The Voting Premium

Finance Working Paper N° 720/2021
January 2021

Doron Levit
University of Washington and ECGI

Nadya Malenko
University of Michigan, CEPR and ECGI

Ernst Maug
University of Mannheim and ECGI

ECGI Working Paper Series in Finance

The Voting Premium

Working Paper N° 720/2021
January 2021

Doron Levit
Nadya Malenko
Ernst Maug

Abstract

This paper develops a theory of blockholder governance and the voting premium. A blockholder and dispersed shareholders first trade in a competitive market and then vote at a shareholder meeting. A positive voting premium emerges only if the blockholder is not the median voter, since he is then willing to pay a higher price to move the median voter in his preferred direction. Hence, the voting premium does not emerge from exercising control, but from influencing who exercises control. Empirical measures of the voting premium generally do not reflect the economic value of voting rights to the blockholder, and the voting premium is unrelated to measures of voting power, such as the probability of being pivotal. A negative voting premium can emerge in situations when dispersed shareholders could free-ride on the blockholder's trades.

Keywords: Voting, trading, voting premium, blockholders, ownership structure, shareholder rights, corporate governance

JEL Classifications: D74, D82, D83, G34, K22

Doron Levit
Associate Professor of Finance and Business Economics
University of Washington, Foster School of Business
434 Paccar Hall, 4273 E Stevens Way NE
Seattle, WA 98195, United States
phone: +1 206-543-3021
e-mail: dlevit@uw.edu

Nadya Malenko
Associate Professor of Finance
Ross School of Business, University of Michigan
701 Tappen Street
Ann Arbor, MI 48109, United States
e-mail: nmalenko@umich.edu

Ernst Maug*
Professor of Corporate Finance
University of Mannheim, Business School
L9 1-2
68161 Mannheim, Germany
phone: +49 621 181 1952
e-mail: maug@uni-mannheim.de

*Corresponding Author

The Voting Premium

Doron Levit[*] **Nadya Malenko**[†] **Ernst Maug**[‡]

December 21, 2020

Abstract

This paper develops a theory of blockholder governance and the voting premium. A blockholder and dispersed shareholders first trade in a competitive market and then vote at a shareholder meeting. A positive voting premium emerges only if the blockholder is not the median voter, since he is then willing to pay a higher price to move the median voter in his preferred direction. Hence, the voting premium does not emerge from exercising control, but from influencing who exercises control. Empirical measures of the voting premium generally do not reflect the economic value of voting rights to the blockholder, and the voting premium is unrelated to measures of voting power, such as the probability of being pivotal. A negative voting premium can emerge in situations when dispersed shareholders could free-ride on the blockholder's trades.

Keywords: Voting, trading, voting premium, blockholders, ownership structure, shareholder rights, corporate governance

JEL classifications: D74, D82, D83, G34, K22

[*]University of Washington and ECGI. Email: dlevit@uw.edu.

[†]University of Michigan, CEPR, and ECGI. Email: nmalenko@umich.edu.

[‡]University of Mannheim and ECGI. Email: maug@uni-mannheim.de

1 Introduction

Blockholders are pervasive in all developed economies. La Porta et al. (1999) show that only 17% of large firms in countries with strong shareholder protection qualify as widely-held, whereas all others have a blockholder who controls at least 10% of the voting rights.[1] Blockholders use multiple channels to influence corporate policies, and voting is arguably the most important of these channels, as it empowers shareholders to elect directors, approve major corporate transactions, and decide on governance issues.[2] Since shares bundle voting rights with cash flow rights, blockholders' desire to accumulate voting power and move the voting outcome in their preferred direction may give rise to a voting premium on the price of the firm's shares.

This paper develops a theory of blockholder governance and the voting premium. We study a setting in which blockholders and dispersed shareholders trade and then vote. Our key focus is on the conditions under which blockholders are willing to pay higher prices for the firm's shares to exercise more voting control, when their trades give rise to a voting premium, and how estimates of the voting premium should be interpreted. A large empirical literature estimates the value of voting rights using different methods, and often arrives at conflicting conclusions about the magnitude and significance of the voting premium.[3] We show that existing measures of the voting premium may underestimate the economic value of voting rights, and some measures of voting power, such as the probability of being pivotal, are likely unrelated to the magnitude of the observed voting premium. Moreover, a voting premium may arise even in the absence of takeovers. Overall, blockholder governance affects asset prices, and

[1] Holderness (2009) and Edmans and Holderness (2017) analyze a random sample of 375 U.S. firms and find that 96% of them have blockholders who own at least 5%, and that the largest blockholder owns on average 26% of the stock. See Edmans (2014), Edmans and Holderness (2017), and Dasgupta, Fos, and Sautner (2020) for surveys of the large theoretical and empirical literature on blockholders.

[2] While some blockholders exert control directly, e.g., though board representation and majority control, many others, such as financial institutions, instead rely on voting as the key channel of influence. For example, Edmans and Holderness (2017) show that mutual funds, pension funds, and other financial institutions obtain board representation in only 5%, 18%, and 25% of the firms they own blocks in, respectively. According to the survey of institutional investors by McCahery, Sautner, and Starks (2016), voting and discussions with management are the two most frequently used channels that institutions use to influence portfolio companies.

[3] Section 7 reviews 40 studies that apply five different methodologies to estimate the voting premium, relates them to our model, and offers reflections on the variation of estimates of the voting premium across studies.

share prices depend critically on the bundling of cash flow rights with voting rights.

We analyze a model with a continuum of dispersed shareholders and a blockholder. All shareholders first trade with each other in a competitive stock market, and shareholders who own the firm after trading then vote on a proposal at a shareholder meeting. Each shareholder's valuation of the firm and the proposal depends on a common value, which is unknown; shareholders observe a public signal about it before they vote. In addition, shareholders are heterogeneous, and their attitudes to the proposal depend on a private value, which biases some of them in favor of the proposal and others against it. Such heterogeneity may arise because investors differ in their time horizons; tax status; ownership of other firms; and in their attitudes to risk, corporate governance issues, and social and political issues.[4] The model accommodates a range of blockholders, which differ regarding their initial toeholds and their preferences, particularly, how they differ from dispersed shareholders.

In this setting, the blockholder trades for two different reasons. First, he has incentives to trade if his private valuation of the shares differs from the market price. The more extreme are the blockholder's views relative to those of the average dispersed shareholder, the stronger is this *cash flow motive* to trade. Second, the blockholder trades to accumulate voting rights. We refer to the blockholder's marginal incentive to trade because of this *voting motive* as the marginal propensity to buy votes, or the MPV.

We show that the MPV results in a voting premium. In particular, we compare the equilibrium price in the baseline model, where the blockholder trades for both of these motives, to the equilibrium price in the benchmark setting in which the blockholder has no ability to influence the votes with his trades but the voting rule is the same. We show that the MPV is proportional to the difference in prices between these two settings and thereby captures the market price of voting rights. Furthermore, in an extension to a dual-class setting, we show that the MPV is proportional to the price differential between voting and non-voting shares. Hence, the MPV captures two measures of the voting premium that have been widely used in

[4] See the following literature on each of these issues: Investor time horizons: Bushee (1998) and Gaspar, Massa, and Matos (2005); tax status: Desai and Jin (2011); conflicts of interest and common ownership: Cvijanovic, Dasgupta, and Zachariadis (2016) and He, Huang, and Zhao (2019); attitudes to corporate governance: Bolton et al. (2019) and Bubb and Catan (2019). Hayden and Bodie (2008) provide a comprehensive overview of different sources of shareholder heterogeneity.

empirical studies: the dual class share premium, as well as the price differential between the stock and the nonvoting synthetic stock constructed using derivatives (see Section 7).

Therefore, we focus on the MPV as our key measure to capture the price effects of voting rights. We show that the main factor that determines the MPV is the identity of the marginal voter, defined as the shareholder whose vote coincides with the final decision on the proposal. The marginal voter could be either the blockholder or a dispersed shareholder; for example, the marginal voter is the median voter under a simple majority rule. Intuitively, by buying additional shares, the blockholder moves the marginal voter, and thereby the voting outcome, in his preferred direction. In doing so, he has two considerations. The first is the effect of the voting outcome on the long-term value of his endowment. Based on this consideration alone, the blockholder prefers to move the marginal voter as much as possible to match his own preferences regarding the proposal. The blockholder's second consideration is the effect of the anticipated voting outcome on the stock price at which he trades and thereby on his short-term trading profits. For example, having a marginal voter who does not maximize the long-term value of the blockholder's endowment may nevertheless benefit the blockholder if it decreases the valuation of other shareholders and allows the blockholder to buy shares at a discount. Both considerations jointly determine the voting motive for trading and are reflected in the voting premium.

The blockholder can either acquire a sufficiently large stake to make himself the marginal voter, or he can acquire a smaller stake and then let a dispersed shareholder be the marginal voter. Since his trades in the second case affect the identity of the marginal voter, they can still move the voting outcome in his preferred direction. We show that the MPV is equal to zero if the blockholder is the marginal voter, since then any further trades do not affect the marginal voter and the voting outcome. A positive voting premium emerges only if the blockholder affects the position of the marginal voter, but he is not the marginal voter himself. Hence, the voting premium does not emerge from exercising control, but from influencing who exercises control. Moreover, we need to distinguish the value of votes to the blockholder from the voting premium, since the latter does not reflect the former. The reason is that the equilibrium MPV and the voting premium capture only the blockholder's marginal value from an additional vote, evaluated at his optimal ownership level. In contrast, the blockholder's total value from voting rights is determined by his average propensity to buy votes, which includes all the infra-marginal shares he trades from his initial endowment to his equilibrium ownership.

Another implication of our analysis is that the marginal voter is usually different from the pivotal voter, i.e., the shareholder whose individual vote decides the voting outcome. In our model, only the blockholder can be the pivotal voter, since dispersed shareholders are atomistic, whereas both the blockholder and a dispersed shareholder can be the marginal voter. Importantly, it is the change in the marginal voter, and not in the pivotal voter, which determines the MPV. Hence, the voting premium is generally unrelated to the voting power of the blockholder, as measured by the likelihood that he is pivotal. In particular, when the blockholder has a large voting stake and is pivotal with a high probability, then his voting power is large, whereas the MPV and the voting premium are often zero. Overall, our results emphasize that common measures of the voting premium may underestimate the actual value of voting rights to their owners.

We also show that although the blockholder has the power to gain influence over the voting outcome by buying additional shares, he may nevertheless choose to do the opposite: sell shares to dispersed shareholders and thereby give up his influence over the voting outcome, while demanding a premium from the dispersed shareholders. Thus, the tension between exit and voice (e.g., Hirschman (1970)) also exists in our model, which demonstrates that the incentives to exit can prevail even when the voting premium is positive. Hence, a positive voting premium does not necessarily indicate a more concentrated ownership structure.

Our model can rationalize a negative voting premium, which has been documented in empirical studies (see Section 7.2). In particular, when the blockholder's initial stake is small and he is more activist than the average dispersed shareholder, then buying shares and accumulating voting control benefits the average dispersed shareholder more than it benefits the blockholder himself. In this case, the blockholder's purchases increase the stock price by more than his own valuation. Then he buys fewer shares because of the voting considerations, to prevent dispersed shareholders from free-riding on his trades. Such a scenario gives rise to a negative voting premium.

Our baseline model focuses on the case where voting and cash flow rights are bundled in one security, which may limit the blockholder's ability to accumulate votes and implement his agenda. We therefore study whether allowing for separate trading of voting rights, e.g., through share lending, can help the blockholder achieve his objectives through voting. Our analysis shows that while a moderately biased blockholder can use the market for votes to successfully pursue his agenda, a blockholder with a strong bias cannot. Thus, blockholders with strong views about corporate policies can benefit from combining voting with other channels of corporate influence, such as engaging with management behind the scenes, lobbying other shareholders and proxy advisors, and running media campaigns. This result is consistent with the evidence in McCahery, Sautner, and Starks (2016) that institutional investors use a broad range of governance mechanisms to influence their portfolio companies.

We extend the model in a number of ways to explore additional questions. First, we consider the case with multiple blockholders. We show that if they share the same preferences toward the proposal, then the voting premium declines as the number of blockholders increases. In contrast, if the blockholders are sufficiently heterogeneous, their trades pull the marginal voter in opposite directions. Then, as the blockholders' biases become more extreme, each blockholder tries harder to gain influence over the voting outcome, which results in a higher voting premium. Second, we examine a case in which, initially, no blockholder exists. Then a blockholder, such as a hedge fund, can emerge from trading with dispersed shareholders. It turns out that only investors who have a stronger preference for the proposal than the average existing shareholder will find it optimal to acquire a block in this way. Finally, we endogenize the blockholder by introducing a first stage in which new investors can acquire the block from an incumbent, as in Burkart, Gromb, and Panunzi (2000), or through a private placement, in which the firm auctions off a block and places it with the highest bidder (e.g., Hertzel and Smith (1993)). We show that block trading may increase the difference in preferences between blockholders and dispersed shareholders, and characterize when the block premium is negative or positive.

Overall, our paper makes two contributions. First, it examines the trading between small and large shareholders and the ownership structure of the firm in a context in which blockholders exercise control through the voting process. Second, it contributes to our understanding of asset prices by showing how and when a voting premium emerges when blockholders can acquire voting control only through securities that bundle cash flow rights and voting rights.

2 Discussion of the literature

Our paper is related to the literature on trading and voting. A first, earlier strand of this literature analyzes the existence of equilibrium and the objectives of the firm in a context with incomplete markets and shareholders with heterogeneous preferences.[5] In particular, Drèze (1985) and DeMarzo (1993) develop models with the board of directors as a group of controlling blockholders. To this literature, we contribute by analyzing the voting premium and a richer characterization of the interplay between small shareholders and blockholders. This also distinguishes our paper from Levit, Malenko, and Maug (2020), who analyze trading and voting by atomistic shareholders – a setting in which the voting premium does not arise.

A second part of the literature on voting and trading focuses on a setting in which votes can be unbundled and traded separately from cash flow rights. Blair, Golbe, and Gerard (1989), Kalay and Pant (2009), and Dekel and Wolinsky (2012) examine the role of vote trading in control contests, while Speit and Voss (2020) show that it can make shareholders vulnerable to value-decreasing activism.[6] Neeman and Orosel (2006), Brav and Mathews (2011), and Esö, Hansen, and White (2014) emphasize the informational role of vote trading.[7] Differently from these papers, our key focus is on the case in which the stock combines the right for cash flows with the right to vote, and so blockholders have to buy cash flow rights if they want to accumulate voting power. Importantly, in our model, the voting premium can be positive even if dispersed shareholders are never pivotal and do not value voting rights per se.[8] We also

[5] See Gevers (1974), Drèze (1985), DeMarzo (1993), and Kelsey and Milne (1996).

[6] Another related strand of the literature studies the optimal security-voting structure, and dual-class companies in particular (e.g., Grossman and Hart (1988) and Harris and Raviv (1988)). See the survey of Burkart and Lee (2008) for the theoretical literature on deviations from the one-share-one-vote principle, and Adams and Ferreira (2008) for a survey of the empirical studies.

[7] A related literature in political science examines how vote trading allows agents with a higher intensity of preferences to buy votes from those who care about the decision less. See, e.g., Casella, Llorente-Saguer, and Palfrey (2012) and the literature surveyed in that paper.

[8] The papers that obtain a positive voting premium in a setting in which the marginal trader is a dispersed

examine how the ability to separately trade voting and non-voting shares affects stock prices, and how different measures of the voting premium are related to each other.

Our paper also contributes to the literature on the equilibrium ownership structure of firms and the strategies of blockholders to exercise control. A large strand of this literature is on direct intervention by blockholders ("voice") and investigates whether trading by blockholders in public markets can overcome the collective action problem among shareholders, who capture the benefits but not the costs of other shareholders' monitoring and governance interventions.[9] Another strand of this literature analyzes how trading by blockholders affects governance through its impact on stock prices and managers' incentives ("exit").[10] By contrast, in our setting, the blockholder exercises influence through the voting process in that his trades affect the identity of the marginal voter. This is empirically important because many blockholders, notably financial institutions, rely on voting to influence firms' policies.[11] Bar-Isaac and Shapiro (2019), Meirowitz and Pi (2020), and Dhillon and Rossetto (2015) also consider blockholder models with voting, but differently from our paper, they do not study the voting premium and focus on the effects of blockholders on information aggregation and the risk taking of the firm, respectively.

A part of the literature on firms' ownership structures considers models with multiple

shareholder who is never pivotal are those that rely on oceanic Shapley values. This method was pioneered by Rydqvist (1987) based on Milnor and Shapley (1978), and the resulting empirical measure is widely used (e.g., Zingales (1994); Nenova (2003)). However, this theory relies on cooperative game theory in which a continuum of atomistic shareholders is pivotal *collectively*, but leaves it open how atomistic shareholders resolve their collective action problem.

[9] See Admati, Pfleiderer, and Zechner (1994), Bolton and von Thadden (1998), Kahn and Winton (1998), and Maug (1998) for earlier contributions to this literature. Shleifer and Vishny (1986) address the free-rider problem but do not endogenize blockholders' trades. An important part of this literature assumes that blockholders either assume control themselves by launching a takeover (Kyle and Vila (1991)), or use their influence on the firm to facilitate a takeover by another party (Corum and Levit (2019), Burkart and Lee (2020)). For other, more recent contributions to this large theoretical literature see the surveys of Edmans (2014), Edmans and Holderness (2017), and Dasgupta, Fos, and Sautner (2020).

[10] See Admati and Pfleiderer (2009), Edmans (2009), Edmans and Manso (2011), as well as the surveys cited in the previous footnote. A part of the blockholder literature addresses the specific situation of institutional blockholders who are concerned about the flows of investors into their funds (e.g., Dasgupta and Piacentino (2015); Brav, Dasgupta, and Mathews (2019); Cvijanovic, Dasgupta, and Zachariadis (2019)). These aspects are outside of the scope of our model.

[11] Thus, our paper contributes to the broader literature on corporate voting (e.g., Maug and Rydqvist, 2009; Levit and Malenko, 2011; Van Wesep, 2014; Malenko and Malenko, 2019; and Cvijanovic, Groen-Xu, and Zachariadis, 2020).

blockholders. This literature analyzes two additional problems that do not occur in single-blockholder models. The first is that multiple blockholders who share the same goal regarding the firm's policies have to coordinate their actions and solve the collective-action problems between them.[12] The second is that blockholders may use their influence on the firm to extract private benefits, and they may either monitor each other when they compete for these private benefits (Pagano and Roell (1998)), or cooperate and form coalitions to extract private benefits (Zwiebel (1995)). Importantly, our framework accommodates both, settings in which multiple blockholders share the same objective, and settings in which they conflict and compete for influence; we show that these settings have opposite predictions for the size of the voting premium.

To the literature on the voting premium, we contribute a new theory about how the voting premium may originate in an equilibrium model of trading. This literature often views the voting rights as being particularly valuable in takeover contests, in which bidders pay an acquisition premium to the holders of the voting shares (e.g., Grossman and Hart (1988); Harris and Raviv (1988); Bergström and Rydqvist (1992)). Yet, the voting premium appears to be largest in those economies in which firms are well-protected against takeovers and control contests hardly ever take place.[13] Alternatively, this literature often relies on the notion of private benefits, while our theory is based on the assumption that shareholders have private values.

3 Model

Consider a publicly traded firm, which is initially owned by a continuum of measure one of dispersed shareholders and one large blockholder. The blockholder is endowed with $\alpha \in [0, 1)$ shares, and each dispersed shareholder is endowed with $e = 1 - \alpha$ shares, so the total number of outstanding shares is 1. In the baseline setting, each share has one vote. There is a proposal on which shareholders vote. The agenda of the proposal could relate to director elections,

[12] See Noe (2002), Brav, Dasgupta, and Mathews (2019), and Edmans and Manso (2011).

[13] Many countries have also enacted provisions that require bidders to pay equal prices to all classes of shares (Nenova (2003)). We discuss the arguments related to takeovers in more detail in Section 7 below.

M&As, executive compensation, corporate governance, or social and environmental policies. The proposal can either be approved ($d = 1$) or rejected ($d = 0$).

Preferences. Dispersed shareholders' preferences over the proposal depend on two components, which reflect a common value and private values. The common value component depends on an unknown state $\theta \in \{-1, 1\}$: if $\theta = -1$ ($\theta = 1$), accepting the proposal and changing the status quo is value-decreasing (increasing). In other words, the common value is maximized if the policy matches the state ($d = 1$ if $\theta = 1$), as common in the strategic voting literature, e.g., Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1996).

Dispersed shareholders also have private values over the firm's policies, which reflect the heterogeneity in their preferences. For simplicity, we refer to these private values as biases and denote them by b. A shareholder with bias $b > 0$ ($b < 0$) receives additional (dis)utility if the proposal is accepted. The distribution of biases b among the initial dispersed shareholders is given by a publicly known differentiable cdf G, which has full support with positive density g on $\left[-\bar{b}, \bar{b}\right]$, where $\bar{b} \in (0, 1)$ measures the heterogeneity among dispersed shareholders. Differences in shareholders' preferences can stem from time horizons, private benefits, different social or political views, common ownership, risk aversion, or tax considerations. As noted in the introduction, the evidence for preference heterogeneity is pervasive.[14]

The value of a share from the perspective of a dispersed shareholder b is

$$v\left(d, \theta, b\right) = v_0 + \left(\theta + b\right) d, \tag{1}$$

where $v_0 \geq 0$ ensures that shareholder value is always non-negative. Notice that because of heterogeneous preferences, shareholders apply different hurdle rates for accepting the proposal: a shareholder with bias b would like the proposal to be accepted if and only if his expectation of $\theta + b$ is positive. We will refer to shareholders with a higher b as being "more activist".

The blockholder has the same preference structure as dispersed shareholders and a bias β towards the proposal. Hence, we denote the value of a share from the perspective of a

[14] With minor modifications, our modeling approach can also capture differences of opinions (priors), when investors agree to disagree.

blockholder by $v(d, \theta, \beta)$.

Timeline. All shareholders are initially uninformed about the state θ and have the same prior about its distribution, which we specify below. Shareholders first trade and then vote on the proposal. This timing allows us to focus on how trading affects the composition of the voter base, which is crucial for the analysis of the voting premium. At the trading stage, each dispersed shareholder can buy any number of shares $x \geq -e$, where $x < 0$ corresponds to the shareholder selling shares and $x \geq -e$ implies that short sales are not allowed. A shareholder's utility from buying x shares is

$$u(d, \theta, b, x; \gamma, e) = (e + x) v(d, \theta, b) - \frac{\gamma}{2} x^2, \tag{2}$$

where $\gamma > 0$ can be motivated by risk aversion or as capturing trading frictions (e.g., illiquidity, transaction costs, wealth constraints), which limit shareholders' ability to build large positions in the firm. Similarly, the blockholder can buy any number of shares $y \geq -\alpha$, and his utility from buying y shares is $u(d, \theta, \beta, y; \eta, \alpha)$, where $u(\cdot)$ is given by (2) and $\eta > 0$. We assume that neither dispersed shareholders nor the blockholder find it in their best interest to short sell, i.e., to sell e or more, and α or more shares, respectively.[15] For simplicity, we assume that the blockholder submits his order y first, and dispersed shareholders observe y and submit their orders next.

