-q$ for R and L, respectively. The choice of R as the favorite among regular voters (i.e., $q > 1/2$) is without loss of generality.

Discretionary voters can choose to vote or not. They base their choice on: an incremental benefit $v > 0$, which represents the difference in \$/share accruing to a discretionary voter when her pre-determined type wins vs. loses; and an opportunity cost $c > 0$, which they face when they vote, regardless of the outcome. We assume all discretionary voters are risk neutral and share the

[7] The number of voting shares n can be thought of as the market capitalization of the firm divided by the average holdings in that firm; for example, in a firm with \$10M market capitalization and \$10K average holdings, we have $n = 1000$. For comparison, the average number of non-N-PX institutions in our sample is 829 (see Table 4).

same v and c, regardless of their preference type R or L.[8]

The benefit v is private to a discretionary voter. Such so called 'private values' can still be reflected in the firm's stock market price depending on the preferences of the marginal post-voting trader vis-à-vis those of the marginal voter.[9] Possible sources of private values amongst investors are different incentives (Chevalier and Ellison (1999)), portfolio concerns (Cohen and Schmidt (2009)), or diverse ideologies (Bolton, Li, Ravina, and Rosenthal (2018)). Moreover, private values are observationally equivalent to a common value (e.g., caring only for the stock price) when investors: either i) 'agree to disagree', that is, have heterogeneous priors and do not learn, as in asset pricing models (Harrison and Kreps (1978)); or ii) receive 'extreme' information, so they again do not learn from the voting contest (Feddersen and Pesendorfer (1997); Yilmaz (2000)).

The cost c captures: i) the administrative cost of voting (e.g., time and effort, see Section 7 for a discussion); and ii) the opportunity cost to keeping one's shares and voting. A significant example of the latter is loan fees earned in the shares lending market. As Porras Prado, Saffi, and Sturgess (2016, pp. 3212-3213) mention "*...[i]nvestors may trade off the income from lending with the potential risks of losing monitoring control through transferring shares to the equity lending market...*". Indeed, loan fees are higher around voting record dates, see Aggarwal, Saffi, and Sturgess (2015, Figure 1). Our model captures these fees via cost c.

Now, among discretionary voters, option R has ex-ante popularity $p \in (0,1)$. The crux of the model is that p is unknown (in contrast to q), with strictly positive density f in $(l,h) \subseteq (0,1)$, and mean $\overline{p}$. Discretionary voters also face an 'availability' shock: even if they decide to vote they will ultimately cast a vote with random probability a, which has density g in $(0,1]$ and a mean of $\overline{a}$; p and a are independent random variables, while q is known.

Upon voting, the outcome is the simple-majority of the votes cast, and in the case of a tie, a fair coin toss is the tie-breaker. All the information above is common knowledge. The only choice variable (strategy) is whether a discretionary voter votes. We look for symmetric strategies across preference types R or L of discretionary voters, and the solution is determined by the Bayesian

[8]This latter assumption primarily assists identification in the estimation. As it will become clear in the analysis, what really matters is the ratio v/c. Hence, our extension with idiosyncratic costs in Section C of the Appendix addresses also (indirectly) any idiosyncrasies in v.

[9]A full analysis of the trading game is out of the scope of this paper; for coverage of post-voting trading —in a model that, however, does not feature abstention in the voting game— see Levit, Malenko, and Maug (2019, Section 7.3). Our model assumes that any information on future actions, including trading, is common knowledge and already embedded in the parameters discretionary voters use to decide on their participation.

Nash Equilibrium. Note that for $\gamma = 0$ (i.e., no regular voters) we coincide with the model of Myatt (2015). For a further discussion of the model's assumptions, particularly, in relation to our estimation, see Section 8. We now proceed to the model's solution.

Primitives. For discretionary participation to be possible, we rule out the case in which either type of regular voter can decide the outcome unilaterally. Since $q > 1/2$, we need only assume:

A1: $\gamma < 1/(2q)$.

Consider a focal discretionary voter (shareholder) of type $i \in \{R, L\}$. Let b_R and b_L be the votes of nonfocal discretionary voters for each option. Then, the total votes for R are $b_R + q\gamma n$, and for L, they are $b_L + (1-q)\gamma n$. The focal shareholder is pivotal if either: i) her type is losing by one vote; she pushes the score to a tie and the coin toss is favorable (with a probability of 1/2); or ii) there is already a tie; the coin toss is against her type and with her vote, she gives a clear majority to her type; that is,

$$\Pr[\text{Pivotal}|R] = \frac{\Pr[b_R + q\gamma n = b_L + (1-q)\gamma n] + \Pr[b_R + q\gamma n - 1 = b_L + (1-q)\gamma n]}{2},$$

$$\Pr[\text{Pivotal}|L] = \frac{\Pr[b_R + q\gamma n = b_L + (1-q)\gamma n] + \Pr[b_R + q\gamma n + 1 = b_L + (1-q)\gamma n]}{2}.$$

The shareholder votes if $v\Pr[\text{Pivotal}|i] > c$ or $\Pr[\text{Pivotal}|i] > c/v$ and does not vote otherwise, for $i \in \{R, L\}$. Hence, for any participation to be possible, we also assume that the cost should not be higher than the benefit:[10]

A2: $v \geq c$.

Now, if

$$\Pr[\text{Pivotal}|i] = \frac{c}{v} \tag{1}$$

for either type $i \in \{L, R\}$, then that type is indifferent between voting or not and follows a mixed

[10]We can strengthen this requirement to $v \geq 2c$ as a "benevolent dictator" would enforce it, but this would not significantly change our subsequent calculations.

strategy, and we have partial participation for i.

Large Elections. As Myatt (2015) notes, the pivotal probabilities are cumbersome to calculate unless n is large. Let t_R and t_L denote discretionary voter participation rates, depending on the shareholders' type. Below, we present the pivotal probabilities as approximated for large elections and the case in which a is equal to $\overline{a}$ (i.e., g is degenerate). The proof appears in Appendix A.

Lemma 1 (**Pivotal Probabilities**). *Assume that $g(a) = \delta(a - \overline{a})$ —that is, the Dirac function— and A1, then the pivotal probabilities for L and R in large elections are approximately:*

$$\Pr[\text{Pivotal}|L] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(t_R+t_L)} f(p^*)(1-p^*), \tag{2}$$

$$\Pr[\text{Pivotal}|R] \approx \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}\overline{p}(t_R+t_L)} f(p^*)\, p^*, \tag{3}$$

where

$$p^* \equiv \frac{t_L}{t_R+t_L} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}(t_R+t_L)}. \tag{4}$$

The value p^* is the average probability of support for R among the discretionary voters, for which the total average support for R and L are equal; that is,

$$\overline{a}(1-\gamma)p^* t_R + \gamma q = \overline{a}(1-\gamma)(1-p^*)t_L + \gamma(1-q). \tag{5}$$

Although (2) and (3) are approximations, we use them as equalities in what follows. In that sense, we are looking at *approximate equilibria*, as defined in Myatt (2015, p. 10). Note that since p^* is a probability, it should be in $(0,1)$, and, hence, we can see from (4) that t_L cannot be zero:

Corollary 1. *There is no equilibrium where discretionary voters of type L do not vote, so $t_L \neq 0$.*

Hence, ruling out trivial equilibria (with $t_L = 0$ where R wins), there are six possible equilibria to compute $t_L \in \{(0,1),1\}, t_R \in \{0,(0,1),1\}$.

For each equilibrium, we follow the steps: i) if t_L and/or t_R are a 'corner' solution (i.e., equal to one or also zero for t_R), then we derive regions in terms of parameters n, γ, and $v/(cn)$ by imposing the relevant condition for the corresponding pivotal probability (i.e., $\Pr[\text{Pivotal}|i] \lesseqgtr c/v$, $i \in \{L, R\}$); ii) if t_L and/or t_R are internal (i.e., strictly between zero and one), then we solve for the rate(s) using (1); iii) we supplement the parameter regions so that the computed internal rates (if any) are well defined; assumptions A1 and A2 are satisfied; and the probability in (4) $p^* \in (l, h)$. To illustrate, in the proposition below, we present the equilibrium with incomplete participation by both types; that is, both L and R employ a mixed strategy, and so we designate such an equilibrium as mm (the proof appears in Section A in the Appendix). We single out equilibrium mm, as it is the centerpiece of the analysis in political elections (see Myatt (2015, Proposition 1)).

Proposition 1 (**Equilibrium** mm)**.** *Assume that $g(a) = \delta(a - \overline{a})$ —that is, the Dirac function— and A1–A2. If*

$$n \in N_{mm} \equiv \left(\frac{f(\overline{p})\left(\overline{a}(1-\overline{p}) + 2q - 1\right)}{\overline{a}(2q-1)}, \infty \right), \tag{6}$$

$$\gamma \in \Gamma_{mm} \equiv \left(\frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1} \right), \tag{7}$$

$$\frac{v}{cn} \in V_{mm} \equiv \left(\frac{\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}, \min\left\{ \frac{\left(\overline{a} - \gamma\left(\overline{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{\left(\overline{a} - \gamma\left(\overline{a} + 1 - 2q\right)\right)}{f(\overline{p})(1-\overline{p})} \right\} \right) \tag{8}$$

then there exists an equilibrium with incomplete participation by both types; that is, t_L, $t_R \in (0, 1)$ given by

$$t_L = \frac{f(\overline{p})\overline{p}}{(1-\gamma)\overline{a}} \frac{v}{cn} + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{9}$$

$$t_R = \frac{f(\overline{p})(1-\overline{p})}{(1-\gamma)\overline{a}} \frac{v}{cn} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}. \tag{10}$$

Furhtermore, in equilibrium, the probability that either L or R is pivotal is equal to the common cost-to-benefit ratio c/v (1), and the total expected votes for L and R are equal (5), i.e., $p^* = \overline{p}$, so that the expected outcome is a tie.

Our approximations work well for large n. Therefore, we assume that there are many voters and that the restriction imposed by $n \in N_{mm}$ in (6) is innocuous. Then, the incomplete participation equilibrium exists for a set of points (region) in the two-dimensional space $(\gamma, v/(cn))$ —that is, the space of the fraction of regular voters (henceforth, the regular block size) and the benefit-to-cost ratio per voter. Conditional on a large n, the infimum of Γ_{mm} in (7) is essentially zero. Hence, we cover all plausible scenarios in the data. The interval Γ_{mm} depends on parameters q, $\overline{p}$, $f(\overline{p})$, and $\overline{a}$. The interval V_{mm} in (8) depends on all the above plus γ. The region $\Gamma_{mm} \times V_{mm}$ is depicted in Figure 1, together with the corresponding regions of all other possible equilibria that we discuss in Section 3.2. We pick parameters, primarily, motivated from our data in Table 4 and estimations in Table 2: $p \sim \mathcal{U}[l, h]$, $h - l = 0.5$, $q = 0.8$, $\overline{a} = 1$, and values $\overline{p} = 0.3$ and 0.7; the first choice of $\overline{p}$ corresponds to disagreement between the (majority of) regular and the (majority of) discretionary voters and the second to agreement.[11]

Where do the parameter restrictions (6)–(8) in Proposition 1 stem from? A large n guarantees that the lower bound on γ does not exceed the upper bound, and, hence, an equilibrium with incomplete participation exists. The lower bound on γ guarantees that v/c is higher than one (see A2; equivalently $v/(cn) \geq 1/n$), and, hence, some participation is possible. The upper bound on γ guarantees that the lower bound on $v/(cn)$ does not exceed the upper bound. The lower bound on $v/(cn)$ guarantees that the participation rate of discretionary voters, who support the favourite of the regulars, i.e., t_R, is positive. The upper bound on $v/(cn)$ guarantees that neither of the types participates fully. In summary, for (large enough n and) $(\gamma, v/(cn)) \in \Gamma_{mm} \times V_{mm}$, discretionary participation is strictly between zero and one for both types L and R.

3.1 Comparative Statics

Before we begin our discussion, note that, in equilibrium mm, average participation of discretionary voters $\bar{t} \equiv \overline{\alpha}\,(t_R\overline{p} + t_L(1 - \overline{p}))$ and average total participation $\bar{t}_{\text{total}} \equiv \gamma + (1-\gamma)\bar{t}$ are given by

$$\bar{t}_{\text{disc}} = \frac{2\overline{p}(1-\overline{p})f(\overline{p})}{1-\gamma}\frac{v}{cn} + \frac{(2q-1)(1-2\overline{p})}{1-\gamma}\gamma, \tag{11}$$

$$\bar{t}_{\text{total}} = 2\overline{p}(1-\overline{p})f(\overline{p})\frac{v}{cn} + \left((2q-1)(1-2\overline{p}) + 1\right)\gamma. \tag{12}$$

Now, the main selection effects —the underdog and free-riding effects— are visible in the formulas for the rates (9)–(12). All rates are the sum of two terms. An intragroup term, also present in Myatt (2015) (i.e., the instance of our model where $\gamma = 0$), captures the interactions among discretionary voters. An intergroup term, unique to our corporate setup, captures interactions between regular and discretionary voters.

[11] In both graphs, the maximum value of γ is $1/(2q)$ (see A1) and we set the maximum of $v/(cn)$, arbitrarily, to 2.5 (i.e., the blue region theoretically extends to plus infinity).

We focus on the intergroup terms first. Note that L is the least populous option —that is, the underdog among regular voters given our innocuous assumption that $q > 1/2$. Then, the stronger the favoritism is for R among the regular voters, the more the supporters of the underdog L among discretionary voters will participate (i.e., $\partial t_L/\partial q > 0$ in (9)). This is the *intergroup* (in contrast to intragroup) underdog effect. In turn, the stronger the favoritism is for R among regular voters, the less the supporters of that favorite R among the discretionary voters will participate (i.e., $\partial t_R/\partial q < 0$ in (10)) —that is, the (intergroup) free-rider effect. Both of these effects are combined in discretionary participation and in total participation. The effect that dominates depends on the identity of the underdog/favorite among the discretionary voters. If the discretionary voters also, on average, prefer R, there is *agreement* between the discretionary and regular voters. Then, discretionary and total participation decrease, as the free-riding effect dominates the underdog effect (i.e., $\partial \bar{t}_{\text{disc}}, \bar{t}_{\text{total}}/\partial q < 0$ in (11) and (12) if $\overline{p} > 1/2$). In contrast, discretionary and total participation increase if there is *disagreement* and the underdog effect dominates (i.e., $\partial \bar{t}_{\text{disc}}, \bar{t}_{\text{total}}/\partial q > 0$ in (11) and (12) if $\overline{p} < 1/2$).

Now, we turn to the intragroup terms (present also in the incomplete participation equilibrium of Myatt (2015, Proposition 1)). More average $\overline{p}$ (support for R among the discretionary voters) increases t_L and decreases t_R; this is the intergroup underdog effect, which is standard in political elections. Moreover, $\bar{t}_{\text{disc}}$ and $\bar{t}_{\text{total}}$ increase with (a measure of) closeness among discretionary voters —that is, $-|\overline{p} - 1/2|$— capturing the fact that (ignoring regular voters) close elections command more participation. All rates: i) decrease with the size of the electorate n, since in a larger pool of shareholders each voter has a smaller probability of being pivotal; ii) increase with the concentration of the ex-ante beliefs around the mean $f(\overline{p})$; and iii) increase with the benefit-to-cost ratio v/c. As expected, the average availability shock $\overline{a}$ decreases both t_L and t_R and does not affect $\bar{t}_{\text{disc}}$ and $\bar{t}_{\text{total}}$. Finally, the first parts of all rates increase with γ, capturing the fact that more regular voters means fewer discretionary voters.

The parameters of our model also affect the region $\Gamma_{mm} \times V_{mm}$ of the incomplete participation equilibrium. For example, for a large regular block size γ (or support for R among regular voters q), the length of the interval of permissible values for the benefit-to-cost ratio per voter diminishes. Intuitively, for high γ, high values of $v/(cn)$ (which were permissible for lower values of γ) lead to full participation for voters of L, while low values of $v/(cn)$ (which were permissible for lower values of γ) lead to no participation for voters of R. Geometrically, this result means that the region of equilibrium mm (which is not orthogonal) becomes narrower as γ (or q) increases, see Figure 1 and expression (7). Moreover, the region is larger when there is disagreement (i.e., $\overline{p} < 1/2$), capturing the greater likelihood of a tie in this case.

[Insert Figure 1 about here]

3.2 Equilibria

Table 1: **List of All Possible Equilibria**

Eqm.	Prop.	t_L	t_R	$\gamma \in$	$v/(cn) \in$	Avg. outcome
mm	1	$\in (0,1)$, (9)	$\in (0,1)$, (10)	Γ_{mm}, (7)	V_{mm}, (8)	Tie
$1m$	3	$=1$	$\in (0,1)$, (OA.4)	Γ_{1m}, (OA.2)	V_{1m}, (OA.3)	Tie/Right
10	4	$=1$	$=0$	Γ_{10}, (OA.6)	V_{10}, (OA.7)	Right
11	5	$=1$	$=1$	Γ_{11}, (OA.9)	V_{11}, (OA.10)	Left/Tie/Right
$m1$	6	$\in (0,1)$, (OA.14)	$=1$	Γ_{m1}, (OA.12)	V_{m1}, (OA.13)	Left
$m0$	7	$\in (0,1)$, (OA.18)	$=0$	Γ_{m0}, (OA.16)	V_{m0}, (OA.17)	Right

We now proceed to all other possible equilibria, which we cover in detail in the Online Appendix.[12] Table 1 reports for each equilibrium the pair (t_L, t_R) (in columns 2 and 3, respectively) and the corresponding regions in the space $(\gamma, v/(cn))$ (in columns 4 and 5, respectively), referring the reader to the explicit formulas in the relevant propositions (column 2) and equations. It can be shown that regions $\Gamma \times V$ are non-overlapping (a pictorial representation appears in Figure 1); hence, given specific parameter values, the equilibrium prediction is unique (if an equilibrium exists at all for those parameters). Moreover, outside of the parameter regions of the equilibria in Table 1, there are no equilibria in which the two types use symmetric strategies (pure or mixed).

The name of the equilibrium (column 1) denotes the participation by L and R, where m stands for mixed, 1 for full, and 0 for no discretionary participation of that type.[13] For equilibria in which only one side uses a mixed strategy (i.e., $1m$, $m1$, and $m0$), we need an additional assumption regarding the distribution of p to obtain closed-form expressions for the participation rate of that type. In particular, we assume that $p \sim \mathcal{U}[l, h]$, i.e., uniform; throughout, we also maintain the assumption that $g(a) = \delta(a - \overline{a})$.

[12] Available at `https://bit.ly/2txlqgk`.

[13] For all equilibria, we posit that n is large enough so that the equilibrium requirements are met, which is sensible since our approximations work well for large n, and empirically, the number of voters is rarely small.

Now, define the voting outcome with discretionary participation:

$$O_{\text{disc}}(p) \equiv \overbrace{\gamma q + \overline{a}(1-\gamma)pt_R}^{\text{total support for } R} - \overbrace{[\gamma(1-q) + \overline{a}(1-\gamma)(1-p)t_L]}^{\text{total support for } L} = \overbrace{\gamma(2q-1)}^{\text{regular voters}} + \overbrace{\overline{a}(1-\gamma)\left[t_R p - t_L(1-p)\right]}^{\text{discretionary voters}}. \tag{13}$$

$O_{\text{disc}}(p)$ is the difference between total support for R and L when the ex-ante support for R among discretionary voters is p. Alternatively, in the second equality, we decompose the outcome to the contribution of regular and discretionary voters. For $O_{\text{disc}}(p) > 0$, R wins; for $O_{\text{disc}}(p) = 0$, there is a tie; and for $O_{\text{disc}}(p) < 0$, L wins. From the definition of p^* (see 5), we know that $O_{\text{disc}}(p^*) = 0$. Moreover, we can compute the average outcome of a voting contest as $O_{\text{disc}}(\overline{p})$ (i.e., the outcome for the average p). As we show in Proposition 1, for all $(\gamma, v/(cn)) \in \Gamma_{mm} \times V_{mm}$ the average outcome in equilibrium mm is a tie, since $\overline{p} = p^*$. However, this is not true for the other equilibria listed in Table 1. For example, there are $(\gamma, v/(cn)) \in \Gamma_{1m} \times V_{1m}$, where in equilibrium $1m$ we have a tie and others where R wins, both on average. To highlight this, for each equilibrium in Table 1, we mention the possible average outcomes (column 7), with the added simplifying assumption that $\overline{a} = 1$. Hence, each average outcome can be consistent with several equilibria: L with equilibria 11 and $m1$; R with equilibria $1m$, 10, 11, and $m0$; and a tie with equilibria mm, $1m$, and 11; so observing the average outcome of a voting contest does not lead to a unique equilibrium prediction.

Note that in Myatt (2015, Proposition 2) —that is, the instance of our model for $\gamma = 0$— we have only equilibria mm, $1m$ (or $m1$ depending on whether $\overline{p}$ is smaller or larger than $1/2$) and 11. So if we focus on the line $\gamma = 0$, then the equilibria intervals in terms of $v/(cn)$ are identical between disagreement and agreement —with the change that $m1$ is replaced by $1m$ (see Figure 1). Hence, equilibrium $1m$ does not exist for $\gamma = 0$ and disagreement. The intuition is that for $\gamma = 0$ and disagreement, R supporters are the (intergroup) underdogs (since there are no regular voters); alas, there cannot be an equilibrium in which the supporters of the "underdog" participate incompletely, while those of the favorite (L in this case) participate fully. In general, the inclusion of regular voters (i.e., $\gamma > 0$) not only enhances the space where certain equilibria exist, but also results in a richer set of strategies for the discretionary voters (e.g., equilibrium $m0$).

Aggregate uncertainty regarding p plays a crucial role. First, it is essential for sustaining equilibria with some participation; that is, it is necessary that $l < h$, otherwise, the regions $\Gamma \times V$

are empty. The intuition is that if voters were certain of the ex-ante preferences, then their perceived probability of being pivotal would almost always be zero, and, hence, their incentives to participate would be diminished. Furthermore, even when $l < h$, not all equilibria necessarily exist (see Propositions 1 and 3–7): i) if $l > 1/2$ (strong agreement, omitted for brevity from Figure 1), then equilibria 11 and $m1$ do not exist (all others do); ii) if $l < 1/2$ and $\overline{p} > 1/2$ (agreement in Figure 1), then equilibrium $m1$ does not exist (all others do); while iii) if $\overline{p} < 1/2$ (disagreement in Figure 1), then all equilibria exist. The intuition is that for supporters of the regular voters' favourite R to show up fully (i.e., $t_R = 1$) it is necessary that at least some (positive mass of) discretionary voters prefer L. In general, more disagreement (i.e., $\overline{p} < 1/2$) should result in less of the free-riding effect (e.g., with disagreement, equilibrium $1m$ occupies a smaller region, see Figure 1) and (OA.2)).

Importantly, (intergroup) underdog and free-riding effects are weakly present in all equilibria; that is, t_R (weakly) decreases in q, while t_L increases. This is also the case for the intragroup underdog effect since across all equilibria, t_R (weakly) decreases in $\overline{p}$, while t_L increases. Finally, the rest of the effects manifest, as well: all rates (weakly) increase in the benefit-to-cost per voter ratio $v/(cn)$ and decrease in the number of voters n and in dispersion $h - l$. However, note that although all these effects are monotonic (for a given equilibrium), they are not linear. All the aforementioned observations guide our estimation process, which we discuss in the next section.

4 Estimation

4.1 Identification

For each proposal, we observe the following parameters in ex-post voting data: γ, the fraction of regular voters; q, the fraction of regular voters in support of their favorite option R; $dSuL$, the discretionary support of type L voters among those who vote; and $dSuR$, the discretionary support of type R voters among those who vote.[14] In the data, we standardize as R the direction (for or against) that is most popular, on average, among the regular voters for a given proposal type. Table 4 reports univariate statistics for these input variables.

We would like to estimate the following unobservable parameters: $v/(cn)$ (the benefit-to-cost ratio per voter); $\overline{p}$ (the average fraction of discretionary voters in support of R); and std(p) (the

[14]Note that $dSuL$ and $dSuR$ correspond to $t_L(1-p)$ and $t_R p$ in the model.

standard deviation of the fraction of discretionary voters in support of R).[15] However, we can only obtain a point estimate of $v/(nc)$ in absence of 'corner' participation (i.e., participation strictly in $(0,1)$), which is true for equilibria mm, $m1$, $m0$, and $1m$. Instead, for equilibria 11 and 10, we reach a set estimate $V = \left[v/\left(nc\right)_{\text{lower}}, v/\left(nc\right)_{\text{upper}}\right]$. This is because all $v/(nc) \in V$ in those equilibria lead to the same estimates, as the particular value of $v/\left(cn\right)$ does not affect participation rates t_L, t_R. Furthermore, we assume, throughout, that the average availability is $\overline{a} = 1$ (we show that our results are robust to varying $\overline{a}$ in Section E.2 'Availability' in the Appendix). Therefore, overall, we are estimating four parameters: $v/\left(nc\right)_{\text{lower}}$, $v/\left(nc\right)_{\text{upper}}$, $\overline{p}$, and $\text{std}(p)$.

We sort the data in terciles of γ × terciles of q × terciles of n (as an approximation, the number of non-N-PX institutions filing 13F forms) × proposal types (see Section D in the Appendix). Our unit of estimation is a bin in this quadruple-sort. For each bin we compute the average $\overline{\gamma}$; the average $\overline{q}$; and the moments $\overline{dSuL}$, $\overline{dSuR}$, $\overline{dSuL^2}$, $\overline{dSuR^2}$. Hence, given the possibility of set estimates, we use four moments to estimate four parameters; thus, our system is exactly identified.[16]

The bins are necessary because the observables $dSuR$, $dSuL$ are in a firm×year×proposal type dimension. Therefore, to compute meaningful averages for a given proposal type, we have to 'fix' the firm×year parameters of the model: γ, q and n. Hence, our *identifying assumption* is that within each bin (i.e., a tercile of γ, a tercile of q, a tercile of n, and a proposal type), unobservable $\{v/\left(cn\right), \overline{p}, \text{std}(p)\}$ are constant and the averages $\overline{\gamma}, \overline{q}$ are representative (we do not use parameter n in the estimation as it is 'absorbed' in the ratio $v/\left(cn\right)$). We, essentially, postulate that variation in p (the discretionary support for R) across proposals is the only variation that allows us to identify the bin-specific parameters. Finally, we face the following tradeoff in choosing the size of each bin: more observations within a bin make our computed moments more accurate, but also reduce the 'representativeness' of the computed $\overline{\gamma}$ and $\overline{q}$. This tradeoff does not affect our results qualitatively in robustness tests with respect to the bin size (see Section E.2 'Alternative Bins' in the Appendix).

4.2 Algorithm

The algorithm performs an exhaustive search for *every* bin. We consider a dense grid of points in the permissible space of the unobservable parameters $\{v/\left(cn\right), \overline{p}, \text{std}(p)\}$: $v/\left(cn\right)$ is positive,

[15] Given our assumption on $p \sim \mathcal{U}[l,h]$, we have $\overline{p} = (h+l)/2$ and $\text{std}(p) = (h-l)/\sqrt{12}$.

[16] Results are very similar when we include the third and fourth moments of $dSuL$, $dSuR$ (see Section E.2 'Alternative Moments' in the Appendix).

while from the above (see footnote 11), for $t_L, t_R \leq 1$, we have $\overline{p} \in \left[\overline{dSuR}, 1-\overline{dSuL}\right]$ and $\text{std}(p) \in \left[\max\left\{\text{std}(dSuR), \text{std}(dSuL)\right\}, 1/\sqrt{12}\right]$, where $\text{std}(dSuR) \equiv \sqrt{\overline{dSuR^2} - \overline{dSuR}^2}$ and $\text{std}(dSuL) \equiv \sqrt{\overline{dSuL^2} - \overline{dSuL}^2}$.[17] Given a point in the grid, the algorithm performs the following (sub)steps for *each* possible equilibrium:

i) Calculates the interval Γ and asks if $\overline{\gamma}$ belongs in it; if it does, then the calculations continue for that equilibrium. Otherwise, we proceed to the following equilibrium.

ii) If $\overline{\gamma} \in \Gamma$, then the algorithm calculates the interval V and asks if the $v/(cn)$ under consideration belongs in it; if it does, then calculations continue for that equilibrium. Otherwise, we proceed to the following equilibrium.

iii) If $\overline{\gamma} \in \Gamma$ and $v/(cn) \in V$, then the algorithm calculates t_L and t_R —using the corresponding formulas for the equilibrium under consideration— and creates the estimates

$$\overline{dSuL}_{\text{est}} = t_L(1-\overline{p}), \qquad \overline{dSuR}_{\text{est}} = t_R\overline{p},$$
$$\overline{dSuL^2}_{\text{est}} = (t_L\text{std}(p))^2 + (t_L(1-\overline{p}))^2, \qquad \overline{dSuR^2}_{\text{est}} = (t_R\text{std}(p))^2 + (t_R\overline{p})^2.$$

iv) Using these estimates, the algorithm calculates the error:

$$\begin{aligned}\text{Estimation Error} \quad = & \quad \left(\overline{dSuL}_{est} - \overline{dSuL}\right)^2 + \left(\overline{dSuR}_{est} - \overline{dSuR}\right)^2 \\ + & \quad \left(\overline{dSuL^2}_{est} - \overline{dSuL^2}\right)^2 + \left(\overline{dSuR^2}_{est} - \overline{dSuR^2}\right)^2.\end{aligned}$$

After we go through all the equilibria for all the points in the grid, in the final step, for the bin under consideration, the algorithm picks the point in the grid with the lowest estimation error. Hence, since we take an identity weighting matrix for our errors, we perform a single-step GMM (see Hansen (1982); Hansen and Singleton (1982)), referred to as 'Baseline' henceforth.[18] Recall that for fixed parameters $\{\gamma, q, v/(cn), \overline{p}, \text{std}(p)\}$, the model predicts a unique equilibrium. In addition, the algorithm picks the parameter values that minimize the estimation error using an exhaustive

[17]Note that $1/\sqrt{12}$ is the standard deviation of a uniform in $[0, 1]$.

[18]Single-step estimates are consistent but not efficient. However, as Parker and Julliard (2005, bottom of p. 193) and references therein note: "*...GMM with a pre-specified weighting matrix has superior small-sample* [as our bins are] *properties...*". Results are very similar for the two-step (efficient) GMM, simply 'GMM', estimation (see Section E.2 'Two-Step GMM' in the Appendix).

search. Hence, we can be certain that no other parameter values (in the grid) and equilibrium would result in a lower estimation error, given the data.

5 Results

5.1 Model Fit

Table 5 reports parameter estimates and quality-of-fit statistics.[19] In terms of model fit, Panel A provides the proposal-weighted model mean absolute error (MAE) for each of the moments. The baseline MAE equals 0.9% for the first moment of $dSuL$ and 0.6% for the first moment of $dSuR$. The second moment MAE is higher, with an average of 2.2% for $dSuL$ and 0.6% for $dSuR$. The MAE from a two-step (efficient) GMM, to the right, is yet higher: 3.6% (4.5%) for the first (second) moment of $dSuL$ and 3.4% (3.2%) for $dSuR$. For comparison, the proposal-weighted mean $dSuL$ is 26% and the one of $dSuR$ is 40%.

Panel B shows parameter estimates for $v/(nc)$, $\overline{p}$ and std(p). The single-step and two-step GMM estimates deliver qualitatively similar estimates with overlapping confidence intervals. In the baseline single-step estimation, the benefit-to-cost ratio per voter, $v/(cn)$, is 1.08 with a 95% confidence interval of 1.03 to 1.14. To put these numbers into context, we can estimate v, the benefit of voting, using rather primitive assumptions on c and n (see Section E.4 in the Appendix). For an average ownership stake of \$1.5 million —that is, the average holding size of insiders convicted by the SEC (Ahern (2017))— and a cost of \$1, the "return" is 2.1%. This result compares to an average return of 1.6% for the passing of governance proposals by Cuñat, Gine, and Guadalupe (2012). Assuming a higher cost linearly translates into lower returns.

Moreover, we estimate a 0.72 mean and a 0.17 standard deviation for the distribution of p (Panel B in Table 5), the fraction of discretionary voters for R. The 95% confidence intervals are 0.67 to 0.73 for the mean and 0.15 to 0.18 for the standard deviation. To set these parameters in context, we compare these estimates to q in Panel B of Table 5. The popularity of R among regular voters differs from the one of discretionary voters, on average, by 12% in absolute distance. The signed difference is 8% (i.e., $\overline{p}$ is on average greater than q), with a range of -15% to 68%. This compares in magnitude to a standard deviation of q of 15%, and a mean of 80% (recall that q is

[19] For univariate statistics for the inputs to the algorithm see Table 4.

by definition above 50%, since it measures the regular voters' preference for their favourite option, which we denote by R). The small difference between regular and discretionary voter preferences is consistent with the literature (Aggarwal, Erel, and Starks (2014)).

Our estimates imply a high turnout by L of 98% and a turnout by R of 57% (Panel C of Table 5). The large difference between the two sides implies that selection moves the voting outcome away from the population preference. We revisit this result in detail below.

We also evaluate the fit of the model by comparing the precision of out-of-sample predictions with that of a reduced form model with a parsimonious set of fixed effects (firm, year, and proposal type) and explanatory variables based on the previous literature. To be more precise, our set of explanatory variables is based on Table 3 of Malenko and Shen (2016) who link ISS recommendations to voting support. To ensure that this model is directly comparable to ours, we add the information that we use in our baseline estimation as explanatory variables: γ, q, and n. We use this model to predict total participation, $dSuL$, $dSuR$, and the outcome index $O_{\text{disc}}(p)$ in (13). We provide the estimates in Table 12. Similar to Malenko and Shen (2016), negative ISS recommendation (NegRec) predicts the outcome significantly.

To compare the prediction accuracy of our baseline model to the reduced-form model, we calculate model parameters with data up to 2010. We then use these parameters (i.e., $v/(cn)$ and the distribution of p) to predict total participation, $dSuL$, $dSuR$, and $O_{\text{disc}}(p)$ for 2011 proposals, using 2011 data for any other input needed (i.e, the proposal type, γ, n, and q). The baseline model produces smaller mean squared errors (MSE) than does the reduced-form model (Panel E in Table 5), by a factor of around two. The difference is significant according to a Diebold and Mariano (1995) test; for this test, we treat the predictions as a time series with the meeting order as a time stamp. The differences in MSE are the highest for total participation and $dSuL$. For $dSuL$, the aforementioned reduced-form model obtains MSEs twice as high as for $dSuR$, while the difference among our estimates are more comparable. This discrepancy is consistent with a prevalence of corner equilibrium outcomes on the L side (i.e., equilibrium $1m$).

5.2 Equilibria

Equilibria differ in terms of the expected outcomes and the relationship between the parameters and participation rates (see Table 1). Graphically, this means that moving in any direction leads to different consequences in each region in Figure 1. Therefore, distinguishing current and possible equilibria is important for any counterfactuals, campaigns, or regulatory actions.

Panel D in Table 5 reports the number of proposals that correspond to each equilibrium in Table 1.[20] The vast majority correspond to the $1m$ equilibrium (2,181 out of 2,305), in which the intergroup underdogs (discretionary voters for L) participate fully, and intergroup free-riders (discretionary voters for R) participate partially. In other words, discretionary voters against regular voters are more likely to participate. In contrast, only 62 proposals correspond to the $m1$ equilibrium in which the R supporters participate fully and the L supporters partially. No proposals correspond to equilibrium 10 or 11 (i.e., full participation only for L or full participation on both sides, respectively).

Recall that in political elections, the typical equilibrium is the incomplete participation equilibrium mm, in which the expected outcome is a tie. This equilibrium is the most appropriate for only 62 of our sample proposals. The predominance of the $1m$ equilibrium differentiates the corporate from the political context. The presence of differences in the ownership structure enables the corner equilibrium with complete turnout by the underdogs. For outcomes, the low incidence of the mm equilibrium means that ties are not as probable —and, in return, the absence of ties does not imply that voters are irrational or do not follow pivotality arguments (we will return to this argument below). For a campaign planner or regulator, this means that actions to increase turnout by the minority side are unlikely to shift the outcome; campaign money is better spent on changing preferences.

5.3 Selection Effects

In Table 6, we use the parameter estimates to quantify selection effects.

Full-participation and zero-participation benchmarks. In Panel A of Table 6, we compare the average estimated outcome with discretionary participation ($O_{\text{disc}}(\overline{p})$ in (32)) with the (average) outcomes under two counterfactual benchmarks: i) when only regular voters participate (that is, no discretionary participation; $O_{\text{only-reg}}$ in (34)); and ii) when all voters participate (that is, full discretionary participation; $O_{\text{full}}(\overline{p})$ in (33)). Participation by discretionary voters increases, in

[20] We 'project' our algorithm's results from the bins, where they were estimated, to the proposals within each bin.

absolute terms, the support for R (the regular voters' favorite option) by 10% relative to the zero-participation benchmark ($O_{\text{disc}}(\overline{p})$ vs. $O_{\text{only-reg}}$). This is consistent with the average agreement between regular and discretionary voters (the average $q - \overline{p}$ is 8%). In contrast, discretionary participation reduces the support for R by 23% from the benchmark with full participation. The substantial 'loss' of support for R is due to the free-riding and underdog effects: discretionary supporters of R free-ride on regular voters and participate less than the underdogs L (the regular voters' minority option).

Non-representative voting outcomes. How often can selection swing the vote? In other words, when is the outcome different to the benchmark under full discretionary participation? We compute the probability of a non-representative outcome, denoted as $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$, using Monte-Carlo simulations over p under the per-bin parameter estimates for $\overline{p}$ and $\text{std}(p)$. Any difference between these probabilities stems from the (equilibrium) participation rate t_L and t_R. According to our estimations, supporters of L —the underdogs— are over-represented (t_L is on average 98%), while supporters of R, the free-riders, are under-represented (t_R is on average 56%) relative to their popularity in the entire population of shareholders. Ultimately, our simulations reveal that free-riders and underdogs lead to outcomes that with average probability 13% and maximum 47% do not represent the majority of the entire shareholder base (Panel A of Table 6).[21] Hence, non-representative outcomes occur substantially often and may lead to the adoption of policies, which do not benefit the majority of shareholders; thus widening even more the gap between ownership and control. The probability of non-representative outcomes exhibits substantial heterogeneity —we explore this in greater detail in the next subsection.

Selection effects per equilibrium. The most stark case of over-representation of L occurs in equilibrium $1m$. In contrast, in the mm and $m1$ equilibria, the R side receives more support under (some) discretionary participation than under full participation. Overall, the probability of non-representative outcomes is 12% in $1m$, 18% in mm, and 33% in $m1$ (Panel B of Table 6. Because most of our proposals correspond to the $1m$ equilibrium, the sample average probability of non-representative outcomes is close to the average in this equilibrium.

[21] We can do a similar exercise relative to the case where only regulars participate (see Section B.2 in the Appendix). Then the outcome is different (relative to the data) with an average probability of 15%.

Probability of closeness. Given the substantial rates of discretionary participation, surprisingly few votes are close to the passing threshold —25% in the [-10%, +10%] range as reported by Cuñat, Gine, and Guadalupe (2013). Why do shareholders vote if their ex-post probability of overturning the outcome is so low? The predominance of the $1m$ equilibrium provides one explanation for the low rates of close outcomes. Our estimates allow us to compute the ex-ante probability of voting support for R, let $R_{\text{support}}(p)$, within a given range $[-x, +x]$ around the passing threshold, denoted by $\mathbb{P}\Big[R_{\text{support}}(p) \in (50\% - x, 50\% + x)\Big]$, and to compare them to their ex-post counterparts. Indeed, the ex-ante probability for the [-10%, +10%] range is higher: 38% (Panel C in Table 6) than the ex-post incidence of 33%. For a [-1%, +1%] range, the ex-ante probability is similarly higher, with 5%, than the ex-post incidence of 4%. Note that the probability of being in the [-10%, +10%] range is highest for the $m1$ equilibrium, at 50%, while it is 38% and 35% for $1m$ and mm, respectively.

Success probabilities in close votes. Understanding ex-ante odds for close votes is important for the interpretation of regression discontinuity tests that rely on the contrast between just-won and just-lost contests (e.g., Cuñat, Gine, and Guadalupe (2012, 2015, 2019); Bach and Metzger (2015); Babenko, Choi, and Sen (2018)). Our estimates imply that the probability of, say a win by R conditional on an ex-ante close contest in the range $[-x, x]$, denoted by $\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big|\, R_{\text{support}}(p) \in (50\% - x, 50\% + x)\Big]$ (see Panel C of Table 6), differs between equilibria. In the most prevalent $1m$ equilibrium, the ex-ante outcome probabilities are 78:22 (R:L) for close proposals in the [-10%,10%] range and 66:44 for those in the [-1%,1%] range. In contrast, the odds are roughly 50:50 in the mm equilibrium, in which both sides expect a tie. Taking these differences into account can improve the precision of regression discontinuity estimates.

5.4 Heterogeneity in Preferences

In this section, we recompute the probability for non-representative outcomes for different subsamples (Table 7, Panel A). The probability of a non-representative outcome differs across proposal types, ranging between an average of 8% for takeover-defense proposals, 13% for board and executive compensation proposals and 16% for other governance proposals. We compare these numbers to proposals outside our sample: management proposals as well as non-governance shareholder proposals (for example, CSR or finance). These proposals face small probabilities of outcome-preference misalignment between 3% and 4%. For all management proposals we estimate a probability of a non-representative outcome of 4%, while for say-on-pay proposals the corresponding estimate is close to zero.

The discrepancy between the shareholder proposals and the management proposals indicates that the sponsor matters. Indeed, Table 7, Panel B shows the variation in the probability of a non-representative outcome, ranging from an average of zero and 2% for corporate sponsors and employee sponsors, respectively, to 16% for coalitions and union sponsors. Proposals by individual activists also face a high probability of a non-representative outcome of 12%.

In Table 7, Panel C we report the outcomes of the estimation algorithm using bins of years instead of proposal types. Non-representative outcome probabilities appear somewhat cyclical, with higher probabilities in 2005, 2006 (13%, 12%) and 2009, 2010 (14%, 14%) and lower ones in 2003–4, 2005–6, and 2011 (in the range 6–9%).

Industries also vary according to their selection-driven non-representative outcome probabilities, as Panel D in Table 7 shows. The probability of a non-representative outcome is highest for the Energy and Information Technology (IT) sectors and lowest for Communication Services, Industrials, and Consumer Staples.

Panel E in Table 7 shows how selection effects vary with past returns. Non-representative outcome probabilities do not vary much when we we compare the population preference to actual outcomes (11% in the lowest vs. 13% in the highest tercile). However, the actual outcome differs more from that of the equilibrium with only regular voting participation for better performing firms (13% in the lowest vs. 18% in the highest tercile).

Finally, selection effects display significant variation with respect to ownership structure (Panel F of Table 7). Outcomes are more likely to be non-representative for firms with lower N-PX ownership. Accordingly, proposals in the lower tercile of N-PX ownership are more likely to be in the mm or $m1$ equilibrium. The probability of non-representative outcomes (13% on average in our whole sample) is also smaller for firms with a large (i.e., holdings of over 5%) private blockholder (9%) and those with more institutional ownership (10% for the highest, 13% for the lowest tercile). In contrast, activist ownership does not seem to matter much for any of the selection estimates.

6 Closeness and Participation

Closeness is central for voting participation decisions in pivotal voter models since it affects the likelihood of being pivotal. However, realized voting outcomes are often far from close (Cuñat, Gine, and Guadalupe (2012); Bach and Metzger (2015); Cuñat, Gine, and Guadalupe (2015)) and close votes do not have higher participation rates. These observations cast doubts on the validity of rational choice models in the context of corporate voting.

Our model provides an explanation for the flat relation between ex-post (realized) closeness and participation rates: differential effects among free-riders and underdogs. More precisely, the model predicts a positive relation between closeness and participation by underdogs, but a negative one between closeness and participation by free-riders. In particular, close votes accentuate the underdog effect (underdogs participate more in the hope of swaying the vote), while at the same time, closeness is made possible by the free-riding effect. We now test these predictions empirically.

First, we confirm that close votes do not vary significantly with participation rates. Controlling for γ, q, n, and proposal type fixed effects (Table 8, column 1), and firm as well as year fixed effects (column 2), proposals with voting support (for the favorite) of 40% to 60% are actually decreasing in discretionary participation per proposal $t_{\text{disc}} \equiv dSuR + dSuL$. The magnitudes are small, with -0.5% without firm or year fixed effects and 0.1% with them. For a narrower band between 49% and 51%, the coefficient is of similar magnitude (column 3). Total participation per proposal (column 4) $t_{\text{total}} \equiv (1-\gamma)t_{\text{disc}} + \gamma$ follows a similar pattern with negative coefficients.

However, the flat relation between closeness and participation masks the two competing forces that the model highlights. Columns 5–7 of Table 8 show a significantly negative relation between closeness and $dSuR$, the discretionary support for the favorite option of the regular voters. That is, potentially close votes exhibit more free-riding (less participation). In contrast, the relation between closeness and $dSuL$ is positive: discretionary voters participate over-proportionally against regular voters, an underdog effect. These two effects are of similar magnitudes, with 11.3% less participation by the R side in close votes (column 5) and 11.7% more participation by the L side (column 8). These coefficients, essentially, sum up to the coefficients of total discretionary participation.

The asymmetric relations between majority and minority participation and closeness are consistent with our model but difficult to reconcile with other models of voting. Information aggregation models (Feddersen and Pesendorfer (1996)) or ethical voting models (Feddersen and Sandroni (2006)) do not predict any relationship between closeness and participation rates. Thus, these results provide evidence in support of the empirical importance of our framework.

7 Counterfactuals on Cost

Here, we consider variations to the cost of voting c. Reducing the administrative cost of voting is an objective of regulators around the world (e.g., see the Shareholder Rights Directive in the European Union).[22] Notable examples of cumbersome and costly voting procedures include pre-registration requirements (e.g., in Switzerland), Power of Attorney requirements (e.g., in Sweden), the non-availability of electronic voting outside the US and Europe (Eckbo, Paone, and Urheim (2010, 2011); Council of Institutional Investors (2011)). In contrast, administrative costs of voting are likely to be small in the US, as reflected in the high participation rate among L voters that we estimate in the US data. Evidently, most firms in the US support electronic voting and do not require cumbersome paperwork to prove ownership or pre-register for voting.

The cost in our model enters through the benefit-to-cost ratio per voter $v/(cn)$. Hence, an increase in the cost, keeping v and n constant, leads to a decrease in the $v/(cn)$. Of course, our counterfactuals are equivalent to changing these ratios directly, for example, by changing v and keeping c and n constant. In Figure 3 (top panel) and Table 9, Panel A, we report $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$, that is, the probability of a non-representative outcome; and in Figure 3 (bottom panel) and Table 9, Panel B, we report the equilibrium incidence for different multiples of the cost of voting, keeping all other parameters constant.

First, for cost levels of 0.25 and 0.5 times the current US level, full participation (i.e., equilibrium 11) is the only viable equilibrium. The probability of a non-representative outcome is 34% for the 55 proposals that qualify for the full participation equilibrium.

For voting costs above the US level, the probability of a non-representative outcome peaks at 41%, for costs of three times the US level. Beyond the peak, the probability of a non-representative outcome decreases with costs. This is because costs of three times the US level lead to a high incidence of the mm equilibrium: 1,104 out of 2,305 proposals (Panel B of Table 9). Recall that in

[22] Available online at http://bit.ly/2rtcZA5.

that equilibrium, both types of discretionary voters use a mixed strategy and the average outcome is a tie. The ultimate decision occurs by a coin flip, so conditional on an mm equilibrium, the favorite loses with 50% probability. This randomness produces voting outcomes that are non-representative of the full population in 46% of all proposals. At three times the US cost level, we also start seeing more incidences of the 10 and $m0$ equilibria instead of the $1m$ equilibrium, so that discretionary support for R even falls to zero. In other words, we obtain maximum free-riding on regular voters from the discretionary voters who agree with their favorite outcome.

With increasing costs of voting, participation moves from full towards none. That is, we move from the 11 equilibrium with full participation to the $m0$ equilibrium, in which almost only regular voters and a few underdogs tend to participate. The representativeness of such high-cost elections depends on the degree to which discretionary voters agree with regular voters. Regular voters that perfectly agree with discretionary voters can represent all shareholders, even if voting is prohibitively costly for discretionary voters. Such perfect alignment of preferences would support regulations that force institutional investors to vote. However, our estimations above show that this is not always the case. The average distance between q and $\overline{p}$ is 12%, which translates into a 15% overall likelihood of an outcome different from the equilibrium with only regular voter participation (see Table 2, Panel B). In summary, the relationship between $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$ and c has an inverted-U shape.

8 Discussion and Robustness

To facilitate the interpretation of the estimations, we now point out the limitations imposed by the model's assumptions. Our model focuses on the strategic decision of whether to participate in voting taking other potential decisions as given. In particular, the model abstracts from: why shareholders make certain proposals; how they obtain their votes; choose their preference type; and arrive at their common knowledge of the model's parameters (i.e., the fraction and preferences of the regular voters and the distribution of preferences among the discretionary voters). In reality, however, these decisions are likely to be endogenous, and, hence, the participation decision may depend on factors outside our model.