We denote the market clearing share price by p. After the market clears, but before voting takes place, all shareholders observe a public signal about the state θ, which may stem from disclosures by management, proxy advisors, or analysts. Let $q = \mathbb{E}[\theta|\text{public signal}]$ be shareholders' posterior expectation of the state following the signal. For simplicity, we assume that the public signal is q itself, and that q is distributed according to a differentiable cdf F with mean zero and full support with positive density f on $[-\Delta, \Delta]$, where $\Delta \in (\bar{b}, 1)$. Thus, the ex-ante expectation of θ is zero. In what follows, we always refer to $H(q^*) \equiv \Pr[q > q^*]$ rather than to the cdf. The symmetry of the support of q around zero is not necessary for any of the

[15] The exact conditions that guarantee this are formulated in Lemma 3 in the Online Appendix. If $\alpha = 0$, which we analyze as a special case and separately from Proposition 2, the no-short-selling constraint can bind for the blockholder, but it does not change our main results.

main results.

After observing the public signal q, each shareholder votes the shares he owns after the trading stage in favor or against the proposal. Hence, we assume that the record date, which determines who is eligible to participate in the vote, is after the trading stage. This timeline applies well to important votes, such as the votes on M&As, proxy fights, and high-profile shareholder proposals, which are typically known well ahead of the record date. The proposal is accepted if a fraction of more than $\tau \in (0,1)$ of all shares are cast in favor; otherwise, the proposal is rejected. We assume that the blockholder's initial stake and ability to buy shares are not large enough to grant him the power to accept the proposal unilaterally, as well as to veto the proposal solely with his own votes. In particular, this assumption implies that the post-trade shareholder base always includes dispersed shareholders. Lemma 3 in the Online Appendix shows that if $\alpha < \min\{\tau, 1-\tau\}$ and η is sufficiently large, then $\alpha + y < \min\{\tau, 1-\tau\}$ in any equilibrium.

We analyze subgame perfect Nash equilibria in undominated strategies of the voting game. The restriction to undominated strategies is common in voting games, which usually impose the equivalent restriction that dispersed shareholders vote as-if-pivotal.[16] This implies that an investor with bias b, whether he is a dispersed shareholder or the blockholder, votes in favor of the proposal if and only if

$$b + q > 0. \tag{3}$$

4 Analysis

We begin by showing that for any trading outcome, proposal approval at the voting stage takes the form of a cutoff rule:

Lemma 1. *In any equilibrium, there exists q^* such that the proposal is approved by shareholders if and only if $q > q^*$.*

[16] See, e.g., Baron and Ferejohn (1989) and Austen-Smith and Banks (1996). This restriction helps rule out trivial equilibria, in which shareholders are indifferent between voting for and against because they are never pivotal.

Intuitively, this is because all shareholders value the proposal more if it is more likely to be value-increasing, i.e., if $\theta = 1$ is more likely.

We solve the model in several steps. We denote the blockholder's trade by y. In Section 4.1, we characterize the trading of dispersed shareholders and the composition of the post-trade shareholder base conditional on y and on dispersed shareholders' expectation of the decision rule at the voting stage, which we denote by q_e^*. In Section 4.2, we build on this characterization and find the decision rule q^* at the voting stage as a function of y as the fixed point of $q^* = q_e^*$. In Section 4.3, we solve for the optimal trading strategy of the blockholder, y^*, and for the equilibrium share price. Section 4.4 discusses the determinants of the equilibrium voting premium and Section 4.5 provides interpretations of the results.

4.1 Trading of dispersed shareholders

Motivated by Lemma 1, we assume that dispersed shareholders expect the proposal to be accepted if and only if $q > q_e^*$ for some cutoff q_e^*. Let $v(b, q_e^*)$ denote the valuation of a shareholder with bias b prior to the realization of q, as a function of the cutoff q_e^*. Then

$$v(b, q_e^*) = \mathbb{E}\left[v\left(\mathbf{1}_{q>q_e^*}, \theta, b\right)\right], \tag{4}$$

where the indicator function $\mathbf{1}_{q>q_e^*}$ equals one if $q > q_e^*$ and zero otherwise, and $v(d, \theta, b)$ is defined by (1). Then (4) can be rewritten as

$$v(b, q_e^*) = v_0 + (b + \mathbb{E}[\theta | q > q_e^*])\, H(q_e^*), \tag{5}$$

which increases in b.

For any expected share price p, each dispersed shareholder solves

$$\max_x \left\{(e + x)\, v(b, q_e^*) - xp - \frac{\gamma}{2}x^2\right\} \tag{6}$$

and optimally chooses

$$x(b,q_e^*,p)=\frac{v(b,q_e^*)-p}{\gamma}. \tag{7}$$

Thus, shareholder b buys shares if his valuation exceeds the market price, $v(b,q_e^*)>p$, sells shares if $v(b,q_e^*)<p$, and does not trade otherwise. Given the blockholder's order y, the market clears if and only if

$$\int_{-\bar{b}}^{\bar{b}} x(b,q_e^*,p)\,g(b)\,db+y=0 \tag{8}$$

$$\Leftrightarrow \quad p^*(y,q_e^*)=\gamma y+v(\mathbb{E}[b],q_e^*). \tag{9}$$

It follows that the equilibrium share price increases in y, and the price impact of the blockholder's trade is larger if γ is larger. Therefore, we can interpret γ as measuring the illiquidity of the market, i.e., the inverse of γ reflects market depth. Equation (9) shows that the price equals the sum of the valuation of the average dispersed shareholder, $v(\mathbb{E}[b],q_e^*)$, and the price impact of the blockholder, γy. From (7) and (9), dispersed shareholders' demand as a function of the blockholder's trade can be written as

$$x(b,y,q_e^*)=\frac{1}{\gamma}(b-\mathbb{E}[b])\,H(q_e^*)-y. \tag{10}$$

The post-trade ownership structure. Next, we characterize the post-trade ownership structure. After the trading stage, the blockholder owns $\alpha+y$ shares, a dispersed shareholder with bias b owns $1-\alpha+x(b,y,q_e^*)$ shares, and all dispersed shareholders collectively own $\int_{-\bar{b}}^{\bar{b}} g(b)(1-\alpha+x(b,y,q_e^*))\,db=1-\alpha-y>0$ shares. Thus, the proportion of shares owned post-trade by a shareholder with bias b, conditional on the expected decision rule q_e^* and blockholder's trade y, is given by

$$\begin{aligned} r(b;y,q_e^*) &\equiv g(b)\frac{1-\alpha+x(b,y,q_e^*)}{1-\alpha-y} \\ &= g(b)\left(1+\frac{b-\mathbb{E}[b]}{\gamma}\frac{H(q_e^*)}{1-\alpha-y}\right), \end{aligned} \tag{11}$$

where the second equality follows from (10). Note that $r(b; y, q_e^*)$ is a density function, i.e., $\int_{-\bar{b}}^{\bar{b}} r(b; y, q_e^*)\, db = 1$. Thus, the post-trade shareholder base is characterized by the cdf $R(b; y, q_e^*)$ given by

$$\begin{aligned} R(b'; y, q_e^*) &= \int_{-\bar{b}}^{b'} r(b; y, q_e^*)\, db \\ &= G(b') \left(1 - \frac{\mathbb{E}[b] - \mathbb{E}[b|b < b']}{\gamma} \frac{H(q_e^*)}{1 - \alpha - y}\right), \end{aligned} \tag{12}$$

where the second equality follows from (11). The cdf R characterizes the post-trade shareholder base, whereas G characterizes the pre-trade shareholder base. Note that $R(b) < G(b)$ for any b, i.e., R dominates G in the sense of first-order stochastic dominance. Hence, trading shifts the shareholder base in such a way that more activist shareholders own a larger proportion of the firm after trading. Moreover, $R(b'; y, q_e^*)$ increases in q_e^*; hence, a more activist decision rule (lower q_e^*) makes the post-trade shareholder base more activist. Intuitively, shareholders' heterogeneous attitudes towards the proposal create gains from trade, so the shareholder base moves in the direction of the expected outcome. Finally, $R(b'; y, q_e^*)$ decreases in the trade y of the blockholder, i.e., larger trades by the blockholder lead to stronger shifts in the distribution of dispersed shareholders in the direction of the expected outcome. This is because higher demand by the blockholder raises the price, and only shareholders with a sufficiently large bias towards the expected outcome are willing to buy shares at this increased price (see (7)).

4.2 Voting

The composition of the post-trade shareholder base determines the voting outcome. To derive the conditions under which the proposal is approved, we characterize the identity of the *marginal voter*, who is defined as the shareholder whose individual vote always coincides with the collective decision on the proposal. In other words, whenever the marginal voter votes in favor (against), the proposal is accepted (rejected).

Denote by $s(q; y, q_e^*)$ the number of votes cast in favor of the proposal by dispersed shareholders if signal q is realized, the blockholder traded y shares, and the expected decision rule is q_e^*. Then,

$$s(q; y, q_e^*) = (1 - \alpha - y)(1 - R(-q; y, q_e^*)), \tag{13}$$

which is the number of shares owned by dispersed shareholders, $1 - \alpha - y$, multiplied by the proportion of dispersed shareholders for whom $b > -q$. Consistent with intuition, this function is increasing in q. The blockholder is pivotal for the outcome whenever the proposal fails without his support but is accepted with his support, i.e., whenever

$$s(q; y, q_e^*) < \tau < s(q; y, q_e^*) + \alpha + y. \tag{14}$$

Denote

$$\beta_l(y, q_e^*) \equiv -s^{-1}(\tau; y, q_e^*), \tag{15}$$

$$\beta_h(y, q_e^*) \equiv -s^{-1}(\tau - \alpha - y; y, q_e^*), \tag{16}$$

where $s^{-1}(\cdot; y, q^*)$ is the inverse function of $s(q; y, q^*)$ with respect to q, which does not depend on β and is monotonically increasing. Therefore, $\beta_l(y, q_e^*) < \beta_h(y, q_e^*)$, and condition (14) can be rewritten as

$$\beta_l(y, q_e^*) < -q < \beta_h(y, q_e^*). \tag{17}$$

To simplify the exposition, we will often refer to $\beta_l(y, q_e^*)$ and $\beta_h(y, q_e^*)$ as β_l and β_h, respectively.

To understand the relation between the pivotality of the blockholder and the identity of the marginal voter, recall that the blockholder votes for the proposal if and only if $\beta + q > 0$. Consider three separate cases. First, if $\beta \leq \beta_l$, then the blockholder always votes against the proposal whenever he is pivotal. Hence, the proposal is accepted if and only if $s(q; y, q_e^*) \geq \tau$. Since $s(q; y, q_e^*) \geq \tau \Leftrightarrow -q \leq \beta_l$, the marginal voter in this case is a dispersed shareholder with bias β_l. Second, if $\beta > \beta_h$, then the blockholder always votes for the proposal whenever he is pivotal, so the proposal is accepted if and only if $s(q; y, q_e^*) + \alpha + y \geq \tau$ or, equivalently, $-q \leq \beta_h$. Hence, the marginal voter in this case is a dispersed shareholder with bias β_h.

Finally, if $\beta_l < \beta \leq \beta_h$, then the blockholder votes for the proposal whenever he is pivotal and $q > -\beta$. When the blockholder is not pivotal, he votes as if he is pivotal, like any dispersed shareholder. Thus, the proposal is accepted if and only if the blockholder votes in its favor, so the blockholder is the marginal voter. Overall, the proposal is approved by the post-trade shareholder base if and only if $q > q^*(y, q_e^*)$, where

$$-q^*(y, q_e^*) = \begin{cases} \beta_l(y, q_e^*) & \text{if } \beta \leq \beta_l(y, q_e^*) \\ \beta & \text{if } \beta_l(y, q_e^*) < \beta \leq \beta_h(y, q_e^*) \\ \beta_h(y, q_e^*) & \text{if } \beta_h(y, q_e^*) < \beta \end{cases} \tag{18}$$

is the identity of the marginal voter.

An important insight from expression (18) is that it distinguishes the marginal voter and the voter who is pivotal for the vote outcome. Recall that the marginal voter is defined as the shareholder whose vote always coincides with the decision on the proposal, so characterizing the identity of the marginal voter is crucial for understanding the voting outcome. By contrast, the pivotal voter is defined as the voter who is decisive for the outcome. In our case, dispersed shareholders are atomistic and only the blockholder can be pivotal (condition (14)), but dispersed shareholders can be marginal voters. For example, if $\beta \leq \beta_l$, the blockholder is pivotal if $q \in (-\beta_h, -\beta_l)$, but the marginal voter is always a dispersed shareholder with bias $b = \beta_l$. This distinction is important for the value of voting rights: as will become clear in Section 4.3, it is the incentive to change the marginal voter, rather than the pivotal voter, that drives the voting premium.

Our discussion above shows that if shareholders anticipate the decision rule q_e^* at the trading stage, then the decision rule at the voting stage is $q^*(y, q_e^*)$ given by (18). In equilibrium, shareholders' expectations at the trading stage have to be consistent with the actual decision rule at the voting stage. Hence, an equilibrium can be characterized as a fixed point of the decision rule q_e^* such that $q^*(y, q_e^*) = q_e^*$. Note that $-q^*$ is the bias of the marginal voter, so in equilibrium, the actual marginal voter at the voting stage must coincide with the marginal voter expected at the trading stage. Using this logic, the equilibrium at the voting stage can be characterized as follows.

Proposition 1 (Voting stage). *Suppose the blockholder trades y shares. Then, the proposal is approved if and only if $q > q^*(y)$, where*

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } \beta < \beta_L(y) \\ \beta & \text{if } \beta_L(y) < \beta < \beta_H(y) \\ \beta_H(y) & \text{if } \beta_H(y) < \beta \end{cases} \tag{19}$$

$$= \begin{cases} \max\{\beta, \beta_L(y)\} & \text{if } \beta < \beta^* \\ \min\{\beta, \beta_H(y)\} & \text{if } \beta > \beta^*, \end{cases} \tag{20}$$

is the identity of the marginal voter, $\beta_L(y)$ and $\beta_H(y)$ are the solutions of

$$s(-\beta_L; y, -\beta_L) = \tau, \tag{21}$$

$$s(-\beta_H; y, -\beta_H) = \tau - \alpha - y, \tag{22}$$

and $\beta^* \equiv \beta_H(-\alpha) = \beta_L(-\alpha)$. Moreover, there exists $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, the solutions of (21) and (22) are unique. In those cases, $\beta_L(y)$ is decreasing and $\beta_H(y)$ is increasing in y.

In general, there may be multiple solutions to equations (21) and (22), and, as such, there may be multiple equilibria at the voting stage. This is because for small γ, the trades of dispersed shareholders and, accordingly, the shifts in the shareholder base can be large, which may give rise to self-fulfilling expectations. In particular, as discussed above, the cdf of the post-trade shareholder base, given by (12), increases in q_e^*, and hence a more activist *expected* decision rule (lower q_e^*) makes the post-trade shareholder base more activist. A more activist shareholder base, in turn, is more likely to approve the proposal for any given signal, leading to a lower *realized* cutoff for approving the proposal, confirming ex-ante expectations. However, as Proposition 1 shows, if γ is sufficiently large, then the dispersed shareholders trade less aggressively, the distribution of the post-trade shareholder base is less sensitive to expected q_e^*, and the equilibrium is unique. To characterize the optimal trading strategy of the blockholder, from this point on, we focus on parameterizations for which the equilibrium at the voting stage is unique.

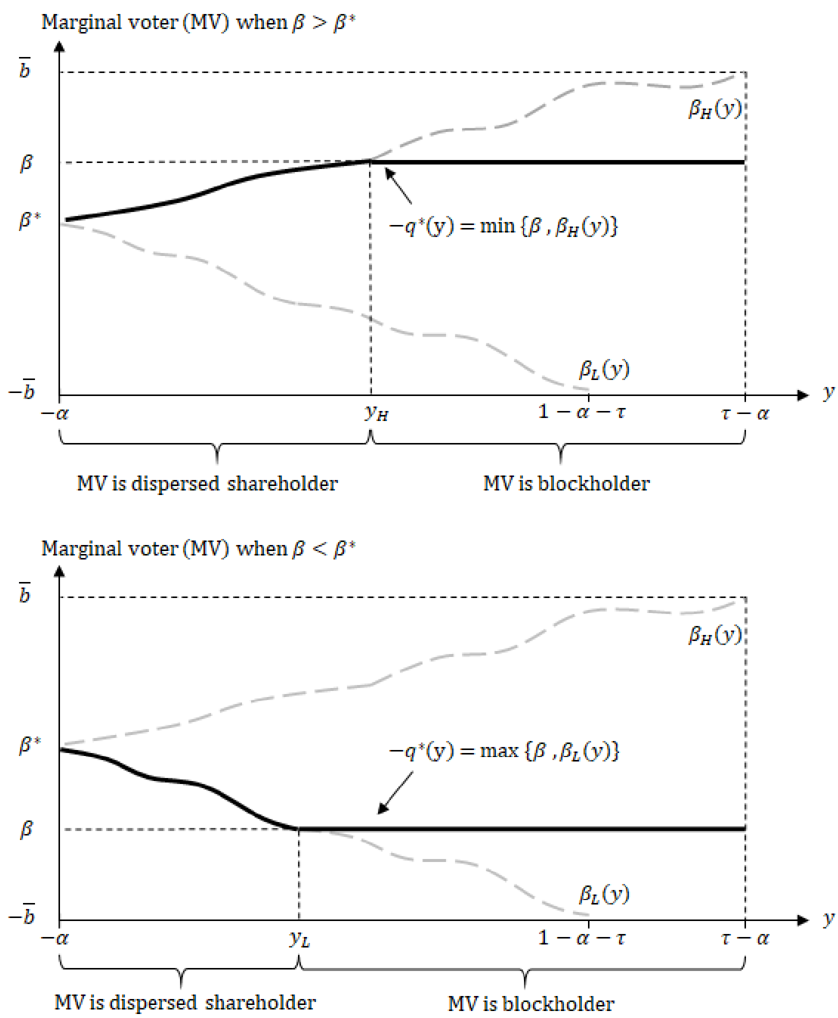

Figure 1 - Marginal voter as a function of the blockholder's trade

Figure 1 illustrates Proposition 1 and plots the marginal voter (vertical axis) as a function of the blockholder's trade y (horizontal axis). The figure shows that function $\beta_H(y)$ ($\beta_L(y)$) is upward (downward) sloping, starting at β^* for $y = -\alpha$ and reaching a maximum of $\bar{b}$ (minimum of $-\bar{b}$) as y approaches $\tau - \alpha$ $(1 - \alpha - \tau)$; this property is shown in the proof of Proposition 1.

The upper panel of Figure 1 considers the case in which the blockholder's bias is $\beta > \beta^*$. For any trade $y < y_H \equiv \beta_H^{-1}(\beta)$, the marginal voter is a dispersed shareholder whose bias is strictly increasing in y, whereas for any $y \geq y_H$, the marginal voter is the blockholder himself. Hence, the marginal voter is given by $\min\{\beta, \beta_H(y)\}$, shown as the bold line in Figure 1. It follows that by choosing the appropriate trade in the interval $[-\alpha, y_H]$, the blockholder can change the identity of the marginal voter to be any point in the interval $[\beta^*, \beta]$. In particular, by buying more (or selling fewer) shares, the blockholder can push the bias of the marginal voter closer to β. However, the blockholder cannot choose a marginal voter outside of the interval $[\beta^*, \beta]$. Intuitively, the marginal voter cannot be larger than β because the blockholder's optimal voting strategy (to support the proposal whenever $q > -\beta$) prevails when his stake becomes sufficiently large. Likewise, the marginal voter cannot be lower than β^* because the collective preferences of dispersed shareholders prevail when the blockholder exits his position.

The lower panel of Figure 1 considers the case in which the blockholder's bias is $\beta < \beta^*$, which is a mirror image of the upper panel. In particular, the blockholder can influence the identity of the marginal voter to be any point in the interval $[\beta, \beta^*]$ by choosing the appropriate trade in the interval $[-\alpha, y_L]$.

4.3 Blockholder trading

Given the blockholder's trade y, all shareholders correctly anticipate that the decision rule at the voting stage will be $q^*(y)$, as given by (19), and that the share price that clears the market will be

$$p^*(y) = \gamma y + v(\mathbb{E}[b], q^*(y)) \tag{23}$$

from (9). Therefore, in equilibrium, the blockholder chooses y to maximize

$$\Pi(y) \equiv (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2} y^2. \tag{24}$$

The marginal effect of buying additional shares on the blockholder's expected payoff is

$$\frac{\partial \Pi(y)}{\partial y} = \underbrace{(\beta - \mathbb{E}[b]) H(q^*(y)) - (2\gamma + \eta) y}_{\text{marginal propensity to buy cash flow rights, } MPC(y)} \tag{25}$$

$$+ \underbrace{\frac{\partial(-q^*(y))}{\partial y} \times [\alpha(q^*(y) + \beta) + y(\beta - \mathbb{E}[b])] f(q^*(y))}_{\text{marginal propensity to buy voting rights, } MPV(y)}, \tag{26}$$

and can be rewritten as

$$\frac{\partial \Pi \left(y\right)}{\partial y} = MPC\left(y\right) + MPV\left(y\right). \tag{27}$$

The term $MPC\left(y\right)$ is the blockholder's marginal benefit from buying additional cash flow rights that are embedded in each share, whereas $MPV\left(y\right)$ is the blockholder's marginal benefit from trading due to buying additional voting rights; it captures the additional incentives to trade in order to shift the marginal voter $-q^{*}\left(y\right)$.[17]

The next proposition characterizes the equilibrium of the game, including the blockholder's optimal trading strategy. As before, we focus on the case when the equilibrium is unique and, accordingly, assume that γ is large enough.

Proposition 2 (Equilibrium). *Suppose the blockholder has an endowment $\alpha > 0$ in the firm. There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, the equilibrium exists and is unique. In this equilibrium:*

(i) *The blockholder's trade satisfies*

$$y^{*} = \frac{1}{2\gamma + \eta}\left(\beta - \mathbb{E}\left[b\right]\right) H(q^{*}\left(y^{*}\right)) + \frac{1}{2\gamma + \eta} MPV\left(y^{*}\right), \tag{28}$$

and the trade of a dispersed shareholder with bias b is given by

$$x^{*}\left(b\right) = \frac{1}{\gamma}\left(b - b_{MT}\right) H(q^{*}\left(y^{*}\right)) - \frac{1}{2\gamma + \eta} MPV\left(y^{*}\right), \tag{29}$$

where

$$b_{MT} = \mathbb{E}\left[b\right] + \frac{\gamma}{2\gamma + \eta}\left(\beta - \mathbb{E}\left[b\right]\right). \tag{30}$$

[17] Note that $\frac{\partial(-q^{*}(y))}{\partial y}$ and $\frac{\partial \Pi(y)}{\partial y}$ do not exist when $\beta = \beta_L\left(y\right)$ or $\beta = \beta_H\left(y\right)$, which correspond to values y_L and y_H in Figure 1. In those cases, we interpret $\frac{\partial(-q^{*}(y))}{\partial y}$ as the right derivative of $-q^{*}\left(y\right)$, which is zero, and $\frac{\partial \Pi(y)}{\partial y}$ as the right derivative of $\Pi\left(y\right)$, which is $MPC\left(y\right)$.