Of particular interest is how shareholders obtain decision-relevant information in practice.

In the model discretionary voters have heterogeneous preferences and observe the parameters: $\{\gamma, q, \overline{p}, \text{std}(p), v, c, n, \overline{a}\}$.[23] Much of the relevant information is fairly easy to access today due to disclosure regulations: the ownership structure (i.e., γ, n), which is disclosed in the invitation to vote; voting manifestos by institutional investors (i.e., q); and recommendations of proxy advisors such as the ISS and Glass Lewis (Iliev and Lowry (2014); Malenko and Shen (2016)), which can be a proxy for average investor preferences $\overline{p}$. Also, it is reasonable to assume that voters know their cost of voting c.

Nevertheless, information can still be costly to acquire. For example, subscription services such as Proxy Insights provide aggregate voting predictions using the past voting behavior of institutional investors. Hence, shareholders may still need to acquire costly information about the benefits of the proposals (i.e., v) and/or the (dispersion of) preferences of the other shareholders (i.e., $\text{std}(p)$). From the perspective of our model any information acquisition cost is sunk at the stage of the participation decision. Moreover, our key assumption is not that discretionary voters have perfect information but that they are homogeneous in the amount of information they have. Absent heterogeneity in information, the channel we are 'shutting down' is information aggregation. In order to evaluate the robustness of our results to this channel, we perform our estimations separately in more and less information demanding proposals: early vs. late meetings; and high vs. low standard deviation of analysts' forecasts (see Section E.3 'Importance of information' in the Appendix). We show that our qualitative results remain similar in each of these subsamples.

The cost of voting c is of crucial importance in the model. Although we take it as fixed for all in our analysis, there can be idiosyncrasies even across discretionary voters (i.e., not 'too big' institutions), for example, hedge funds vs. investor groups. To capture this, we extend the model to idiosyncratic voting costs (see Section C for the analysis and Section E.1 for the estimation results, both in the Appendix). This allows us to accommodate investors even with zero cost of voting. We focus on the mm equilibrium for this extension and show that parameter estimates are qualitatively similar to our baseline ones, albeit with higher errors; thus reinforcing the robustness of the baseline setup.

Another issue of practical relevance is equity lending. Aggarwal, Saffi, and Sturgess (2015)

[23] Our estimates already take into account any actions by major stakeholders to affect those very parameters. For example investors buying shares to vote or managerial actions to affect the outcome (including campaigning, picking the location of the Annual General Meeting, etc). In essence, our identifying assumption also requires that when discretionary voters decide whether to vote these parameters are fixed and constant up until voting is over.

report a 1.9 percentage points rise in recalls by institutions around record dates, leading to a reduced supply and higher fees. In the presence of such a market, discretionary voters may decide to lend their shares instead of voting, effectively increasing the opportunity cost component of c. If voting constitutes a significant factor in the equity lending market, the demand, supply, and interest should be endogenous to shareholder preferences and the voting decision. Given that the expected benefits of voting are greater for shareholders with a preference for the underdog, they should be willing to pay more to borrow shares. This will allow them to acquire more votes, which will lead to closer votes than would occur in a world without equity lending. Closer voting outcomes can lead to more incidences of the incomplete participation equilibrium mm, which is rare in our data. However, the additional borrowing demand around the record date is small, with estimates around 0.2% (Christoffersen, Geczy, Musto, and Reed (2007)) to 0.3% (Aggarwal, Saffi, and Sturgess (2015)). Consistent with these magnitudes, our estimates do not change qualitatively even in the top quintiles of equity demand and supply (see Appendix E.3 'Equity lending').

We also make procedural assumptions that are standard in the literature (see Heard and Sherman (1987); McGurn (1989); Monks and Minow (2003) for further details about the mechanics of proxy voting). First, we assume that voting occurs simultaneously. In practice, there might be instances of early access to tabulations (Bach and Metzger (2015)). However, most voters and brokers submit their votes at the deadline to prevent access to such information and to avoid having to change their votes should they change their opinion. Second, we assume that regular voters never abstain from voting, which is consistent with the data with fewer than 1% of empty votes cast within our sample. Third, our model assumes that the vote is for a single issue/proposal —i.e., abstracts from bundling, which occurs in reality. To address this, in Appendix E.3 'Number of Proposals' we show that our results hold qualitatively even in meetings with more proposals. In Appendix E.2, we also show that our estimations are robust to alternative estimation choices: 'Excluding the $1m$ equilibrium', which is the most popular in our baseline estimations; 'Two-step GMM', which is efficient; and 'Medians instead of means' for the representative γ and q per bin.

9 Conclusion

We study the voting participation decision as a trade-off between the costs and benefits of voting. The expected benefits of voting depend on the probability that a shareholder's vote matters, which in turn depends on the action of other shareholders. In a rational choice model, we show how: shareholders with popular preferences participate less (free-rider effect) relative to those with unpopular preferences (underdog effect). As a consequence, the less popular side is over-represented in the voting results, and the outcome itself can be non-representative of the shareholder base. Based on the model, we provide a tool to estimate the preferences of the underlying shareholder base using US data. On average, support for the majority is 23% less relative to its popularity among the entire shareholder base. This results in a considerable average probability of 13% of a non-representative outcome.

Our estimation algorithm can help firms and regulators to identify which voting outcomes are representative, and which proposals are more important to shareholders. The algorithm performs very well, producing significantly smaller estimation errors than comparable models. The lower prediction power in reduced form models reflects the non-linearity (with respect to key parameters such as γ and $v/(cn)$) of the two main effects —free-riders and underdogs— that affect participation rates in opposite ways. Taking these two competing effects into account can improve the prediction power of parameters such as the voting support.

Regulators worry about potentially non-representative decisions and aim to reduce the costs of voting. Our estimates of the model show how much such reductions could affect the likelihood of non-participation effects. We show that the likelihood of non-representative outcomes is an inverted-U-shaped function of the cost of voting given the current preference parameters in the US data. Thus, decreasing the cost of voting from a very high level (above $3\times$ the US level) will first increase the probability of non-representative outcomes. However, these numbers assume that regular and discretionary voters have similar preferences, as they have in the US: for diverging interests, decreasing the cost of voting may still decrease the probability of non-representative outcomes. We locate the current US cost of voting at $4\times$ the full participation benchmark and far below the peak of the non-representation likelihood.

Most of US voting features full participation on the side of the minority, contrary to the standard intuition of the political elections literature in which both sides participate partially to the extent that the expected outcome is a tie. This is because in political elections, each voter has one vote, whereas the voting power in corporate elections depends on the ownership structure. As we exhibit, the existence and, in the case of the US, dominance of such corner solutions affects the comparative statics and characteristics of counterfactuals. For example, campaigns to increase voting participation are unlikely to sway the vote any further towards the underdog —if anything, persuading more of the majority to vote less often could.

Some investors worry that voting results only represent certain parties such as mutual funds, or the opinion of certain proxy advisors. We show how voting outcomes can be non-representative, but also document that regular and discretionary voters in US firms have similar preferences (with an average difference of 12%). Thus, mandating more investor types to vote or allowing any of them to refrain from voting would not have a significant impact via the participation channel. Indeed, both turnout and the likelihood of non-representative outcomes is lowest for firms with more N-PX ownership (regular voters in our context).

Ours is a first attempt to infer information about underlying voter preferences from participation rates —and to provide a guideline for interpreting voting support. It is a stylized and static attempt that ignores many important forces such as information aggregation, the selection into ownership structures, or the selection into the proposals that make it to the ballot. It is our hope that this study provides the foundation for future work on the interaction of these forces with the participation decision.

References

Aggarwal, R., I. Erel, and L. S. Starks, 2014, "Influence of Public Opinion on Investor Voting and Proxy Advisors," *Fisher College of Business Working Paper Series*, WP 2014-12.

Aggarwal, R., P. A. C. Saffi, and J. Sturgess, 2015, "The Role of Institutional Investors in Voting: Evidence from the Securities Lending Market," *Journal of Finance*, 70, 2309–2346.

Ahern, K. R., 2017, "Information Networks: Evidence from Illegal Insider Trading Tips," *Journal of Financial Economics*, 125, 26–47.

Babenko, I., G. Choi, and R. Sen, 2018, "Management (of) Proposals," Available at SSRN: https://ssrn.com/abstract=3155428.

Bach, L., and D. Metzger, 2015, "Why Do Shareholder Votes Matter?," *Swedish House of Finance Research Paper*.

Bar-Isaac, H., and J. D. Shapiro, 2017, "Blockholder Voting," Rotman School of Management Working Paper No. 2928820. Available at SSRN: https://ssrn.com/abstract=2928820.

Bebchuk, L. A., A. Cohen, and A. Ferrell, 2009, "What Matters in Corporate Governance?," *Review of Financial Studies*, 22, 783–827.

Bebchuk, L. A., and J. M. Fried, 2009, *Pay without Performance: The Unfulfilled Promise of Executive Compensation*, Harvard University Press.

Becht, M., J. Franks, C. Mayer, and S. Rossi, 2009, "Returns to Shareholder Activism: Evidence from A Clinical Study of the Hermes UK Focus Fund," *Review of Financial Studies*, 22, 3093–3129.

Becht, M., A. Polo, and S. Rossi, 2016, "Does Mandatory Shareholder Voting Prevent Bad Acquisitions?," *Review of Financial Studies*, 29, 3035–3067.

Bolton, P., T. Li, E. Ravina, and H. Rosenthal, 2018, "Investor Ideology," Working Paper. Available at https://ssrn.com/abstract=3119935.

Bond, P., and H. Eraslan, 2010, "Strategic Voting over Strategic Proposals," *Review of Economic Studies*, 77, 459–490.

Brav, A., M. D. Cain, and J. Zytnick, 2019, "Retail Shareholder Participation in the Proxy Process: Monitoring, Engagement, and Voting," Available at SSRN: https://ssrn.com/abstract=3387659.

Bubb, R., and E. Catan, 2018, "The Party Structure of Mutual Funds," *Available at SSRN 3124039*.

Cai, J., J. L. Garner, and R. A. Walkling, 2009, "Electing Directors," *The Journal of Finance*, 64, 2389–2421.

Cai, J., J. L. Garner, and R. A. Walkling, 2013, "A Paper Tiger? An Empirical Analysis of Majority Voting," *Journal of Corporate Finance*, 21, 119 – 135.

Campello, M., and P. A. C. Saffi, 2015, "The Rise of the Equity Lending Market: Implications for Corporate Policiess," Available at SSRN: `https://ssrn.com/abstract=2703318`.

Chevalier, J., and G. Ellison, 1999, "Career Concerns of Mutual Fund Managers," *Quarterly Journal of Economics*, 114, 389–432.

Christoffersen, S. E. K., C. C. Geczy, D. K. Musto, and A. V. Reed, 2007, "Vote Trading and Information Aggregation," *Journal of Finance*, 62, 2897–2929.

Cohen, L., and B. Schmidt, 2009, "Attracting Flows by Attracting Big Clients," *Journal of Finance*, 64, 2125–2151.

Council of Institutional Investors, 2011, "Everything You Wanted to Know About International Proxy Voting But Were Afraid to Ask," Available from: `https://www.cii.org`.

Cuñat, V., M. Gine, and M. Guadalupe, 2012, "The Vote Is Cast: The Effect of Corporate Governance on Shareholder Value," *Journal of Finance*, 67, 1943–1977.

Cuñat, V., M. Gine, and M. Guadalupe, 2013, "Corporate Governance and Value: Evidence from "Close Calls" On Shareholder Governance Proposals," *Journal of Applied Corporate Finance*, 25, 44–54.

Cuñat, V., M. Gine, and M. Guadalupe, 2015, "Say Pays! Shareholder Voice and Firm Performance," *Review of Finance*, 20, 1799–1834.

Cuñat, V., M. Gine, and M. Guadalupe, 2019, "Price and Probability: Decomposing the Takeover Effects of Anti-takeover Provisions," *forthcoming in the Journal of Finance.*

Cvijanović, D., A. Dasgupta, and K. E. Zachariadis, 2016, "Ties that Bind: How Business Connections Affect Mutual Fund Activism," *The Journal of Finance*, 71, 2933–2966.

Diebold, F. X., and R. S. Mariano, 1995, "Comparing Predictive Accuracy," *Journal of Business & Economic Statistics*, 13, 253–263.

Eckbo, B. E., G. Paone, and R. Urheim, 2010, "Efficiency of Share-Voting Systems: Report on Sweden," Available at SSRN: `https://ssrn.com/abstract=1651582`.

Eckbo, B. E., G. Paone, and R. Urheim, 2011, "Efficiency of Share-Voting Systems: Report on Italy," Available at SSRN: `https://ssrn.com/abstract=1431733`.

Evren, Ö., 2012, "Altruism and Voting: A Large-Turnout Result That Does not Rely on Civic Duty or Cooperative Behavior," *Journal of Economic Theory*, 147, 2124–2157.

Feddersen, T. J., 2004, "Rational Choice Theory and the Paradox of Not Voting," *Journal of Economic Perspectives*, 18, 99–112.

Feddersen, T. J., and W. Pesendorfer, 1996, "The Swing Voter's Curse," *American Economic Review*, 86, 408–424.

Feddersen, T. J., and W. Pesendorfer, 1997, "Voting Behavior and Information Aggregation in Elections With Private Information," *Econometrica*, 65, 1029–1058.

Feddersen, T. J., and A. Sandroni, 2006, "A Theory of Participation in Elections," *American Economic Review*, 96, 1271–1282.

Ferri, F., and D. A. Maber, 2012, "Say on Pay Votes and CEO Compensation: Evidence from the UK," *Review of Finance*, 17, 527–563.

Gantchev, N., and M. Giannetti, 2019, "The costs and benefits of shareholder democracy," *SMU Cox School of Business Research Paper*, pp. 18–15.

Geys, B., 2006, ""Rational" Theories of Voter Turnout: A Review," *Political Studies Review*, 4, 16–35.

Gompers, P. A., J. L. Ishii, and A. Metrick, 2003, "Corporate Governance and Equity Prices," *Quarterly Journal of Economics*, 118, 107–155.

Gordon, L. A., and J. Pound, 1993, "Information, Ownership Structure, and Shareholder Voting: Evidence from Shareholder-Sponsored Corporate Governance Proposals," *The Journal of Finance*, 48, 697–718.

Grossman, S. J., and O. D. Hart, 1980, "Takeover Bids, the Free-Rider Problem, and the Theory of the Corporation," *Bell Journal of Economics*, 11, 42–64.

Grullon, G., and R. Michaely, 2002, "Dividends, Share Repurchases, and the Substitution Hypothesis," *Journal of Finance*, 57, 1649–1684.

Guercio, D. D., L. Seery, and T. Woidtke, 2008, "Do Boards Pay Attention When Institutional Investor Activists "Just Vote No"?," *Journal of Financial Economics*, 90, 84 – 103.

Hansen, L. P., 1982, "Large Sample Properties of Generalized Method of Moments Estimators," *Econometrica: Journal of the Econometric Society*, pp. 1029–1054.

Hansen, L. P., and K. J. Singleton, 1982, "Generalized Instrumental Variables Estimation of Nonlinear Rational Expectations Models," *Econometrica: Journal of the Econometric Society*, pp. 1269–1286.

Harrison, J. M., and D. M. Kreps, 1978, "Speculative Investor Behavior in a Stock Market with Heterogeneous Expectations," *Quarterly Journal of Economics*, 92, 323–336.

Hart, O., and L. Zingales, 2017, "Companies Should Maximize Shareholder Welfare Not Market Value," *Journal of Law, Finance, and Accounting*, 2, 247–275.

He, Y., B. Kahraman, and M. Lowry, 2019, "ES Risks and Shareholder Voice," *Available at SSRN 3284683*.

Heard, J. E., and H. D. Sherman, 1987, *Conflicts of Interest in the Proxy Voting System*, Investor Responsibility Research Center.

Iliev, P., and M. Lowry, 2014, "Are Mutual Funds Active Voters?," *Review of Financial Studies*, 28, 446.

Kalay, A., O. Karakaş, and S. Pant, 2014, "The Market Value of Corporate Votes: Theory and Evidence from Option Prices," *Journal of Finance*, 69, 1235–1271.

Knoeber, C. R., 1986, "Golden Parachutes, Shark Repellents, and Hostile Tender Offers," *American Economic Review*, 76, 155–167.

Krishna, V., and J. Morgan, 2012, "Voluntary Voting: Costs and Benefits," *Journal of Economic Theory*, 147, 2083–2123.

LaPorta, R., F. L. de Silanes, A. Shleifer, and R. W. Vishny, 1997, "Legal Determinants of External Finance," *Journal of Finance*, 52, 1131–1150.

LaPorta, R., F. L. de Silanes, A. Shleifer, and R. W. Vishny, 1998, "Law and Finance," *Journal of Political Economy*, 106, 1113–1155.

Lee, C., and M. Souther, 2018, "Managerial Reliance on the Retail Shareholder Vote: Evidence from Proxy Delivery Methods," *Management Science, Forthcoming.*

Levit, D., and N. Malenko, 2011, "Nonbinding Voting for Shareholder Proposals," *Journal of Finance*, 66, 1579–1614.

Levit, D., N. Malenko, and E. G. Maug, 2019, "Trading and Shareholder Voting," *European Corporate Governance Institute–Finance Working Paper.*

Li, S. Z., E. G. Maug, and M. Schwartz-Ziv, 2019, "When Shareholders Disagree: Trading After Shareholder Meetings," Finance Working Paper 594/2019, European Corporate Governance Institute.

Malenko, N., and Y. Shen, 2016, "The Role of Proxy Advisory Firms: Evidence from a Regression-Discontinuity Design," *Review of Financial Studies*, 29, 3394–3427.

Matvos, G., and M. Ostrovsky, 2010, "Heterogeneity and Peer Effects in Mutual Fund Proxy Voting," *Journal of Financial Economics*, 98, 90–112.

Maug, E., and K. Rydqvist, 2009, "Do Shareholders Vote Strategically? Voting Behavior, Proposal Screening, and Majority Rules," *Review of Finance*, 13, 47–79.

Maug, E., and B. Yilmaz, 2002, "Two-Class Voting: A Mechanism for Conflict Resolution," *American Economic Review*, 92, 1448–1471.

McGurn, P. S., 1989, *Confidential Proxy Voting*, Investor Responsibility Research Center.

Moeller, S. B., F. P. Schlingemann, and R. M. Stulz, 2007, "How Do Diversity of Opinion and Information Asymmetry Affect Acquirer Returns?," *Review of Financial Studies*, 20, 2047–2078.

Monks, R. A. G., and N. Minow, 2003, *Corporate Governance*, Wiley-Blackwell.

Myatt, D. P., 2015, "A Theory of Voter Turnout," Working Paper. Available at: `http://dpmyatt.org/uploads/turnout-2015.pdf`.

Parker, J. A., and C. Julliard, 2005, "Consumption Risk and the Cross Section of Expected Returns," *Journal of Political Economy*, 113, 185–222.

Pinto, A. R., 2009, "The European Union's Shareholder Voting Rights Directive from an American Perspective: Some Comparisons and Observations," *Fordham International Law*, pp. 587–623.

Porras Prado, M., P. A. C. Saffi, and J. Sturgess, 2016, "Ownership Structure, Limits to Arbitrage, and Stock Returns: Evidence from Equity Lending Markets," *The Review of Financial Studies*, 29, 3211–3244.

The European Parliament and the Council of the European Union, 2017, "Directive (EU) 2017/828 of the European Parliament and of the Council," amending Directive 2007/36/EC as regards the encouragement of long-term shareholder engagement.

Thomas, R. S., and J. F. Cotter, 2007, "Shareholder Proposals in the New Millennium: Shareholder Support, Board Response, and Market Reaction," *Journal of Corporate Finance*, 13, 368–391.

Thomas, S., 2002, "Firm Diversification and Asymmetric Information: Evidence from Analysts' Forecasts and Earnings Announcements," *Journal of Financial Economics*, 64, 373 – 396.

U.S. Securities and Exchange Commission, 2015, "Proxy Voting Roundtable," `https://www.sec.gov/spotlight/proxy-voting-roundtable/proxy-voting-roundtable-transcript.txt`.

Van der Elst, C., 2011, "Revisiting Shareholder Activism at AGMs: Voting Determinants of Large and Small Shareholders," *ECGI-Finance Working Paper*.

Wooldridge, J. M., 2010, *Econometric Analysis of Cross Section and Panel Data*, MIT press.

Yilmaz, B., 2000, "Strategic Voting and Proxy Contests," Working Paper 05-00, Rodney L. White Center for Financial Research.

APPENDIX

A. Proofs

Proof of Lemma 1 [Pivotal Probabilities]. Define $u_R \equiv apt_R$ and $u_L \equiv a(1-p)t_L$, the actual voting probabilities for R and L among discretionary voters, respectively, and the probability of absentee votes $u_0 \equiv 1 - u_R - u_L$. Vector $u \equiv (u_R, u_L, u_0)$ lives in the two-dimensional unit simplex simplex Λ and we, further, assume that beliefs regarding u are represented by continuous and bounded density $h(\cdot|i)$, for $i \in \{R, L\}$. Then, the number of discretionary votes b_R, b_L, and $(1-\gamma)n - b_R - b_L$ follow a multinomial distribution, with probabilities $u \in \Lambda$. Hence, by adapting Myatt (2015, Eqs (4) & (5)), for our purposes, we calculate the probability of a tie with x discretionary votes for R, by taking the following expectation over u:

$$\Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| \, h(\cdot|i)\right] =$$

$$\int_{\Lambda} \frac{((1-\gamma)n)! \, {u_R}^{x} {u_L}^{x+(2q-1)\gamma n} {u_0}^{(1-2q\gamma)n-2x}}{x!(x+(2q-1)\gamma n)!((1-2q\gamma)n-2x)!} h(u|i) du \approx$$

$$h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n} \middle| i\right) \frac{\Gamma\left((1-\gamma)n+1\right)}{\Gamma\left((1-\gamma)n+3\right)} \approx$$

$$\frac{1}{(1-\gamma)n} \frac{h\left(\frac{x}{(1-\gamma)n}, \frac{x}{(1-\gamma)n} + \frac{(2q-1)\gamma}{(1-\gamma)}, \frac{1-2q\gamma}{1-\gamma} - 2\frac{x}{(1-\gamma)n} \middle| i\right)}{(1-\gamma)n},$$

for $i \in \{R, L\}$, where the first equality follows from the multinomial distribution; the first approximation relies on the observation that, for n large, most of the distribution will be concentrated at its mode, and uses properties of the Dirichlet density; the second approximation follows from the definition of $\Gamma(y+1) = y!$, for $y \in \mathbb{N}$, and that, for n large, $(1-\gamma)n+1 \approx (1-\gamma)n$, $(1-\gamma)n+2 \approx (1-\gamma)n$. Then, by summing over all the possible x, the overall probability of a tie is:

$$\sum_{x=0}^{n/2-q\gamma n} \Pr\left[b_R = x, b_L = x + (2q-1)\gamma n \middle| \, h(\cdot|i)\right],$$

where we assume in the context of this proof, primarily for simplicity, that both $n/2$ and γqn are integers. Now, given the above approximations, for n large, the sum can be approximated by the integral

$$\frac{1}{(1-\gamma)n}\int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|i)dy.$$

Therefore, by employing Myatt (2015, Lemma 2), the probability of a tie and a near tie are equal for a large n and, hence, for $i \in \{R, L\}$:

$$\Pr[\text{Pivotal}|i] \quad \approx \quad \frac{1}{(1-\gamma)n}\int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|i)dy. \tag{14}$$

Below, we revert the above expressions from vector u to vector (p, a) so that we can transition from density h to densities f and g. Recall that $u_R = apt_R$, and $u_L = a(1-p)t_L$; hence, the Jacobian $\partial(u_R, u_L)/\partial(p, a)$ has a determinant equal to at_Rt_L. Moreover, note that each shareholder updates her beliefs based on her own availability and so

$$h(x, y, 1-x-y|i) = \frac{f\,(p(x,y)|i)\, g\,(a(x,y)|\text{available})}{a(x,y)t_Lt_R},$$

for any $x, y \in (0, 1)$ and $i \in \{R, L\}$, where

$$g\,(a|\text{available}) = \frac{g(a)a}{\overline{a}},\; f(p|L) = f(p)\frac{1-p}{1-\overline{p}},\; f(p|R) = f(p)\frac{p}{\overline{p}}.$$

Hence, for $u_R = y$ and $u_L = y + (2q-1)\gamma/(1-\gamma)$ from (14), after some simple algebra, we define

$$\begin{aligned} p(a) &\equiv \frac{t_L}{t_R+t_L} - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{a(t_R+t_L)}, \\ a(y) &\equiv \frac{y(t_R+t_L)}{t_Rt_L} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L}, \end{aligned}$$

so that $p(\overline{a}) = p^*$ (see (4)). By substituting all the above in the integrand of (14), we have

$$\begin{aligned} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|R) &= \frac{f\left(p(a(y))\,g\left(a(y)\right)a(y)p\left(a(y)\right)\right)}{a(y)t_R t_L \overline{ap}} \\ &= p\left(a(y)\right) f\left(p(a(y))\,g\left(a(y)\right)\right) \frac{1}{t_R t_L \overline{ap}} \end{aligned} \tag{15}$$

and, similarly,

$$h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|L) = (1-p\left(a(y)\right)) f\left(p(a(y))\,g\left(a(y)\right)\right) \frac{1}{t_R t_L \overline{a}(1-\overline{p})}. \tag{16}$$

Now, to calculate the integral in (14), we change the variable from y to $a = a(y)$. We have $da = dy(t_R + t_L)/(t_R t_L)$ and

$$\begin{aligned} a(0) &= \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L}, \\ a\left((1/2-q\gamma)/(1-\gamma)\right) &= \frac{t_R\left(1/2-\gamma(1-q)\right)+t_L\left(1/2-\gamma q\right)}{(1-\gamma)t_R t_L}. \end{aligned}$$

Then, we have that:

$$\begin{aligned} \Pr[\text{Pivotal}|R] &\approx \frac{1}{(1-\gamma)n}\int_0^{\frac{1/2-q\gamma}{1-\gamma}} h(y, y+(2q-1)\gamma/(1-\gamma), (1-2q\gamma)/(1-\gamma)-2y|R)dy.dy \\ &= \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}(t_R+t_L)}\int_{a(0)}^{a((1/2-q\gamma)/(1-\gamma))} f(p(a))p(a)g(a)da, \end{aligned} \tag{17}$$

where in the second line we substituted from (15); and also using (16)

$$\Pr[\text{Pivotal}|L] = \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(t_R+t_L)}\int_{a(0)}^{a((1/2-q\gamma)/(1-\gamma))} f(p(a))(1-p(a))g(a)da. \tag{18}$$

Since we are seeking to develop a simple formula that we can use with the data, we assume that a follows a degenerate distribution around its mean; that is, $g(a) = \delta(a-\overline{a})$, where δ is the Dirac function. Then, to have a strictly positive probability of being pivotal, we need:

$$a(0) < \overline{a} < a\left((1/2 - q\gamma)/(1-\gamma)\right) \quad \Longleftrightarrow \quad \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{t_L} < \overline{a} < \frac{t_R\left(1/2 - \gamma(1-q)\right) + t_L\left(1/2 - \gamma q\right)}{(1-\gamma)t_R t_L}. \tag{19}$$

The above is a restriction on equilibrium t_R, t_L. Let us start with the lower bound on $\overline{a}$ in (19). This restriction can be, equivalently, written as:

$$\overline{a}t_L(1-\gamma) + (1-q\gamma) > q\gamma.$$

In words this says that if all expected support for R is zero (i.e., $\overline{p} = 0$), then there is t_L so that L wins. This has to be true in any equilibrium for otherwise discretionary supporters of L would never turn out. Now, the restriction imposed by the upper bound in (19) can be, equivalently, written as:

$$t_R\left(1/2 - \gamma(1-q)\right)\left(\overline{a}t_L - 1\right) + t_L\left(1/2 - \gamma q\right)\left(\overline{a}t_R - 1\right) < 0,$$

which is always true as (from $q > 1/2$ and A1): $\gamma(1-q) < \gamma q < 1/2$; $\overline{a} \leq 1$; and $t_{R,L} \leq 1$. Hence, (19) will always be satisfied in equilibrium. Also, note that for (17) and (18) not to be zero we need $p(\overline{a}) = p^* \in (l, h)$, which we will check for each equilibrium, separately. Therefore, given $g(a) = \delta(a - \overline{a})$ and A1, we have that (18) leads to (2), and, similarly, (17) becomes (3), where p^* is given by (4). ■

In Proposition 2, below, we present the necessary and sufficient conditions for the existence of equilibrium mm for any number of voters n. Then, Proposition 1 in the main text is the restriction of Proposition 2 to large n (case (b), which is the only one that survives as $n \to \infty$).

Proposition 2 (Equilibrium mm for any n). *Assume that $q \in (1/2, 1)$, $\overline{p} \in (l, h)$, $\overline{a} \in (0, 1]$, A1-A2, $g(a) = \delta(a - \overline{a})$ and*

$$\frac{v}{c} \; < \; \min\left\{\frac{n\left(\overline{a} - \gamma\left(\overline{a} - 1 + 2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{n\left(\overline{a} - \gamma\left(\overline{a} + 1 - 2q\right)\right)}{f(\overline{p})(1-\overline{p})}\right\}.$$

In addition, consider the following disjoint parameter regions:

(a) *Small regular block size:*

$$\begin{aligned} \frac{v}{c} &\geq 1, \\ n &> \frac{f(\overline{p})\overline{p}}{\overline{a}}, \text{ and} \end{aligned}$$

(i) *Many voters, high availability:*

$$n > \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} > 2q-1, \text{and } 0 \leq \gamma < \min\left\{\frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\,n}, \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\,n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{or}$$

(ii) *Many voters, low availability:*

$$n > \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} < 2q-1, \text{and } 0 \leq \gamma < \min\left\{\frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\,n}, \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}\right\}, \text{ or}$$

(iii) *Few voters, low availability:*

$$\begin{aligned} \frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)} < n < \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}, \overline{a} < 2q-1, \overline{p} < \frac{1}{2}, \quad \text{and} \\ \frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\,n} < \gamma < \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\,n}. \end{aligned}$$

(b) *Large regular block size (Many voters, any availability):*

$$\begin{aligned} \frac{v}{c} &> \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}, \\ n &> \frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)}, \text{ and} \\ \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)} < \gamma &< \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}. \end{aligned}$$

These conditions are necessary and sufficient for the existence of an incomplete participation equilibrium by both types —that is, $t_L, t_R \in (0,1)$*, which are given by equations (9) and (10). Furthermore, the average participation among discretionary voters* $\overline{t}_{disc}$ *and the average total participation* $\overline{t}_{total}$ *are given by (11) and (12). Finally, in such an equilibrium, the probability of being pivotal for either R of L is equal to the common cost-to-benefit ratio* c/v *(1), and the expected votes for R and L are equal (5), i.e.,* $p^* = \overline{p}$*.*

Proof. We proceed in two steps. In the first, we derive the expressions for the rates, and in the second, we determine the feasible parameter regions.

Step 1: Derivation of Rates Given the expressions for the pivotal probabilities, we now seek to determine whether an equilibrium exists with incomplete participation for both L and R (i.e., t_L, $t_R \in (0,1)$). From (1), we know that since the cost-to-benefit ratio is the same for both types, the pivotal probabilities should also be the same for both types. Hence, using (3) and (2), in equilibrium, we must have:

$$p^* \quad = \quad \overline{p}, \tag{20}$$

which, given (5), means that at equilibrium, the total average supports for L and R are equal. Hence, the expected outcome is a tie. From (20), the pivotal probabilities in equilibrium are:

$$\Pr[\text{Pivotal}|R] = \Pr[\text{Pivotal}|L] \quad = \quad \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(t_R+t_L)}f\left(p^*\right).$$

Moreover, the pivotal probability for type R is equal to her cost-to-benefit ratio (1) in the equilibrium with incomplete participation; hence,

$$t_R + t_L \quad = \quad \frac{1}{(1-\gamma)n\overline{a}}f(\overline{p})\frac{v}{c}. \tag{21}$$

Furthermore, according to the definition of p^* (4) and the fact that it is equal to $\overline{p}$ (20), after some simple algebra, we have

$$t_L \quad = \quad (t_R+t_L)\overline{p} + \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}}. \tag{22}$$

Using (21) with (22), we derive the equilibrium t_L and t_R, as given in (9) and (10).

Step 2: Bounds on the Parameters Incomplete participation means that $(t_L, t_R) \in (0,1)$. In order to ensure this, we now derive the restrictions on the parameters of the model —in particular, n, γ and v/c or, equivalently, $v/(cn)$. By definition, what we need for incomplete participation is $(t_L, t_R) \in (0,1)$. According to (9), it is evident that $t_L > 0$ for all parameter values. The condition

$t_L < 1$ is equivalent to:

$$\gamma \;<\; \frac{\overline{a}}{2q-1+\overline{a}}, \text{ and} \tag{23}$$

$$\frac{v}{c} \;<\; \frac{n\left(\overline{a}-\gamma\left(\overline{a}-1+2q\right)\right)}{f(\overline{p})\overline{p}}. \tag{24}$$

Given our assumption that $q > 1/2$, the upper bound on γ in (23) takes precedence over A1 (i.e., $\gamma < 1/(2q)$). Now, the condition $t_R > 0$ is equivalent to:

$$\frac{v}{c} \;>\; \frac{n\gamma(2q-1)}{f(\overline{p})(1-\overline{p})}. \tag{25}$$

For (25) to define the relevant lower bound on v/c given A2 (i.e., $v/c \geq 1$), we need a lower bound on the regular block size and the number of voting shares,

$$\gamma \;>\; \frac{f(\overline{p})(1-\overline{p})}{n(2q-1)}, \text{ and} \tag{26}$$

$$n \;>\; \frac{2qf(\overline{p})(1-\overline{p})}{2q-1}; \tag{27}$$

otherwise, we just need to impose $v/c \geq 1$ from A2. Finally, the condition $t_R < 1$ is equivalent to:

$$\frac{v}{c} \;<\; \frac{n\left(\overline{a}-\gamma\left(\overline{a}+1-2q\right)\right)}{f(\overline{p})(1-\overline{p})}. \tag{28}$$

Hence, for the benefit-to-cost ratio v/c, we have two possible upper bounds: (24) and (28). Either can be relevant, depending on the parameter values. Therefore, the upper bound on v/c is

$$\frac{v}{c} \;<\; \min\left\{\frac{n\left(\overline{a}-\gamma\left(\overline{a}-1+2q\right)\right)}{f(\overline{p})\overline{p}}, \frac{n\left(\overline{a}-\gamma\left(\overline{a}+1-2q\right)\right)}{f(\overline{p})(1-\overline{p})}\right\}. \tag{29}$$

For v/c, we also have a lower bound, which is either (25), if γ satisfies (26), or one. In either case, we need to make sure that the lower bound is smaller than the upper bound on v/c in (29). Let us look at each case in turn:

Case A: If (26) holds, then to ensure that the upper bounds on v/c in (29) are larger than the lower bound in (25), we have another restriction on γ,

$$\gamma \quad < \quad \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}. \tag{30}$$

From the two possible upper bounds of γ in (23) and (30), we can show that for $q > 1/2$, the relevant bound is condition (30). Finally, we need to make sure that the lower bound of γ in (26) is lower than the upper bound in (30). This puts a lower bound on the number of voting shares:

$$n \quad > \quad \frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)},$$

which supersedes the other lower bound of (27).

Case B: If (26) does not hold, then to ensure that the upper bound on v/c in (29) is higher than the lower bound imposed by A2, we add the following two restrictions on γ,

$$\begin{aligned} n(\overline{a}-(2q-1))\gamma &\quad < \quad \overline{a}n - f(\overline{p})(1-\overline{p}), \text{ and} \\ n(\overline{a}+(2q-1))\gamma &\quad < \quad \overline{a}n - f(\overline{p})\overline{p}. \end{aligned} \tag{31}$$

For these to be satisfied, we need

$$n \quad > \quad \frac{f(\overline{p})\overline{p}}{\overline{a}}.$$

Observe, also, that the second restriction in (31) implies the upper bound in (23), so this latter bound can be ignored in what follows. Then, we have the following subcases:

i) If $n > f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} < 2q-1$, then the only other restriction on γ (i.e., other than $\gamma < f(\overline{p})(1-\overline{p})/(n(2q-1))$) can be written as

$$\gamma < \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\, n}.$$

ii) If $n > f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} > 2q-1$, then the added restriction on γ can be written as

$$\gamma < \min\left\{\frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\, n}, \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\, n}\right\}.$$

iii) If $n < f(\overline{p})(1-\overline{p})/\overline{a}$ and $\overline{a} > 2q-1$, then there is no equilibrium with incomplete participation.

iv) If $n < f(\overline{p})(1-\overline{p})/\overline{a}$, $\overline{a} < 2q-1$, and $\overline{p} < 1/2$, then we need

$$\begin{aligned} \frac{\overline{a}n - f(\overline{p})(1-\overline{p})}{(\overline{a}+1-2q)\,n} < \quad \gamma \quad < \frac{\overline{a}n - f(\overline{p})\overline{p}}{(\overline{a}-1+2q)\,n}, \text{ for} \\ \frac{f(\overline{p})\left(\overline{a}(1-\overline{p})+2q-1\right)}{\overline{a}(2q-1)} < \quad n \quad < \frac{f(\overline{p})(1-\overline{p})}{\overline{a}}. \end{aligned}$$

We summarize all the above in the statement of Proposition 2. ■

B. Notes on the Estimation

B.1 Delta Method

This subsection describes how we compute standard errors for our estimates using the Delta Method approach (see Wooldridge (2010, pp. 44-45)). Given the moments $m \equiv [\overline{dSuL}, \overline{dSuR}, \overline{dSuL^2}, \overline{dSuR^2}]$, denote the vector of estimated parameters by

$$\theta \equiv [v/\left(nc\right)_{\text{lower}}, v/\left(nc\right)_{\text{upper}}, \overline{p}, \text{std}(p)].$$

First, we use the data to numerically compute the sensitivity of these estimates to changes in the moments; that is, $\partial\theta_i/\partial m_j$, for $i, j \in \{1, 2, 3, 4\}$. Second, we estimate the variance-covariance matrix —denoted by S— of the four errors that we base our estimation on (see Section 4.2); i.e.,

$$\overline{dSuL}_{est} - dSuL,\ \overline{dSuR}_{est} - dSuR,\ \overline{dSuL^2}_{est} - dSuL^2,\ \overline{dSuR^2}_{est} - dSuR^2.$$

Then, the variance of our error in estimating parameter θ_i is given by

$$\Delta_i \times S \times \Delta_i^T,$$

where vector $\Delta_i \equiv [\partial\theta_i/\partial m_1, \partial\theta_i/\partial m_2, \partial\theta_i/\partial m_3, \partial\theta_i/\partial m_4]$, for $i = \{1, 2, 3, 4\}$. These are reported in Table 2.

B.2 Probabilities of Misalignment and Closeness

This subsection describes how we compute the probabilities of non-representative outcomes and closeness. First, *per bin* given our estimates for $\overline{p}, \mathrm{std}(p)$, we simulate proposals $p \sim \mathcal{U}[l, h]$. Second, for each p given our estimates for t_L, t_R in the corresponding bin, we compute:

i) The outcome index under discretionary participation:[24]

$$O_{\mathrm{disc}}(p) \equiv \overbrace{\overline{\gamma}\overline{q} + (1-\overline{\gamma})\, t_R p}^{\text{total support for} R} - \overbrace{\overline{\gamma}\,(1-\overline{q}) + (1-\overline{\gamma})\, t_L(1-p)}^{\text{total support for} L}. \tag{32}$$

ii) The outcome index under full participation:

$$O_{\mathrm{full}}(p) \equiv \overbrace{\overline{\gamma}\overline{q} + (1-\overline{\gamma})\, p}^{\text{total support for} R} - \overbrace{\overline{\gamma}\,(1-\overline{q}) + (1-\overline{\gamma})\,(1-p)}^{\text{total support for} L}. \tag{33}$$

The signs of O_{disc} and O_{full} determine whether the proposal passes or fails under discretionary and full participation, respectively. Hence, a measure of the difference in the decision between discretionary and full participation is the indicator

$$\mathbb{I}\left(O_{\mathrm{disc}}(p) O_{\mathrm{full}}(p) \leq 0\right).$$

Then, we average for all p in our simulation, and this gives us, a per bin estimate of the *probability of misalignment with respect to full participation*, $\mathbb{P}\left[O_{\mathrm{disc}}(p) O_{\mathrm{full}}(p) \leq 0\right]$.

We would like to highlight the following for this measure. First, O_{disc} for every p is computed using estimations of t_R, t_L and the bin-specific $\overline{\gamma}$, $\overline{q}$, and not the actual $dSuR$, $dSuL$. This is because we would like our non-representativeness measure to be 'free' from estimation error. Second, the only difference between O_{disc} and O_{full} is the appearance of rates t_R and t_L —which exactly capture the selection effect we aim to quantify— in the former but not the latter.

Moreover, we can perform exactly the same exercise as above, but instead of full participation as our benchmark, use the case of no discretionary (i.e., only regular) participation. To this end,

[24] The difference between (13) in the main text and (32) below is that in the latter we use the per bin 'representative' $\overline{q}$, instead of the per proposal q in the former.

define:

$$O_{\text{only-reg}} \equiv \overbrace{\overline{\gamma q}}^{\text{regular support for} R} - \overbrace{\overline{\gamma}\left(1-\overline{q}\right)}^{\text{regular support for} L} = \overline{\gamma}(2\overline{q}-1). \tag{34}$$

Note that: i) $O_{\text{only-reg}}$ does not depend on p, which is relevant only to discretionary voters; and ii) given our assumption that $q > 1/2$ (i.e., R wins by definition if only regulars vote), $O_{\text{only-reg}}$ is always positive. Hence, when we average for all p, we estimate the per bin *probability of misalignement with respect to no discretionary participation* as $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right] = \mathbb{P}\left[O_{\text{disc}}(p) \leq 0\right]$.

Now, under discretionary participation define the ratio

$$R_{\text{support}}(p) \equiv \frac{\overbrace{\overline{\gamma q} + \left(1-\overline{\gamma}\right)t_R p}^{\text{total support for} R}}{\underbrace{\overline{\gamma q} + \left(1-\overline{\gamma}\right)t_R p}_{\text{total support for} R} + \underbrace{\overline{\gamma}\left(1-\overline{q}\right) + \left(1-\overline{\gamma}\right)t_L(1-p)}_{\text{total support for} L}},$$

of total support for R to total participation $t_{\text{total}}(p)$ (note that in Proposition 1, $\bar{t}_{\text{total}} \equiv t_{\text{total}}(\overline{p})$). The fraction of supporters for R, $R_{\text{support}}(p)$ allows us to define closeness of level $x \in \{1\%, 2\%, 5\%, 10\%\}$ for every p as

$$\mathbb{I}\left(R_{\text{support}}(p) \in (50\% - x, 50\% + x)\right).$$

Given that the voting outcome is determined by a simple majority: if $R_{\text{support}} < 50\%$ (or, equivalently, O_{disc} is negative), then L wins, while if $R_{\text{support}} > 50\%$ (or, equivalently, O_{disc} is positive), then R wins. Hence, this measure captures how close a vote is expected to be, a key determinant of participation in any pivotal voter model. Averaging over all simulated proposals p, within each bin, provides the *ex-ante closeness probability* $\mathbb{P}\Big[R_{\text{support}}(p) \in (50\% - x, 50\% + x)\Big]$.

Finally, also averaging over all simulated proposals p, within each bin, gives the *probability of an R win conditional on ex-ante closeness* of level $x \in \{1\%, 2\%, 5\%, 10\%\}$ as

$$\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big|\, R_{\text{support}}(p) \in (50\% - x, 50\% + x)\Big] = \frac{\mathbb{P}\Big[R_{\text{support}}(p) \in (50\%, 50\% + x)\Big]}{\mathbb{P}\Big[R_{\text{support}}(p) \in (50\% - x, 50\% + x)\Big]},$$

where the equality is due to Bayes' rule.

C. Idiosyncratic Voting Costs Model

In this section, we derive equilibrium mm under the alternative assumption that the cost of voting is not constant but, rather, a random variable in $[0, \overline{c}]$ (see Myatt (2015, p. 19) for the case $\gamma = 0$). This allows us to capture idiosyncrasies in the cost among discretionary voters, allowing a positive mass of them to even have zero cost of voting. We maintain the structure and all other assumptions in the model, including A1 (i.e., regular voters cannot decide the outcome unilaterally) and a variant of A2 that $v > \overline{c}$ to allow for even very-high-cost voters to have a meaningful pivotal probability.

Now, for $t \in [0, 1]$, let $C(t)$ be the inverse of the distribution function of voting costs so that $t = \mathbb{P}[c \leq C(t)]$. In particular, for $c \sim \mathcal{U}[0, \overline{c}]$, we have $C(t) = \overline{c}t$. As we know in the mm equilibrium:

$$\mathbb{P}[\text{pivotal}|i] = \frac{C(t_i)}{v} = \frac{\overline{c}t_i}{v}, \text{ for } i \in \{L, R\}.$$

Clearly, this imposes the restriction $t_i < v/\overline{c}$ for $i \in \{L, R\}$. Now, using Lemma 1 and the expressions for the pivotal probabilities (3) and (2), we have

$$\frac{1}{(1-\gamma)n} \frac{1}{\overline{a}\overline{p}(t_R + t_L)} f(p^*)\, p^* = \frac{\overline{c}t_R}{v}, \tag{35}$$

$$\frac{1}{(1-\gamma)n} \frac{1}{\overline{a}(1-\overline{p})(t_R + t_L)} f(p^*)\,(1 - p^*) = \frac{\overline{c}t_L}{v}. \tag{36}$$

Dividing (35) by (36), also using the definition of p^* in (4), we arrive at:

$$\frac{t_L - (2q-1)\gamma/\left((1-\gamma)\overline{a}\right)}{t_R + (2q-1)\gamma/\left((1-\gamma)\overline{a}\right)} = \frac{t_R\overline{p}}{t_L(1-\overline{p})}. \tag{37}$$

Now, adding (35) and (36), also using the definition of p^* in (4) and additionally assuming that $p \sim \mathcal{U}[l, h]$, after some algebra, we have:

$$\frac{1}{(1-\gamma)n} \frac{1}{\overline{a}(t_R + t_L)^2} \frac{1}{d} \left[t_L/\overline{p} + t_R/(1-\overline{p}) + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \left(1/\overline{p} - 1/(1-\overline{p})\right) \right] = (t_R + t_L)\overline{c}/v, \tag{38}$$

where $d = h - l$, as before. Unfortunately, the system of equations (37) and (38) for t_L, t_R does not have a closed-form solution in the general case with regular voters (i.e., $\gamma > 0$). For our estimation (presented in Section E.1 below), we seek numerical solutions in which we also need to ensure that $t_L, t_R \in (0, 1)$ and the resulting $p^* \in [l, h]$.

Our coverage of the idiosyncratic costs case focuses only on the mm equilibrium. This is for two reasons. First, equilibrium mm is the centrepiece of the analysis in political elections (e.g., Myatt (2015)). Second, in view of the first reason, one may find the prevalence of the $1m$ equilibrium in our data curious and may attribute it to the specifics of the model, including the constant cost assumption. Hence, as, a robustness test, we also derive the mm equilibrium with idiosyncratic costs and show (in Section E.1 in the Appendix) that even then, the $1m$ equilibrium with constant cost fits the data better. Note that in the mm equilibrium with idiosyncratic costs, we have a partial underdog effect in that, on average, R wins; contrast this with the full underdog effect of the mm equilibrium under constant costs, which results in a tie, on average.

D. Proposal Classification

The ISS's functional classification of proposals into 257 types is fine enough to risk obscuring the economic meaning of each proposal type. For example, ISS assigns a different proposal type for "Amend Articles/Bylaws/Charter to Remove Antitakeover Provisions (S0326)," "Approve/Amend Terms of Existing Poison Pill(S0322)," and "Submit Shareholder Rights Plan (Poison Pill) to Shareholder Vote (S0302)," even though these proposals address the same economic issue —takeover defense. For this reason, it is useful to work with a coarser, more economically meaningful, classification. Our classification groups corporate governance proposals into four economically relevant types. We list these types along with their frequency in our sample in Table 3. The set of types is chosen to reflect leading issues arising in the literature on voting and corporate governance (see, for example, Knoeber (1986); LaPorta, de Silanes, Shleifer, and Vishny (1998); Grullon and Michaely (2002); Gompers, Ishii, and Metrick (2003); Bebchuk, Cohen, and Ferrell (2009); Becht, Franks, Mayer, and Rossi (2009); Bebchuk and Fried (2009); Ferri and Maber (2012)). Once the set of types is chosen, the proposals are classified in a straightforward way, based on their description, as illustrated in the example above. In Table 11, we list the top three proposals per type.

E. Robustness

Now, we conduct several robustness checks of our estimation, which are reported in Table 10.