(ii) The bias of the marginal voter is

$$-q^*(y^*) = \begin{cases} \beta_L(y^*) > \beta & \text{if } \beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) \\ \beta & \text{if } \beta \in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right) \\ \beta_H(y^*) < \beta & \text{if } \beta > G^{-1}(\frac{1-\tau}{1-\alpha}). \end{cases} \tag{31}$$

(iii) The share price is given by

$$p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*), \tag{32}$$

where $MPV(y^*) = 0$ if $\beta \in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right)$ and $MPV(y^*) > 0$ otherwise.

The blockholder's optimal trade of voting shares y^* consists of two terms. The first term reflects the incentives to buy shares for cash flow considerations, and it is positive if and only if $\beta > \mathbb{E}[b]$. That is, the blockholder has incentives to buy (sell) shares whenever his intrinsic valuation of the proposal is higher (lower) than the average dispersed shareholder's valuation. The second term reflects the blockholder's additional incentives to acquire shares in order to utilize the embedded voting rights, and it is proportional to $MPV(y^*)$. Importantly, Proposition 2 implies that in equilibrium the $MPV(y^*)$ is always (weakly) positive. Hence, the blockholder's incentives to buy (sell) shares are weakly stronger (weaker) if voting rights are bundled with cash flow rights.

To explain the intuition behind Proposition 2, Figure 2 plots the equilibrium bias of the marginal voter as a function of the blockholder's bias. There are two distinct scenarios. First, if $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ ($\beta > G^{-1}(\frac{1-\alpha}{1-\alpha})$), then the blockholder's preferences over the proposal are extreme relative to dispersed shareholders' preferences. Consequently, the equilibrium marginal voter has a larger (smaller) bias toward the proposal than the blockholder. This can be seen from the bold black curve in the figure, which plots the bias of the marginal voter against the bias of the blockholder and is above (below) the dashed 45-degree line.[18] In this region, a strictly positive $MPV(y^*)$ reflects the blockholder's marginal benefit from the additional

[18]The figure shows that the position of the marginal voter is increasing in β for $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. This is for illustrative purposes only and not a general feature of the model.

power to tilt the voting outcome in his preferred direction by casting a larger number of votes against (for) the proposal whenever his vote is pivotal. Second, if $\beta \in \left(G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right), G^{-1}\left(\frac{1-\tau}{1-\alpha}\right)\right)$, then the blockholder's preferences regarding the proposal are moderate and roughly aligned with those of dispersed shareholders, and the black curve coincides with the dashed 45-degree line. An $MPV(y^*)$ of zero arises in this region, because acquiring additional voting rights would not change the voting outcome as we explain in greater details below.

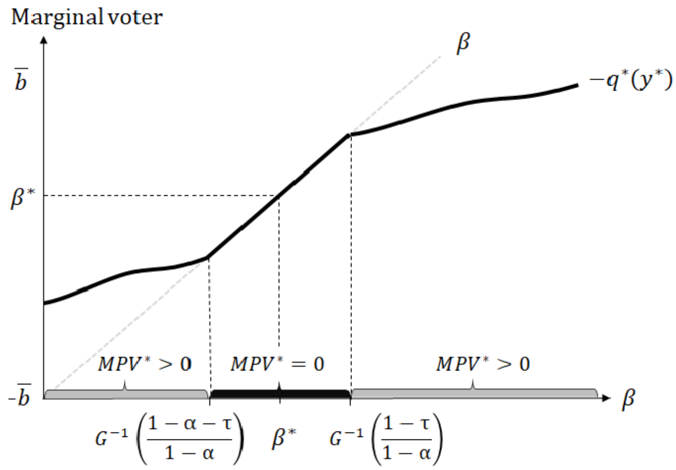

Figure 2 - Equilibrium marginal voter, $-q^*(y^*)$.

4.4 The determinants of the voting premium

To better understand the determinants of the voting premium and expression (32) for the equilibrium share price, consider a hypothetical scenario in which the decision rule is set exogenously at the level $q^*(y^*)$. For example, this could describe a scenario in which the decision on the proposal is made by a board of directors with bias $-q^*(y^*)$, rather than by a shareholder vote. In this scenario, the decision rule is not affected by the blockholder's trades, so (25)-(26) imply that the marginal effect of buying additional shares on the blockholder's expected payoff is simply $MPC(y)$, whereas $MPV(y) = 0$. Hence, (32) implies that the stock price in this hypothetical scenario is exactly $v(b_{MT}, q^*(y^*))$, the valuation of the marginal trader, who is indifferent between buying and selling (see (29)), and has bias b_{MT}.[19]

[19] Expression (30) shows that the bias b_{MT} of the marginal trader is a weighted average of the average bias of the dispersed shareholders, $\mathbb{E}[b]$, and that of the blockholder, β, where the weights reflect the relative trading intensities: If the trading frictions of dispersed shareholders are small relative to those of the blockholder (γ is

Therefore, equation (32) implies that the difference between the share price if the proposal is decided by shareholder voting and the share price if the proposal were decided exogenously by the same decision rule, is proportional to the $MPV(y^*)$. In other words, $MPV(y^*)$ not only measures the blockholder's marginal willingness to pay for additional votes, but also translates into a price differential for voting shares, namely, the *voting premium*. In Section 5.1 below, we show that $MPV(y^*)$ is also proportional to the dual-class share premium. For this reason, we next focus on understanding the determinants of $MPV(y^*)$. To this end, we decompose $MPV(y)$ as follows:

$$MPV(y) = \underbrace{\frac{\partial(-q^*(y))}{\partial y}}_{\text{ability to influence MV}} \times \underbrace{\left[\underbrace{\alpha(q^*(y)+\beta)}_{\text{endowment benefit}} + \underbrace{y(\beta - \mathbb{E}[b])}_{\text{net trading benefit}}\right] f(q^*(y))}_{\text{incentive to influence MV}}, \tag{33}$$

where MV stands for the marginal voter. The first observation is that $MPV(y)$ can be decomposed into the blockholder's *ability* to influence the marginal voter, $\frac{\partial(-q^*(y))}{\partial y}$, and his *incentive* to influence the marginal voter's identity, which is the remainder of the expression in (33).

Consider first the blockholder's ability to influence the marginal voter. From (19), $\frac{\partial(-q^*(y))}{\partial y} = 0$ if and only if $\beta_L(y) < \beta < \beta_H(y)$. This happens to the right of y_H (y_L) in the upper (lower) panel of Figure 1: In this region, $-q^*(y) = \beta$, i.e., the blockholder is the marginal voter regardless of marginal changes in y; hence, he cannot influence the voting outcome by buying additional shares. Therefore, his marginal propensity to buy voting rights in this region is zero, i.e., $MPV(y) = 0$. This observation further highlights the difference between the pivotal voter and the marginal voter discussed in Section 4.2: it is the latter and not the former who affects the blockholder's payoff, and hence, the voting premium.

Next, we consider the blockholder's *incentives* to change the location of the marginal voter, which are given by the expression in square brackets in (33). From this expression, these incentives can be broken down into the *endowment benefits* and the *net trading benefits*. (Note

small relative to η), then the bias of dispersed shareholders has a larger weight in the identity of the marginal trader.

that both of these are *net* benefits, which can be negative.) The former captures the marginal benefits on the valuation of the blockholder's endowment from influencing the marginal voter:

$$\alpha \frac{\partial v(\beta, q^*)}{\partial(-q^*)} = \alpha (q^* + \beta) f(q^*). \tag{34}$$

Endowment benefits are positive only if the marginal voter is less activist than the blockholder, $-q^* < \beta$: the value of the endowment increases as the blockholder buys additional shares and the marginal voter approaches β, and is maximized when the marginal voter's bias is β.

The net trading benefits capture the marginal profitability of the blockholder's trades y, which depend on the spread between his valuation and the stock price p. Using (9) and holding y constant (since we evaluate only a shift in the marginal voter), an incremental change in the marginal voter affects the stock price by

$$\frac{\partial p^*}{\partial(-q^*)} = \frac{\partial v(\mathbb{E}[b], q^*)}{\partial(-q^*)} = (\mathbb{E}[b] + q^*) f(q^*). \tag{35}$$

Hence, the blockholder's net benefit from moving the marginal voter on the shares he trades is

$$y \left(\frac{\partial v(\beta, q^*)}{\partial(-q^*)} - \frac{\partial p}{\partial(-q^*)} \right) = y (\beta - \mathbb{E}[b]) f(q^*), \tag{36}$$

which is positive if and only if $y(\beta - \mathbb{E}[b]) > 0$. Intuitively, the blockholder benefits from a more activist marginal voter whenever $y > 0$ and $\beta > \mathbb{E}[b]$ ($y < 0$ and $\beta < \mathbb{E}[b]$), since dispersed shareholders who on average dislike (like) the proposal more than the blockholder, would be willing to sell (buy) shares for a lower (higher) price.

Overall, the MPV is positive if and only if the purchase of an additional voting share increases (decreases) the likelihood that the proposal is approved by shareholders, and in turn, this change increases (decreases) the blockholder's total benefit as reflected by the sum of the endowment and the net trading benefits.

4.5 Interpretation

In this section, we discuss the implications of our results and several important special cases, which allow us to further isolate the forces that determine the voting premium.

4.5.1 Zero voting premium

While a zero $MPV(y^*)$ implies a zero voting premium, it does not imply that the blockholder does not value voting rights, or that he would not benefit from further influencing the voting outcome. In particular, from (33), the sum of the endowment and net trading benefits is generally not zero when the blockholder is the marginal voter (i.e., $\beta_L(y) < \beta < \beta_H(y)$). A zero $MPV(y^*)$ only implies that the blockholder cannot influence the position of the marginal voter through additional trades of voting shares, an issue we follow up on in the discussion of a separate market for votes in Section 5 below.

Note also that the blockholder's overall benefits from accumulating voting rights can be positive even if the marginal benefits are zero. Specifically, $MPV(y^*) = 0$ does not necessarily imply $\int_0^{y^*} |MPV(y)| \, dy = 0$ whenever $y^* > 0$. In this respect, the voting premium is unlikely to correctly estimate the overall value of voting rights.

4.5.2 Exit and a positive voting premium

Even if the blockholder has the power to increase his influence over the voting outcome and move the marginal voter closer to his bias β by buying additional shares, he may nevertheless choose to do the opposite: sell shares to dispersed shareholders and give up his influence over the voting outcome.

Corollary 1. *Suppose $\beta < \mathbb{E}[b]$ and $\alpha \in (0, \alpha^*)$ for some $\alpha^* > 0$. There exist $\bar{\gamma}$ and $\bar{\eta}$ such that if $\gamma > \bar{\gamma}$ and $\eta > \bar{\eta}$, then $y^* < 0$ and $MPV(y^*) > 0$.*

Hence, the voting premium may be positive even when the blockholder sells shares, i.e., when the ownership structure becomes less concentrated. This can happen if $\beta < \mathbb{E}[b]$, so cash flow considerations ($MPC(y) < 0$) and the net trading benefits from moving the marginal voter (see (36)) lead the blockholder to sell. However, selling also implies that the blockholder moves the marginal voter further away from his own bias β, which leads to a loss on his endowment α. Hence, when selling, the blockholder demands a premium from the dispersed shareholders, because selling diminishes his ability to influence the vote outcome. For small α, this countervailing effect on the blockholder's endowment is too small to outweigh the benefits from selling. By contrast, for large α the blockholder may buy shares, forego the profits from selling, and use his voting rights to make sure the proposal is rejected more often in order to protect his endowment.

4.5.3 Free-riding and a negative voting premium

Under the assumptions of Proposition 2, i.e., when the blockholder has a positive endowment and γ is large, the voting premium is always positive, $MPV(y^*) \geq 0$. That is, the blockholder's incentives to trade are always stronger due to the bundling of voting and cash flow rights. However, if the blockholder has no endowment, then $MPV(y^*) < 0$ is possible, resulting in a negative voting premium.

Proposition 3. *Suppose $\alpha = 0$. There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. In equilibrium, $MPV(y^*) < 0$ if and only if $\mathbb{E}[b] < \beta < G^{-1}(1-\tau)$. In this case, the blockholder buys shares ($y^* > 0$) and the share price exhibits a negative voting premium: $p^* < v(b_{MT}, q^*(y^*))$.*

Intuitively, if the blockholder is less activist than the marginal voter, $\beta < G^{-1}(1-\tau)$, then as the blockholder buys more shares, the marginal voter becomes less activist and closer to the blockholder's own bias. However, if the blockholder is more activist than the average dispersed shareholder, $\beta > \mathbb{E}[b]$, then this change in the marginal voter increases the valuation of the average dispersed shareholder, and thereby the stock price, even more than the valuation of the blockholder. This free-rider problem creates negative net trading benefits (see (36)), which may dominate the positive endowment benefits.

The assumptions of Proposition 2 guarantee that the endowment benefits always dominate the net trading benefits, and hence, $MPV(y^*)$ is non-negative. In contrast, under the assumptions of Proposition 3, $\alpha = 0$, so the endowment benefits are zero and the negative net trading benefits are the only force. Hence, the blockholder buys fewer shares than he would have if cash flow rights were not bundled with voting rights. Thus, free-riding by dispersed shareholders results in a negative voting premium.

While Propositions 2 and 3 imply that for large γ, a negative voting premium arises only when the blockholder has no initial endowment ($\alpha = 0$), the existence of a negative voting premium is more general: If γ is not too large, a negative MPV can also arise for small but strictly positive values of α.

4.5.4 Conflicts of interest and homogeneous shareholders

It is often argued that the voting premium reflects private benefits of control, whereas our model is based on private values. We interpret private benefits as implying a conflict between the blockholder and *all* dispersed shareholders, so that the blockholder gains what the dispersed shareholders lose. By contrast, with private values, those dispersed shareholders whose preferences are sufficiently close to the blockholder benefit from his trades and his influence on the marginal voter.

In the context of our model, a conflict of interest between the blockholder and all dispersed shareholders does not by itself generate a voting premium. To see this, we adapt the model to reflect private benefits by assuming that the distribution G of dispersed shareholders' private values is concentrated around its mean, and this mean is different from the bias of the blockholder, $\mathbb{E}[b] \neq \beta$. In such a setting, there are no dispersed shareholders who are close to the blockholder, so any move of the marginal voter towards the blockholder moves the marginal voter away from all dispersed shareholders.

Proposition 4 (Conflicts of interest). *Suppose $\mathbb{E}[b] \neq \beta$. Consider a mean-preserving parametrization δ such that as $\delta \to 0$, the cdf $G(b;\delta)$ becomes more concentrated around the mean $\mathbb{E}[b]$. Then, for any $y \in (-\alpha, \min\{\tau, 1-\tau\} - \alpha)$, $\lim_{\delta\to 0} \beta_H(y) = \lim_{\delta\to 0} \beta_L(y) = \mathbb{E}[b]$. Moreover, $\lim_{\delta\to 0} MPV(y^*(\delta)) = 0$.*

In Proposition 4, the dispersion of private values among dispersed shareholders becomes second order relative to the difference between them and the blockholder. Then, as the heterogeneity among dispersed shareholders vanishes, the blockholder loses his ability to influence the identity of the marginal voter through his trades. Accordingly, his control motive for buying the shares vanishes, and so does the voting premium. This argument clarifies that a setting in which private benefits create a conflict of interest between the blockholder and all dispersed shareholders does not automatically give rise to a voting premium.

5 Unbundling votes and cash-flow rights

In the baseline model, votes are always bundled with cash flow rights in a fixed proportion. In this section, we relax this assumption. Section 5.1 introduces a dual-class share structure, which allows investors to build portfolios with different proportions of cash flow to voting rights in markets with identical trading frictions. In Section 5.2, we ask how the blockholder would choose the marginal voter if he could make this choice without any constraints. Finally, Section 5.3 asks whether the blockholder can achieve his desired voting outcome in a setting in which votes can be traded without any frictions and be costlessly separated from cash flow rights.

5.1 Dual-class shares

In this section, we investigate how the MPV is related to measures of the voting premium estimated in the empirical literature (see Section 7). To do so, we extend our model to a setting with two classes of shares with different voting rights. Specifically, suppose that in addition to the traded voting shares, investors can also trade non-voting shares. This setting can either capture companies with a dual-class share structure (e.g., Zingales (1995); Nenova (2003)), or the ability of investors to trade synthetic shares in derivative markets (e.g., Kalay, Karakas, and Pant (2014)).

We assume that the blockholder and each dispersed shareholder are endowed with $\hat{\alpha} \in [0, 1]$ and $\hat{e} \in [0, 1 - \hat{\alpha}]$ non-voting shares, respectively, so that the total number of outstanding non-voting shares lies in the interval $[0, 1]$. Notice that we allow for the supply of non-voting shares to be zero (i.e., $\hat{\alpha} = \hat{e} = 0$), which could capture the creation of non-voting securities in derivatives markets. Similarly, we denote by $\hat{x}$ and $\hat{y}$ the trades of dispersed shareholders and the blockholder in the non-voting shares, respectively. The utility of dispersed shareholders is given by

$$\hat{u}\left(d, \theta, b, x, \hat{x}; \gamma, e, \hat{e}\right) = \left(e + x\right) v\left(d, \theta, b\right) - \frac{\gamma}{2} x^2 + \left(\hat{e} + \hat{x}\right) v\left(d, \theta, b\right) - \frac{\gamma}{2} \hat{x}^2, \tag{37}$$

which means that in this extension, γ is best interpreted as capturing trading costs, rather than as risk aversion. To make sure that the price differential between voting and non-voting shares does not stem from differences in the microstructure of these markets, we assume that the trading costs are the same for these two securities. Similarly, the blockholder's utility is given by $u\left(d, \theta, \beta, y, \hat{y}; \eta, \alpha, \hat{\alpha}\right)$. Notice that in principle, shorting of non-voting shares is feasible. However, we assume that γ and η are large enough, so that $e + x + \hat{e} + \hat{x} > 0$ and $\alpha + y + \hat{\alpha} + \hat{y} > 0$, i.e., the net cash-flow exposure of each investor is always non-negative.

Consider first the case in which the decision rule q^* is exogenous, e.g., if decisions are taken by the board of directors. With an exogenous decision rule q^*, the trading strategies of all investors in each market are given by the expressions in Proposition 2, assuming that $\frac{\partial(-q^*(y))}{\partial y} = 0$, and hence, $MPV = 0$. Indeed, if the decision rule is not affected by trading, then the existence of the market for non-voting shares does not affect trading in the market for voting shares, and vice versa. This is because investors have no budget constraints and the trading costs apply to each market separately. Moreover, although the endowment of non-voting shares could be different from the endowment of voting shares, the trading quantities are the same as in our model, since they are invariant to the level of the endowment.

This observation implies that with an exogenous cutoff q^*, the prices of voting and non-voting shares must be identical. Indeed, given $(y, \hat{y})$ and q^*, the difference in prices is

$$p\left(y, q^*\right) - p\left(\hat{y}, q^*\right) = \gamma y + v\left(\mathbb{E}\left[b\right], q^*\right) - \left(\gamma \hat{y} + v\left(\mathbb{E}\left[b\right], q^*\right)\right) = \gamma\left(y - \hat{y}\right),$$

and since $y = \hat{y}$, the two prices are the same.

Next, consider the model with voting. By assumption, the net positions of dispersed shareholders and the blockholder are always non-negative. Therefore, a dispersed shareholder with bias b votes for the proposal if and only if $q+b>0$, and the blockholder votes for the proposal if and only if $q+\beta>0$. Note also that for a given q^* and $(y,\hat{y})$, the trading strategies of dispersed shareholders in the voting and non-voting shares are the same as in the baseline model. Thus, the identity of the marginal voter as a function of the blockholder's trade, namely $q^*(y)$, is determined as in the baseline model (see Proposition 1). This implies that given y, the marginal voter is unaffected by $\hat{y}$, i.e., the trades that take place in the market for non-voting shares. However, the presence of non-voting shares changes the blockholder's trades of voting shares, because he internalizes the effect of the voting outcome on the value of his non-voting shares. The objective of the blockholder becomes:

$$\max_{y,\hat{y}} \Pi(y,\hat{y}) = (\alpha+y)\, v(\beta, q^*(y)) - y p^*(y) - \frac{\eta}{2} y^2 \tag{38}$$

$$+ (\hat{\alpha}+\hat{y})\, v(\beta, q^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2. \tag{39}$$

We obtain the following result:

Proposition 5 (Dual-class shares) *If the blockholder and dispersed shareholders can trade in voting and non-voting shares, the voting premium is:*

$$p^*_{voting} - p^*_{non-voting} = \gamma(y^* - \hat{y}^*) = \frac{\gamma}{2\gamma+\eta} MPV(y^*, \hat{y}^*), \tag{40}$$

where

$$MPV(y,\hat{y}) = \frac{\partial(-q^*(y))}{\partial y} f(q^*(y)) \left[(\alpha+\hat{\alpha})(q^*(y)+\beta) + (y+\hat{y})(\beta - \mathbb{E}[b])\right]. \tag{41}$$

Proposition 5 shows that the dual-class voting premium is proportional to the MPV. Thus, the blockholder's increased willingness to pay for additional shares to affect the voting outcome directly translates into an actual price difference between voting and non-voting shares. Note also that $MPV(y^*, \hat{y}^*)$ depends on $\hat{y}^*$, which means that the volume of trades in the market for non-voting shares affects the blockholder's incentives to buy voting shares, and hence the voting premium. Intuitively, the blockholder's position in non-voting shares gives him additional incentives to change the marginal voter for the same reasons as his position in voting shares – the endowment benefits ($\hat{\alpha}(q^*(y)+\beta)$), and the net trading benefits ($\hat{y}(\beta - \mathbb{E}[b])$).

5.2 The ideal marginal voter

In this section, we analyze how the blockholder would choose the marginal voter, who is denoted by $-q_B^*$. This ideal marginal voter is obtained when the blockholder has no incentives to further influence the marginal voter. Based on (33), this happens whenever the sum of the endowment and net trading benefits is zero, which gives

$$-q_B^*(y) = \beta + \frac{y}{\alpha}(\beta - \mathbb{E}[b]). \tag{42}$$

The expression has two components. If the blockholder does not trade ($y = 0$), his ideal marginal voter is $-q_B^* = \beta$, which maximizes the value of his endowment. However, the more important are the blockholder's trading considerations relative to his endowment, $\frac{y}{\alpha}$, the further away from β is his ideal marginal voter. The wedge between the blockholder's bias and the average dispersed shareholder's bias, which captures the potential gains from trade, determines the deviation of the blockholder's ideal marginal voter from β.

In our baseline model, where voting and cash flow rights are always bundled in one security, the blockholder's ideal marginal voter cannot be obtained in equilibrium. For example, according to Proposition 1, if $\beta > \max\{\beta^*, \mathbb{E}[b]\}$, then the blockholder can only choose a marginal voter in the interval $[\beta^*, \beta]$; yet, his ideal marginal voter is $-q_B^*(y) > \beta$, which is outside that interval. Moreover, even if the blockholder could choose the ideal marginal voter by picking the appropriate y, he would generally choose not to do so because of cash flow considerations, so the optimal trade y^* generally does not lead to the ideal marginal voter, $-q_B^*(y^*) \neq -q^*(y^*)$. Therefore, in the next section, we explore whether the blockholder can obtain his ideal marginal voter if votes can be costlessly separated from cash flow rights.

5.3 Vote trading

To examine whether the separation of voting and cash flow rights could allow a blockholder to obtain his ideal marginal voter, we add a separate market for voting rights (e.g., share lending) to our baseline model. (The full analysis is shown in the Online Appendix.) Since dispersed shareholders are never pivotal for the voting outcome, they are willing to supply their votes for an arbitrarily small price. Therefore, we assume that the price of a vote is zero, that vote trading involves no transaction costs, and that the votes are sold by dispersed shareholders in proportion to their ownership of the voting shares. These assumptions likely overstate the blockholder's ease of access to the market for votes.[20] Nevertheless, the next result shows that the blockholder cannot always obtain his ideal marginal voter in equilibrium.

Proposition 6. *Suppose the blockholder has access to the market of votes. There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the blockholder obtains his ideal marginal voter if and only if $\beta < G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$.*

To understand the intuition, first note that the blockholder's ideal marginal voter is typically more activist than the blockholder, $-q_B^*(y^*) > \beta$. Indeed, if $\beta > \mathbb{E}[b]$ ($\beta < \mathbb{E}[b]$), then dispersed shareholders on average dislike (like) the proposal more than the blockholder, and the blockholder would benefit from buying (selling) shares. By pushing the marginal voter to have a bias greater than β, the blockholder decreases (increases) the valuation of dispersed shareholders, and therefore, the price at which he can buy (sell) shares. Thus, the net trading benefits push $-q_B^*(y)$ to be greater than β.

However, as Proposition 6 demonstrates, the market for votes does not allow the blockholder to achieve this ideal marginal voter if the blockholder is already sufficiently activist, $\beta > G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$. Intuitively, while vote-buying increases the blockholder's influence on the identity of the marginal voter, it does so in a very specific way: it always pushes the marginal voter closer to the blockholder. This is because when the blockholder casts his vote, his gains from trade are sunk, so he always votes to maximize the value of his position. Without a commitment

[20] See Christoffersen et al. (2007) for a discussion of the market for equity lending. Their results suggest that the ease of access assumed here is realistic.

to do otherwise, the accumulation of disproportional voting rights can only push the bias of the marginal voter even closer to β. When the blockholder is less activist than the dispersed shareholders, he can obtain a more activist ideal marginal voter by rationing the amount of votes he buys from the dispersed shareholders, who are more activist. However, when the blockholder is relatively activist, the best he can do is to buy enough votes to ensure that he is the marginal voter, but he cannot use the market for votes to select a marginal voter who is more activist than himself.

Proposition 6 has the following interesting implication. A blockholder whose bias toward the proposal is relatively small can use shareholder voting to fulfill his agenda. However, if the blockholder is highly motivated and his bias toward the proposal is relatively large, the market for votes would be insufficient, and the blockholder could potentially benefit from exercising corporate influence by means other than voting. These other channels of influence could be lobbying institutional investors, proxy advisory firms, and regulators; engaging in behind-the-scenes discussions with management and board members; or launching media campaigns to put pressure on the firm. Overall, our analysis highlights that even when shareholder voting is amplified by the market for votes, other mechanisms of corporate governance continue to play a key role when blockholders are highly motivated, an observation that is consistent with the evidence that institutional investors use a variety of channels to exert corporate influence (e.g., McCahery, Sautner, and Starks (2016)).

6 Extensions

In this section, we discuss several extensions of the baseline model. In Section 6.1 we analyze the case of multiple blockholders, and in Section 6.2 we extend the model by an initial stage in which blockholders trade with each other.

6.1 Multiple blockholders

In this section, we consider an extension of the baseline model to the case of multiple blockholders. Specifically, assume there are $N \geq 2$ blockholders and each blockholder is endowed with $\alpha_i = \frac{\alpha}{N}$ shares. All N blockholders face the same trading cost η and have the same bias β. Hence, blockholders compete when trading shares, but they apply the same decision rule when voting on the proposal. We solve for the symmetric equilibrium in which the optimal trades for all blockholders are the same, y^*. Furthermore, we maintain the same assumptions as in Proposition 2 to ensure that the equilibrium exists and is unique.[21] We provide the full analysis of this extension in Section C.1 of the Online Appendix and summarize the main conclusions here.

The share price with N blockholders is given by

$$p^*(N) = v\left(b_{MT}(N), q^*(Ny^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny^*), \tag{43}$$

where $q^*(\cdot)$ is given by (19), $MPV(\cdot)$ is given by (33), and $b_{MT}(N)$ is defined in the Online Appendix.

We show that, as the number of blockholders increases, the share price puts an increasingly smaller weight on voting considerations and a larger weight on cash flow considerations. Moreover, the marginal shareholder $b_{MT}(N)$ converges to β, the bias of the blockholders. Intuitively, the blockholders compete with each other on buying shares. As in Cournot competition, they do not fully internalize the effect of their own trades on their peers, and as N increases, this effect becomes more and more significant (see Kyle (1989) and Edmans and Manso (2011) for similar effects). Each blockholder fully bears the costs of his trades, but the benefits from affecting the marginal voter accrue to all other blockholders as well. In the limit, competition among blockholders reduces their valuation of voting power and their marginal propensity to buy votes to zero.

The conclusion that the voting premium decreases with the number of blockholders crucially depends on the assumption that blockholders are homogeneous and, accordingly, free ride on each other's efforts to move the marginal voter. Hence, in another extension, we consider the case in which the blockholders have conflicting interests. Suppose μN blockholders have bias $\beta_c \approx -\Delta$, and $(1-\mu)N$ blockholders have bias $\beta_a \approx \Delta$, where $\mu \in (0,1)$ is such that

[21] We impose conditions on γ and η to guarantee that the trade y_i of each blockholder satisfies $y_i > -\frac{\alpha}{N}$.

both μN and $(1-\mu)N$ are integers. Blockholders with bias β_c always (i.e., regardless of the realization of signal q) vote against the proposal, and blockholders with bias β_a always vote for the proposal.

We again relegate the complete analysis of the symmetric equilibrium to Section C.2 in the Online Appendix. We show that blockholders with bias β_a and those with bias β_c have different MPVs, which depend on their biases and also result in different optimal trades. As the number N of blockholders increases, the bias of the marginal shareholder $b_{MT}(N)$ now converges to $\overline{\beta} \equiv (1-\mu)\beta_a + \mu\beta_c$, i.e., the weighted average of the biases of the two groups of blockholders.

The key observation from this extension is that if blockholders have stronger conflicts of interests, they value their voting rights more. Specifically, as blockholders become more extreme such that β_a increases by ε and β_c decreases by ε, then both the aggregate MPV, $\mu MPV_c^{**} + (1-\mu) MPV_a^{**}$, and the voting premium increase. This is true even if the average bias of the blockholders remains unchanged (e.g., $\mu = \frac{1}{2}$ and the biases become more extreme by the same amount). Intuitively, the marginal propensity to buy votes is positive both for blockholders biased in favor and those biased against the proposal, since both are trying to move the marginal voter in their preferred direction, which opposes the preferred direction of the other type. As the biases become more extreme, the incentives to do so increase, which is reflected in a higher voting premium embedded in the share price.

Overall, the important conclusion of these two extensions is that the voting premium crucially depends on whether blockholders have the same or conflicting objectives.

6.2 Block trading

The block trading premium has been analyzed by financial economists at least since Barclay and Holderness (1989) and has been used to measure the private benefits of control enjoyed by large shareholders. For example, Dyck and Zingales (2004) regard it as an alternative and sometimes advantageous empirical strategy to the dual-class share premium for measuring private benefits.

We therefore augment our baseline model with a pre-stage of block trading in the spirit of Burkart, Gromb, and Panunzi (2000) or Albuquerque and Schroth (2010). Specifically, at the first stage, an incumbent blockholder (I) with bias β_I and Nash bargaining power δ negotiates a trade of the block α with a rival blockholder (R) with bias β_R and Nash bargaining power $1-\delta$. After that, the investor who emerges as the owner of the block trades with dispersed shareholders and then votes his shares as in the baseline model. The other investor exits the game.

We let $\Pi^*(\beta) \equiv \Pi^*(y^*(\beta);\beta)$ be the blockholder's expected profit in equilibrium, given endowment α, optimal trade $y^*(\beta)$, and bias β. If $\Pi^*(\beta_I) \geq \Pi^*(\beta_R)$, then I and R do not trade with each other, so I owns the block and realizes payoff $\Pi^*(\beta_I)$. Otherwise, I and R trade, and R pays a price per share of

$$p_B = \frac{1}{\alpha}\left(\Pi^*(\beta_I) + \delta\left(\Pi^*(\beta_R) - \Pi^*(\beta_I)\right)\right) \tag{44}$$

for the block. In Section D in the Online Appendix, we provide a formal analysis of this extended model. There, we first establish that for sufficiently large γ and η, the profit $\Pi^*(\beta)$ is strictly increasing in β. The reason is that more activist blockholders value the proposal more. Hence, the blockholder with the larger, more activist bias ends up owning the controlling block. If we allowed for multiple potential bidders for the controlling block, the model would predict that the most activist bidder would end up owning the block.

Second, we show that the block premium is always negative if $\beta < \mathbb{E}[b]$. However, it is always positive if $\beta > \mathbb{E}[b]$, the block is sufficiently small, and the market for block trades is sufficiently competitive, which in our setup means that the incumbent has sufficient bargaining power, i.e., δ is large enough. Hence, the model can accommodate positive as well as negative block premiums, which is consistent with the empirical evidence.[22]

The discussion above also applies to private placements of blocks, in which a firm issues equity and places the new shares as a block with a single investor. Wruck (1989), Hertzel

[22] See Albuquerque and Schroth (2010), Table 2, which reports a negative median block premium, and Albuquerque and Schroth (2015), Figure 1, which shows that 53 of 114 block trades in their sample have a negative premium.

and Smith (1993), and Barclay, Holderness, and Sheehan (2007) study samples of such private placements. Our analysis implies that the firm would always auction of the block to the most activist blockholder, and that the blockholder might pay a premium or accept a discount relative to the price paid by dispersed shareholders.

7 Empirical implications and measures of the voting premium

There is a large empirical literature that provides measurements of the voting premium and analyses of the cross-sectional and time-series variation of the voting premium. The purpose of this section is to locate the model developed above in the context of the existing empirical evidence and, conversely, shed some light on the empirical discussion by exploring the implications of our model. Specifically, it is not the purpose of this section to offer a comprehensive survey of empirical studies and methodologies and their potential strengths and shortcomings.[23]

Broadly, there are five major strategies that have been developed in the literature to measure the voting premium and the economic value of voting power. We survey 40 studies in more detail in Table 1 in the Appendix and provide a summary in the table below. Of these studies, 15 use data on the US, 4 on Germany, 3 on Italy, 3 are cross-country studies, and 17 studies provide evidence on 11 other countries.

Methodology	**Avg. (%)**	**Median (%)**	**Min (%)**	**Max (%)**	**Studies**
Dual-class shares	23.59	14.53	5.44	81.50	23
Block-trade premium	41.50	29.55	13.00	130.90	14
Option replication	0.20	0.16	0.09	0.37	5
Equity lending	0.01				3
Record-day trading	0.09				1

[23] Some papers already contain surveys of different strands of this literature. Rydqvist (1992) provides an early survey of studies on dual-class shares and Dittmann (2004), Adams and Ferreira (2008), and Kind and Poltera (2013) provide more recent updates.

The most salient feature of these studies is that they report very divergent estimates of the voting premium. Below, we first discuss why estimates of the voting premium vary across methodologies (Section 7.1) and then the cross-sectional variation of voting premiums within methodologies (Section 7.2) and relate them to our model.

7.1 Marginal values vs. block values

Most methods to estimate the voting premium measure the value of a marginal vote. This applies to all methods that rely on stock market prices, i.e., all methods except for the block-trade premium. By contrast, block trades reveal the average valuation of a voting right for the entire block. The table above shows that block trades are associated with significantly larger premiums (average: 41.50%; median: 29.55%) than found in studies of dual-class share premiums (average: 23.73%; median: 13.85%) or those using the three other methods. Based on our model, we would expect the blockholder's willingness to pay for an entire block of shares to be larger than his willingness to pay for an additional voting share. In particular, MPV in our model may equal zero in equilibrium if the blockholder is the marginal voter (Proposition 2) at his equilibrium trading amount y^*, resulting in a zero dual-class share premium (Proposition 5, equation (40)). However, his average, per-share willingness to pay for a block of votes of size y^* in addition to his endowment α equals $\int_0^{y^*} MPV(y)\, dy$ and may be much larger.

In addition, it is salient from the table that studies relying on dual-class shares and block-trades obtain much larger estimates than the other three methods. We attribute this to the fact that the former two methods capitalize the value of the voting right over longer time horizons, which span potentially infinitely many future shareholder meetings. In contrast, the three other studies estimate the voting yield, which captures a period of one year or less. In the Online Appendix, we calibrate a simple valuation model and show that once the difference in the time horizon is accounted for, the estimates from these two sets of methods are in fact consistent with each other.

7.2 The cross-sectional variation in the voting premium

This section offers further observations on the cross-sectional variation of the voting premium and discusses them in the context of our model.

Negative values of the voting premium. One implication from our analysis is that the voting premium can sometimes be negative, which emanates from the free-rider effect (see Proposition 3 and the related discussion). Interestingly, while the estimates of the mean and median of the voting premium reported in Table 1 are always positive, many studies report that the voting premium is negative for some companies.[24] These findings are consistent with our model, but are difficult to interpret in the context of extant theories. Empirical studies often explain them by pointing out that voting shares may suffer from a liquidity discount relative to non-voting shares.[25]

Voting premiums, takeovers, and shareholder meetings. One of the standard explanations for how the blockholder's willingness to pay a premium for voting control is translated into higher prices for voting shares is the takeover mechanism, and several empirical studies find support for this explanation.[26] However, this theory has some limitations. First, since the 1990s, many countries have enacted coattail provisions, which mandate equal treatment of all classes of shares in control changes (Maynes (1996); Nenova (2003)). Second, Dittmann (2003) surveys 12 studies of companies with dual-class share structures and shows that if investors would correctly anticipate the ex-post frequencies of takeovers and takeover premiums paid, then the premium on voting shares in dual-class firms should be smaller by about one order of magnitude compared to the observed premium in most countries.[27] Hence, the takeover

[24] E.g., see Rydqvist (1996), Nenova (2003), and Caprio and Croci (2008) for the dual-class share premium and Albuquerque and Schroth (2010) and Albuquerque and Schroth (2015) for the block trading premium.

[25] Odegaard (2007) separates liquidity effects from control effects in Norway, which used to have three classes of shares that differed in their voting rights and the possibility of foreign ownership.

[26] For models, see Grossman and Hart (1988); Harris and Raviv (1988); Bergström and Rydqvist (1992). For empirical evidence see Bergström and Rydqvist (1992); Zingales (1995); Rydqvist (1996); Smith and Amoako-Adu (1995).

[27] This is acknowledged in the literature. For example, Zingales (1994) writes that the existence of a positive intercept in regressions of the voting premium on variables measuring control contests shows that "there is something other than control value that makes voting shares more (less) valuable." (p. 141).

explanation is probably only a partial explanation of premiums on voting shares.

Differently from this argument, our analysis shows how the voting premium can arise without contests for majority control, and solely as a result of blockholders' desire to influence the voting outcomes at shareholder meetings. This prediction is consistent with the findings of the more recent literature, which analyzes the time-series variation in the voting premium and finds that the voting premium is largest around shareholder meetings compared to other periods of the year (see Kind and Poltera (2013); Kalay, Karakas, and Pant (2014); Kind and Poltera (2017); Fos and Holderness (2020)).

Voting premiums and ownership structure. Studies on the relationship between the voting premium and ownership concentration show that it is often non-monotonic: the value of voting rights is small both if ownership is very dispersed and if it is very concentrated with one blockholder who has majority control (Kind and Poltera (2013)). Therefore, one common methodology uses the probability of being pivotal inferred from oceanic Shapley values instead of ownership concentration to predict the voting premium.[28] Our analysis in Section 6 suggests a new empirical direction by showing that it is not only the concentration of ownership and the probability of being pivotal that matters, but also the preferences of blockholders: If they (dis)agree and have similar (divergent) preferences, ownership concentration is positively (negatively) correlated with the voting premium.

8 Conclusion

We develop a theory of voting and trading in which a blockholder and dispersed shareholders trade with each other and then vote on a proposal. We analyze the trading decisions of blockholders, when they would be willing to pay a higher price in order to accumulate voting power, and how their trades translate into a premium for voting shares. The model generates a number of insights about the voting premium and the equilibrium ownership structure of the

[28] The method was pioneered by Rydqvist (1987) and is based on the theory of oceanic Shapley values of Milnor and Shapley (1978). For applications, see Zingales (1994), Zingales (1995), Chung and Kim (1999), Caprio and Croci (2008), and Nenova (2003).

firm.

If we interpret the voting premium as the market price of a vote, we find that the price of a vote does not reflect the economic value of voting rights to the blockholder, and that it is also unrelated to the voting power of the blockholder. In general, our results indicate that common measures of the voting premium may often underestimate the true value of voting rights to their owners. Our analysis also shows that a positive premium can be consistent with a less concentrated ownership structure, and that a negative voting premium can arise when dispersed shareholders free-ride on the blockholder's trades. We extend the model to explore the role of the market for votes, the interaction between multiple blockholders, and the pricing of block trades. Overall, our analysis emphasizes how asset prices are affected by blockholders' desire to move the voting outcome in their preferred direction when shares bundle voting and cash flow rights.

References

Adams, Renee B. and Daniel Ferreira. 2008. "One Share, One Vote: The Empirical Evidence." *Review of Finance* 12 (1):51–91.

Admati, Anat R. and Paul C. Pfleiderer. 2009. "The 'Wall Street Walk' and Shareholder Activism: Exit as a Form of Voice." *Review of Financial Studies* 22 (7):2445–2485.

Admati, Anat R., Paul C. Pfleiderer, and Josef Zechner. 1994. "Large shareholder activism, risk sharing, and financial market equilibrium." *Journal of Political Economy* 102 (6):1097–1130.

Aggarwal, Reena, Pedro A. C. Saffi, and Jason Sturgess. 2015. "The Role of Institutional Investors in Voting: Evidence from the Securities Lending Market." *The Journal of Finance* 70 (5):2309–2346.

Albuquerque, Rui A. and Enrique Schroth. 2010. "Quantifying private benefits of control from a structural model of block trades." *Journal of Financial Economics* 96 (1):33–55.

———. 2015. "The Value of Control and the Costs of Illiquidity." *Journal of Finance* 70 (4):1405–1455.

Austen-Smith, David and Jeffrey S. Banks. 1996. "Information Aggregation, Rationality, and the Condorcet Jury Theorem." *American Political Science Review* 90 (1):34–45.

Bar-Isaac, Heski and Joel D. Shapiro. 2019. "Blockholder Voting." *Journal of Financial Economics* 136 (3):695–717.

Barak, Ronen and Beni Lauterbach. 2011. "Estimating the private benefits of control from partial control transfers: methodology and evidence." *International Journal of Corporate Governance* 2 (3/4):183–200.

Barclay, Michael J. and Clifford G. Holderness. 1989. "Private benefits from control of public corporations." *Journal of Financial Economics* 25 (2):371–395.

Barclay, Michael J., Clifford G. Holderness, and Dennis P. Sheehan. 2007. "Private placements and managerial entrenchment." *Journal of Corporate Finance* 13 (4):461–484.

Baron, David P. and John A. Ferejohn. 1989. "Bargaining in Legislatures." *American Political Science Review* 83 (4):1181–1206.

Bergström, Clas and Kristian Rydqvist. 1992. "Differentiated Bids for Voting and Restricted Voting Shares in Public Tender Offers." *Journal of Banking and Finance* 16:97–114.

Bigelli, Marco and Ettore Croci. 2013. "Dividend privileges and the value of voting rights: Evidence from Italy." *Journal of Empirical Finance* 24:94–107.

Blair, Douglas H., Devra L. Golbe, and James M. Gerard. 1989. "Unbundling the Voting Rights and Profit Claims of Common Shares." *Journal of Political Economy* 97 (2):420–443.

Bolton, Patrick, Tao Li, Enrichetta Ravina, and Howard Rosenthal. 2019. "Investor Ideology." *Journal of Financial Economics* (forthcoming).

Bolton, Patrick and Ernst-Ludwig von Thadden. 1998. "Blocks, Liquidity, and Corporate Control." *The Journal of Finance* 53 (1):1–25.

Bradley, Michael. 1980. "Interfirm Tender Offers and the Market for Corporate Control." *Journal of Business* 53 (4):345–376.

Brav, Alon, Amil Dasgupta, and Richmond D. Mathews. 2019. "Wolf Pack Activism." *ECGI Finance Working Paper* 501/2017.

Brav, Alon and Richmond D. Mathews. 2011. "Empty Voting and the Efficiency of Corporate Governance." *Journal of Financial Economics* 99 (2):289–307.

Broussard, John Paul and Mika Vaihekoski. 2019. "Time-Variation of Dual-Class Premia." *Working Paper, Rutgers School of Business* .

Bubb, Ryan and Emiliano Catan. 2019. "The Party Structure of Mutual Funds." *Working Paper,* New York University.

Burkart, Mike, Denis Gromb, and Fausto Panunzi. 2000. "Agency Conflicts in Public and Negotiated Transfers of Corporate Control." *Journal of Finance* 55 (2):647–677.

Burkart, Mike and Samuel Lee. 2008. "The One Share - One Vote Debate: A Theoretical Perspective." *Review of Finance* 12 (1):1–49.

———. 2020. "Activism and Takeovers." *ECGI Finance Working Paper* (543/2018).

Bushee, Brian J. 1998. "The Influence of Institutional Investors on Myopic R&D Investment Behavior." *The Accounting Review* 73 (3):305–333.

Caprio, Lorenzo and Ettore Croci. 2008. "The determinants of the voting premium in Italy: The evidence from 1974 to 2003." *Journal of Banking & Finance* 32 (11):2433–2443.

Casella, Alessandra, Aniol Llorente-Saguer, and Thomas R. Palfrey. 2012. "Competitive Equilibrium in Markets for Votes." *Journal of Political Economy* 120 (4):593–658.