E.1 Alternative model with idiosyncratic costs

The estimates of our baseline model imply a predominance of the $1m$ equilibrium with high turnout by the underdog t_L despite an expected win by R. A concern is whether this result holds if we change our constant positive cost assumption to idiosyncratic costs of voting (see Section C in the Appendix and Myatt (2015, p. 19)). This is because under idiosyncratic costs of voting, the expected outcome is a win by R, even in the mm equilibrium, and not necessarily a tie (a so-called partial underdog effect).

In order to address this concern, we estimate the variant of our model with idiosyncratic costs (Section C in the Appendix). In these estimations, we allow some discretionary voters to have very small costs —including zero— and others to have larger costs of voting. Unfortunately, closed-form expressions for the turnout rates are not attainable, which makes the estimations of such a model more susceptible to numerical error and longer in computation time.

To assess the performance of the alternative model with idiosyncratic costs, we compare its estimation errors to those of our baseline estimations. Table 10 Panel A reveals an MSE of 0.051 with idiosyncratic costs, which is significantly higher than our baseline (i.e., with constant costs) MSE of 0.026, as well as an alternative estimation in which we exclude the $1m$ equilibrium (see Section E.2, MSE of 0.042). Despite the higher errors, the alternative model yields qualitative conclusions similar to our baseline estimates. The probability of a misalignment in the outcome between discretionary and full participation, $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$, is almost identical to that in the baseline (12% vs. 13%); the underdog participates almost fully, with a t_L of 92%; the average participation of the majority side is significantly lower, at 48%.

[Insert Table 10 about here]

E.2. Alternative Estimation Methods

In this section, we show how robust our estimations are to alternative estimation choices. For all of the following variations, MSEs are significantly smaller than the comparison model, and the $1m$

equilibrium remains dominant.

Excluding the $1m$ equilibrium. How robust is our allocation of proposals to equilibria, especially the most popular $1m$ equilibrium? If allocating proposals to other equilibria increases errors only marginally but changes our results significantly, we should consider those alternative results more seriously. In Table 10 Panel A, we report MSEs and equilibrium allocations excluding the $1m$ equilibrium. Doing so increases our MSEs by a factor of 2. Out of 2,181 proposals that our baseline estimation allocates to $1m$, 1,155 are now in mm and 1,026 in 11. Despite the different equilibrium allocation, the predicted probability of a minority win is lower, at 8%, and participation rates are similar to those of the baseline estimation. This indicates that the equilibrium allocation does not change the conclusions about our estimates qualitatively.

Two-step GMM. Our baseline estimations use a non-weighted one-step GMM because the two-step optimization does not usually perform well for small numbers of observations, as in our bins. Here, we show that a two-step GMM procedure yields similar MSEs (simply referred to as 'GMM' in Table 10 Panel A). The equilibrium instance distribution is also similar, with more mm estimates (472, vs. 62 in the baseline) but still a predominance of the $1m$ equilibrium. The probability of a misrepresentative outcome is slightly lower than baseline at 9%.

Availability. Our baseline estimations assume that all shareholders are available to vote ($\overline{a} = 1$). For proposals in the $1m$ equilibrium, this implies votes from all shareholders against R. However, Holderness and Pontiff (2016) document that not all shareholders participate in valuable rights offerings, suggesting that some shareholders miss even high-impact corporate events. Assuming an $\overline{a}$ of 0.9 or 0.8, instead, increases estimation errors by 0.0025 and 0.0027, respectively. With a 10% (20%) of shareholders not participating, participation rates within the remaining shareholder base is higher, with 98% (99%) on the left and 76% (64%) on the right. This decreases the probability of a misrepresentative outcome to 10% or 7%, respectively.

Alternative bins. Our baseline estimations use bins per proposal type$\times\gamma$ tercile$\times n$ tercile$\times q$ tercile. In Table 10 Panel A, we report the performance and equilibrium incidence for bins using quartiles of γ, q, and n instead. The estimates are very similar to the baseline ones .

Alternative moments. Our baseline estimations use the first and second moments. In Table 10 Panel A, we report the performance and equilibrium incidence when we also match the third and fourth moments of $dSuL$, $dSuR$. The equilibrium assignments do not change, and the MSE does not improve. Therefore, we maintain the use of only the first two moments as our baseline estimation.

Medians instead of means. Our baseline estimations use the means of γ and q per bin as representative. In Table 10 Panel A, we report the performance and equilibrium incidence using medians instead. Using medians does not change the model performance or equilibrium allocation significantly.

ISS recommendations. Proxy advisors have a large influence on the voting direction of regular voters (Iliev and Lowry (2014); Malenko and Shen (2016)). To reflect this influence, we compute alternative estimates for bins of ISS recommendation$\times$proposal type$\times\gamma$ tercile$\times n$ tercile$\times q$ tercile. Errors are similar, and the $1m$ equilibrium still dominates.

E.3. Sample Splits

In this section, we examine our model's performance and the robustness of the estimates for subsamples in which our assumptions are less likely to hold. All results are reported in Table 10 Panel B.

Equity lending. In the presence of an equity lending market, discretionary voters may decide to lend their shares instead of voting, effectively increasing the opportunity costs of voting. Currently, the US equity lending market operates over-the-counter, and data are available from Markit for the period 2001-2016.[25] The Markit database covers over 90% of that market and contains firm-quarter level information on the supply of lendable shares for the majority of stocks listed in public exchanges. Following Campello and Saffi (2015), we define equity lending supply as the difference between the value of a firm's lendable shares and the number of lendable shares currently on loan, divided by the firm's market capitalization. This calculation gives us a precise measure of the net lendable supply. We define equity lending demand as the value of shares actually borrowed

[25] We are most grateful to Pedro Saffi for providing us with part of this dataset.

divided by the firm's market capitalization. We then compare the firms above and below the median in terms of the demand and supply of equity lending and investigate the effect on our estimation results. We obtain similar estimates for the probability of non-representative outcomes, participation rates, and equilibrium allocation.

Number of proposals. Since our model focuses on one proposal, results for meetings with only one non-routine proposal should be more representative. Indeed, the MSEs for meetings with a bundle of proposals is 0.004 higher than those for meetings with only one proposal. However, neither the estimates nor the equilibrium allocation differ significantly between the two subsamples. The incidence of the $1m$ equilibrium is dominant for both subsamples.

Ownership concentration. Our model assumes equal holdings by dispersed shareholders. To see how relevant this assumption is, we split the sample into terciles of the Herfindahl concentration index of institutional ownership. Indeed, the MSEs for firms with more concentrated ownership are higher by 0.009. However, neither the estimates nor the equilibrium incidence differ significantly across the subsamples.

Importance of information. An important ingredient of the voting process that is not included in our model is information aggregation of dispersed private information. Hence, the algorithm should be more suitable when information aggregation is less important. This should be the case for firms with a lower variation in analyst forecasts (see Thomas (2002); Moeller, Schlingemann, and Stulz (2007)), and for proposals later in the proxy season, after shareholders have already observed the voting preferences in many firms and guidance on the respective proposal types. We split the sample by the terciles of standard deviation of analyst forecasts, of the day and month of the meeting within each year, as well as by whether ISS has already issued recommendations in both directions for the same proposal type in the same season ("Early/Late meeting/ISS"). The MSEs are actually not that much different for low-variance and late meetings in which information should matter less: none of the subsample MSEs are more than 0.004 apart from the baseline MSEs. The estimates and equilibrium incidences are similar across the subsamples and similar to the base algorithm.

E.4. The magnitude of the benefits of voting

The algorithm estimates the benefit-to-cost ratio per voter $v/(cn)$. In this section, we set these estimates into the context of the literature. Because we are the first to estimate the benefit-to-cost ratio, we transform it to a more comparable measure, the return on investment conditional on a win of one's preferred side. This transformation requires assumptions on two unknowns: the number of shareholders n and the cost of voting c. For this section, we will vary n and assume $c = 1$; please see Section 7 for a in-depth discussion of the cost of voting. Assuming a higher cost would linearly translate into lower returns. Under this assumption, one can interpret the ratio between the benefit-to-cost ratio per voter $v/(cn)$ and the average ownership stake in dollars ($) as the average return of winning.

[Insert Figure 2 about here]

Figure 2 depicts the "return on winning" as a function of the ratio of market capitalization and n, that is, the average value of shares held prior to the vote. We exclude estimations in which the benefit-to-cost ratio is lower than 1 (i.e., irrelevant proposals). The graph shows that an average ownership stake over $500,000 can yield realistic estimates of the benefit-to-cost ratios (below 20%). For an average ownership stake of $1.5 million (the average holding size of insider holdings convicted by the SEC (Ahern (2017)), the return is 2.0%. This result compares to an average return of 1.6% for the passing of governance proposals in Cuñat, Gine, and Guadalupe (2012). Hence, as in Ahern (2017), using the average ownership stake yields estimates that are roughly comparable to those of the previous literature.

F. Tables and Figures

Table 2: **Univariate Statistics**

This table shows the univariate statistics for a sample of proposals voted upon in US firms from 2003–2011. Panel A shows the number of proposals and meetings. Panel B shows the firm characteristics (at the firm-year level). Panel C shows the summary statistics for ownership of voting shares (at the meeting level).

Panel A: Number of observations per year

Year	Proposals	Meetings
2003	31	21
2004	511	293
2005	117	76
2006	313	187
2007	287	164
2008	236	162
2009	315	207
2010	461	319
2011	36	31

Panel B: Firm characteristics

Variable	Obs	Mean	Std. Dev.	Min	Max
Assets	1,355	76,374	243,761	23	2,265,792
Leverage	1,355	0.26	0.17	0	1.20
M/B	1,355	1.72	0.97	0.69	10.17
Return (annual)	1,258	0.08	0.39	(1.98)	3.62

Panel C: Summary statistics for ownership

Variable	Obs	Mean	Std. Dev.	Min	Max
% institutional ownership	1,460	71.77	16.15	0.60	99.27
of which: N-PX	1,460	0.25	0.09	0.00	0.56
% >5% ownership	1,363	16.78	13.95	0	95.87
of which: institutional	1,363	16.00	13.32	0	82.10
of which: private	1,363	0.78	3.81	0	53.48
% management ownership	1,460	2.33	5.02	0	50.36
% activists	1,460	4.18	3.82	0	40.56
Shares outstanding	1,444	873M	1,760M	1.607M	29,100M
n	1,460	871.29	915.65	0	5,933

Table 3: **Proposals**

This table shows the univariate statistics for a sample of proposals voted upon in US firms from 2003–2011. Panel B (C) shows the frequency of proposals, per proposal (sponsor) type.

Panel A: Summary Statistics

Variable	Mean	Std. Dev.	Min	Max
Total participation	0.72	0.09	0.09	0.98
Discretionary participation	0.66	0.12	0.09	0.99
Outcome in $(-10, 10)$	0.33	0.47	0	1
Outcome in $(-5, 5)$	0.16	0.37	0	1
Outcome in $(-2, 2)$	0.07	0.25	0	1
Outcome in $(-1, 1)$	0.04	0.19	0	1

Panel B: Proposals per proposal type

			Participation (%)	
Proposal type	Frequency	Support (%)	Total	Discretionary
Board	738	26.50%	72.37%	66.26%
Executive compensation	763	23.04%	70.39%	64.80%
Takeover defense	409	49.24%	73.80%	67.01%
Other governance	397	31.29%	72.52%	66.01%
Total	2,307	30.21%	71.99%	65.87%

Panel C: Proposals per sponsor type

			Participation (%)	
Sponsor type	Frequency	Support (%)	Total	Discretionary
Individual activist	869	29%	71%	64%
Pension fund	461	30%	72%	66%
Union	215	30%	72%	66%
Fund	131	30%	72%	67%
Social group	103	24%	70%	66%
Coalition	26	37%	75%	68%
Employee	9	21%	61%	47%
Corporate	2	16%	73%	72%
Proxy advisor	2	37%	68%	63%
Other	489	33%	73%	67%

Table 4: **Algorithm Input Parameters**

This table shows univariate statistics for the input parameters of the algorithm.

Variable	#N	Mean	Std. Dev.	Min	Max
$dSuL$	2,305	0.26	0.08	0.08	0.42
$dSuR$	2,305	0.40	0.07	0.25	0.61
$dSuL^2$	2,305	0.09	0.04	0.01	0.23
$dSuR^2$	2,305	0.18	0.06	0.07	0.38
γ	2,305	0.25	0.09	0.00	0.56
q	2,305	0.80	0.15	0.55	1.00
n	2,305	829.46	943.51	0	5,933

Table 5: **Estimation Results**

This table shows the univariate statistics per proposal for the estimation results of the algorithm run for the bins of proposal type×tercile of γ×tercile of $n\times$ tercile of q. Panel A shows the mean average errors of our estimated moments (first and second of $dSuL$ and $dSuR$). Panel B shows parameter estimates for $v/(nc)$, $\overline{p}$ and std(p) with their confidence intervals and standard errors, which are computed using the Delta Method (see Section B.1 in the Appendix). Panel C reports participation rates and distances between regular and discretionary voter preferences implied by the parameter estimates. Panel D reports the incidence of equilibria and implied average participation rates and distances between regular and discretionary voter preferences by equilibrium. Panel E reports the mean squared error (MSE) of our baseline estimations and the ones in Table 12, their difference, the test statistic and the p-value of the Diebold-Mariano test for equality of predictive accuracy.

Panel A: Mean average errors

	Baseline	GMM
$\overline{dSuL}_{est}$	0.0086	0.0364
$\overline{dSuR}_{est}$	0.0059	0.0341
$\overline{dSuL^2}_{est}$	0.0217	0.0454
$\overline{dSuR^2}_{est}$	0.0060	0.0322

Panel B: Parameter estimates

	Baseline			**GMM**		
	Estimate	95% Confidence interval		Estimate	95% Confidence interval	
Benefit-to-cost ratio per voter						
$v/(cn)$	1.08	1.03	1.14	0.99	0.94	1.08
[s.e.]		[0.03203]	[0.02622]		[0.04564]	[0.02759]
Popularity of R among discretionary voters p						
$\overline{p}$	0.72	0.67	0.73	0.68	0.60	0.74
[s.e.]		[0.0083]	[0.02546]		[0.03135]	[0.0398]
std(p)	0.17	0.15	0.18	0.17	0.15	0.18
[s.e.]		[0.0053]	[0.00668]		[0.00607]	[0.00743]
$\#N$	2,305			2,213		

Panel C: Implied parameters

	Mean	Std. Dev.	Min	Max
t_L	97.8%	9.8%	49.3%	100%
t_R	56.4%	9.8%	43.8%	100%
$q-\overline{p}$	8%	13.3%	-15.3%	68.2%
$\lvert q-\overline{p}\rvert$	12.3%	9.4%	0.2%	68.2%

Panel D: Equilibria

	mm	$1m$	$m1$
$\#N$	62	2181	62
%	2.7%	94.6%	2.7%
t_L	66.5%	100%	52.7%
t_R	76.1%	54.6%	100%
$q - \overline{p}$	42%	6.4%	29%
$\lvert q - \overline{p} \rvert$	42%	11%	29%
Estimation error	0.02%	0.20%	0.02%

Panel E: Diebold-Mariano comparison between reduced-form and baseline forecasts

	Participation	$dSuL$	$dSuR$	$O_{\text{disc}}(\overline{p})$
In-sample				
MSE algorithm	0.010	0.013	0.015	0.026
MSE Malenko-Shen	0.398	0.182	0.110	0.215
Difference	-0.388	-0.169	-0.094	-0.190
$S(1)$	-171.190	-70.669	-48.203	-48.244
p-value	<0.0000	<0.0000	<0.0000	<0.0000
Out-of-sample				
MSE baseline estimation	0.018	0.025	0.032	0.066
MSE reduced-form	0.403	0.232	0.130	0.318
Difference	-0.385	-0.207	-0.098	-0.251
$S(1)$	-22.500	-7.813	-6.109	-5.637
p-value	<0.0000	<0.0000	<0.0000	<0.0000

Table 6: **Selection**

This table shows implied distances between the actual voting support and the preference estimates for the entire shareholder population. Panel A includes the mean, standard deviation, minimum, and maximum for the whole sample. Panel B shows the mean for each equilibrium. Panel C shows the probability of close votes. For the definitions of O_{disc}, O_{full}, $O_{\text{only-reg}}$, and R_{support} and the corresponding probabilities involving them, see Section B.2 in the Appendix.

Panel A: Distance between underlying preferences and voting outcomes

	Mean	Std. Dev.	Min	Max
$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	12.8%	14.1%	0%	46.6%
$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	15.1%	17.4%	0%	67.9%
$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$	10.1%	11.6%	-19.9%	53.3%
$O_{\text{full}}(\overline{p}) - O_{\text{only-reg}}$	32.7%	19.6%	-43.1%	85.2%
$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	-22.6%	11.4%	-34.0%	31.7%
$\lvert O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})\rvert$	24.9%	4.4%	12.4%	34%

Panel B: By equilibrium

		mm	$1m$	$m1$
$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$		17.5%	12.1%	32.8%
$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$		50%	12.9%	58.6%
$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	signed	9.1%	-24.9%	28.4%
$\lvert O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})\rvert$	unsigned	21%	24.9%	28.4%
$O_{\text{disc}}(\overline{p}) - O_{\text{only-reg}}$		-7.9%	11.2%	-7.8%

Panel C: Probability of close votes

	All	mm	$1m$	$m1$
$\mathbb{P}\Big[R_{\text{support}}(p) \in (40\%, 60\%)\Big]$	38.4%	35.4%	38.1%	50.1%
$\mathbb{P}\Big[R_{\text{support}}(p) \in (45\%, 55\%)\Big]$	20.8%	17.7%	20.8%	25.1%
$\mathbb{P}\Big[R_{\text{support}}(p) \in (48\%, 52\%)\Big]$	8.8%	7.1%	8.8%	10.1%
$\mathbb{P}\Big[R_{\text{support}}(p) \in (49\%, 51\%)\Big]$	4.6%	3.5%	4.6%	5.1%
$\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big\vert\, R_{\text{support}}(p) \in (40\%, 60\%)\Big]$	76.1%	50.7%	77.5%	52.9%
$\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big\vert\, R_{\text{support}}(p) \in (45\%, 55\%)\Big]$	72.8%	50.3%	74.0%	51.7%
$\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big\vert\, R_{\text{support}}(p) \in (48\%, 52\%)\Big]$	69.2%	49.9%	70.2%	50.8%
$\mathbb{P}\Big[R_{\text{support}}(p) > 50\% \,\Big\vert\, R_{\text{support}}(p) \in (49\%, 51\%)\Big]$	65.5%	49.6%	66.4%	50.4%

Table 7: **Heterogeneity**

This table shows average parameter estimates by proposal type (Panel A), sponsor type (Panel B), year (Panel C), GICS 1-digit industry (Panel D), performance in the previous year (Panel E), and ownership structure (Panel F). For the definitions of O_{disc}, O_{full}, $O_{\text{only-reg}}$ and the corresponding probabilities involving them, see Section B.2 in the Appendix.

Panel A: Shareholder preferences by proposal type

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Proposal type	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
Board	13%	16%	98%	56%	-23%	5%	0	709	29	738
Compensation	13%	14%	100%	54%	-26%	5%	28	380	0	408
Defense	8%	11%	97%	59%	-21%	18%	0	141	0	141
Other governance	16%	21%	94%	60%	-18%	9%	34	330	33	397
Non-sample proposals for comparison										
Non-governance shareholder proposals										
CSR	3%	3%	100%	58%	-27%	7%	0	762	0	762
Finance	3%	3%	100%	56%	-27%	5%	0	141	0	141
Management proposals										
All	4%	7%	100%	71%	-20%	7%	26	12,273	0	12,299
Say-on-pay	0%	0%	100%	70%	-22%	6%	0	2,116	0	2,116

Panel B: Shareholder preferences by sponsor type

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Sponsor type	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
Coalition	0.07	16%	94%	62%	-16%	12%	0	25	6	31
Corporate	0.00	0%	100%	60%	-33%	-7%	0	1	0	1
Employee	0.02	2%	100%	33%	-39%	4%	0	9	0	9
Fund	0.08	15%	98%	57%	-23%	11%	29	262	16	307
Individual activist	0.12	13%	100%	53%	-27%	7%	0	1,017	0	1,017
Pension fund	0.14	18%	100%	54%	-24%	6%	0	659	0	659
Proxy advisor	0.06	6%	100%	52%	-30%	-17%	0	2	0	2
Social group	0.09	10%	99%	56%	-24%	6%	15	430	0	445
Union	0.13	16%	97%	60%	-19%	8%	0	271	22	293

Panel C: Shareholder preferences by year

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Year	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(\overline{p}) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
2003	0.07	11%	98%	56%	-25%	15%	0	52	3	55
2004	0.09	11%	98%	56%	-24%	10%	8	386	15	409
2005	0.13	19%	94%	59%	-22%	11%	5	107	17	129
2006	0.12	15%	98%	57%	-23%	6%	6	267	9	282
2007	0.06	9%	99%	56%	-22%	8%	0	245	8	253
2008	0.08	10%	99%	53%	-23%	9%	0	217	7	224
2009	0.14	21%	91%	61%	-16%	11%	7	248	56	311
2010	0.14	22%	92%	65%	-15%	13%	28	320	58	406
2011	0.08	12%	95%	70%	-15%	10%	0	28	4	32

Panel D: Shareholder preferences by industry

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Industry	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
Energy	12%	18%	92%	60%	-18%	9%	8	132	19	159
Materials	10%	15%	96%	59%	-20%	11%	0	94	11	105
Industrials	8%	17%	96%	63%	-17%	12%	3	276	36	315
Consumer discretionary	11%	16%	97%	60%	-20%	9%	5	273	13	291
Consumer staples	8%	19%	94%	67%	-15%	12%	6	134	21	161
Health care	10%	16%	93%	62%	-17%	11%	5	191	26	222
Financials	11%	16%	94%	60%	-19%	14%	16	325	38	379
IT	12%	15%	96%	52%	-26%	5%	14	168	6	188
Communication services	7%	10%	96%	54%	-25%	8%	0	81	6	87
Utilities	9%	10%	99%	50%	-29%	6%	0	201	5	206

Panel E: Shareholder preference by performance

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
Stock return										
Tercile 1	11%	13%	99%	56%	-23%	7%	4	647	13	664
Tercile 2	12%	16%	97%	59%	-21%	9%	15	605	42	662
Tercile 3	13%	18%	97%	56%	-22%	9%	12	607	39	658
Market-to-book ratio										
Tercile 1	12%	16%	97%	57%	-22%	10%	6	656	52	714
Tercile 2	11%	15%	96%	59%	-20%	10%	25	646	42	713
Tercile 3	12%	15%	99%	56%	-24%	7%	0	691	21	712

Panel F: Shareholder preference by ownership structure

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	$O_{\text{disc}}(\overline{p}) - O_{\text{full}}(\overline{p})$	$q - \overline{p}$	mm	$1m$	$m1$	Total
N-PX ownership γ										
Tercile 1	17%	22%	94%	59%	-22%	10%	46	662	62	770
Tercile 2	13%	13%	100%	54%	-26%	6%	0	767	0	767
Tercile 3	8%	10%	99%	56%	-20%	8%	16	752	0	768
No-private blockholder										
	13%	14%	99%	55%	-24%	8%	0	1953	59	2012
Private blockholder										
	9%	19%	94%	65%	-16%	13%	8	238	45	291
Institutional ownership										
Tercile 1	13%	20%	94%	55%	-24%	10%	35	658	73	766
Tercile 2	11%	14%	98%	56%	-22%	8%	25	725	18	768
Tercile 3	10%	13%	98%	61%	-18%	10%	2	738	28	768
Activist ownership										
Tercile 1	12%	16%	96%	57%	-22%	9%	0	695	73	768
Tercile 2	11%	14%	99%	56%	-23%	9%	17	734	15	766
Tercile 3	12%	16%	97%	58%	-21%	8%	21	713	34	768

Table 8: **Closeness**

This table shows OLS regression estimates. The dependent variables are in the table caption. Independent variables are indicators for ex-ante or ex-post closeness —that is, the probability or the proportion of proposals to receive voting support within 10% or 1% away from the threshold, respectively. R_{support} is the (realized) support for R (the preferred option of regular voters); t_{disc} is discretionary participation; t_{total} is total participation (i.e., including regular voters); $dSuR$ is discretionary participation of R voters; and $dSuL$ is discretionary participation of L voters. Standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Dependent variable	t_{disc}	t_{disc}	t_{disc}	t_{total}	$dSuR$	$dSuR$	$dSuR$	$dSuL$	$dSuL$	$dSuL$
$R_{\text{support}} \in (40, 60)$	-0.005	0.001		-0.002	-0.113***	-0.106***		0.117***	0.107***	
	(-0.81)	(0.15)		(-0.69)	(-17.76)	(-15.35)		(20.28)	(16.47)	
$R_{\text{support}} \in (49, 510)$			0.001				-0.056***			0.058***
			(0.11)				(-6.82)			(6.83)
γ FE, q FE, n-tercile FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Proposal type FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Firm FE		Y	Y	Y		Y	Y		Y	Y
Year FE		Y	Y	Y		Y	Y		Y	Y
R^2	0.03	0.64	0.64	0.70	0.64	0.70	0.25	0.48	0.39	0.44
$\#N$	2,283	1,909	1,909	1,909	1,909	1,909	2,180	1,816	1,816	1,824

Table 9: **Counterfactuals**

This table shows $\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$ (see Section B.2), the probability of a non-representative outcome (Panel A) and the number of proposals per equilibrium (Panel B) for varying values of c, holding all other parameters constant. The column caption states the value of c as a multiple of the estimates presented in Table 2.