Christoffersen, Susan E. K., Christopher Geczy, David K. Musto, and Adam V. Reed. 2007. "Vote Trading and Information Aggregation." *Journal of Finance* 62 (6):2897–2929.

Chung, Kee H. and Jeong-Kuk Kim. 1999. "Corporate ownership and the value of a vote in an emerging market." *Journal of Corporate Finance* 5 (1):35–54.

Corum, Adrian A and Doron Levit. 2019. "Corporate Control Activism." *Journal of Financial Economics* 133 (1):1–17.

Cox, Steven R. and Dianne M. Roden. 2002. "The source of value of voting rights and related dividend promises." *Journal of Corporate Finance* 8 (4):337–351.

Cvijanovic, Dragana, Amil Dasgupta, and Konstantinos E. Zachariadis. 2016. "Ties That Bind: How Business Connections Affect Mutual Fund Activism." *The Journal of Finance* 71 (6):2933–2966.

———. 2019. "The Wall Street Stampede: Exit as Governance with Interacting Blockholders." *Working Paper, University of North Carolina* .

Cvijanovic, Dragana, Moqi Groen-Xu, and Konstantinos E. Zachariadis. 2020. "Free-Riders and Underdogs: Participation in Corporate Voting." *ECGI - Finance Working Paper* (649).

Dasgupta, Amil, Vyacheslav Fos, and Zacharias Sautner. 2020. "Institutional Investors and Corporate Governance." *Working Paper, London School of Economics* .

Dasgupta, Amil and Giorgia Piacentino. 2015. "The Wall Street Walk when Blockholders Compete for Flows." *The Journal of Finance* 70 (6):2853Ű2896.

Daske, Stefan and Olaf Ehrhardt. 2002. "Kursunterschiede und Renditen deutscher Stamm- und Vorzugsaktien." *Financial Markets and Portfolio Management* 16 (2):179–207.

DeAngelo, Harry and Linda DeAngelo. 1985. "Managerial Ownership of Voting Rights - A Study of Public Corporations with Dual Classes of Common Stock." *Journal of Financial Economics* 14 (1):33–69.

Dekel, Eddie and Asher Wolinsky. 2012. "Buying shares and/or votes for corporate control." *The Review of Economic Studies* 79 (1):196–226.

DeMarzo, Peter. 1993. "Majority Voting and Corporate Control: The Rule of the Dominant Shareholder." *Review of Economic Studies* 60 (204):713–734.

Desai, Mihir A. and Li Jin. 2011. "Institutional tax clienteles and payout policy." *Journal of Financial Economics* 100 (1):68–84.

Dhillon, Amrita and Silvia Rossetto. 2015. "Ownership Structure, Voting, and Risk." *Review of Financial Studies* 28 (2):521–560.

Dittmann, Ingolf. 2003. "Measuring Private Benefits of Control from the Returns of Voting and Non-Voting Shares." *Working Paper, Erasmus University* .

———. 2004. "Block Trading, Ownership Structure, and the Value of Corporate Votes." *Working Paper, Erasmus University* .

Drèze, Jacques H. 1985. "(Uncertainty and) The Firm in General Equilibrium Theory." *The Economic Journal* 95:1–20.

Dyck, I. J. Alexander and Luigi Zingales. 2004. "Private benefits of control: An international comparison." *Journal of Finance* 59 (2):537–600.

Edmans, Alex. 2009. "Blockholder Trading, Market Efficiency, and Managerial Myopia." *Journal of Finance* 64 (6):2481–2513.

———. 2014. "Blockholders and Corporate Governance." *Annual Review of Financial Economics* 6 (1):23–50.

Edmans, Alex and Clifford G. Holderness. 2017. *Chapter 8 - Blockholders: A Survey of Theory and Evidence.* in: Benjamin E. Hermalin and Michael S. Weisbach, The Handbook of the Economics of Corporate Governance, North-Holland, 541–636.

Edmans, Alex and Gustavo Manso. 2011. "Governance Through Trading and Intervention: A Theory of Multiple Blockholders." *Review of Financial Studies* 24 (7):2395–2428.

Esö, Peter, Stephen Hansen, and Lucy White. 2014. "A Theory of Vote-trading and Information Aggregation." *Working Paper,* University of Oxford.

Fama, Eugene F. and Kenneth R. French. 2002. "The Equity Premium." *Journal of Finance* 57 (2):637–660.

Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *American Economic Review* 86 (3):408–424.

Fos, Vyacheslav and Clifford G. Holderness. 2020. "The Price of a Marginal Vote: What Happens when Stocks Go Ex Vote." *Working Paper, Boston College* .

Franks, Julian and Colin Mayer. 2001. "Ownership and Control of German Corporations." *Review of Financial Studies* 14 (4):943–977.

Gaspar, José-Miguel, Massimo Massa, and Pedro Matos. 2005. "Shareholder Investment Horizons and the Market for Corporate Control." *Journal of Financial Economics* 76 (1):135–165.

Gevers, Louis. 1974. *Competitive Equilibrium of the Stock Exchange and Pareto Efficiency.* London: in: Jacques H. Drèze, Allocation under Uncertainty: Equilibrium and Optimality, Proceedings from a workshop sponsored by the International Economic Association, Palgrave Macmillan (UK), 167–191.

Goetzmann, William N., Andrey Ukhov, and Matthew I. Spiegel. 2002. "Modeling and Measuring Russian Corporate Governance: The Case of Russian Preferred and Common Shares (English Version)." *Working Paper, Yale School of Management* .

Grossman, Sanford J. and Oliver D. Hart. 1988. "One Share-One Vote and the Market for Corporate Control." *Journal of Financial Economics* 20:175–202.

Gurun, Umit and Oguzhan Karakas. 2020. "Earnings and the Value of Voting Rights." *Working Paper, University of Texas at Dallas* .

Harris, Milton and Artur Raviv. 1988. "Corporate Governance: Voting Rights and Majority Rules." *Journal of Financial Economics* 20:203–236.

Hayden, Grant M and Matthew T Bodie. 2008. "One Share, One Vote and the False Promise of Shareholder Homogeneity." *Cardozo Law Review* 30:445–505.

He, Jie, Jiekun Huang, and Shan Zhao. 2019. "Internalizing Governance Externalities: The Role of Institutional Cross-Ownership." *Journal of Financial Economics* 134 (2):400–418.

Hertzel, Michael and Richard L. Smith. 1993. "Market Discounts and Shareholder Gains for Placing Equity Privately." *Journal of Finance* 48 (2):459–485.

Hirschman, Albert O. 1970. *Exit, voice, and loyalty: Responses to decline in firms, organizations, and states*, vol. 25. Harvard university press.

Hoffman-Burchardi, Ulrike. 1999. "Corporate governance rules and the value of control - A study of German dual-class shares." *FMG Discussion Paper* (315).

Holderness, Clifford G. 2009. "The Myth of Diffuse Ownership in the United States." *Review of Financial Studies* 22 (4):1377–1408.

Horner, Melchior R. 1988. "The Value of the Corporate Voting Right: Evidence from Switzerland." *Journal of Banking and Finance* 12:69–83.

Jang, In Ji, Hwagyun Kim, and Mahdi Mohseni. 2019. "What Does the Value of Corporate Votes Tell Us About Future Stock Returns?" *Working Paper, Bentley University* .

Kahn, Charles and Andrew Winton. 1998. "Ownership Structure, Speculation, and Shareholder Intervention." *Journal of Finance* 53 (1):99–129.

Kalay, Avner, Oguzhan Karakas, and Shagun Pant. 2014. "The Market Value of Corporate Votes: Theory and Evidence from Option Prices." *Journal of Finance* (3):1235–1271.

Kalay, Avner and Shagun Pant. 2009. "One Share-One Vote is Unenforceable and Sub-Optimal." *Working Paper,* Tel Aviv University.

Kelsey, David and Frank Milne. 1996. "The Existence of Equilibrium in Incomplete Markets and the Objective Function of the Firm." *Journal of Mathematical Economics* 25 (2):229–245.

Kind, Axel and Marco Poltera. 2013. "The value of corporate voting rights embedded in option prices." *Journal of Corporate Finance* 22:16–34.

———. 2017. "Shareholder proposals as governance mechanism: Insights from the market value of corporate voting rights." .

Kunz, Roger M. and James J. Angel. 1996. "Factors Affecting the Value of the Stock Voting right: Evidence from the Swiss Equity Market." *Financial Management* 25:7–20.

Kyle, Albert S. 1989. "Informed Speculation with Imperfect Competition." *Review of Economic Studies* 56 (3):317–356.

Kyle, Albert S. and Jean-Luc Vila. 1991. "Noise Trading and Takeovers." *Rand Journal of Economics* 22 (1):54–71.

La Porta, Rafael, Florencio Lopez-de Silanes, Andrei Shleifer, and Robert Vishny. 1999. "Corporate Ownership Around the World." *Journal of Finance* 54 (2):471–517.

Lease, Ronald C., John J. McConnell, and Wayne H. Mikkelson. 1983. "The market value of control in publicly-traded corporations." *Journal of Financial Economics* 11 (1):439–471.

Levit, Doron and Nadya Malenko. 2011. "Nonbinding Voting for Shareholder Proposals." *Journal of Finance* 66 (5):1579–1614.

Levit, Doron, Nadya Malenko, and Ernst Maug. 2020. "Trading and Shareholder Democracy." *ECGI Finance Working Paper* (631).

Levy, Haim. 1983. "Economic Evaluation of Voting Power of Common Stock." *Journal of Finance* 38 (1):79–93.

Malenko, Andrey and Nadya Malenko. 2019. "Proxy Advisory Firms: The Economics of Selling Information to Voters." *Journal of Finance* 74 (5):2441–2490.

Maug, Ernst. 1998. "Large Shareholders as Monitors: Is there a trade-off between liquidity and control?" *Journal of Finance* 53 (1):65–98.

Maug, Ernst and Kristian Rydqvist. 2009. "Do Shareholders Vote Strategically? Voting Behavior, Proposal Screening, and Majority Rules." *Review of Finance* 13 (1):47–79.

Maynes, Elizabeth. 1996. "Takeover rights and the value of restricted shares." *Journal of Financial Research,* 19 (2):157–173.

McCahery, Joseph A, Zacharias Sautner, and Laura T Starks. 2016. "Behind the scenes: The corporate governance preferences of institutional investors." *Journal of Finance* 71 (6):2905–2932.

Megginson, William L. 1990. "Restricted Voting Stock, Acquisition Premiums, and the Market Value for Corporate Control." *Financial Review* 25:175–198.

Meirowitz, Adam and Shaoting Pi. 2020. "The Shareholder's Dilemma: Voting and Trading." *Working Paper, University of Utah* .

Milnor, J. W. and L. S. Shapley. 1978. "Values of Large Games II: Oceanic Games." 3 (4):290–307.

Muravyev, Alexander. 2004. "The puzzle of dual class stock in Russia: Explaining the price differential between common and preferred shares." *MPRA Paper* (27726).

Muus, Christian. 1998. "Non-voting shares in France: an empirical analysis of the voting premium." *Johann Wolfgang Goethe-Universität Frankfurt am Main Working Paper Series: Finance & Accounting* (22).

Neeman, Zvika and Gerhard O. Orosel. 2006. "On the Efficiency of Vote Buying when Voters Have Common Interests." *International Review of Law and Economics* 26 (4):536–556.

Nenova, Tatiana. 2003. "The value of corporate votes and control benefits: A cross-country analysis." *Journal of Financial Economics* 68 (3):325–351.

Neumann, Robert. 2003. "Price Differentials between Dual-class Stocks: Voting Premium or Liquidity Discount?" *European Financial Management* 9 (3):315 – 332.

Noe, Thomas H. 2002. "Investor Activism and Financial Market Structure." *Review of Financial Studies* 15 (1):289–318.

Odegaard, Bernt Arne. 2007. "Price differences between equity classes. Corporate control, foreign ownership or liquidity?" *Journal of Banking & Finance* 31 (12):3621–3645.

Pagano, Marco and Ailsa Roell. 1998. "The Choice of Stock Ownership structure: Agency Costs, Monitoring, and the Decision to Go Public." *Quarterly Journal of Economics* 113 (1):187–225.

Rydqvist, Kristian. 1987. *The Pricing of Shares with Different Voting Power and the Theory of Oceanic Games*. Stockholm School of Economics, 1st ed. ed.

———. 1992. "Dual-Class Shares: A Review." *Oxford Review of Economic Policy* 8:45–57.

———. 1996. "Takeover Bids and the Relative Prices of Shares That Differ in Their Voting Rights." *Journal of Banking and Finance* 20 (8):1407–25.

Shleifer, Andrei and Robert W. Vishny. 1986. "Large shareholders and corporate control." *Journal of Political Economy* 94 (3):176–216.

Smith, Brian F. and Ben Amoako-Adu. 1995. "Relative Prices of Dual Class Shares." *Journal of Financial and Quantitative Analysis* 30 (2):223–239.

Speit, Andre and Paul Voss. 2020. "Shareholder Votes on Sale." *Working Paper, University of Bonn* .

Van Wesep, Edward D. 2014. "The Idealized Electoral College Voting Mechanism and Shareholder Power." *Journal of Financial Economics* 113 (2):90–108.

Wruck, Karen Hopper. 1989. "Equity ownership concentration and firm value: Evidence from private equity financings." *Journal of Financial Economics* 23 (1):3–28.

Zingales, Luigi. 1994. "The Value of the Voting Right: A Study of the Milan Stock Exchange Experience." *Review of Financial Studies* 7 (1):125–148.

———. 1995. "What Determines the Value of Corporate Votes?" *Quarterly Journal of Economics* 110:1047–1073.

Zwiebel, Jeffrey. 1995. "Block Investment and Partial Benefits of Corporate Control." *The Review of Economic Studies* 62 (2):161–185.

Appendix - Proofs

Proof of Lemma 1. Given the realization of q, a shareholder indexed by b votes his shares for the proposal if and only if $q > -b$. Denote the fraction of post-trade shares voted to approve the proposal by $\Lambda(q)$. Note that $\Lambda(q)$ is weakly increasing. If we have $\Lambda(\Delta) \leq \tau$ for the highest possible $q = \Delta$, then q^* in the statement of the lemma is equal to Δ. Similarly, if we have $\Lambda(-\Delta) > \tau$ for the lowest possible $q = -\Delta$, , then q^* in the statement of the lemma is equal to $-\Delta$. Finally, if $\Lambda(-\Delta) \leq \tau < \Lambda(\Delta)$, there exists $q^* \in [-\Delta, \Delta)$ such that the fraction of votes voted in favor of the proposal is greater than τ if and only if $q > q^*$. Hence, the proposal is approved if and only if $q > q^*$. ■

Proof of Proposition 1. Condition (21) can be rewritten from (13) as

$$\beta_L = \beta_l(y, -\beta_L) \Leftrightarrow R(\beta_L; y, -\beta_L) = 1 - \frac{\tau}{1-\alpha-y}. \tag{45}$$

Similarly, condition (22) can be rewritten as

$$\beta_H = \beta_h(y, -\beta_H) \Leftrightarrow R(\beta_H; y, -\beta_H) = 1 - \frac{\tau-\alpha-y}{1-\alpha-y}. \tag{46}$$

From (12), $R(b'; y, q^*)$ is a cdf and lies in the unit interval. Moreover,

$$\lim_{\beta \to -\bar{b}} R(\beta; y, -\beta) = 0 \text{ and } \lim_{\beta \to \bar{b}} R(\beta; y, -\beta) = 1. \tag{47}$$

Hence, solutions to (45) and (46), and, therefore, of (21) and (22), must exist. For $y = -\alpha$, the right hand sides of (45) and (46) are identical and β^* is defined from $R(\beta^*; y, -\beta^*) = 1 - \tau$.

The derivative of $R(\beta; y, -\beta)$ with respect to β is :

$$\begin{aligned} \frac{\partial R(\beta; y, -\beta)}{\partial \beta} &= g(\beta)\left(1 + \frac{\beta - \mathbb{E}[b]}{\gamma}\frac{H(-\beta)}{1-\alpha-y}\right) \\ &\quad -G(\beta)\frac{f(-\beta)}{1-\alpha-y}\frac{\mathbb{E}[b] - \mathbb{E}[b|b<\beta]}{\gamma}. \end{aligned} \tag{48}$$

The first line of (48) equals $r(\beta; y, -\beta) > 0$. Since, $\mathbb{E}[b] > \mathbb{E}[b|b<\beta]$, the second line is negative. Hence, $R(\beta; y, -\beta)$ and $s(-\beta, y, -\beta)$ may be non-monotonic in β. From (48), $\frac{\partial R(\beta;y,-\beta)}{\partial \beta} > 0$ if and only if

$$\frac{\frac{G(\beta)}{g(\beta)} f(-\beta)(\mathbb{E}[b] - \mathbb{E}[b|b<\beta]) + H(-\beta)(\mathbb{E}[b] - \beta)}{1-\alpha-y} < \gamma,$$

and thus, there exists $\overline{\gamma} < \infty$ such that if $\gamma > \overline{\gamma}$, then $\frac{\partial R(\beta;y,-\beta)}{\partial \beta}$ for every $y \geq -\alpha$. In this case, (47) implies that the solutions to (21)-(22) exist and are unique. The proof of the properties of $\beta_H(y)$ and $\beta_L(y)$ follows from Lemma 2 below. ■

Lemma 2 (Properties of the marginal voter) *Suppose that the solutions $\beta_L(y)$ and $\beta_H(y)$ of (21) and (22) are unique for all y. Then:*

(i) $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L} > 0$ *and*

$$\frac{\partial \beta_L(y)}{\partial y} = -\frac{\frac{1-G(\beta_L)}{1-\alpha-y}}{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}} < 0. \tag{49}$$

Moreover, $lim_{y \nearrow 1-\tau-\alpha} \beta_L(y) = -\underline{b}$ *and* $lim_{y \nearrow 1-\tau-\alpha} \frac{\partial \beta_L}{\partial y} = -\left[\tau \frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}\right]^{-1}$.

(ii) $\frac{\partial R(\beta_H;y,-\beta_H)}{\partial \beta_H} > 0$ *and*

$$\frac{\partial \beta_H(y)}{\partial y} = \frac{\frac{G(\beta_H)}{1-\alpha-y}}{\frac{\partial R(\beta_H;y,-\beta_H)}{\partial \beta_H}} > 0. \tag{50}$$

Moreover, $lim_{y \nearrow \tau-\alpha} \beta_H(y) = \underline{b}$ *and* $lim_{y \nearrow \tau-\alpha} \frac{\partial \beta_H(y)}{\partial y} = \left[(1-\tau) \frac{\partial R(\beta_H;y,-\beta_H)}{\partial \beta_H}\right]^{-1}$.

(iii) If the blockholder sells all her shares ($y = -\alpha$), then $\beta_L(-\alpha) = \beta_H(-\alpha) \equiv \beta^*$.

(iv) As γ becomes large, we have:

$$\lim_{\gamma\to\infty} \beta_L(y) = G^{-1}\left(1 - \frac{\tau}{1-\alpha-y}\right), \ \lim_{\gamma\to\infty} \beta_H(y) = G^{-1}\left(\frac{1-\tau}{1-\alpha-y}\right), \tag{51}$$

and

$$\lim_{\gamma\to\infty} \frac{\partial \beta_L(y)}{\partial y} = -\frac{1 - G(\lim_{\gamma\to\infty} \beta_L(y))}{g(\lim_{\gamma\to\infty} \beta_L(y))(1-\alpha-y)}, \tag{52}$$

$$\lim_{\gamma\to\infty} \frac{\partial \beta_H(y)}{\partial y} = \frac{G(\lim_{\gamma\to\infty} \beta_H(y))}{g(\lim_{\gamma\to\infty} \beta_H(y))(1-\alpha-y)}, \tag{53}$$

$$\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau). \tag{54}$$

Proof of Lemma 2. To simplify the expressions, define

$$X(b';y,q^*) = \frac{(\mathbb{E}[b] - \mathbb{E}[b \,|b \le b'])\, H(q^*)}{\gamma(1-\alpha-y)}. \tag{55}$$

With this definition, we have

$$R(b';y,q^*) = G(b')(1 - X(b';y,q^*)) \Leftrightarrow -G(b')\,X(b';y,q^*) = R(b';y,q^*) - G(b') \tag{56}$$

and

$$\frac{\partial R(b';y,q^*)}{\partial y} = -\frac{X(b';y,q^*)\,G(b')}{1-\alpha-y}. \tag{57}$$

(i) If $\beta_L(y)$ is the unique solution to (21), then $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L} > 0$. We apply the implicit function theorem to condition (21), which requires

$$\begin{aligned}
&\left[(1-\alpha-y)\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+1-R(\beta_L;y,-\beta_L)\right]dy \\
+\quad &(1-\alpha-y)\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}d\beta_L=0,
\end{aligned} \tag{58}$$

where $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}$ is given by the same expression as above and $R(\beta_L;y,-\beta_L)$ is again given from (45) so that

$$1-R(\beta_L;y,-\beta_L)=\frac{\tau}{1-\alpha-y}. \tag{59}$$

Substituting for $1-R(\beta_L;y,-\beta_L)$ and dividing by $1-\alpha-y$ allows us to rewrite (58) as

$$\left[\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+\frac{\tau}{(1-\alpha-y)^2}\right]dy+\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}d\beta_L=0. \tag{60}$$

Hence,

$$\frac{\partial \beta_L}{\partial y}=-\frac{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+\frac{\tau}{(1-\alpha-y)^2}}{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}}. \tag{61}$$

We next use (57) and (59) to rewrite the numerator of (61) as

$$\begin{aligned}
\frac{\partial R(\beta_L;y,-\beta_L)}{\partial y}+\frac{\tau}{(1-\alpha-y)^2} &= \frac{1}{1-\alpha-y}\left(-G(\beta_L)X(\beta_L,y,-\beta_L)+1-R(\beta_L;y,-\beta_L)\right) \\
&= \frac{1-G(\beta_L)}{1-\alpha-y}>0,
\end{aligned}$$

where the second transformation uses (56). Hence, $\frac{\partial \beta_L}{\partial y}<0$ if $\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}>0$.