Panel A: Probability of non-representative outcome across cost multiple

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	0.25×	0.5×	0.75×	1×	1.25×	1.5×	1.75×	2×	3×	4×	5×	10×	20×	30×
All	34.3%	34.3%	19%	12.8%	23.3%	29.9%	34.4%	37.4%	40.9%	38.1%	34.8%	19.6%	6.2%	4.2%
Equilibrium														
mm				17.5%	42.4%	44.6%	46.2%	46%	46%	45%	44%	45%	41%	41%
$1m$			12.7%	12.1%	18.1%	22.2%	23.5%	27%						
10									28%	27%	24%	21%		
11	34%	34%												
$m1$			69.5%	32.8%										
$m0$									45%	38%	34%	17%	6%	3%

Panel B: Equilibrium Incidence across cost multiple

	0.25×	0.5×	0.75×	1×	1.25×	1.5×	1.75×	2×	3×	4×	5×	10×	20×	30×
Equilibrium														
mm	0	0	0	62	494	789	1,112	1,225	1,104	910	678	204	80	62
$1m$	0	0	848	2,181	1,811	1,516	1,193	1,080	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	644	606	462	183	0	0
11	55	55	0	0	0	0	0	0	0	0	0	0	0	0
$m1$	0	0	106	62	0	0	0	0	0	0	0	0	0	0
$m0$	0	0	0	0	0	0	0	0	557	789	1,165	1,918	2,215	2,051

Table 10: **Robustness**

This table reports alternative estimations. In Panel A we maintain the whole sample and change some attribute of our algorithm. The first column describes whether the algorithm matches the moments of terciles, the moments of quartiles, the median instead of the mean, or, instead of a proposal type×tercile γ×tercile n×tercile q bin, an ISS recommendation×proposal type×tercile γ×tercile n×tercile q bin. In Panel B we maintain the baseline algorithm and look into different (sub)samples. The first column reports whether the entire sample or a subsample was used, where "High/Low forecast standard deviation" refers to the highest/lowest tercile of analyst forecast standard deviation); "Early/Late meetings" refer to meetings held before/after the median proxy meeting date; "Early/Late meetings–ISS" refer to meetings held before/after ISS has issued recommendations for the respective proposal type in both directions; and "High/low equity lending supply/demand" refers to the quarterly highest/lowest quintile in equity lending supply/demand. In both panels we report: the MSE (column (1)); the probability of an outcome non-representative of the population preference or the preference of regular voters (columns (2) and (3), respectively); the expected turnout against and for the majority of regular voters (columns (4) and (5), respectively); and the incidence of each equilibrium, as well as the total, for these estimations (columns (6)–(12)).

Panel A: Whole sample

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Algorithm	MSE ($O_{\text{disc}}(\overline{p})$)	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	mm	$1m$	10	11	$m1$	$m0$	Total
Base	0.026	13%	15%	98%	56%	62	2,181	0	0	62	0	2,305
Malenko-Shen	0.215											
Idiosyncratic costs	0.051	12%	21%	92%	48%	2,305	0	0	0	0	0	2,305
No $1m$	0.042	8%	30%	77%	86%	1,217	0	0	1,026	62	0	2,305
GMM	0.031	9%	19%	93%	56%	472	1,730	0	0	11	0	2,213
$\overline{a} = 0.9$	0.028	10%	15%	98%	64%	44	2,199	0	0	62	0	2,305
$\overline{a} = 0.8$	0.028	7%	16%	99%	76%	28	2,197	0	0	80	0	2,305
Quartiles	0.026	11%	15%	97%	57%	79	2,136	0	0	90	0	2,305
Six moments	0.028	13%	15%	98%	57%	62	2,181	0	0	62	0	2,305
Eight moments	0.028	13%	15%	98%	57%	62	2,181	0	0	62	0	2,305
Median	0.028	13%	15%	98%	56%	62	2,181	0	0	62	0	2,305
ISS	0.024	13%	16%	98%	56%	44	2,192	0	0	66	0	2,302

Panel B: Baseline algorithm

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Sample	MSE $(O_{\text{disc}}(\bar{p}))$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{full}}(p) \leq 0\right]$	$\mathbb{P}\left[O_{\text{disc}}(p)O_{\text{only-reg}} \leq 0\right]$	t_L	t_R	mm	$1m$	10	11	$m1$	$m0$	Total
Low equity lending supply	0.025	13%	23%	90%	63%	12	212	0	0	59	0	283
High equity lending supply	0.028	12%	14%	99%	56%	29	1,681	0	0	22	0	1,732
Low equity lending demand	0.032	12%	14%	99%	56%	22	1,139	0	0	50	0	1,211
High equity lending demand	0.022	13%	18%	97%	57%	7	778	0	0	34	0	819
Single proposal	0.024	11%	17%	96%	60%	8	286	0	0	26	0	320
Bundle	0.028	12%	14%	98%	55%	31	1,413	0	0	37	0	1,481
Low ownership concentration	0.021	14%	15%	98%	56%	0	733	0	0	38	0	771
High ownership concentration	0.030	12%	17%	96%	58%	37	693	0	0	35	0	765
Low forecast standard deviation	0.029	14%	15%	98%	56%	0	733	0	0	38	0	771
High forecast standard deviation	0.022	12%	17%	96%	58%	37	693	0	0	35	0	765
Early meeting	0.025	12%	15%	97%	57%	20	969	0	0	47	0	1,036
Late meeting	0.026	11%	16%	98%	56%	37	623	0	0	23	0	683
Early meeting–ISS	0.027	11%	17%	92%	66%	7	253	0	0	50	0	310
Late meeting–ISS	0.028	13%	15%	99%	55%	28	1,933	0	0	34	0	1,995

Table 11: **Proposal type classification**

Proposal type	Most frequent ISS proposal descriptions
Board	Separate Chairman and CEO, Require a Majority Vote for the Election of Board, Require Independent Board Chairman
Executive compensation	Advisory Vote to Ratify Named Executive Officers' Compensation, Limit Executive Compensation, Submit Severance Agreement to Shareholder Vote
Takeover defense	Declassify the Board of Directors, Submit Shareholder Rights Plan (Poison Pill), Reduce Supermajority Vote Requirement
Other governance	Amend Articles/Bylaws/Charter, Provide Right to Act by Written Consent

Table 12: **Comparison of Estimation Methods**

This table compares the prediction accuracy of the algorithm to reduced-form models for each proposal. For the dependent variable and the relevant sample see the table caption. Independent variables are the input variables to the algorithm γ, q, n, and the dependent variables from Table 3 of Malenko and Shen (2016), most importantly: NegRec, which equals one if ISS gives a negative recommendation, and zero otherwise; BelowCutoff, which equals one if the firm is below the cutoff ($MaxTSR < 0$), and zero otherwise; and the interaction of these variables. Standard errors are in parentheses. *, **, and *** represent significance at the 10%, 5%, and 1% level, respectively.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$
Sample	All	All	All	All	Excl. 2011	Excl. 2011	Excl. 2011	Excl. 2011
Regression	OLS	OLS	OLS	OLS	OLS	OLS	OLS	OLS
γ	0.355***	0.152***	0.044	-0.159***	0.152***	0.048	-0.165***	0.348***
	(4.820)	(4.508)	(0.825)	(-2.746)	(4.484)	(0.901)	(-2.851)	(4.759)
q	0.705***	0.025***	-0.141***	0.132***	0.025***	-0.142***	0.133***	0.709***
	(18.721)	(2.674)	(-5.440)	(4.930)	(2.698)	(-5.401)	(4.898)	(18.539)
n	0	0	0	0	0	0	0	0
	(-0.700)	(-0.445)	(-0.249)	(-1.250)	(-0.166)	(-0.122)	(-1.165)	(-0.740)
NegRec	16.510	-1.505	-16.44	14.743	-1.679	-16.243	14.525	16.277
	(0)	(0)	(0.000*)	(0)	(0)	(0.000*)	(0)	(0)
MaxTSR	-0.950	0.350	1.916	-0.401	0.644	1.862	-0.169	-0.843
	(0.002***)	(0.001**)	(-0.001)	(0.002***)	(0.001**)	(-0.001)	(0.002***)	(0.002***)
BelowCutoffMaxTSR	0.002***	0.001**	-0.001	0.002***	0.001**	-0.001	0.002***	0.002***
	(2.789)	(2.453)	(-1.333)	(3.688)	(2.515)	(-1.135)	(3.451)	(2.594)
Total compensation	0.000*	0	0	0.000**	0	0	0.000**	0.000*
	(1.745)	(0.900)	(-0.740)	(2.185)	(0.954)	(-0.764)	(2.268)	(1.820)
TDC1 change	0	0	0	0	0	0	0	0
	(-0.221)	(-0.419)	(0.413)	(-0.028)	(-0.390)	(0.589)	(-0.127)	(-0.372)

Continued on next page

Continued from previous page

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent variable	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$	Total participation	$dSuL$	$dSuR$	$O_{\text{disc}}(p)$
% Stock compensation	-0.578	0.068	0.812	0.159	0.090	0.697	0.231	-0.455
	(-0.004*)	(0)	(0.003*)	(-0.003*)	(0)	(0.003*)	(-0.003*)	(-0.004)
Director holdings %	-1.752	0.682	1.889	-1.933	0.782	1.735	-1.796	-1.578
	(-0.015)	(0.002)	(0.002)	(-0.01)	(0.003)	(0)	(-0.01)	(-0.013)
Log equity	-0.015	0.002	0.002	-0.01	0.003	0	-0.01	-0.013
	(-1.205)	(0.465)	(0.195)	(-1.045)	(0.486)	(0.047)	(-1.012)	(-1.090)
ROA	0.102	0.071	0.008	0.174**	0.067	0.016	0.161**	0.086
	(1.039)	(0.961)	(0.0978)	(2.432)	(0.908)	(0.203)	(2.264)	(0.877)
Market-to-book ratio	-0.009	-0.008*	0	-0.01	-0.008*	-0.001	-0.01	-0.008
	(-0.700)	(-1.774)	(0.00022)	(-1.183)	(-1.808)	(-0.091)	(-1.119)	(-0.620)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Proposal type FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.709	0.704	0.559	0.51	0.705	0.559	0.509	0.709
$\#N$	1,666	1,737	1,672	1,667	1,715	1,652	1,647	1,646

Figure 1: Equilibria regions under disagreement and agreement across the group of voters ($\overline{a} = 1$, $h - l = 0.5$, $q = 0.8$, $p \sim \mathcal{U}[l, h]$).

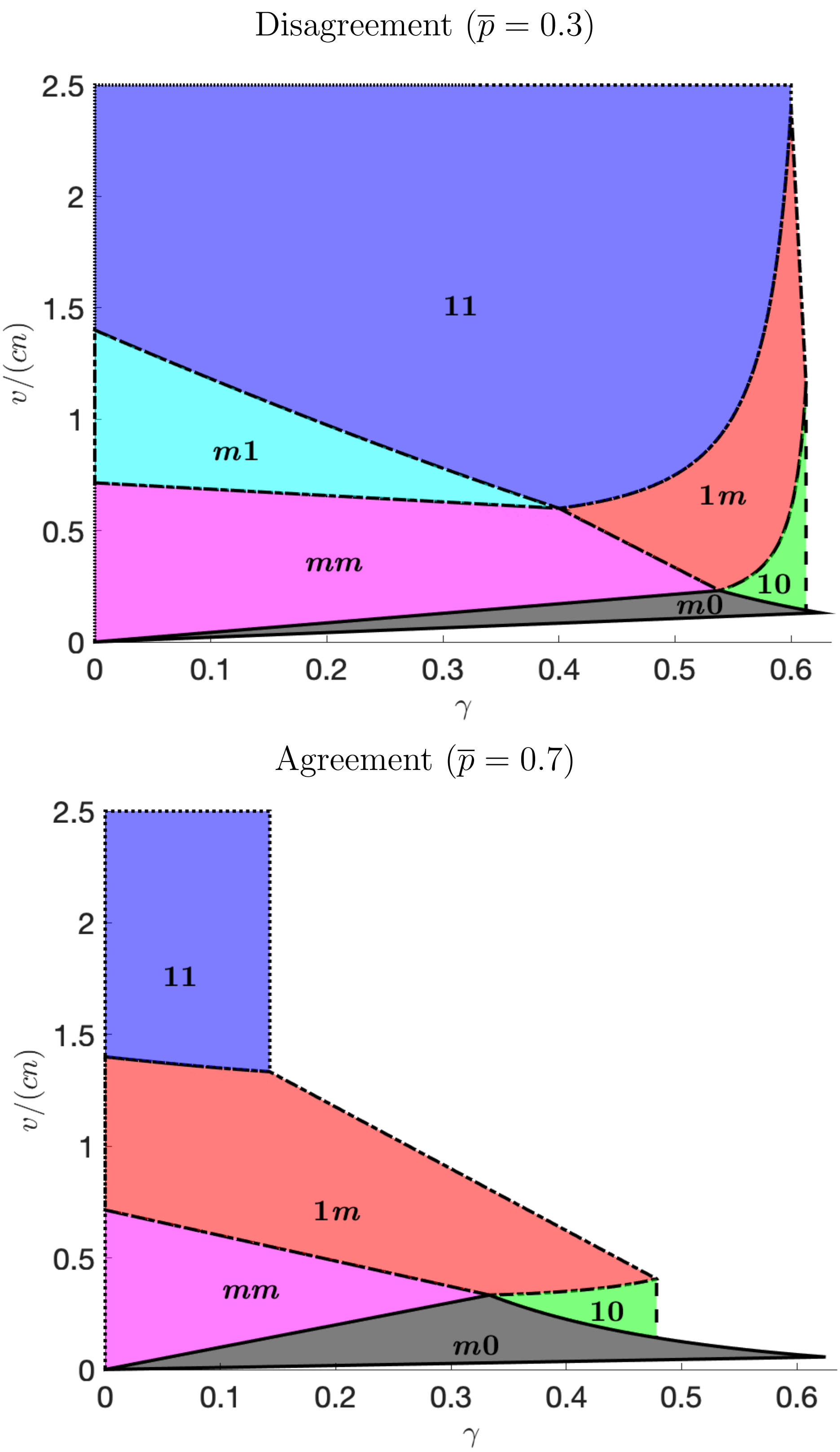


Figure 2: **Benefit-to-cost ratio vs. average ownership stake.**

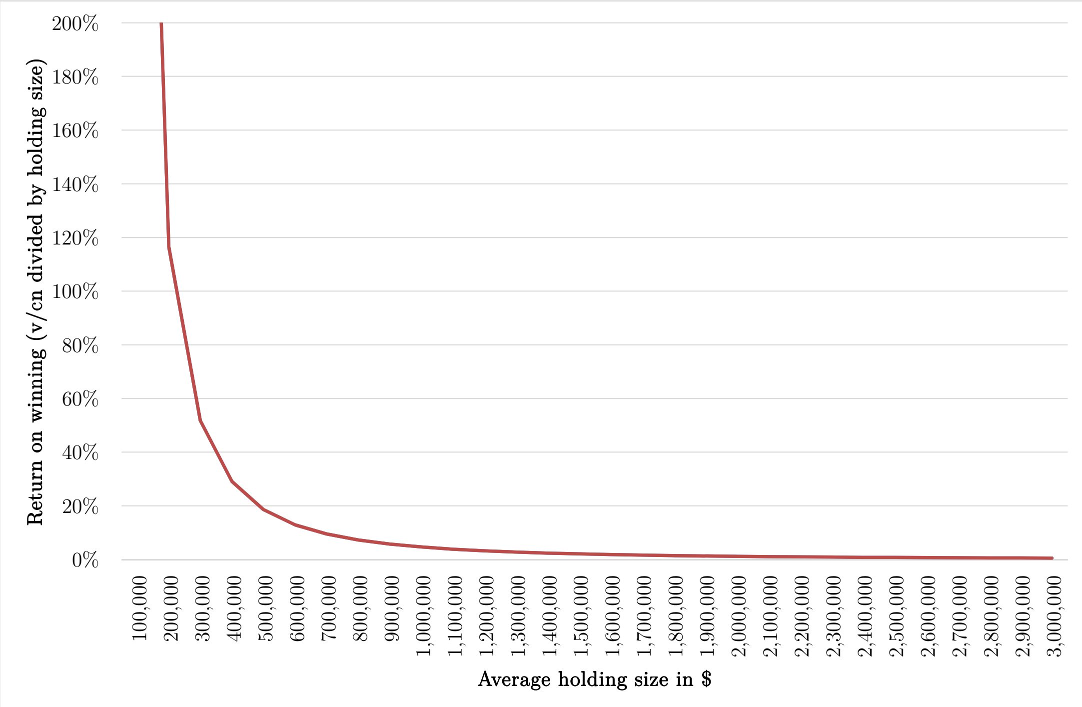
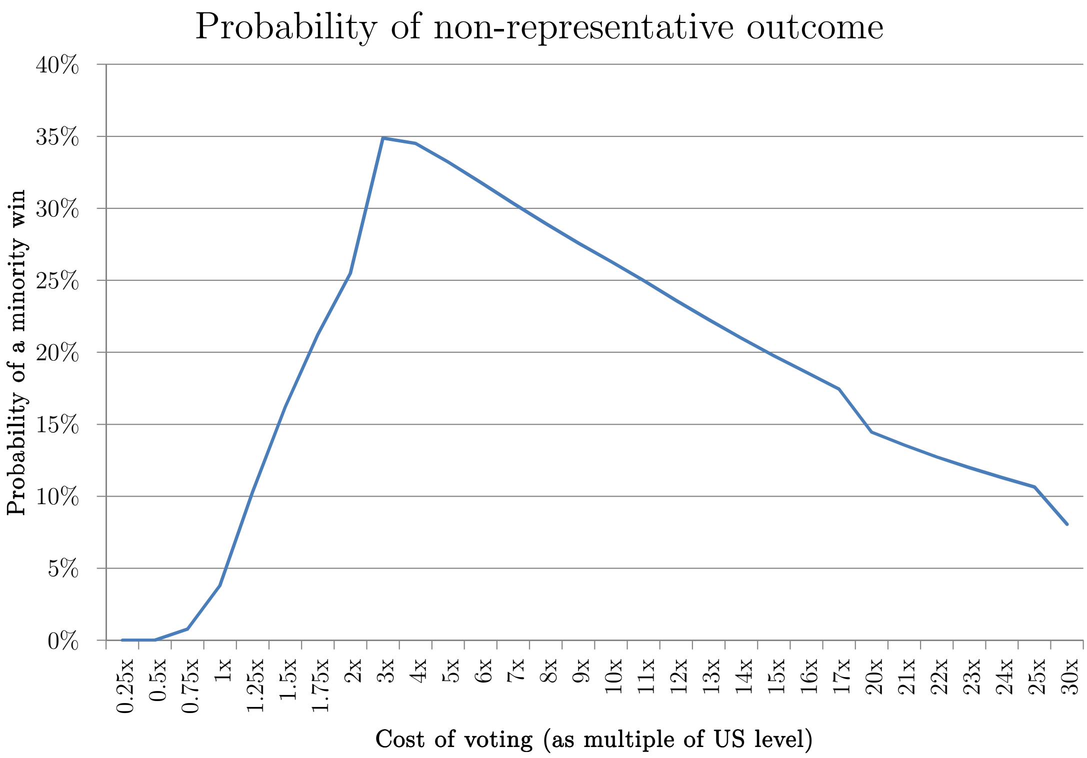
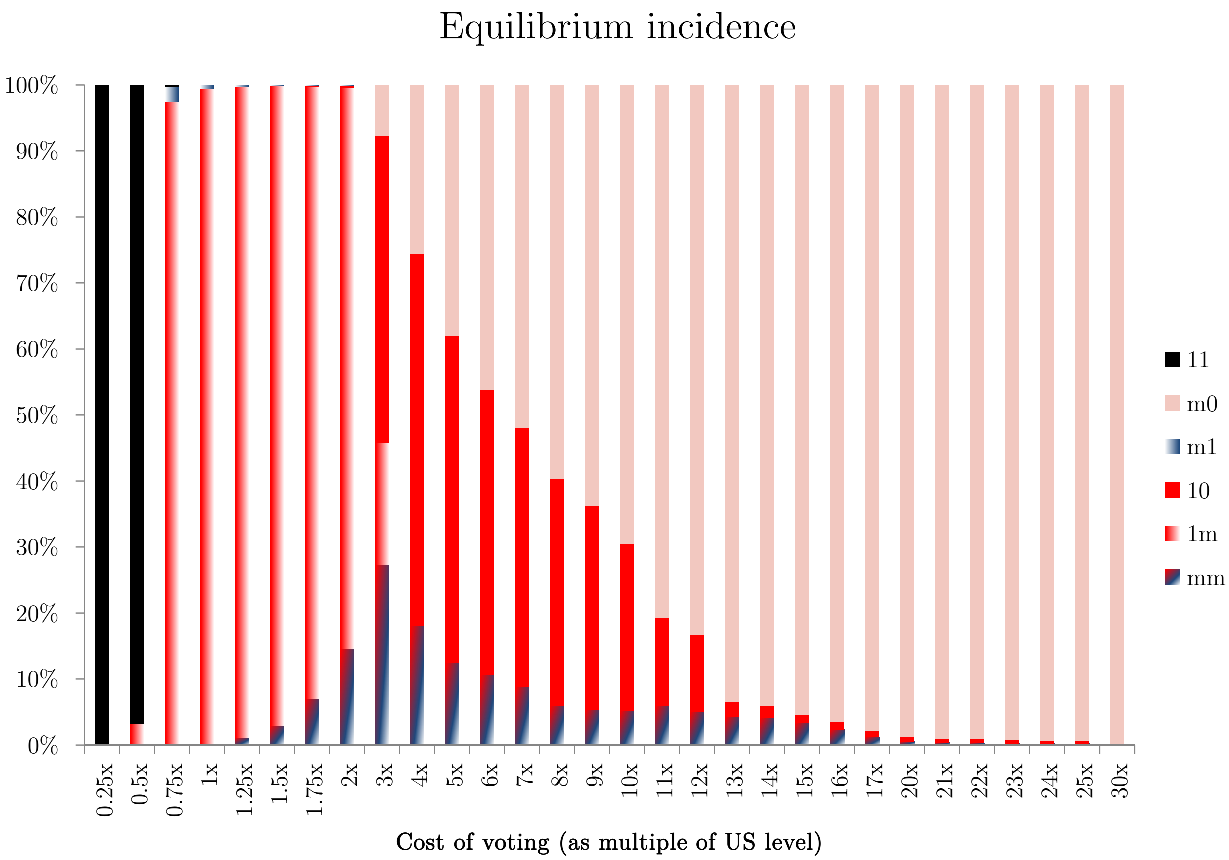


Figure 3: **Counterfactuals**





Online Appendix

"Freeriders and Underdogs: Participation in Corporate Voting"

Konstantinos E. Zachariadis, Dragana Cvijanović, and Moqi Groen-Xu

In this Online Appendix, we present the propositions and corresponding proofs for the equilibria where at least one type of discretionary voter has a 'corner' participation rate (Section OA.1).

OA.1 Equilibria with 'Corner' Participation

Throughout, assume that $q \in (1/2, 1)$, $p \in (l, h) \subseteq (0,1)$, $\overline{a} \in (0,1]$, $g(a) = \delta(a - \overline{a})$, where δ is the Dirac function, A1–A2, and let $d \equiv h - l$.