For $y \nearrow 1-\tau-\alpha$, almost all dispersed shareholders are required to pass the proposal without the blockholder. Then $R\to 0$ and $\beta_L \to -\bar{b}$, $G(\beta_L)\to 0$, and $\frac{1-G(\beta_L)}{1-\alpha-y}\to\frac{1}{\tau}$. Then

$$\frac{\partial \beta_L}{\partial y}\to -\frac{1}{\frac{\partial R(\beta_L;y,-\beta_L)}{\partial \beta_L}\tau}<0. \tag{62}$$

(ii) If $\beta_H(y)$ is the unique solution to (22), then $\frac{\partial R(\beta_H,y,-\beta_H)}{\partial \beta_H}>0$. We apply the implicit function theorem to condition (46), which requires

$$\begin{aligned}
&\left[(1-\alpha-y)\frac{\partial R(\beta_H;y,-\beta_H)}{\partial y}-R(\beta_H,y,-\beta_H)\right]dy \\
+\quad &(1-\alpha-y)\frac{\partial R(\beta_H;y,-\beta_H)}{\partial \beta_H}d\beta_H=0.
\end{aligned} \tag{63}$$

Substituting for $R(\beta_H, y, -\beta_H)$ from (46)and dividing by $1-\alpha-y$ gives

$$\left[\frac{\partial R(_H, y, -\beta_H)}{\partial \beta_H} - \frac{1-\tau}{(1-\alpha-y)^2}\right] dy + \frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} d\beta_H = 0. \tag{64}$$

Hence,

$$\frac{\partial \beta_H}{\partial y} = -\frac{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial y} - \frac{1-\tau}{(1-\alpha-y)^2}}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H}}. \tag{65}$$

We use (57) and (46) to rewrite the numerator of (65) as

$$\begin{aligned}\frac{\partial R(\beta_H, y, -\beta_H)}{\partial y} - \frac{1-\tau}{(1-\alpha-y)^2} &= \frac{1}{1-\alpha-y}\left(-G(\beta_H) X(\beta_H, y, -\beta_H) - R(\beta_H, y, -\beta_H)\right) \\ &= -\frac{G(\beta_H)}{1-\alpha-y} < 0,\end{aligned}$$

where the second line uses (56). Hence, $\frac{\partial \beta_H}{\partial y} > 0$ in any equilibrium in which $\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H} > 0$.

For $y \nearrow \tau - \alpha$, the number of dispersed shareholders needed to pass the proposal becomes negligible. Then (46) implies $R \to 1$ and (47) implies $\beta_H \to \bar{b}$ and $\mathbb{E}[b \,|b \leq \beta_h] \to \mathbb{E}[b]$. Then $\frac{-G(\beta_h)}{1-\alpha-y} \to \frac{-1}{1-\tau}$ and (65) simplifies to

$$\frac{\partial \beta_H}{\partial y} \to \frac{1}{\frac{\partial R(\beta_H, y, -\beta_H)}{\partial \beta_H}(1-\tau)} > 0. \tag{66}$$

(iii) If $y = -\alpha$, $\beta_h(y, q_e^*) = \beta_l(y, q_e^*)$ from (15) and (16), hence $\beta_L(y) = \beta_H(y)$, which are both assumed to be unique. Hence, β^* can be obtained as the unique solution to (21).

(iv) From (12), $\lim_{\gamma\to\infty} R(-q^*; y, q^*) = G(-q^*)$. Substituting into (45) and (46) gives (51). From (48), $\lim_{\gamma\to\infty} \frac{\partial R(-q^*; y, q^*)}{\partial(-q^*)} = g(-q^*)$. Substituting into (49) and (50) gives (52). ■

Proof of Proposition 2. We start by noting that given (4) and (9), we can rewrite

$$\begin{aligned}\Pi(y) &= (\alpha + y) v(\beta, q^*(y)) - yp^*(y) - \frac{\eta}{2}y^2 \\ &= \alpha v(\beta, q^*(y)) + y(\beta - \mathbb{E}[b]) H(q^*) - (\gamma + \eta/2)y^2 \\ &= \alpha v_0 + \alpha \mathbb{E}[\theta | q > q^*(y)] H(q^*) + ((\alpha + y)\beta - yE[b]) H(q^*) - (\gamma + \eta/2)y^2,\end{aligned}$$

which explains the derivation of

$$\frac{\partial \Pi(y)}{\partial y} = (\beta - \mathbb{E}[b]) H(q^*) - (2\gamma + \eta)y + \frac{\partial(-q^*(y))}{\partial y}\left[\alpha(q^*(y) + \beta) + y(\beta - \mathbb{E}[b])\right] f(q^*(y)),$$

as claimed in the main text.

Recall that by assumption, the blockholder's trade in equilibrium is in the interval $(-\alpha, 1-\alpha)$. So hereafter we assume $y \in (-\alpha, 1-\alpha)$. We start by giving sufficient conditions under which $\Pi(y)$ is "well-behaved," namely, continuous, concave, and has a unique maximizer. From Proposition 1, there exists a $\overline{\gamma}_1$ such that, if $\gamma > \overline{\gamma}_1$, then $\beta_L(y)$ and $\beta_H(y)$ are uniquely determined and both are continuous functions of y. If so, $\Pi(y)$ is a continuous function of y as well. In addition, in the online appendix, we show that there exists $\overline{\eta}_1 < \infty$ such that if $\eta > \overline{\eta}_1$, then $\Pi(y)$ is a concave function. Combined, if $\gamma > \overline{\gamma}_1$ and $\eta > \overline{\eta}_1$, then $\Pi(y)$ is a continuous and concave function, and hence, it has a unique maximizer. We denote the unique maximizer by y^*.

Next, we define y^{**}. If y^* is such that $\beta_L(y^*) < \beta < \beta_H(y^*)$, then it must be $q_a^*(y^*) = -\beta$ and $\frac{\partial(-q_a^*(y))}{\partial y} = 0$. Therefore, using (25)-(26), y^* must solve

$$\begin{aligned}(\beta - \mathbb{E}[b])\Pr[q > -\beta] - (2\gamma+\eta)y^* &= 0 \Leftrightarrow \\ y^* &= y^{**} \equiv \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])(1 - F(-\beta)).\end{aligned} \tag{67}$$

Notice that

$$\lim_{\gamma\to\infty} y^{**} = 0.$$

Second, recall that $\beta_L(-\alpha) = \beta_H(-\alpha) = \beta^*$ from Proposition 1, and note that

$$\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau) \in \left(-\bar{b}, \bar{b}\right).$$

Next, we consider two cases:

1. Suppose $\beta \in [-\bar{b}, \beta^*)$. We argue that there exists a unique $\underline{y} \in (-\alpha, 1-\alpha-\tau)$ such that: (i) $\beta = \beta_L(\underline{y})$, (ii) if $y \in (-\alpha, \underline{y})$ then $\beta \in [-\bar{b}, \beta_L(y))$, and (iii) if $y > \underline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. To see why, recall that: (1) $\beta_H(y)$ is an increasing function of y, (2) $\beta_L(y)$ is a decreasing function of y, (3) $\beta < \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$, and (4) $\lim_{y\nearrow 1-\alpha-\tau}\beta_L(y) = -\bar{b}$. Combined, these four facts prove the arguments above. Moreover, these arguments imply that the marginal voter is given by

$$-q_a^*(y) = \begin{cases} \beta_L(y) & \text{if } -\alpha < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < 1-\alpha, \end{cases}$$

Notice that by the definition of $\beta_L(\cdot)$, $\underline{y}$ is given by the solution of

$$\begin{aligned} R(\beta, \underline{y}, -\beta) &= 1 - \frac{\tau}{1-\alpha-\underline{y}} \Leftrightarrow \\ \underline{y} &= 1 - \alpha - \frac{\tau}{1-G(\beta)} + \frac{1}{\gamma}\frac{G(\beta)}{1-G(\beta)}(\mathbb{E}[b] - \mathbb{E}[b|b<\beta])(1-F(-\beta)),\end{aligned}$$

where $\lim_{\gamma\to\infty}\underline{y} = 1-\alpha-\frac{\tau}{1-G(\beta)}$ and $\lim_{\gamma\to\infty}\underline{y} < 0 \Leftrightarrow \beta > G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Also notice that

$$\Pi'(y) = (\beta - \mathbb{E}[b])\Pr[q > q_a^*(y)] - (2\gamma+\eta)y + \begin{cases} MPV(y) & \text{if } -\alpha < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for $y = \underline{y}$. For $-\alpha < y < \underline{y}$, we have

$$MPV(y) = \frac{\partial\beta_L(y)}{\partial y} f(-\beta_L(y))[\alpha(\beta - \beta_L(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\lim_{\gamma\to\infty} MPV(y) = -\frac{1-G(\lim_{\gamma\to\infty}\beta_L(y))}{g(\lim_{\gamma\to\infty}\beta_L(y))} \times \frac{f(-\lim_{\gamma\to\infty}\beta_L(y))}{1-\alpha-y}\left[\alpha(\beta-\lim_{\gamma\to\infty}\beta_L(y))+y(\beta-\mathbb{E}[b])\right], \tag{68}$$

where $\lim_{\gamma\to\infty}\beta_L(y) = G^{-1}(1-\frac{\tau}{1-\alpha-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y<1-\alpha$). We consider two subcases.

(a) First, suppose $\beta > G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Then for a large γ we have $\underline{y} < 0 \approx y^{**}$. Fix any $\varepsilon \in (0, -\lim_{\gamma\to\infty}\underline{y})$, and notice that for any $y \in (-\alpha, \lim_{\gamma\to\infty}\underline{y}+\varepsilon)/\{\lim_{\gamma\to\infty}\underline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y)=\infty$. Thus, there exists $\overline{\gamma}_3<\infty$ such that if $\gamma>\overline{\gamma}_3$ then $\Pi'(y)>0$ for any $y\in(-\alpha,\underline{y}+\varepsilon)/\{\underline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is greater than $\underline{y}$, that is, $y^*>\underline{y}$. Recall that if $y>\underline{y}$ then $\beta\in(\beta_L(y),\beta_H(y))$, which implies $MPV(y)=0$. Therefore, it must be $MPV(y^*)=0$, $-q_a^*(y^*)=\beta$, and $y^*=y^{**}$.

(b) Second, suppose $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$. Then, for a large γ we have $1-\alpha>\underline{y}>0\approx y^{**}$. Fix any $\varepsilon\in(0,\lim_{\gamma\to\infty}\underline{y})$, and notice that for any $y\in(\lim_{\gamma\to\infty}\underline{y}-\varepsilon, 1-\alpha)/\{\lim_{\gamma\to\infty}\underline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y)=-\infty$. Thus, there exists $\overline{\gamma}_4<\infty$ such that if $\gamma>\overline{\gamma}_4$ then $\Pi'(y)<0$ for any $y\in(\underline{y}-\varepsilon,1-\alpha)/\{\underline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is smaller than $\underline{y}$, that is, $y^*<\underline{y}$. In particular, since $\Pi(y)$ is continuous and concave, and since $\Pi'(y)|_{y=-\alpha}>0$ for a large γ, there is a unique $\hat{y}\in(-\alpha,\underline{y})$ such that $\Pi'(y)|_{y=\hat{y}}=0$. Therefore, the optimizer of $\Pi(y)$ is $y^*=\hat{y}$ and the marginal voter is a dispersed shareholder with a bias $\beta_L(\hat{y})>\beta$.
Since $\lim_{\gamma\to 0}\hat{y}=0$ and $\lim_{\gamma\to\infty}\beta_L(y)=G^{-1}(1-\frac{\tau}{1-\alpha-y})$, (68) implies

$$\lim_{\gamma\to\infty} MPV(\hat{y}) = \frac{\tau}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})-\beta)>0. \tag{69}$$

In addition,

$$\lim_{\gamma\to\infty} MPC(\hat{y}) = (\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right)-2\lim_{\gamma\to\infty}[\gamma\hat{y}].$$

Since $\Pi'(y)|_{y=\hat{y}}=0$ in this case, it must be $\lim_{\gamma\to\infty}MPC(\hat{y})+\lim_{\gamma\to\infty}MPV(\hat{y})=0$, that is

$$\lim_{\gamma\to\infty}[2\gamma\hat{y}] = \lim_{\gamma\to\infty}MPV(\hat{y})+(\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right).$$

This implies that $\hat{y}$ and $MPV(\hat{y})$ could have different signs. For example, if α is small, $\beta<\mathbb{E}[b]$ (which is likely given $\beta<G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$), then $\lim_{\gamma\to\infty}MPV(\hat{y})>0$ but is close to zero, whereas $(\beta-\mathbb{E}[b])\,H\left(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})\right)<0$ and is bounded from zero, so $\lim_{\gamma\to\infty}[2\gamma\hat{y}]<0$, which implies that $\hat{y}$ converges to 0 from below, i.e., $\hat{y}<0$ while $MPV(\hat{y})>0$.

2. Suppose $\beta \in (\beta^*, \overline{b}]$. We argue that there exists a unique $\overline{y} \in (-\alpha, \tau - \alpha)$ such that: (i) $\beta = \beta_H(\overline{y})$, (ii) if $y \in (-\alpha, \overline{y})$ then $\beta \in (\beta_H(y), \overline{b}]$, and (iii) if $y > \overline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. To see why, recall that: (1) $\beta_H(y)$ is an increasing function of y, (2) $\beta_L(y)$ is a decreasing function of y, (3) $\beta > \beta^* = \beta_L(-\alpha) = \beta_H(-\alpha)$, and (4) $\lim_{y \nearrow \tau-\alpha} \beta_H(y) = \overline{b}$. Combined, these four facts prove the arguments above. Moreover, these arguments imply that the marginal voter is given by

$$-q_a^*(y) = \begin{cases} \beta_H(y) & \text{if } -\alpha < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1-\alpha, \end{cases}$$

Notice that by the definition of $\beta_H(\cdot)$, $\overline{y}$ is given by the solution of

$$\begin{aligned} R(\beta, \overline{y}, -\beta) &= 1 - \frac{\tau - \alpha - \overline{y}}{1 - \alpha - \overline{y}} \Leftrightarrow \\ \overline{y} &= 1 - \alpha - \frac{1-\tau}{G(\beta)} - \frac{1}{\gamma}\left(\mathbb{E}[b] - \mathbb{E}[b|b<\beta]\right)(1 - F(-\beta)), \end{aligned}$$

where $\lim_{\gamma\to\infty} \overline{y} = 1 - \alpha - \frac{1-\tau}{G(\beta)}$ and $\lim_{\gamma\to\infty} \overline{y} < 0 \Leftrightarrow \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$. Also notice that

$$\Pi'(y) = (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (2\gamma + \eta)y + \begin{cases} MPV(y) & \text{if } -\alpha < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for $y = \overline{y}$. For $-\alpha < y < \overline{y}$, we have

$$MPV(y) = \frac{\partial \beta_H(y)}{\partial y} f(-\beta_H(y))[\alpha(\beta - \beta_H(y)) + y(\beta - \mathbb{E}[b])]$$

and

$$\begin{aligned} \lim_{\gamma\to\infty} MPV(y) &= \frac{G(\lim_{\gamma\to\infty}\beta_H(y))}{g(\lim_{\gamma\to\infty}\beta_H(y))} \\ &\quad \times \frac{f(-\lim_{\gamma\to\infty}\beta_H(y))}{1-\alpha-y}\left[\alpha(\beta - \lim_{\gamma\to\infty}\beta_H(y)) + y(\beta - \mathbb{E}[b])\right], \end{aligned} \tag{70}$$

where $\lim_{\gamma\to\infty} \beta_H(y) = G^{-1}(\frac{1-\tau}{1-\alpha-y})$. Thus $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1-\alpha$). We consider two subcases.

(a) First, suppose $\beta < G^{-1}(\frac{1-\tau}{1-\alpha})$. Then for a large γ we have $\overline{y} < 0 \approx y^{**}$. Fix any $\varepsilon \in (0, -\lim_{\gamma\to\infty}\overline{y})$, and notice that for any $y \in (-\alpha, \lim_{\gamma\to\infty}\overline{y} + \varepsilon)/\{\lim_{\gamma\to\infty}\overline{y}\}$ we have $\lim_{\gamma\to\infty}\Pi'(y) = \infty$. Thus, there exists $\overline{\gamma}_5 < \infty$ such that if $\gamma > \overline{\gamma}_5$ then $\Pi'(y) > 0$ for any $y \in (-\alpha, \overline{y} + \varepsilon)/\{\overline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is greater than $\overline{y}$, that is, $y^* > \overline{y}$. Recall that if $y > \overline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$, which implies $MPV(y) = 0$. Therefore, it must be $MPV(y^*) = 0$, $-q_a^*(y^*) = \beta$, and $y^* = y^{**}$. Moreover, since $y > \overline{y} \Rightarrow MPV(y) = 0$, it must be $-q_a^*(y^*) = \beta$, $MPV(y^*) = 0$, and $y^* = y^{**}$.

(b) Second, suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Then, for a large γ we have $1-\alpha > \overline{y} > 0 \approx y^{**}$. Fix any $\varepsilon \in (0, \lim_{\gamma\to\infty}\overline{y})$, and notice that for any $y \in (\lim_{\gamma\to\infty}\overline{y} - \varepsilon, 1-\alpha)/\{\lim_{\gamma\to\infty}\overline{y}\}$

we have $\lim_{\gamma\to\infty}\Pi'(y) = -\infty$. Thus, there exists $\overline{\gamma}_6 < \infty$ such that if $\gamma > \overline{\gamma}_6$ then $\Pi'(y) < 0$ for any $y \in (\overline{y} - \varepsilon, 1-\alpha)/\{\overline{y}\}$. Therefore, the maximizer of $\Pi(y)$ is smaller than $\overline{y}$, that is, $y^* < \overline{y}$. In particular, since $\Pi(y)$ is continuous and concave, and since $\Pi'(y)|_{y=-\alpha} > 0$ for a large γ, there is a unique $\hat{y} \in (-\alpha, \overline{y})$ such that $\Pi'(y)|_{y=\hat{y}} = 0$. Therefore, the optimizer of $\Pi(y)$ is $y^* = \hat{y}$ and the marginal voter is a dispersed shareholder with a bias $\beta_H(\hat{y}) < \beta$.
Since $\lim_{\gamma\to 0}\hat{y} = 0$ and $\lim_{\gamma\to\infty}\beta_H(y) = G^{-1}(\frac{1-\tau}{1-\alpha-y})$, (70) implies

$$\lim_{\gamma\to\infty} MPV(\hat{y}) = \frac{1-\tau}{1-\alpha} \times \frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})) > 0. \tag{71}$$

In addition,

$$\lim_{\gamma\to\infty} MPC(\hat{y}) = (\beta - \mathbb{E}[b])\, H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right) - 2\lim_{\gamma\to\infty}[\gamma\hat{y}].$$

Since $\Pi'(y)|_{y=\hat{y}} = 0$ in this case, it must be $\lim_{\gamma\to\infty} MPC(\hat{y}) + \lim_{\gamma\to\infty} MPV(\hat{y}) = 0$, that is

$$\lim_{\gamma\to\infty}[2\gamma\hat{y}] = \lim_{\gamma\to\infty} MPV(\hat{y}) + (\beta - \mathbb{E}[b])\, H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right).$$

Potentially, $\hat{y}$ and $MPV(\hat{y})$ could again have different signs. For example, if α is small, $\beta < E[b]$ (which is possible if $G^{-1}(\frac{1-\tau}{1-\alpha}) < E[b]$), then $\lim_{\gamma\to\infty} MPV(\hat{y}) > 0$ but is close to zero, whereas $(\beta - \mathbb{E}[b])\, H\left(-G^{-1}(\frac{1-\tau}{1-\alpha})\right) < 0$ and is bounded from zero, so $\lim_{\gamma\to\infty}[2\gamma\hat{y}] < 0$, which implies that $\hat{y}$ converges to 0 from below, i.e., $\hat{y} < 0$ while $MPV(\hat{y}) > 0$.

Notice that $\lim_{\gamma\to\infty}\beta^* = G^{-1}(1-\tau) \in \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right)$. According to part 1.a and 2.a, for a large γ, if $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < \beta^*$ or $\beta^* < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$, then $-q_a^*(y^*) = \beta$, $MPV(y^*) = 0$, and $y^* = y^{**}$. This establishes part (i) in the statement. According to part 1.b, if $\beta < \min\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), \beta^*\} = G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, then the marginal voter is a dispersed shareholder with bias $\beta_L(y^*) > \beta$. According to part 2.b, if $\beta > \max\{G^{-1}(\frac{1-\tau}{1-\alpha}), \beta^*\} = G^{-1}(\frac{1-\tau}{1-\alpha})$, then the marginal voter is a dispersed shareholder with bias $\beta_H(y^*) < \beta$. Combined, this establishes part (ii).

To see expression (32) for the share price, we simply plug in y^* and $q^*(y^*)$ into (23).

Finally, given (69) and (71), there exists $\overline{\gamma}_2 > 0$ such that if $\gamma > \overline{\gamma}_2$, then $MPV(y^*) > 0$ for $\beta \notin \left(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right)$.

Letting $\overline{\gamma} = \max\{\overline{\gamma}_1, \overline{\gamma}_2, \overline{\gamma}_3, \overline{\gamma}_4, \overline{\gamma}_5, \overline{\gamma}_6\}$ completes the proof. ■

Proof of Corollary 1. Based on Proposition 2 and its proof, if $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then $MPV(y^*) = 0$. If $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then based on case 1.b in the proof of Proposition 2, $MPV(y^*) > 0$ and the FOC holds and in the limit,

$$(\beta - \mathbb{E}[b])\, H(q^*(0)) + \frac{\tau}{1-\alpha}\frac{\alpha}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) - \beta) = \lim_{\gamma\to\infty} 2\gamma y^*.$$

Notice that if $\beta < \mathbb{E}[b]$ and α is sufficiently small, then the LHS is negative, and hence $\lim_{\gamma\to\infty} 2\gamma y^* < 0$. This implies that for large enough γ it must be $y^* < 0$. If $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$ then based on case 2.b in the proof of Proposition 2, $MPV(y^*) > 0$ and the FOC holds and in the limit,

$$(\beta - \mathbb{E}[b]) H(q^*(0)) + \frac{1-\tau}{1-\alpha}\frac{\alpha}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})) = \lim_{\gamma\to\infty} 2\gamma y^*.$$

Notice that if $\beta < \mathbb{E}[b]$ and α is sufficiently small, then the LHS is negative, and hence $\lim_{\gamma\to\infty} 2\gamma y^* < 0$. This implies that for large enough γ it must be $y^* < 0$, which completes the proof. ■

Proposition 3 is a special case of the following result.

Proposition 7. *Suppose $\alpha = 0$. There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. In equilibrium, the marginal voter is a dispersed investor with bias*

$$-q^*(y^*) = \begin{cases} \beta_L(y^*) > \beta & \text{if } \beta < G^{-1}(1-\tau) \\ \beta_H(y^*) < \beta & \text{if } \beta > G^{-1}(1-\tau). \end{cases} \tag{72}$$

Moreover:

(*i*) *If $\mathbb{E}[b] < \beta$, then the blockholder's equilibrium trade satisfies $y^* > 0$ and*

$$y^* = \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma+\eta} MPV(y^*), \tag{73}$$

the share price is given by

$$p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*), \tag{74}$$

where $MPV(y^*) < 0$ if and only if $\beta < G^{-1}(1-\tau)$.