Proposition 3 (Equilibrium $1m$)**.** *Assume that $p \sim \mathcal{U}[l,h]$. If and only if*

$$n \in N_{1m} \equiv \left(\max\left\{ \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}, \frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a} - (2q-1)\gamma} \right\}, \infty \right), \quad \text{(OA.1)}$$

$$\gamma \in \Gamma_{1m} \equiv \left(\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p}) + 2q - 1} \mathbb{I}\left(\overline{p} < \frac{1}{2}\right), \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \quad \text{(OA.2)}$$

$$\begin{aligned} \frac{v}{cn} \in V_{1m} \equiv & \left(\max\left\{ \frac{(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}} d, \frac{1}{n} \right\}, \right. \\ & \left. \min\left\{ \frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma}, \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{l^2}\overline{p}d \right\} \right), \end{aligned} \quad \text{(OA.3)}$$

where $\mathbb{I}$ is the indicator function, there exists an equilibrium with $t_L = 1$ and $t_R \in (0,1)$ given by

$$t_R = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{d\overline{p}}\frac{v}{cn}} - 1. \quad \text{(OA.4)}$$

Proposition 4 (Equilibrium 10)**.** *If and only if*

$$n \in N_{10} \equiv \left(\frac{f(p^*)\,p^*}{(1-\gamma)\overline{ap}}, \infty \right), \quad \text{(OA.5)}$$

$$\gamma \in \Gamma_{10} \equiv \left(\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}, \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \right), \quad \text{(OA.6)}$$

$$\frac{v}{cn} \in V_{10} \equiv \left(\max\left\{ \frac{1}{n}, \frac{(1-\gamma)\overline{a}(1-\overline{p})}{f(p^*)\,(1-p^*)} \right\}, \frac{(1-\gamma)\overline{ap}}{f(p^*)\,p^*} \right), \quad \text{(OA.7)}$$

there exists an equilibrium with $t_L = 1$ *and* $t_R = 0$*, where from (4)* $p^* = 1 - (2q-1)\gamma / ((1-\gamma)\overline{a})$.

Proposition 5 (Equilibrium 11)**.** *If and only if* $l < 1/2$ *and*

$$\begin{aligned}
n \in N_{11} &\equiv (0, \infty), && \text{(OA.8)}\\
\gamma \in \Gamma_{11} &\equiv \left(\max\left\{0, \frac{\overline{a}(1-2h)}{\overline{a}(1-2h)+2q-1}\mathbb{I}\left(h<\frac{1}{2}\right)\right\}, \frac{\overline{a}(1-2l)}{\overline{a}(1-2l)+2q-1}\right), && \text{(OA.9)}\\
\frac{v}{cn} \in V_{11} &\equiv \left(\max\left\{\frac{(1-\gamma)\overline{ap}}{f(p^*)\frac{p^*}{2}}, \frac{(1-\gamma)\overline{a}(1-\overline{p})}{f(p^*)\frac{1-p^*}{2}}, \frac{1}{n}\right\}, \infty\right), && \text{(OA.10)}
\end{aligned}$$

where $\mathbb{I}$ *is the indicator function, there exists an equilibrium with* $t_L = 1$ *and* $t_R = 1$*, where from (4)* $p^* = 1/2 - (2q-1)\gamma / (2(1-\gamma)\overline{a})$.

Proposition 6 (Equilibrium $m1$)**.** *Assume that* $p \sim \mathcal{U}[l,h]$*. If and only if* $\overline{p} < 1/2$ *and*

$$n \in N_{m1} \equiv \left(\max\left\{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})}, \frac{(1-h)^2}{d(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}\right\}, \infty\right), \tag{OA.11}$$

$$\begin{aligned}
\gamma \in \Gamma_{m1} &\equiv \left(0, \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}\right), && \text{(OA.12)}\\
\frac{v}{cn} \in V_{m1} &\equiv \left(\max\left\{\frac{1}{n}, d\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})}\right\},\right.\\
&\quad \left.\min\left\{d\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma}, d\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}\right\}\right), && \text{(OA.13)}
\end{aligned}$$

there exists an equilibrium with $t_L \in (0,1)$ *given by*

$$t_L = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{d(1-\overline{p})}\frac{v}{cn}} - 1, \tag{OA.14}$$

and $t_R = 1$.

Proposition 7 (Equilibrium m0)**.** *Assume that* $p \sim \mathcal{U}[l,h]$*. If and only if*

$$\begin{aligned}
n \in N_{m0} &\equiv \left(\max\left\{\frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}, \frac{1-\overline{p}}{d(2q-1)\gamma}\right\}, \infty\right), && \text{(OA.15)}\\
\gamma \in \Gamma_{m0} &\equiv \left(0, \frac{\overline{a}}{2q-1+\overline{a}}\right), && \text{(OA.16)}
\end{aligned}$$

$$\frac{v}{cn} \in V_{m0} \equiv \left(\max\left\{\frac{1}{n}, d(1-\overline{p})(2q-1)\gamma\right\}, \min\left\{d\frac{(1-\gamma)^2\overline{a}^2(1-p)}{(2q-1)\gamma}, \frac{d(2q-1)\gamma}{1-\overline{p}}\right\}\right), \tag{OA.17}$$

there exists an equilibrium with $t_L \in (0,1)$ *given by*

$$t_L \quad = \frac{1}{(1-\gamma)\overline{a}}\sqrt{\frac{(2q-1)\gamma}{d(1-\overline{p})}\frac{v}{cn}} \tag{OA.18}$$

and $t_R = 0$.

OA.1.1 Proofs of Propositions 3–7

Note that in equilibrium, it cannot be that $t_L = t_R = 0$ (see Assumptions 1 and 2); also, recall that, $t_L = 0$ (and $t_R \in (0.1]$) can, also, not occur in any equilibrium (see Corollary (1)), as this would imply that the probability p^* in (4) is negative. Hence, the equilibria we need to inquire about are (in addition to equilibrium mm, which we presented in Proposition 1 in the main text):

$$t_L = 1, t_R \in (0,1), \tag{1m}$$

$$t_L = 1, t_R = 0, \tag{10}$$

$$t_L = 1, t_R = 1, \tag{11}$$

$$t_L \in (0,1), t_R = 1, \tag{m1}$$

$$t_L \in (0,1), t_R = 0. \tag{m0}$$

We begin with equilibria $1m$, 10, and 11 where $t_L = 1$. Let:

$$K \equiv 1 - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{OA.19}$$

so from (4), in those equilibria, $p^* = K/(1+t_R)$. Given this, we need $K > 0$, otherwise $p^* < 0$, i.e.,

$$1 > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \overline{a} - \overline{a}\gamma > (2q-1)\gamma \iff \overline{a} > (2q-1+\overline{a})\gamma \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}}. \tag{OA.20}$$

This upper bound on γ takes precedent over the one required by assumption A1 (i.e., $\gamma < 1/(2q)$), since $\overline{a} < 1$.

We also need $p^* < 1$, i.e.,

$$K\frac{1}{1+t_R} < 1 \iff t_R > K - 1,$$

which is always true since $K<1$ (see (OA.19)).

Moreover, for $t_L=1$, we must have:

$$\mathbb{P}\left[\text{pivotal}|L\right]>\frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}\frac{f(p^*)(1-p^*)}{1+t_R}>\frac{c}{v} \iff \frac{f(p^*)(1-p^*)}{1+t_R}>\frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p})$$
$$\iff f\left(K\frac{1}{1+t_R}\right)\frac{1+t_R-K}{(1+t_R)^2}>\frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}), \tag{OA.21}$$

where in the second inequality we substituted for the pivotal probability from (2).

Now, we focus on each of the equilibria with $t_L=1$ in turn. The main goal of the proofs is to derive the regions of n, γ, and $v/(cn)$ so that, given $t_L=1$ we have that $t_R\in(0,1)$ and probability $p^*\in(l,h)$ (while also respecting assumptions A1 and A2).

Equilibrium 1*m*: Deriving the expression for t_R: For $t_R\in(0,1)$, we must have:

$$\mathbb{P}\left[\text{pivotal}|R\right]=\frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{1+t_R}=\frac{c}{v} \iff f\left(K\frac{1}{1+t_R}\right)\frac{K}{(1+t_R)^2} \quad =\frac{c}{v}(1-\gamma)n\overline{ap}, \tag{OA.22}$$

where in the second equality we substituted for the pivotal probability from (3) and in the third for $p^*=K/(1+t_R)$. Let us assume that $p\sim\mathcal{U}[l,h], 0\leq l<h\leq 1$, and $p^*\in(l,h)$, which we check below; then, $\overline{p}=(h+l)/2, f(p^*)=1/(h-l)=1/d$, and (OA.22) becomes:

$$\frac{1}{d}\frac{K}{(1+t_R)^2}=\frac{c}{v}(1-\gamma)n\overline{ap} \iff (1+t_R)^2=\frac{K}{\frac{dc}{v}(1-\gamma)n\overline{ap}}=\frac{1-\dfrac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\frac{dc}{v}(1-\gamma)n\overline{ap}},$$

which if we solve yields (OA.4); where in the last equality we substituted for K from (OA.19).

Checking that $t_R\in(0,1)$: We need to ensure that

$$t_R<1 \iff \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}}\frac{1}{(1-\gamma)^2\overline{a}^2}<4 \iff \frac{v}{c}<nd\frac{4(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \tag{OA.23}$$

where in the second inequality we substituted from (OA.4); and, similarly,

$$t_R>0 \iff \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\frac{dc}{v}n\overline{p}}>(1-\gamma)^2\overline{a}^2 \iff \frac{v}{c}>nd\frac{(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}. \tag{OA.24}$$

In addition, from (OA.21), for $p \sim \mathcal{U}[l,h]$ we have that:

$$
\begin{aligned}
&\frac{1}{d}\frac{1+t_R-K}{(1+t_R)^2} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \iff \frac{1}{1+t_R} - \frac{K}{(1+t_R)^2} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
&\iff \sqrt{\frac{\frac{dc}{v}n\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}\overline{p} > \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \\
&\iff \sqrt{\frac{v}{dcn}\frac{\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}} > \overline{p}+1-\overline{p} \iff \frac{v}{c} > nd\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}, \qquad \text{(OA.25)}
\end{aligned}
$$

where in the third inequality we substituted for t_R from (OA.4).

Now, we compare the two lower bounds on v/c (OA.24) and (OA.25), to see if they either of them can be relevant:

$$
(1-\gamma)^2\overline{a}\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2 \iff \overline{a}(1-\overline{p}) < \gamma\left[\overline{a}(1-\overline{p})+2q-1\right] \iff \gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1}.
$$

Is this last requirement consistent with the current lower bound on γ (OA.20)?

$$
\frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p})+2q-1} < \frac{\overline{a}}{2q-1+\overline{a}} \iff (1-\overline{p})(2q-1)+(1-\overline{p})\overline{a} < \overline{a}(1-\overline{p})+2q-1 \iff \overline{p} > 0,
$$

which is always true, hence, either of (OA.24) and (OA.25) can be relevant.

Moreoever, for existence of equilibrium $1m$, we need to check that the upper bound on v/c in (OA.23) is larger than both the two possible lower bounds in (OA.24) and (OA.25). First, from (OA.23) and (OA.25) we have existence if and only if

$$
\begin{aligned}
&4(1-\gamma)^2\overline{a}^2\overline{p}^2 > [(1-\gamma)\overline{a}-(2q-1)\gamma]^2 \iff 2(1-\gamma)\overline{a}\overline{p} > (1-\gamma)\overline{a}-(2q-1)\gamma \\
&\iff (1-\gamma)\overline{a}(1-2\overline{p})-(2q-1)\gamma < 0 \iff \overline{a}(1-2\overline{p}) < \gamma\left[\overline{a}(1-2\overline{p})+2q-1\right]. \qquad \text{(OA.26)}
\end{aligned}
$$

We inquire on the validity of (OA.26) by considering the following cases:

Case A: If $1-2\overline{p} > 0 \iff \overline{p} < 1/2$, then from (OA.26) we need:

$$
\gamma > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}. \qquad \text{(OA.27)}
$$

For this to be consistent with the current upper bound on γ (OA.20), we further need:

$$\frac{\overline{a}}{2q-1+\overline{a}} > \frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \iff \overline{a}(1-2\overline{p})+2q-1 > (1-2\overline{p})(2q-1)+\overline{a}(1-2\overline{p}) \iff \overline{p} > 0,$$

which is always true.

Case B: If $1-2\overline{p} < 0 \iff \overline{p} > 1/2$ and $\overline{a}(1-2\overline{p})+2q-1 > 0 \iff \overline{p} < (2q-1)/(2\overline{a})+1/2$, that is, if $1/2 < \overline{p} < (2q-1)/(2\overline{a})+1/2$, then (OA.26) is true without any further parameter restrictions.

Case C: If $\overline{p} > (2q-1)/(2\overline{a})+1/2$ for (OA.26) to be true we need:

$$\overline{a}(2\overline{p}-1) > \gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \iff \gamma < \frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}. \tag{OA.28}$$

We need to check how the above compares with the current upper bound on γ (OA.20):

$$\frac{2\overline{p}-1}{\overline{a}(2\overline{p}-1)-(2q-1)} < \frac{1}{2q-1+\overline{a}} \iff (2\overline{p}-1)(2q-1)+(2\overline{p}-1)\overline{a} < (2\overline{p}-1)\overline{a}-(2q-1)$$
$$\iff 2\overline{p}-1 < -1 \iff \overline{p} < 0,$$

which is false; so (OA.28) is weaker than (OA.20) and does not take precedence (i.e., is irrelevant).

Second, trivially, the lower bound on v/c in (OA.24) is smaller than the upper bound in (OA.23).

Let us summarize our findings up to this point: An equilibrium with $t_L = 1, t_R \in (0,1)$ given by (OA.4) exists if:

$$\max\left\{\frac{(1-\gamma)^2\overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}nd\right\} \tag{OA.29}$$

$$< \frac{v}{c} <$$

$$nd\frac{4(1-\gamma)^2\overline{a}^2\overline{p}}{(1-\gamma)\overline{a}-(2q-1)\gamma}, \tag{OA.30}$$

and $$\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}\mathbb{I}(\overline{p} < 1/2) < \gamma < \frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.31}$$

where $\mathbb{I}$ is the indicator function.

Checking assumptions A1 and A2: As mentioned, for all equilibria with $t_L = 1$ the upper bound in (OA.20) takes precedent over the one in A1. We continue with the requirement of A2 that $v/c \geq 1$. Hence, for an equilibrium to exist we need that expression (OA.30) (the current upper bound) is higher than one:

$$d4(1-\gamma)^2\overline{a}n\overline{p} > (1-\gamma)\overline{a} - (2q-1)\gamma \iff n > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d}. \tag{OA.32}$$

This implies a lower bound on the number of voters n. Next, we compare the lower bound imposed by A2 with the current ones in (OA.29). In turn we have:

i) For the lower bound (OA.24) to be the relevant one we need:

$$\text{relative to `1'} \rightarrow \quad n > \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d},$$
$$\text{and relative to (OA.25)} \rightarrow \quad \gamma > \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}.$$

ii) For the lower bound (OA.25) to be the relevant one we need:

$$\text{relative to `1'} \rightarrow \quad n > \frac{\overline{p}}{d\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]},$$
$$\text{and relative to (OA.24)} \rightarrow \quad \gamma < \frac{\overline{a}(1-\overline{p})}{\overline{a}(1-\overline{p}) + 2q - 1}.$$

iii) For the lower bound in A2 (i.e., '1') to be the relevant one we need:

$$\text{relative to (OA.24)} \rightarrow \quad n < \frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{(1-\gamma)^2\overline{a}^2\overline{p}d},$$
$$\text{and relative to (OA.25)} \rightarrow \quad n < \frac{\overline{p}}{\left[(1-\gamma)\overline{a} - (2q-1)\gamma\right]d}.$$

Hence, depending on the parameters any of the three lower bounds on v/c can be the relevant one.

Checking that $p^* \in (l, h)$: Since $p \in [l, h]$ we also need to check that $l < p^* < h$. Recall that for the equilibrium under consideration (i.e., $1m$), $p^* = K/(1 + t_R)$, where K is given by (OA.19) and t_R

by (OA.4). Hence, after some algebra we write the requirement on p^* as:

$$l < p^* < h \iff \frac{1}{h^2} < \frac{1}{p^{*2}} < \frac{1}{l^2} \iff \frac{1}{h^2} < \frac{v}{nc}\frac{1}{d\overline{p}}\frac{1}{[(1-\gamma)\overline{a} - (2q-1)\gamma]} < \frac{1}{l^2}$$

$$\iff \frac{nd\overline{p}[(1-\gamma)\overline{a} - (2q-1)\gamma]}{h^2} \tag{OA.33}$$
$$< \frac{v}{c} <$$
$$\frac{nd\overline{p}[(1-\gamma)\overline{a} - (2q-1)\gamma]}{l^2}. \tag{OA.34}$$

How does these reconcile with the existing bounds (OA.29) and (OA.30)? In turn we have:

i) (OA.33) vs the second part of the maximum in the lower bound (OA.29) (i.e., the right-hand-side (RHS) of (OA.25)):

$$\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{\overline{p}} d > d\overline{p}\frac{(1-\gamma)\overline{a} - (2q-1)\gamma}{h^2} \iff \frac{1}{\overline{p}} > \frac{\overline{p}}{h^2} \iff \overline{p} < h,$$

which is always true; hence, (OA.33) is never the relevant lower bound.

ii) (OA.34) vs the upper bound in (OA.30):

$$\frac{d\overline{p}\,[(1-\gamma)\overline{a} - (2q-1)\gamma]}{l^2} > \frac{4(1-\gamma)^2\overline{a}^2\overline{p}d}{(1-\gamma)\overline{a} - (2q-1)\gamma} \iff [(1-\gamma)\overline{a} - (2q-1)\gamma]^2 > 4(1-\gamma)^2\overline{a}^2 l^2$$

$$\iff (1-\gamma)\overline{a} - (2q-1)\gamma > 2(1-\gamma)\overline{a}l \iff (1-2l)\overline{a} > (2q-1+\overline{a}(1-2l))\gamma. \tag{OA.35}$$

To inquire on the validity of (OA.35) we need to consider the following cases:

Case A: If $1-2l > 0 \iff l < 1/2$, then we need $\gamma < (1-2l)\overline{a}(2q-1+\overline{a}(1-2l))$ for the current upper bound on v/c in (OA.30) to be the relevant one. Otherwise, (OA.34) takes precedence. Note that the above upper bound on γ is lower than our current one in (OA.31), and so, is in the feasible region.

Case B: If $1-2l < 0 \iff l > 1/2$, then:

B-1) If $2q-1+\overline{a}(1-2l) > 0 \iff \overline{a} < (2q-1)/(2l-1)$ then (OA.34) is the relevant upper bound.

B-2) If $2q-1+\overline{a}(1-2l) < 0 \iff \overline{a} > (2q-1)/(2l-1)$, then we need $\gamma > (2l-1)\overline{a}/(1-2q+\overline{a}(2l-1))$ for the current upper bound on v/c in (OA.30) to be the relevant one. Otherwise, (OA.34) takes precedence. Note that, again, the above upper bound on γ is lower than our current one in (OA.31), and so is in the feasible region.

Therefore, (OA.34) can be the relevant upper bound and, hence, we need to ensure that it is larger than the existing possible lower bounds on v/c in (OA.29) and one (from A2). We have, in turn:

i) Comparing (OA.34) vs '1', leads to

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d > 1 \iff n > \frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a}-(2q-1)\gamma}. \tag{OA.36}$$

How does (OA.36) compare with the existing lower bound on n in (OA.32)?

$$\frac{1}{\overline{p}d}\frac{l^2}{(1-\gamma)\overline{a}-(2q-1)\gamma} > \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{4(1-\gamma)^2\overline{a}^2\overline{p}d} \iff 2l(1-\gamma)\overline{a} > (1-\gamma)\overline{a}-(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(1-2l) < (2q-1)\gamma.$$

Hence, if $1-2l<0 \iff l>1/2$, then the above is always true and (OA.36) takes precedent. While if $1-2l>0 \iff l<1/2$, then we need: $\gamma > \overline{a}(1-2l)/(2q-1+\overline{a}(1-2l))$, which is within the feasible for γ interval since $l>0$. Therefore, for $l<1/2$ either of (OA.36) and (OA.32) can be the relevant lower bound on n. This results in region N_{1m} defined in (OA.1).

ii) Comparing (OA.34) vs the first term in the maximum for the lower bound in (OA.29), leads to

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d > \frac{(1-\gamma)^2\overline{a}^2 n\overline{p}d}{(1-\gamma)\overline{a}-(2q-1)\gamma} \iff (1-\gamma)\overline{a}-(2q-1)\gamma > (1-\gamma)\overline{a}l$$
$$\iff (1-\gamma)\overline{a}(1-l) > (2q-1)\gamma \iff \overline{a}(1-l) > [2q-1+\overline{a}(1-l)]\,\gamma \iff \gamma < \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}. \tag{OA.37}$$

Now, the upper bound in (OA.37) is lower than the one in (OA.31) so it takes precedent (and it is also larger than the current lower bound in (OA.31), since $\overline{p}>l$). This, together with the lower bound in (OA.31), results in region Γ_{1m} defined in (OA.2).

iii) Comparing (OA.34) vs the second term in the maximum for the lower bound in (OA.30):

$$\frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{l^2}n\overline{p}d > \frac{(1-\gamma)\overline{a}-(2q-1)\gamma}{\overline{p}}nd \iff \overline{p} > l,$$

which is always true.

Hence, the upper bound in (OA.30) needs to be amended to include (OA.34). While, as we mentioned before, the lower bounds in (OA.29) need to be amended to include the bound imposed by A2. These result in region V_{1m} defined in (OA.3) (where we further divide all expressions by n). This conclude the proof of Proposition 3. ■

Equilibrium 10. Drawing inferences from the values of t_L, t_R: For $t_L = 1$ and $t_R = 0$, we have that $p^* = K/(1+t_R) = K$, where K is defined in (OA.19). Moreover, no discretionary participation of R means that (adapting (OA.22)) we have:

$$\mathbb{P}\left[\text{pivotal}|R\right] < \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{1+0} < \frac{c}{v} \iff \frac{v}{c} < \frac{(1-\gamma)n\overline{ap}}{f(K)K}. \tag{OA.38}$$

While for full discretionary participation of L we need from (OA.21):

$$\frac{v}{c} > \frac{(1-\gamma)n\overline{a}(1-\overline{p})}{f(K)(1-K)}. \tag{OA.39}$$

For existence of equilibrium we need the lower bound on v/c in (OA.39) to be smaller than the upper bound implied by (OA.38). Hence,

$$\frac{(1-\gamma)n\overline{a}(1-\overline{p})}{(1-\gamma)n\overline{ap}} < \frac{f(K)(1-K)}{f(K)K} \iff 1-\overline{p}-\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} < 0 \iff \gamma > \frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}}, \tag{OA.40}$$

where in the second inequality we substituted for K from (OA.19) and cancelled common terms. Is this new lower bound smaller than the current upper bound in (OA.20)? That is, for existence we need:

$$\frac{(1-\overline{p})\overline{a}}{2q-1+(1-\overline{p})\overline{a}} < \frac{\overline{a}}{2q-1+\overline{a}} \iff (1-\overline{p})\overline{a}\left[2q-1+\overline{a}\right] < (2q-1+(1-\overline{p})\overline{a})\overline{a}$$
$$\iff (1-\overline{p}-1)\overline{a}(2q-1) < 0 \iff -\overline{pa}(2q-1) < 0,$$

which is always true, so the lower bound in (OA.40) is in the feasible region of γ without further parameter restrictions.

Checking assumptions A1 and A2: As mentioned for all the equilibria with $t_L = 1$ the upper bound in (OA.20) takes precedent over the one in A1. What about A2 that $v/c \geq 1$? For existence, we need '1' to be smaller than the current upper bound in (OA.38):

$$1 \leq \frac{(1-\gamma)n\overline{a}\overline{p}}{f(K)K} \iff n \geq \frac{f(K)K}{(1-\gamma)\overline{a}\overline{p}}, \tag{OA.41}$$

which imposes a lower bound on the number of voters. This bound defines N_{10} in (OA.5). Then the lower bound of v/c in (OA.39) together with '1', and the upper bound in (OA.38) define V_{10} in (OA.7) (where we further divide all expressions by n).

Checking that $p^* \in (l, h)$: Since $p \in [l, h]$ we also need to check that $l < p^* < h$ or, for equilibrium 10, $l < K < h$. Substituting for K from (OA.19) and after a bit of algebra we arrive at:

$$\frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} \tag{OA.42}$$

$$< \gamma <$$

$$\frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)}. \tag{OA.43}$$

How does (OA.42) compare with the current lower bound in (OA.40)?

$$\frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)} < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})} \iff \frac{2q-1}{\overline{a}(1-h)} > \frac{2q-1}{\overline{a}(1-\overline{p})} \iff \overline{p} < h,$$

which is always true and therefore, the current lower bound stands.