(*ii*) *If $\beta < \mathbb{E}[b]$, then the blockholder does not trade in equilibrium (i.e., $y^* = 0$), the no-short-selling constraint binds, $MPV(0) = 0$, and*

$$\begin{aligned} p^* &= v(\mathbb{E}[b], q^*(0)) \\ &= v(b_{MT}, q^*(0)) + \frac{\gamma}{2\gamma+\eta}(\mathbb{E}[b] - \beta) H(q^*(0)). \end{aligned}$$

Proof. We build on the proof of Proposition 2, and adjust to the special case with $\alpha = 0$. Showing the existence of a unique maximizer, which we denote by y^*, follows the same arguments and hence is omitted. Recall that $\beta_L(0) = \beta_H(0) = \beta^*$ from Proposition 1, and note that

$$\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau) \in \left(-\bar{b}, \bar{b}\right).$$

Next, we consider two cases:

1. Suppose $\beta \in [-\bar{b}, \beta^*)$. As in the proof of Proposition 2, there exists a unique $\underline{y} \in (0, 1-\tau)$ such that: (i) $\beta = \beta_L(\underline{y})$, (ii) if $y \in (0, \underline{y})$ then $\beta \in [-\bar{b}, \beta_L(y))$, and (iii) if $y > \underline{y}$ then $\beta \in (\beta_L(y), \beta_H(y))$. Thus, the marginal voter is given by

$$-q^*(y) = \begin{cases} \beta_L(y) & \text{if } 0 < y < \underline{y} \\ \beta & \text{if } \underline{y} < y < 1. \end{cases}$$

Moreover, as in the proof of Proposition 2, and by the definition of $\beta_L(\cdot)$ we have $\lim_{\gamma\to\infty} \underline{y} = 1 - \frac{\tau}{1-G(\beta)}$, and notice that $\lim_{\gamma\to\infty} \underline{y} > 0 \Leftrightarrow \beta < G^{-1}(1-\tau)$. Also notice that

$$MPV(y) = \begin{cases} \frac{\partial \beta_L(y)}{\partial y} f(-\beta_L(y)) y (\beta - \mathbb{E}[b]) & \text{if } 0 < y < \underline{y} \\ 0 & \text{if } y > \underline{y}, \end{cases}$$

and it is not defined for $y = \underline{y}$. Notice that

$$\lim_{\gamma\to\infty} MPV(y) = -\frac{1 - G(\lim_{\gamma\to\infty} \beta_L(y))}{g(\lim_{\gamma\to\infty} \beta_L(y))} \frac{f(-\lim_{\gamma\to\infty} \beta_L(y))}{1-y} y (\beta - \mathbb{E}[b]),$$

where $\lim_{\gamma\to\infty} \beta_L(y) = G^{-1}(1 - \frac{\tau}{1-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1$). Since $\beta < \beta^*$ and $\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau)$, for a large γ we have $1 > \underline{y} > 0$, and for the same reasons as in the proof of Proposition 2, the maximizer of $\Pi(y)$ is smaller than $\underline{y}$ and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma + \eta} MPV(y^*),$$

subject to being non-negative. In particular, notice that $\lim_{\gamma\to\infty} y^* = 0$, and hence $\lim_{\gamma\to\infty} MPV(y^*) = 0$. Suppose the no-short-selling constraint does not bind in the limit, that is, y^* converges to zero from above. Then, the FOC implies

$$\begin{aligned} \lim_{\gamma\to\infty} (\beta - \mathbb{E}[b]) H\left(q^*\left(\lim_{\gamma\to\infty} y^*\right)\right) - \lim_{\gamma\to\infty} 2\gamma\hat{y} + \lim_{\gamma\to\infty} MPV(y^*) &= 0 \Leftrightarrow \\ (\beta - \mathbb{E}[b]) H(q^*(0)) &= \lim_{\gamma\to\infty} 2\gamma y^*. \end{aligned}$$

Thus, y^* converges to zero from above if and only if $\beta > \mathbb{E}[b]$. If $\beta < \mathbb{E}[b]$, then the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade (i.e., $y^* = 0$). If $\beta > \mathbb{E}[b]$, then the no-short-selling constraint does not bind in the limit, and having y^* converging to zero from above implies that $MPV(y^*)$ converges to zero from below. That is, if $\beta > \mathbb{E}[b]$, then for large γ it must be

$$MPV(y^*) < 0 < y^*.$$

Then, the share price is $p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*)$, where $MPV(y^*) < 0$.

2. Suppose $\beta \in (\beta^*, \bar{b},]$. As in the proof of Proposition 2, there exists a unique $\bar{y} \in (0, \tau)$ such that: (i) $\beta = \beta_H(\bar{y})$, (ii) if $y \in (0, \bar{y})$ then $\beta \in (\beta_H(y), \bar{b}]$, and (iii) if $y > \bar{y}$ then

$\beta \in (\beta_L(y), \beta_H(y))$. Thus, the marginal voter is given by

$$-q^*(y) = \begin{cases} \beta_H(y) & \text{if } 0 < y < \overline{y} \\ \beta & \text{if } \overline{y} < y < 1, \end{cases}$$

Moreover, as in the proof of Proposition 2, and by the definition of $\beta_H(\cdot)$ we have $\lim_{\gamma\to\infty} \overline{y} = 1 - \frac{1-\tau}{G(\beta)}$, and notice that $\lim_{\gamma\to\infty} \overline{y} > 0 \Leftrightarrow \beta > G^{-1}(1-\tau)$. Also notice that

$$MPV(y) = \begin{cases} \frac{\partial \beta_H(y)}{\partial y} f(-\beta_H(y)) y (\beta - \mathbb{E}[b])] & \text{if } 0 < y < \overline{y} \\ 0 & \text{if } y > \overline{y}, \end{cases}$$

and it is not defined for $y = \overline{y}$. Notice that

$$\lim_{\gamma\to\infty} MPV(y) = \frac{G(\lim_{\gamma\to\infty} \beta_H(y))}{g(\lim_{\gamma\to\infty} \beta_H(y))} \frac{f(-\lim_{\gamma\to\infty} \beta_H(y))}{1-y} y (\beta - \mathbb{E}[b]),$$

where $\lim_{\gamma\to\infty} \beta_H(y) = G^{-1}(\frac{1-\tau}{1-y})$. Thus, $\lim_{\gamma\to\infty} MPV(y)$ is bounded (recall $y < 1$). Since $\beta > \beta^*$ and $\lim_{\gamma\to\infty} \beta^* = G^{-1}(1-\tau)$, for a large γ we have $1 > \overline{y} > 0$, and for the same reasons as in the proof of Proposition 2, the maximizer of $\Pi(y)$ is smaller than $\overline{y}$ and given by the solution of FOC subject to the no-short-selling constraint that we impose below. That is, the optimal trade solves

$$y^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) H(q^*(y^*)) + \frac{1}{2\gamma + \eta} MPV(y^*),$$

subject to being non-negative. In particular, notice that $\lim_{\gamma\to\infty} y^* = 0$, and hence $\lim_{\gamma\to\infty} MPV(y^*) = 0$. Suppose the no-short-selling constraint does not bind in the limit, that is, y^* converges to zero from above. Then, the FOC implies

$$\begin{aligned} \lim_{\gamma\to\infty} (\beta - \mathbb{E}[b]) H\left(q^*\left(\lim_{\gamma\to\infty} y^*\right)\right) - \lim_{\gamma\to\infty} 2\gamma\hat{y} + \lim_{\gamma\to\infty} MPV(y^*) &= 0 \Leftrightarrow \\ (\beta - \mathbb{E}[b]) H(q^*(0)) &= \lim_{\gamma\to\infty} 2\gamma y^*. \end{aligned}$$

Thus, y^* converges to zero from above if and only if $\beta > \mathbb{E}[b]$. If $\beta < \mathbb{E}[b]$, then the no-short-selling constraint must bind in the limit, and in that case, the blockholder does not trade (i.e., $y^* = 0$). If $\beta > \mathbb{E}[b]$, then the no-short-selling constraint does not bind in the limit, and having y^* converging to zero from above implies that $MPV(y^*)$ converges to zero from above. That is, if $\beta > \mathbb{E}[b]$, then for large γ it must be

$$0 < MPV(y^*) \text{ and } 0 < y^*.$$

Then, the share price is $p^* = v(b_{MT}, q^*(y^*)) + \frac{\gamma}{2\gamma+\eta} MPV(y^*)$, where $MPV(y^*) > 0$.

■

Proof of Proposition 4. Recall

$$s(-z; y, -z) = (1 - G(z))(1 - \alpha - y) + \frac{1}{\gamma} G(z) (\mathbb{E}[b] - \mathbb{E}[b|b < z]) H(-z).$$

The set of equations (21) and (22) can be written as

$$\begin{aligned} s\left(-\beta_L; y, -\beta_L\right) &= \tau \\ s\left(-\beta_H; y, -\beta_H\right) &= \tau - \alpha - y. \end{aligned}$$

Let $z(\delta)$ be a sequence. Then,

$$\lim_{\delta\to 0} s\left(-z(\delta); y, -z(\delta)\right) = (1-\alpha-y)\times\left(1-\lim_{\delta\to 0} G\left(z(\delta);\delta\right)\right).$$

If $\lim_{\delta\to 0} z(\delta) > \mathbb{E}[b]$, then $\lim_{\delta\to 0} G(z(\delta);\delta) = 1$ and $\lim_{\delta\to 0} s(-z(\delta);y,-z(\delta)) = 0$. And if $\lim_{\delta\to 0} z(\delta) < \mathbb{E}[b]$, then $\lim_{\delta\to 0} G(z(\delta);\delta) = 0$ and $\lim_{\delta\to 0} s(-z(\delta);y,-z(\delta)) = 1-\alpha-y$. As long as $\tau\in(0,1)$, the solutions of (21) and (22) do not exist in the limit of $\delta\to 0$. Therefore, it must be $\lim_{\delta\to 0} z(\delta) = \mathbb{E}[b]$. Since $\beta_L(y;\delta)$ solves $s(-\beta_L;y,-\beta_L) = \tau$ and $\lim_{\delta\to 0} s(-z(\delta);y,-z(\delta)) = (1-\alpha-y)\times(1-\lim_{\delta\to 0} G(z(\delta);\delta))$, then $\lim_{\delta\to 0}\beta_L(y;\delta) = \mathbb{E}[b]$ and it converges at a rate that satisfies $1-\frac{\tau}{1-\alpha-y} = \lim_{\delta\to 0} G(\beta_L(y;\delta);\delta)$. Similarly, since $\beta_H(y;\delta)$ solves $s(-\beta_H;y,-\beta_H) = \tau-\alpha-y$ and $\lim_{\delta\to 0} s(-z(\delta);y,-z(\delta)) = (1-\alpha-y)\times(1-\lim_{\delta\to 0} G(z(\delta);\delta))$, then $\lim_{\delta\to 0}\beta_H(y) = \mathbb{E}[b]$ and it converges at a rate that satisfies $1-\frac{\tau-\alpha-y}{1-\alpha-y} = \lim_{\delta\to 0} G(\beta_H(y;\delta);\delta)$.

Finally, since $\lim_{\delta\to 0}\beta_H(y) = \lim_{\delta\to 0}\beta_L(y) = \mathbb{E}[b]$ for any $y\in(-\alpha,\min\{\tau,1-\tau\}-\alpha)$, then $\lim_{\delta\to 0}\frac{\partial(-q^*(y))}{\partial y} = 0$, which implies $MPV(y) = 0$. ■

Proof of Proposition 5. The objective $\Pi(y,\hat{y})$ of the blockholder with dual-class shares can be rewritten as:

$$\begin{aligned} \max_{y,\hat{y}} \Pi(y,\hat{y}) &= (\alpha+y)\,v(\beta,q_a^*(y)) - yp^*(y) - \frac{\eta}{2}y^2 + (\hat{\alpha}+\hat{y})\,v(\beta,q_a^*(y)) - \hat{y}\hat{p}^*(\hat{y}) - \frac{\eta}{2}\hat{y}^2 \\ &= \alpha v(\beta,q_a^*(y)) + y(\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] - (\gamma+\eta/2)y^2 \\ &\quad +\hat{\alpha} v(\beta,q_a^*(y)) + \hat{y}(\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] - (\gamma+\eta/2)\hat{y}^2 \\ &= (\alpha+\hat{\alpha})\,v(\beta,q_a^*(y)) + (y+\hat{y})(\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] - (\gamma+\eta/2)\left(y^2+\hat{y}^2\right) \\ &= (\alpha+\hat{\alpha})\,v_0 + (\alpha+\hat{\alpha})\Pr[q>q_a^*(y)]\,\mathbb{E}[\theta|q>q_a^*(y)] \\ &\quad + ((\alpha+\hat{\alpha}+y+\hat{y})\,\beta - (y+\hat{y})\,\mathbb{E}[b])\Pr[q>q_a^*(y)] - (\gamma+\eta/2)\left(y^2+\hat{y}^2\right). \end{aligned}$$

We rewrite the first-order condition with respect to y, $\frac{\partial\Pi(y,\hat{y})}{\partial y} = 0$, as

$$\begin{aligned} \left[\begin{array}{c} \underbrace{(\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] - (2\gamma+\eta)y}_{\text{marginal propensity to buy cash flows}} \\ +\underbrace{\frac{\partial(-q_a^*(y))}{\partial y} f(q_a^*(y))\left[(\alpha+\hat{\alpha})(q_a^*(y)+\beta) + (y+\hat{y})(\beta-\mathbb{E}[b])\right]}_{\text{marginal propensity to buy votes } MPV(y,\hat{y})} \end{array}\right] &= 0 \Leftrightarrow \\ (\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] - (2\gamma+\eta)y + MPV(y,\hat{y}) &= 0 \Leftrightarrow \end{aligned}$$

$$y^* = \frac{1}{2\gamma+\eta}(\beta-\mathbb{E}[b])\Pr[q>q_a^*(y)] + \frac{1}{2\gamma+\eta}MPV(y,\hat{y}).$$

We rewrite the first-order condition with respect to $\hat{y}$, $\frac{\partial \Pi(y,\hat{y})}{\partial \hat{y}} = 0$, as

$$\underbrace{(\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)] - (2\gamma + \eta)\hat{y}}_{\text{marginal propensity to buy cash flows in non-voting shares}} = 0 \Leftrightarrow$$

$$\hat{y}^* = \frac{1}{2\gamma + \eta} (\beta - \mathbb{E}[b]) \Pr[q > q_a^*(y)].$$

Thus,

$$p^*_{voting} - p^*_{non-voting} = \gamma (y^* - \hat{y}^*) = \frac{\gamma}{2\gamma + \eta} MPV(y^*, \hat{y}^*).$$

■

Appendix - Table 1

The following table lists 40 studies that measure dual-class shares using five different methodologies.[29] Methods of measurement within one methodology may differ slightly. The studies on dual-class tender offers by Bradley (1980) and DeAngelo and DeAngelo (1985) are classified as block trades because tender offers are bids for a block of shares, not for individual shares. Christoffersen et al. (2007) is listed as two separate studies.

Study		Sample			Voting premium		
Authors	Year of publication	Country	Period	Size	Dual-class premium	Block-trade premium	Voting yield
Panel A: Dual-class shares							
Lease et al.	1983	USA	1940-1978	26	5.44%		
Levy	1983	Israel	1974-1980	25	45.50%		
Horner	1988	Switzerland	1973-1983	45	22.40%		
Megginson	1990	UK	1955-1982	152	13.30%	32.10%	
Zingales	1994	Italy	1987-1990	96	81.50%		
Smith and Amoako-Adu	1995	Canada	1981-1986	62	10.40%	57.20%	
Zingales	1995	USA	1984-1990	94	10.47%	81.50%	
Rydqvist	1996	Sweden	1983-1990	65	12.00%		
Kunz and Angel	1996	Switzerland	1990-1991	29	18.00%		
Maynes	1996	Canada	1984	46	6.66%		
Muus	1998	France	1986-1996	25	51.35%		
Chung and Kim	1999	Korea	1992-1993	119	9.60%		
Hoffmann-Burchardi	1999	Germany	1988-1997	84	26.30%	21.70%	
Cox and Roden	2002	USA	1984-1999	98	7.70%		
Daske and Ehrhardt	2002	Germany	1956-1998	101	17.23%	20.60%	
Dittmann	2003	Germany	1960-2001	82	12.62%		
Neumann	2003	Denmark	1992-1999	34	12.31%		
Nenova	2003	cross-country	1997	661	13.85%		
Muravyev	2004	Russia	1997-2003	37	64.72%		
Ødegaard	2007	Norway	1988-2005	226	5.60%		
Caprio and Croci	2008	Italy	1974-2003	116	56.51%		
Bigelli and Croci	2013	Italy	1999-2008	74	20.35%		
Broussard and Vaihekoski	2019	Finland	1982-2018	50	27.20%		

[29] The 40 studies are: Aggarwal, Saffi, and Sturgess (2015); Albuquerque and Schroth (2010); Albuquerque and Schroth (2015); Barak and Lauterbach (2011); Barclay and Holderness (1989); Bergström and Rydqvist (1992); Bigelli and Croci (2013); Bradley (1980); Broussard and Vaihekoski (2019); Caprio and Croci (2008); Christoffersen et al. (2007); Chung and Kim (1999); Cox and Roden (2002); Daske and Ehrhardt (2002); DeAngelo and DeAngelo (1985); Dittmann (2003); Dyck and Zingales (2004); Fos and Holderness (2020); Franks and Mayer (2001); Gurun and Karakas (2020); Hoffman-Burchardi (1999); Horner (1988); Jang, Kim, and Mohseni (2019); Kalay, Karakas, and Pant (2014); Kind and Poltera (2013); Kind and Poltera (2017); Kunz and Angel (1996); Lease, McConnell, and Mikkelson (1983); Levy (1983); Maynes (1996); Megginson (1990); Muravyev (2004); Muus (1998); Nenova (2003); Neumann (2003); Odegaard (2007); Rydqvist (1996); Smith and Amoako-Adu (1995); Zingales (1994); Zingales (1995).

Online Appendix for "The Voting Premium"

by Doron Levit[30], Nadya Malenko[31], and Ernst Maug[32]

A Supplemental Analysis

Lemma 3. *Let* $\underline{v} \equiv v_0 + \min\left\{-\overline{b}, \beta\right\}$ *and* $\overline{v} \equiv v_0 + \max\left\{\overline{b}, \beta\right\} + 1$. *Then,*

(i) If $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ *then no dispersed shareholder short-sells the share in any equilibrium, that is,* $x^*(b) + 1 - \alpha > 0$ *for any* $b \in \left[-\overline{b}, \overline{b}\right]$.

(ii) Suppose $\alpha > 0$. *If* $\eta > \frac{4(\overline{v}-\underline{v})}{\alpha}$ *then the blockholder never short-sells the share in any equilibrium, that is,* $y^* + \alpha > 0$.

(iii) If $\alpha < \min\{\tau, 1-\tau\}$ *and* $\eta > \frac{2(\overline{v}-\underline{v})\min\{\tau,1-\tau\}}{(\min\{\tau,1-\tau\}-\alpha)^2}$ *then the blockholder never obtains a control stake or veto power in equilibrium, that is,* $\alpha + y^* < \min\{\tau, 1-\tau\}$.

Proof. The valuation of the share by any investor is bounded from above by $\overline{v}$ and from below by $\underline{v}$. Therefore, in any equilibrium, $|p^* - v(b, q^*)| < \overline{v} - \underline{v}$ and $|p^* - v(\beta, q^*)| < \overline{v} - \underline{v}$. Based on (7), $x(b, p^*) + 1 - \alpha > 0 \Leftrightarrow \gamma > \frac{p^*-v(b,q^*)}{1-\alpha}$. Therefore, requiring $\gamma > \frac{\overline{v}-\underline{v}}{1-\alpha}$ guarantees that in any equilibrium $x(b, p^*) + 1 - \alpha > 0$ as required by part (i).

Consider the blockholder. First, if the blockholder chooses $y^* < 0$ in equilibrium then it must be

$$\begin{aligned}
\Pi(y^*) &\geq \Pi(0) \Leftrightarrow \\
y^*\left[v(\beta, q^*(y^*)) - p^*(y^*)\right] - \frac{\eta}{2}y^{*2} &\geq \alpha\left[v(\beta, q^*(0)) - v(\beta, q^*(y^*))\right] \Rightarrow \\
-y^*(\overline{v} - \underline{v}) - \frac{\eta}{2}y^{*2} &\geq -\alpha(\overline{v} - \underline{v}) \Leftrightarrow \\
y^* &\geq -\frac{(\overline{v} - \underline{v}) + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta}.
\end{aligned}$$

Therefore, assuming

$$-\frac{(\overline{v} - \underline{v}) + \sqrt{(\overline{v} - \underline{v})^2 + 2\alpha\eta(\overline{v} - \underline{v})}}{\eta} > -\alpha \Leftrightarrow \eta > \frac{4(\overline{v} - \underline{v})}{\alpha}$$

guarantees that the blockholder will sell less than his entire endowment in any equilibrium, that is, $y^* > -\alpha$ as required by part (ii). Second, if the blockholder chooses $y^* > 0$ in equilibrium

[30] University of Washington and ECGI. Email: dlevit@uw.edu.
[31] University of Michigan, CEPR, and ECGI. Email: nmalenko@umich.edu.
[32] University of Mannheim and ECGI. Email: maug@uni-mannheim.de.

then it must be

$$\begin{aligned}
\Pi\left(y^*\right) &\geq \Pi\left(0\right) \Leftrightarrow \\
y^*\left[v\left(\beta, q^*\left(y^*\right)\right) - p^*\left(y^*\right)\right] - \frac{\eta}{2}y^{*2} &\geq \alpha\left[v\left(\beta, q^*\left(0\right)\right) - v\left(\beta, q^*\left(y^*\right)\right)\right] \Rightarrow \\
y^*\left(\overline{v} - \underline{v}\right) - \frac{\eta}{2}y^{*2} &\geq -\alpha\left(\overline{v} - \underline{v}\right) \Leftrightarrow \\
y^* &\leq \frac{\overline{v} - \underline{v} + \sqrt{\left(\overline{v} - \underline{v}\right)^2 + 2\alpha\eta\left(\overline{v} - \underline{v}\right)}}{\eta}.
\end{aligned}$$

Therefore, assuming

$$\frac{\overline{v} - \underline{v} + \sqrt{\left(\overline{v} - \underline{v}\right)^2 + 2\alpha\eta\left(\overline{v} - \underline{v}\right)}}{\eta} < \min\left\{\tau, 1-\tau\right\} - \alpha \Leftrightarrow \eta > \frac{2\left(\overline{v} - \underline{v}\right)\min\left\{\tau, 1-\tau\right\}}{\left(\min\left\{\tau, 1-\tau\right\} - \alpha\right)^2}$$

guarantees that the blockholder will not obtain a stake larger than $\min\{\tau, 1-\tau\}$ in any equilibrium, that is, $y^* + \alpha < \min\{\tau, 1-\tau\}$ as required by part (iii). ■

B Voting yields and capitalized voting premiums

The dual-class share premium is commonly computed as the relative price difference between voting and non-voting shares. Let $P_{t,v}$ be the price per voting share and $P_{t,nv}$ the price per non-voting shares in period t:[33]

$$\text{Dual class premium}_t = \frac{P_{t,v} - P_{t,nv}}{P_{t,v}}. \tag{75}$$

The dual-class premium captures the potentially infinite time horizon over which the owner of a block of voting rights enjoys control rights, which ends only if the firm ceases to exist, e.g., because of acquisitions or insolvencies, or when the two classes of shares are unified into one class. Hence, they represent the capitalized value of a the right to vote at all future shareholder meetings. The same is true for the block-trading premium, which is discussed below.