How does (OA.43) compare with the current upper bound in (OA.20)?

$$\frac{\overline{a}}{2q-1+\overline{a}} < \frac{\overline{a}(1-l)}{2q-1+\overline{a}(1-l)} \iff \frac{2q-1}{\overline{a}} > \frac{2q-1}{\overline{a}(1-l)} \iff l < 0,$$

which is never true, and so, (OA.43) takes precedence. Then this upper bound together with the lower bound in (OA.40) define region Γ_{10} in (OA.6). This concludes the proof of Proposition 4. ■

Equilibrium 11. Drawing inferences from the values of t_L, t_R: For $t_R = 1$ we have $p^* = K/2$, and we need (adapting (OA.22)) that:

$$\mathbb{P}\left[\text{pivotal}|R\right] > \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}}\frac{f(p^*)p^*}{2} > \frac{c}{v}, \iff \frac{v}{c} > \frac{(1-\gamma)n\overline{ap}}{f\left(\frac{K}{2}\right)\frac{K}{4}}. \tag{OA.44}$$

For $t_L = 1$, we have from (OA.21):

$$f\left(\frac{K}{2}\right)\frac{2-K}{4} > \frac{c}{v}(1-\gamma)n\overline{a}(1-\overline{p}) \iff \frac{v}{c} > \frac{(1-\gamma)n\overline{a}(1-\overline{p})}{f\left(\frac{K}{2}\right)\frac{2-K}{4}}. \tag{OA.45}$$

Depending on parameter values either of (OA.44) and (OA.45) can be the relevant lower bound.

Checking assumptions A1 and A2: As mentioned for all the equilibria with $t_L = 1$ the upper bound in (OA.20) takes precedent over the one in A1. Now, lower bounds (OA.44) and (OA.45), together with '1' from A2 define region V_{11} in (OA.10) (where we divide all expressions by n). Note that there is no upper bound on v/c in equilibrium 11.

Checking that $p^* \in (l, h)$: Since $p \in [l, h]$ we also need to check that $l < p^* < h$. Using that now $p^* = K/2$ and (OA.19) for K we arrive at the

$$1 - 2h \leq \frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1 - 2l. \tag{OA.46}$$

Therefore, for equilibrium to exist, we need $1 - 2l \geq 0 \iff l \leq 1/2$, and this is part of the requirements in the statement of Proposition 5. Given this from the second inequality in (OA.46) we have the requirement:

$$\frac{2q-1}{\overline{a}}\frac{\gamma}{1-\gamma} \leq 1 - 2l \iff \gamma \leq \frac{(1-2l)\overline{a}}{(1-2l)\overline{a} + 2q - 1}, \tag{OA.47}$$

which as an upper bound takes precedence over the current one in (OA.20). Now, for $1 - 2h$ we consider two cases:

Case A: If $1 - 2h < 0 \iff h > 1/2$, then the first inequality in (OA.46) is trivially true without further restrictions on parameters.

Case B: If $1 - 2h > 0 \iff h < 1/2$, then we also need:

$$\gamma \geq \frac{(1-2h)\overline{a}}{(1-2h)\overline{a} + 2q - 1}. \tag{OA.48}$$

The lower bound on γ in (OA.48) is always smaller than the current upper bound in (OA.47); putting them together leads to region Γ_{11} is (OA.9). Note, that we do not have any restriction on the number of voters n and hence the region N_{11} in (OA.8) is the entire positive half-line. This concludes the proof of Proposition 5. ■

Below, me move on to the proofs pertaining to equilibria where $t_L < 1$, i.e., $m1$ and $m0$. Again, the main goal of the proofs is to derive the regions of n, γ, and $v/(cn)$ so that, given $t_R = 1$ or 0 we have that $t_L \in (0,1)$ and probability $p^* \in (l,h)$ (while also respecting assumptions A1 and A2).

Equilibrium *m*1. Deriving p^*: Here, we inquire whether $t_L \in (0,1)$, $t_R = 1$ can be an equilibrium. First, from (4) we have that:

$$p^* = \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L} \iff 1 - p^* = \frac{1}{1+t_L}\left[1 + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right]. \tag{OA.49}$$

The term in the square brackets is positive so $p^* < 1$ but we further need that

$$p^* > 0 \iff 1 - p^* < 1 \iff 1 + t_L > 1 + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff t_L > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}. \tag{OA.50}$$

So, for an equilibrium where L play a mixed strategy to exist we must have

$$\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} < 1 \iff (2q-1+\overline{a})\gamma < \overline{a} \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.51}$$

which is the same as (OA.20) and so as there, this new upper bound on γ takes precedent over the one required by A1 (i.e., $\gamma < 1/(2q)$).

Deriving the expression for t_L: For $t_L \in (0,1)$, $t_R = 1$ to be an equilibrium we must have that the pivotal probabilities in (2) and (3), respectively, satisfy:

$$\mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})(1+t_L)}f(p^*)(1-p^*) = \frac{c}{v}, \tag{OA.52}$$

$$\mathbb{P}\left[\text{pivotal}|R\right] > \frac{c}{v} \iff \frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}(1+t_L)}f(p^*)p^* > \frac{c}{v}. \tag{OA.53}$$

Assume that $p \sim \mathcal{U}[l,h]$, so that, provided that $p^* \in (l,h)$ (which we check below), we have $f(p^*) = 1/d$, where $d = h - l$; then (OA.52) becomes:

$$\frac{1}{(1-\gamma)n}\frac{1}{\overline{a}(1-\overline{p})}\frac{1}{(1+t_L)}\frac{1}{d}\frac{1+\dfrac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{1+t_L} = \frac{c}{v} \iff (1+t_L)^2 = \frac{1+\dfrac{(2q-1)\gamma}{(1-\gamma)\overline{a}}}{\dfrac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p})},$$

which, if we solve, yields (OA.14); where in the first equality we also substituted for $1 - p^*$ from (OA.49).

Checking that $t_L \in (0,1)$: We need to ensure that given (OA.14):

$$t_L < 1 \iff \sqrt{\frac{(1-\gamma)\overline{a} + (2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}} < 2 \iff \frac{v}{c} < d\frac{4(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a} + (2q-1)\gamma}, \tag{OA.54}$$

and

$$t_L > 0 \iff \frac{(1-\gamma)\overline{a} + (2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})} > (1-\gamma)^2\overline{a}^2 \iff \frac{v}{c} > d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(1-\gamma)\overline{a} + (2q-1)\gamma}. \tag{OA.55}$$

Clearly, the upper bound on v/c imposed by (OA.54) is larger than the lower bound in (OA.55).

Drawing inferences from the values of t_L, t_R: In addition, from (OA.53), we have for $p \sim \mathcal{U}[l,h]$ that:

$$\begin{aligned}
&\frac{1}{(1-\gamma)n}\frac{1}{\overline{ap}(1+t_L)^2}\frac{1}{d}\left(t_L - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\right) > \frac{c}{v} \iff \left(\frac{1}{1+t_L} - \frac{L}{(1+t_L)^2}\right) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
&\iff \sqrt{\frac{\frac{dc}{v}n(1-\overline{p})}{(1-\gamma)\overline{a} + (2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{dc}{v}(1-\gamma)n\overline{a}(1-\overline{p}) > \frac{dc}{v}(1-\gamma)n\overline{ap} \\
&\iff \sqrt{\frac{\frac{v}{cdn}(1-\overline{p})}{(1-\gamma)\overline{a} + (2q-1)\gamma}} > 1 \iff \frac{v}{c} > nd\frac{(1-\gamma)\overline{a} + (2q-1)\gamma}{1-\overline{p}},
\end{aligned} \tag{OA.56}$$

where in the second line we substituted for the computed t_L from (OA.14). Let us also see how (OA.50) changes in terms of t_L. Substituting we have:

$$\sqrt{\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{\frac{dc}{v}n(1-\overline{p})}}\frac{1}{(1-\gamma)\overline{a}}-1>\frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \quad \frac{v}{c}>nd(1-\overline{p})\left[(2q-1)\gamma+(1-\gamma)\overline{a}\right]. \tag{OA.57}$$

Note that since $1/(1-\overline{p})>1-\overline{p}$, (OA.56) always supersedes (OA.57) as a lower bound on v/c. How about the rank between the lower bounds on v/c imposed by (OA.55) and (OA.56). We have:

$$(1-\gamma)^2\overline{a}^2(1-\overline{p})^2>\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]^2 \iff (1-\gamma)\overline{a}(1-\overline{p})>(1-\gamma)\overline{a}+(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(1-\overline{p}-1)>(2q-1)\gamma \iff -(1-\gamma)\overline{a}\overline{p}>(2q-1)\gamma,$$

which is not true for any parameter value; therefore, (OA.56) supersedes (OA.55) and is the (up to now) relevant lower bound on v/c.

Now, we need to check whether the upper bound on v/c in (OA.54) is larger than the lower bound in (OA.56); otherwise equilibrium $m1$ does not exist. We have:

$$4(1-\gamma)^2\overline{a}^2(1-\overline{p})^2>\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]^2 \iff 2(1-\gamma)\overline{a}(1-\overline{p})>(1-\gamma)\overline{a}+(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}(2-2\overline{p}-1)>(2q-1)\gamma \iff \overline{a}(1-2\overline{p})>\gamma\left[\overline{a}(1-2\overline{p})+2q-1\right]. \tag{OA.58}$$

We consider the following cases to inquire on the validity of (OA.58).

Case A: If $1-2\overline{p}>0 \iff \overline{p}<1/2$; then, we need:

$$\gamma<\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1}. \tag{OA.59}$$

How does this compare with the current upper bound on γ in (OA.51)?

$$\frac{\overline{a}}{2q-1+\overline{a}}>\frac{\overline{a}(1-2\overline{p})}{\overline{a}(1-2\overline{p})+2q-1} \iff \overline{p}>0,$$

which is always true; therefore, (OA.59) supersedes (OA.51) in this case. Hence, for $\overline{p}<1/2$ and for γ less than the bound in (OA.59), inequality (OA.58) holds and equilibrium exists.

Case B: If $1-2\overline{p}<0 \iff \overline{p}>1/2$ and $\overline{a}(1-2\overline{p})+2q-1>0 \iff \overline{p}<1/2+(2q-1)/(2\overline{a})$, then inequality (OA.58) does not hold for any $\gamma>0$. Hence, equilibrium does not exist.

Case C: If $1-2\overline{p}<0 \iff \overline{p}>1/2$ and $\overline{a}(1-2\overline{p})+2q-1<0 \iff \overline{p}>1/2+(2q-1)/(2\overline{a})$, then, we need

$$\overline{a}(2\overline{p}-1)<\gamma\left[\overline{a}(2\overline{p}-1)-(2q-1)\right] \iff \gamma>\frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}.$$

For equilibrium to exist, we need that this lower bound is smaller than the current upper bound of γ in (OA.51), that is,

$$\frac{\overline{a}(2\overline{p}-1)}{\overline{a}(2\overline{p}-1)-(2q-1)}<\frac{\overline{a}}{2q-1+\overline{a}} \iff (2q-1)2\overline{p}<0, \tag{OA.60}$$

which is never true, so also in this case inequality (OA.58) does not hold and equilibrium does not exist.

Therefore, equilibrium $m1$ only exists for $\overline{p}<1/2$ and so we add this restriction in the statement of Proposition 6. Additionally, we need the upper bound on γ in (OA.59) that, hence, defines region Γ_{m1} in (OA.12).

<u>Checking assumptions A1 and A2</u>: As mentioned the upper bound on γ in (OA.59) supersedes the one in (OA.51), which, in turn, supersedes the bound imposed by A1 (i.e., $\gamma<1/(2q)$). Therefore for $\overline{p}<1/2$, where equilibrium exists, (OA.59) is the relevant bound. How about A2 that $v/c\geq 1$? We check with the current upper bound in (OA.54) for existence:

$$1<d\frac{4(1-\gamma)^2\overline{a}^2n(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} \iff n>\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \tag{OA.61}$$

So this imposes a lower bound on the number of voters n. How does '1' compare with the lower bound in (OA.56)? Well if

$$n>\frac{(1-\overline{p})}{d\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]},$$

(OA.56) is relevant; otherwise, '1' is relevant.

<u>Checking that $p^*\in(l,h)$</u>: Since $p\in[l,h]$ we also need to check that $l<p^*<h$. For the case

$t_L \in (0,1), t_R = 1$, we have from (4):

$$p^* = \frac{t_L}{1+t_L} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L} \iff 1-p^* = \frac{1}{1+t_L} + \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{1}{1+t_L}$$
$$\iff 1-p^* = \sqrt{\frac{dcn}{v}(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}, \tag{OA.62}$$

where in the last equality we used expression (OA.14) for t_L and did a bit of algebra. Using (OA.62) the requirement that $l < p^* < h$, becomes:

$$1-h < 1-p^* < 1-l \iff (1-h)^2 < \frac{dcn}{v}(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right) < (1-l)^2$$
$$\iff \frac{nd(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}{(1-l)^2} \tag{OA.63}$$
$$< \frac{v}{c} <$$
$$\frac{nd(1-\overline{p})\left((1-\gamma)\overline{a}+(2q-1)\gamma\right)}{(1-h)^2}. \tag{OA.64}$$

We now compare the lower bound in (OA.63) with the current one in (OA.56):

$$\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{1-\overline{p}} > \frac{(1-\overline{p})\left[(1-\gamma)\overline{a}+(2q-1)\gamma\right]}{(1-l)^2} \iff (1-l)^2 > (1-\overline{p})^2 \iff l < \overline{p},$$

which is always true; therefore, (OA.56) supersedes the bound in (OA.63) and remains the relevant lower bound on v/c.

How about (OA.64) vs the current upper bound on v/c in (OA.54)? We have:

$$\frac{4(1-\gamma)^2\overline{a}^2(1-\overline{p})}{(1-\gamma)\overline{a}+(2q-1)\gamma} < \frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2} \iff 2(1-\gamma)\overline{a}(1-h) < (1-\gamma)\overline{a}+(2q-1)\gamma$$
$$\iff (1-\gamma)\overline{a}\left[2(1-h)-1\right] < (2q-1)\gamma \iff \left[2(1-h)-1\right]\overline{a} < \left[2q-1+(2(1-h)-1)\,\overline{a}\right]\gamma.$$

The sign, of the above inequality, depends on whether: $2(1-h)-1 > 0 \iff h < 1/2$ or not. Hence, either of (OA.64) and (OA.54) can de the relevant bound, depending on parameter values.

We need to ensure that the new possible upper bound in (OA.64) is larger than the existing lower bounds: the one in A2 (i.e., $v/c \geq 1$) and the one in (OA.56). We have in turn:

i) Comparing (OA.64) vs 1, leads to:

$$\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > 1 \iff n > \frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}. \tag{OA.65}$$

How does (OA.65) compare with the existing lower bound on n in (OA.61)? We have:

$$\frac{(1-h)^2}{d(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)} > \frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{4d(1-\gamma)^2\overline{a}^2(1-\overline{p})} \iff (1-\gamma)\overline{a}(1-2h) > (2q-1)\gamma.$$

Hence, if $1-2h<0 \iff h>1/2$, then the current bound (OA.61) is the relevant one.

While, if $1-2h>0 \iff h<1/2$, then the current bound holds if and only if:

$$(1-\gamma)\overline{a}(1-2h) < (2q-1)\gamma \iff \gamma > \frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)}.$$

How does this lower bound on γ compare with our current one in (OA.59)

$$\frac{\overline{a}(1-2h)}{2q-1+\overline{a}(1-2h)} < \frac{\overline{a}(1-2\overline{p})}{2q-1+\overline{a}(1-2\overline{p})} \iff h > \overline{p},$$

which is always true. Therefore, either of the lower bounds on n in (OA.65) and (OA.61) can be relevant for $h<1/2$. And they are necessary for equilibrium to exist. These two lower bounds together define interval N_{m1} in (OA.11).

ii) Comparing (OA.64) vs (OA.56), leads to:

$$\frac{(1-\overline{p})((1-\gamma)\overline{a}+(2q-1)\gamma)}{(1-h)^2}nd > nd\frac{(1-\gamma)\overline{a}+(2q-1)\gamma}{(1-\overline{p})} \iff \overline{p} < h,$$

which is, again, always true, so (OA.64) is larger than (OA.56) without any further restrictions on parameters.

So, either of (OA.64) and (OA.54) can be the relevant upper bound on v/c; consistent with the possible lower bounds from A2 (i.e., '1') and in (OA.56). These bounds define interval V_{m1} in (OA.3). This concludes the proof of Proposition 6. ■

Equilibrium $m0$. Deriving p^*: Now, we inquire about the existence of equilibrium with $t_L \in (0,1)$ and $t_R = 0$. Then, p^* in (4) becomes:

$$p^* = 1 - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_L} \iff 1 - p^* = \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_L}. \tag{OA.66}$$

We have $1 - p^* > 0$ but also need:

$$1 - p^* < 1 \iff (2q-1)\gamma < (1-\gamma)\overline{a}t_L \iff t_L > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}, \tag{OA.67}$$

and for equilibrium $m0$ to exist, the right hand side in (OA.67) cannot exceed one, which yields:

$$\gamma < \frac{\overline{a}}{2q-1+\overline{a}}, \tag{OA.68}$$

which is the same as (OA.51) and so as there, this new upper bound on γ takes precedent over the one required by A1.

Deriving the expression for t_L: For $t_L \in (0,1)$, $t_R = 0$ to be an equilibrium we must have that the pivotal probabilities in (2) and (3), respectively, satisfy:

$$\mathbb{P}\left[\text{pivotal}|R\right] < \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{t_L}f(p^*)p^* < \frac{c}{v}, \tag{OA.69}$$

$$\mathbb{P}\left[\text{pivotal}|L\right] = \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}(1-\overline{p})}\frac{1}{t_L}f(p^*)(1-p^*) = \frac{c}{v}. \tag{OA.70}$$

Assume that $p \sim \mathcal{U}[l,h]$, so that if $p^* \in (l,h)$ (which we check below) then $f(p^*) = 1/d$, where $d = h - l$; then, (OA.70) becomes:

$$\frac{1}{(1-\gamma)^2 n\overline{a}^2(1-\overline{p})}\frac{1}{t_L^2}\frac{1}{d}(2q-1)\gamma = \frac{c}{v} \iff t_L = \sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}}\frac{1}{(1-\gamma)\overline{a}},$$

which coincides with (OA.18).

Checking that $t_L \in (0,1)$: We need to ensure that given (OA.18):

$$t_L < 1 \iff \frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}} < (1-\gamma)^2\overline{a}^2 \iff \frac{v}{c} < d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma}, \tag{OA.71}$$

and $t_L > 0$, which, in this case, is always true.

Drawing inferences from the values of t_L, t_R: In addition, from (OA.69) we have for $p \sim \mathcal{U}[l,h]$:

$$\frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{d}\frac{1}{t_L}p^* < \frac{c}{v} \iff \frac{1}{(1-\gamma)n\overline{a}\overline{p}}\frac{1}{d}\frac{1}{t_L}\left(1 - \frac{(2q-1)\gamma}{1-\gamma}\frac{1}{\overline{a}t_L}\right) < \frac{c}{v}$$
$$\iff \sqrt{\frac{n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma}}(1-\gamma)\overline{a} - \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}}\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})d\frac{c}{v}}{(2q-1)\gamma} < d\frac{c}{v}(1-\gamma)n\overline{a}\overline{p} \iff \frac{v}{c} < d\frac{n(2q-1)\gamma}{1-\overline{p}}, \tag{OA.72}$$

where in the first equality we used $f(p^*) = 1/d$, in the second inequality we substituted for p^* from (OA.66), and in the third inequality we substituted for t_L from (OA.18). Between (OA.71) and (OA.72), which is the relevant upper bound for v/c?

$$d\frac{n(2q-1)\gamma}{1-\overline{p}} < d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} \iff (2q-1)\gamma < (1-\gamma)\overline{a}(1-\overline{p}) \iff \gamma < \frac{\overline{a}(1-\overline{p})}{2q-1+\overline{a}(1-\overline{p})} \tag{OA.73}$$

Hence, if (OA.73) is true, then (OA.72) is relevant; otherwise, (OA.71) is relevant. Note, that (OA.73) is smaller than (OA.68), and hence in the feasible (up until now) region for γ.

Furthermore, let us see how (OA.67) changes in terms of the computed t_L in (OA.18). We have:

$$\sqrt{\frac{(2q-1)\gamma}{n(1-\overline{p})d\frac{c}{v}}}\frac{1}{(1-\gamma)\overline{a}} > \frac{(2q-1)\gamma}{(1-\gamma)\overline{a}} \iff \sqrt{\frac{1}{n(1-\overline{p})d\frac{c}{v}}} > \sqrt{(2q-1)\gamma} \iff \frac{v}{c} > dn(1-\overline{p})(2q-1)\gamma. \tag{OA.74}$$

For the existence of equilibrium $m0$, we need both possible upper bounds on v/c (OA.71) and (OA.72) to be larger than the lower bound in (OA.74). We have in turn:

i) Comparing (OA.71) with (OA.74):

$$d\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} > dn(1-\overline{p})(2q-1)\gamma \iff (1-\gamma)^2\overline{a}^2 > (2q-1)^2\gamma^2 \iff \gamma < \frac{\overline{a}}{2q-1+\overline{a}},$$

which is identical to the current upper bound on γ in (OA.68), and, hence, true in the region we are considering.

ii) Comparing (OA.72) with (OA.74):

$$d\frac{n(2q-1)\gamma}{1-\overline{p}} > dn(1-\overline{p})(2q-1)\gamma \iff \overline{p} > 0,$$

which is also always true.

Checking assumptions A1 and A2: As mentioned, the upper bound on γ in (OA.68) supersedes the bound imposed by A1 (i.e., $\gamma < 1/(2q)$). This upper bound and the generic lower bound of '0' define interval Γ_{m0} in (OA.16). Now, we check the consistency of the upper bounds for v/c in (OA.71) and (OA.72) with respect to A2 (i.e., $v/c \geq 1$):

i) Comparing '1' with the bound in (OA.71):

$$n > \frac{(2q-1)\gamma}{d(1-\gamma)^2\overline{a}^2(1-\overline{p})}. \tag{OA.75}$$

ii) Comparing '1' with the bound in (OA.72):

$$n > \frac{1-\overline{p}}{d(2q-1)\gamma}. \tag{OA.76}$$

Expressions (OA.75) and (OA.76) impose lower bounds on the number of voters n. Either of them can be relevant, depending on parameter values. Together they define interval N_{m0} in (OA.15).

Checking that $p^* \in (l,h)$: Given that $p \in [l,h]$, we need to also ensure that $l < p^* < h$. From the expression for p^* in (OA.66) and t_L given by (OA.18) we have:

$$1-p^* = \sqrt{(2q-1)\gamma\frac{dcn}{v}(1-\overline{p})}$$

so that

$$l < p^* < h \iff 1-h < 1-p^* < 1-l \iff (1-h)^2 < (1-p^*)^2 < (1-l)^2$$

$$\iff \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} \tag{OA.77}$$

$$< \frac{v}{c} <$$

$$\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2}. \tag{OA.78}$$

How does the lower bound in (OA.77) compare with the existing one in (OA.74):

$$\frac{nd(2q-1)\gamma(1-\overline{p})}{(1-l)^2} < nd(1-\overline{p})(2q-1) \iff (1-l)^2 > 0,$$

which is always true, and hence, (OA.77) is never relevant.

Now, how does the upper bound (OA.78) compare with the existing ones in (OA.71) and (OA.72). We have in turn:

i) Comparing (OA.78) with the bound in (OA.71):

$$\frac{(1-\gamma)^2\overline{a}^2 n(1-\overline{p})}{(2q-1)\gamma} < \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2} \iff \gamma > \frac{\overline{a}(1-h)}{2q-1+\overline{a}(1-h)}. \tag{OA.79}$$

Therefore, we need to inquire further. Note, that (OA.79) is smaller than (OA.68), and hence in the feasible region of γ. Hence, either of (OA.71) and (OA.78) can be the relevant upper bound on v/c.

i) Comparing (OA.78) with the bound in (OA.72):

$$\frac{d(2q-1)\gamma n}{1-\overline{p}} < \frac{nd(2q-1)\gamma(1-\overline{p})}{(1-h)^2} \iff \overline{p} < h,$$

which is always true, and, hence, (OA.78) is never the relevant upper bound on v/c.

Putting together the lower bound (OA.77) with the one implied by A2 (i.e., $v/c \geq 1$), and the upper bounds in (OA.71) and (OA.72) defines interval V_{m0} in (OA.17) (where we also divide all parameters by n). This concludes the proof of Proposition 7. ■

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities.*

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

ECGI Working Paper Series in Finance

https://ecgi.global/content/working-papers

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (https://ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html