By contrast, the last three methods in our list measure the value of voting rights only for very limited periods of time ranging from three days to 57 days.[34] These time spans do not capture more than one shareholder meeting. Hence, these methods estimate a voting yield,

[33] This statistic applies only if one class of shares has no voting rights and both classes have the same par value. It is appropriately adjusted when par values differ (Megginson (1990)) or when computing the value of control for firms that have two classes of voting shares, but different ratios of cash flow rights to voting rights; see, e.g., Zingales (1995). Bigelli and Croci (2013) Argue that many studies lack appropriate adjustments for differential dividends.

[34] Table 1 reports annualized figures if such figures are reported by the authors. Kalay, Karakas, and Pant (2014) and Gurun and Karakas (2020) construct non-voting shares synthetically from options with an average maturity of, respectively, 38 days and 57 days. the equity-lending method (Christoffersen et al. (2007); Aggarwal, Saffi, and Sturgess (2015)) investigates fees for lending shares around record dates, and the record-day trading method measures stock price drops in a 3-day trading window around record dates (Fos and Holderness (2020)).

which has the same dimension as a dividend yield. Let V_t represent the per-share dollar value of a voting right and D_t the dollar value of dividends per share, where the subscript indexes time. Then $D_t/P_{t-1,v}$ and $D_t/P_{t-1,nv}$ are the dividend yields of, respectively, the voting and the non-voting shares, and $V_t/P_{t-1,nv}$ is the voting yield. Let r_v be the constant per-period discount rate for the voting shares and r_{nv} the constant per-period discount rate for the non-voting shares, and assume the value of voting rights and dividends both grow at the same constant rate g, which allows us to calculate the value of voting shares and of non-voting shares using the Gordon growth formula:

$$P_{0,v} = \frac{D_1 + V_1}{r_v - g}; \ P_{0,nv} = \frac{D_1}{r_{nv} - g}. \tag{76}$$

Solving both expressions for g, equating them, and rearranging gives:

$$\frac{V_1}{P_{0,v}} = \frac{P_{0,v} - P_{0,nv}}{P_{0,nv}} \frac{D_1}{P_{0,v}} + r_v - r_{nv}. \tag{77}$$

Note, however, that numerous aspects are missing in this simple analysis, e.g., the time horizon for control may be finite because of possible mergers, stock unifications, or regulatory change. See, e.g., Goetzmann, Ukhov, and Spiegel (2002).If we assume that $r_v = r_{nv}$, we obtain:

$$\text{Voting yield} = \text{ Dual-class premium } \times \text{ Dividend yield.}$$

This relationship is broadly consistent with US data. A dual-class share premium of about 5%-10% (see Table 1), multiplied by a dividend yield of 3.7% (see Fama and French (2002) for the period 1951-2000) results in estimates of the voting yield of 0.18% to 0.37%, which compares well to estimates of the annualized voting yield from option replications (Kalay, Karakas, and Pant (2014): 0.16%; Kind and Poltera (2013): 0.37%). If we relax the assumption that $r_v = r_{nv}$, then from (77) we would expect a lower voting yield if the discount rate for voting shares is higher than that for non-voting shares, i.e., if investors consider votes to be riskier than dividends.

C Analysis of multiple blockholders

C.1 Multiple homogenous blockholders

Assume there are $N \geq 2$ blockholders, each blockholder is endowed with $\alpha_i = \frac{\alpha}{N}$ shares. All blockholders face the same trading cost η and have the same bias β.

Proposition 8. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. The equilibrium trade of each blockholder, denoted by y^*, satisfies*

$$Ny^* = \frac{N}{\gamma(N+1) + \eta} (\beta - \mathbb{E}[b]) H(q^*(Ny^*)) + \frac{1}{\gamma(N+1) + \eta} MPV(Ny^*), \tag{78}$$

and the share price is given by

$$p^*(N) = v(b_{MT}(N), q^*(Ny^*)) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny^*),$$

where $q^*(\cdot)$ is given in Proposition 1 and

$$b_{MT}(N) = \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\beta. \tag{79}$$

Proof of Proposition 8. We denote the trade of blockholder $i \in \{1, ..., N\}$ by y_i. Let $y = \sum_{i=1}^{N} y_i$ and $y_{-i} = \sum_{j\neq i} y_j$. Since all blockholders have the same bias, Proposition 1 holds with respect to y. Moreover, given y and $q^*(y)$, trade by dispersed shareholders is also the same as in the baseline model, and in particular, the share price is given by $p^*(y) = \gamma y + v(\mathbb{E}[b], q^*(y))$. The profit of blockholder i is given by

$$\begin{aligned}
\Pi(y_i, y_{-i}) &= (\alpha_i + y_i) v(\beta, q^*(y_{-i}+y_i)) - y_i p^*(y_{-i}+y_i) - \frac{\eta}{2}(y_i)^2 \\
&= \alpha_i v(\beta, q^*(y_{-i}+y_i)) + y_i(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - y_i\gamma y_{-i} - (\gamma+\eta/2)(y_i)^2 \\
&= \alpha_i v_0 + \alpha_i \mathbb{E}[\theta | q > q^*(y_{-i}+y_i)] H(q^*(y_{-i}+y_i)) \\
&\quad ((\alpha_i + y_i)\beta - y_i \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (\gamma+\eta/2)(y_i)^2 - y_i\gamma y_{-i}.
\end{aligned}$$

The derivative of $\Pi(y_i, y_{-i})$ with respect to y_i, $\Pi'(y_i, y_{-i})$, is given by

$$\begin{aligned}
\Pi'(y_i, y_{-i}) &= \underbrace{(\beta - \mathbb{E}[b]) H(q^*(y_{-i}+y_i)) - (2\gamma+\eta) y_i - \gamma y_{-i}}_{\text{marginal propensity to buy cash flows}} \\
&\quad \underbrace{+\frac{\partial(-q^*(y_{-i}+y_i))}{\partial y} f(q^*(y_{-i}+y_i)) [\alpha_i(q^*(y_{-i}+y_i)+\beta) + y_i(\beta - \mathbb{E}[b])]}_{\text{marginal propensity to buy votes}}.
\end{aligned}$$

The symmetry across blockholders requires all of them to trade the same amount, and thus, the equilibrium level of y_i^* satisfies

$$\begin{aligned}
0 &= \begin{bmatrix} (\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) - (2\gamma+\eta) y_i^* - \gamma(N-1) y_i^* \\ +\frac{\partial(-q^*(Ny_i^*))}{\partial y} f(q^*(Ny_i^*)) \left[\frac{\alpha}{N}(q^*(Ny_i^*)+\beta) + y_i^*(\beta - \mathbb{E}[b])\right] \end{bmatrix} \Leftrightarrow \\
Ny_i^* &= \frac{N}{\gamma(N+1)+\eta}(\beta - \mathbb{E}[b]) H(q^*(Ny_i^*)) + \frac{1}{\gamma(N+1)+\eta} MPV(Ny_i^*).
\end{aligned}$$

The share price in equilibrium is

$$\begin{aligned}
p^*(N) &= \gamma N y_i^* + v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\
&= \gamma\left[\frac{N}{\gamma(N+1)+\eta}\left(\beta - \mathbb{E}[b]\right) H\left(q^*(Ny_i^*)\right) + \frac{1}{\gamma(N+1)+\eta} MPV(Ny_i^*)\right] \\
&\quad + v\left(\mathbb{E}[b], q^*(Ny_i^*)\right) \\
&= v_0 + \left(\frac{\gamma N}{\gamma(N+1)+\eta}\left(\beta - \mathbb{E}[b]\right) + \mathbb{E}[b]\right) H\left(q^*(Ny_i^*)\right) \\
&\quad + \mathbb{E}\left[\theta | q > q^*(Ny_i^*)\right] H\left(q^*(Ny_i^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny_i^*) \\
&= v\left(\frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\beta, q^*(Ny_i^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny_i^*) \\
&= v\left(b_{MT}(N), q^*(Ny_i^*)\right) + \frac{\gamma}{\gamma(N+1)+\eta} MPV(Ny_i^*),
\end{aligned}$$

as required. ■

Hence, in the limit, the price is the valuation of the blockholders absent any voting premium, that is, $\lim_{N\to\infty} p^*(N) = v\left(\beta, q^*(y_\infty^*)\right)$, where y_∞^* satisfies $y_\infty^* = \frac{1}{\gamma}\left(\beta - \mathbb{E}[b]\right) H\left(q^*(y_\infty^*)\right)$.

C.2 Heterogenous blockholders

Suppose μN blockholders have bias $\beta_c \approx -\overline{b}$, and $(1-\mu)N$ blockholders have bias $\beta_a \approx \overline{b}$, where $\mu \in (0,1)$ is such that both μN and $(1-\mu)N$ are integers. Thus, blockholders with bias β_c always (i.e., regardless of the realization of signal q) vote against the proposal, and blockholders with bias β_a always vote for the proposal. We let $\overline{\beta} = (1-\mu)\beta_a + \mu\beta_c$ and $q^*(y_c, y_a)$ be the solution of

$$s\left(-q^*; y_c + y_a, q^*\right) + (1-\mu)\alpha + y_a = \tau. \tag{80}$$

The following result holds.

Proposition 9. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the equilibrium exists and is unique. In equilibrium, blockholders biased in favor (against) the proposal trade y_a^* (y_c^*) shares such that*

$$\begin{aligned}
N\left(\mu y_c^* + (1-\mu) y_a^*\right) &= \frac{N}{\gamma(N+1)+\eta}\left(\overline{\beta} - \mathbb{E}[b]\right) H\left(q^{**}\right) \\
&\quad + \frac{1}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**} + (1-\mu) MPV_a^{**}\right),
\end{aligned} \tag{81}$$

and the share price is given by

$$p^*(N) = v\left(b_{MT}(N,\mu), q^{**}\right) + \frac{\gamma}{\gamma(N+1)+\eta}\left(\mu MPV_c^{**} + (1-\mu) MPV_a^{**}\right), \tag{82}$$

where

$$\begin{aligned}
q^{**} &= q^*\left(\mu N y_c^*, (1-\mu) N y_a^*\right), \\
b_{MT}(N) &= \frac{\gamma+\eta}{\gamma(N+1)+\eta}\mathbb{E}[b] + \frac{\gamma N}{\gamma(N+1)+\eta}\overline{\beta}, \\
MPV_a^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_a}|_{q^*=q^{**}} f(q^{**})\left[\alpha\left(q^{**}+\beta_a\right) + N y_a^*\left(\beta_a - \mathbb{E}[b]\right)\right], \\
MPV_c^{**} &= \frac{\partial\left(-q^*(y_c, y_a)\right)}{\partial y_c}|_{q^*=q^{**}} f(q^{**})\left[\alpha\left(q^{**}+\beta_c\right) + N y_c^*\left(\beta_c - \mathbb{E}[b]\right)\right].
\end{aligned}$$

Moreover, as blockholders become more extreme in the sense that β_a increases by ε and β_c decreases by ε, the aggregate MPV, $\mu MPV_c^{**} + (1-\mu) MPV_a^{**}$, increases.

Proof of Proposition 9. We denote the aggregate trade of all blockholders biased against (in favor) of the proposal by y_c (y_a). Given $y = y_c + y_a$, the trade of dispersed investors is as in the baseline model. Given q and the expectations of dispersed shareholders that the proposal will be approved if and only if $q > q_e^*$, the number of affirmative votes by dispersed investors is $s(q; y, q_e^*)$. Thus, the proposal is accepted if and only if

$$s(q; y, q_e^*) + (1-\mu)\alpha + y_a \geq \tau. \tag{83}$$

Thus, in equilibrium, the marginal voter is a dispersed investor whose bias $-q^*$ solves

$$\begin{gathered}
s(-q^*; y, q^*) + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
(1-\alpha-y_c-y_a)\left[1 - G(-q^*)\left(1 - \tfrac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\tfrac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right] + (1-\mu)\alpha + y_a = \tau \Leftrightarrow \\
1 - G(-q^*)\left(1 - \tfrac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\tfrac{H(q^*)}{1-\alpha-y_c-y_a}\right) = \tfrac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}.
\end{gathered}$$

If γ is sufficiently high, the solution is unique, as in the baseline model. Suppose this is the case. Then,

$$\frac{\partial(-q^*)}{\partial y_c} = -\frac{G(-q^*)\left(\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{(1-\alpha-y_c-y_a)^2}\right) - \frac{\tau-(1-\mu)\alpha-y_a}{(1-\alpha-y_c-y_a)^2}}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}}.$$

Notice that the denominator is negative (since the solution is unique for a large γ) and that the numerator can be written as

$$\begin{aligned}
& \frac{1}{1-\alpha-y_c-y_a}\left[G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} - \frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[\begin{array}{c} G(-q^*)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a} \\ -\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)\end{array}\right] \\
= & \frac{1}{1-\alpha-y_c-y_a}\left[-1+G(-q^*)\right] < 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_c} < 0$. That is, the marginal voter becomes more biased against the proposal as blockholders biased against the proposal buy more shares. Next, notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_a} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[G\left(-q^*\right)\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}\left[\frac{\tau-(1-\mu)\alpha-y_a}{1-\alpha-y_c-y_a}-\left(1-G\left(-q^*\right)\right)-\frac{\tau-1+\mu\alpha+y_c}{1-\alpha-y_c-y_a}\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} > 0,
\end{aligned}$$

so $\frac{\partial(-q^*)}{\partial y_a} > 0$. That is, the marginal voter becomes more activist as the activist blockholders buy more shares. Notice that

$$\begin{aligned}
\frac{\partial\left(-q^*\right)}{\partial y_c} &= -\frac{\frac{1}{1-\alpha-y_c-y_a}G\left(-q^*\right)}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{-1+G\left(-q^*\right)}\frac{\frac{1}{1-\alpha-y_c-y_a}\left[-1+G\left(-q^*\right)\right]}{\frac{\partial\left(1-G(-q^*)\left(1-\frac{\mathbb{E}[b]-\mathbb{E}[b|b<-q^*]}{\gamma}\frac{H(q^*)}{1-\alpha-y_c-y_a}\right)\right)}{\partial(-q^*)}} \\
&= -\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{\partial\left(-q^*\right)}{\partial y_a}.
\end{aligned}$$

The profit of an activist blockholder i is given by

$$\begin{aligned}
\Pi_a\left(y_i, y_c, y_{a,-i}\right) &= \left(\alpha_i+y_i\right)v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)-y_i p^*\left(y_c+y_{a,-i}+y_i\right)-\frac{\eta}{2}\left(y_i\right)^2 \\
&= \alpha_i v\left(\beta_a, q^*\left(y_c, y_{a,-i}+y_i\right)\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad -y_i\gamma\left(y_c+y_{a,-i}\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2 \\
&= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_c, y_{a,-i}+y_i\right)\right]H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_a-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_c+y_{a,-i}\right),
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)-\left(2\gamma+\eta\right)y_i-\gamma\left(y_c+y_{a,-i}\right)+ \\ \frac{\partial\left(-q^*\left(y_c, y_{a,-i}+y_i\right)\right)}{\partial y_a}f\left(q^*\left(y_c, y_{a,-i}+y_i\right)\right)\left[\alpha_i\left(q^*\left(y_c, y_{a,-i}+y_i\right)+\beta_a\right)+y_i\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]=0.$$

Similarly, the profit of a blockholder i biased against the proposal is given by

$$\begin{aligned}
\Pi_c\left(y_i, y_{c,-i}, y_a\right) &= \alpha_i v_0+\alpha_i\mathbb{E}\left[\theta|q>q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right]H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right) \\
&\quad \left(\left(\alpha_i+y_i\right)\beta_c-y_i\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_{c,-i}+y_i, y_{a,}\right)\right)-\left(\gamma+\eta/2\right)\left(y_i\right)^2-y_i\gamma\left(y_{c,-i}+y_a\right)
\end{aligned}$$

and thus, the first order condition implies

$$\left[\begin{array}{c}\left(\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)-\left(2\gamma+\eta\right)y_i-\gamma\left(y_{c,-i}+y_a\right)+\\ \frac{\partial\left(-q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)}{\partial y_c}f\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)\right)\left[\alpha_i\left(q^*\left(y_{c,-i}+y_i,y_{a,}\right)+\beta_c\right)+y_i\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]=0.$$

The symmetry across blockholders with bias β_a implies that they all choose y_a^*, and the symmetry across blockholders with bias β_c implies that they all choose y_c^*. Let

$$q^{**}\equiv q^*\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right).$$

Then, the two FOC conditions are reduced to

$$\begin{array}{c}\left(\beta_a-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)-\left(\gamma+\eta\right)y_a^*-\gamma\left(\mu y_c^*+\left(1-\mu\right)y_a^*\right)N\\ +\frac{\partial\left(-q^{**}\right)}{\partial y_a}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_a\right)+y_a^*\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]=0\end{array}$$

and

$$\begin{array}{c}\left(\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)-\left(\gamma+\eta\right)y_c^*-\gamma\left(\mu y_c^*+\left(1-\mu\right)y_a^*\right)N\\ +\frac{\partial\left(-q^{**}\right)}{\partial y_c}f\left(q^{**}\right)\left[\frac{\alpha}{N}\left(q^{**}+\beta_c\right)+y_c^*\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]=0.\end{array}$$

The price is

$$p^*=\gamma\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right)+v\left(\mathbb{E}\left[b\right],q^*\left(\mu N y_c^*,\left(1-\mu\right)N y_a^*\right)\right).$$

Multiplying the FOC of blockholders with bias β_a by $(1-\mu)$ and the FOC of blockholders with bias β_c by μ, and adding the two outcomes, we get

$$\begin{aligned}&\left[\mu y_c^*+\left(1-\mu\right)y_a^*\right]N\\ =\;&\frac{N}{\gamma\left(N+1\right)+\eta}\left(\left(1-\mu\right)\beta_a+\mu\beta_c-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)\\ &+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\begin{array}{c}\left(1-\mu\right)\frac{\partial\left(-q^{**}\right)}{\partial y_a}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_a\right)+N y_a^*\left(\beta_a-\mathbb{E}\left[b\right]\right)\right]\\ +\mu\frac{\partial\left(-q^{**}\right)}{\partial y_c}f\left(q^{**}\right)\left[\alpha\left(q^{**}+\beta_c\right)+N y_c^*\left(\beta_c-\mathbb{E}\left[b\right]\right)\right]\end{array}\right]\\ =\;&\frac{N}{\gamma\left(N+1\right)+\eta}\left(\overline{\beta}-\mathbb{E}\left[b\right]\right)H\left(q^{**}\right)+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\mu MPV_c^{**}+\left(1-\mu\right)MPV_a^{**}\right].\end{aligned}$$

Thus, we can write the share price as

$$p^*=v\left(b_{MT}\left(N\right),q^{**}\right)+\frac{1}{\gamma\left(N+1\right)+\eta}\left[\mu MPV_c^{**}+\left(1-\mu\right)MPV_a^{**}\right].$$

Next, notice that

$$\begin{aligned}\lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_a}&=\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}>0,\\ \lim_{\gamma\to\infty}\frac{\partial\left(-q^*\right)}{\partial y_c}&=-\frac{G\left(-q^*\right)}{1-G\left(-q^*\right)}\frac{1}{1-\alpha}\frac{G\left(-q^*\right)}{g\left(-q^*\right)}<0,\end{aligned}$$

and thus,

$$\lim_{\gamma\to\infty} MPV_a^{**} = \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})}f(q^{**})\frac{\alpha}{N}(q^{**}+\beta_a) > 0,$$
$$\lim_{\gamma\to\infty} MPV_c^{**} = -\frac{G(-q^{**})}{1-G(-q^{**})}\frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})}f(q^{**})\frac{\alpha}{N}(q^{**}+\beta_c) > 0.$$

Also notice that $\lim_{\gamma\to\infty} q^{**}$ solves

$$1-G(-q^{**}) = \frac{\tau-(1-\mu)\alpha}{1-\alpha}.$$

Thus,

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})}f(q^{**})\frac{\alpha}{N}\\ &\times\left[q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G(-q^{**})}\right].\end{aligned}$$

Suppose β_a increases by $\varepsilon>0$ and β_c decreases by ε. Then, we can write

$$\begin{aligned}\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right] &= \frac{1}{1-\alpha}\frac{G(-q^{**})}{g(-q^{**})}f(q^{**})\frac{\alpha}{N}\\ &\times\left[\begin{array}{c} q^{**}+(1-\mu)\beta_a+\mu\beta_c-\mu\frac{q^{**}+\beta_c}{1-G(-q^{**})}\\ +(1-\mu)\varepsilon-\mu\varepsilon+\mu\frac{\varepsilon}{1-G(-q^{**})}\end{array}\right].\end{aligned}$$

The derivative of $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right]$ with respect to ε is proportional to

$$1-2\mu+\frac{\mu}{1-G(-q^{**})} = 1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu}.$$

Notice that

$$1-2\mu+\frac{\mu(1-\alpha)}{\tau-\alpha+\alpha\mu} > 0 \Leftrightarrow \tau-\alpha+(1-2(\tau-\alpha))\mu-2\alpha\mu^2 > 0.$$

This concave expression is positive both when $\mu=0$ and when $\mu=1$ (since $\alpha<1-\tau$), and thus it is positive for any $\mu\in[0,1]$. Therefore, $\lim_{\gamma\to\infty}\left[\mu MPV_c^{**}+(1-\mu)MPV_a^{**}\right]$ increases in ε, as required. ■

D Analysis of block trading

By definition, $\Pi^*(\beta) = \Pi(y^*(\beta),\beta)$, where $\Pi(y,\beta)$ is given by (24), $y^*(\beta)$ by (28), and $p^*(y,\beta) = \gamma y + v(\mathbb{E}[b], q^*(y,\beta))$. To ease the exposition we let $q^*(\beta) = q^*(y^*(\beta),\beta)$ and

$p^*(\beta) = p^*(y^*(\beta), \beta)$. Thus,

$$\begin{aligned}
\Pi^*(\beta) &= (\alpha + y^*(\beta))\, v(\beta, q^*(\beta)) - y^*(\beta)\, p^*(\beta) - \frac{\eta}{2}(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + y^*(\beta)(\beta - E[b])\, H(q^*(\beta)) - (\gamma + \eta/2)(y^*(\beta))^2 \\
&= \alpha v(\beta, q^*(\beta)) + \frac{1}{2}\frac{1}{2\gamma + \eta}\left[(\beta - \mathbb{E}[b])^2 H(q^*(\beta))^2 - MPV(y^*(\beta))^2\right]
\end{aligned}$$

The first term in the blockholder's equilibrium payoff is the value of his endowment given $q^*(\beta)$, whereas the second term represents the gains from trade.

Proposition 10. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then the blockholder's equilibrium payoff strictly increases in β.*

Proof of Proposition 10. In the proof of Proposition 2, we show that if $\beta \neq G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ ($\beta \neq G^{-1}(\frac{1-\tau}{1-\alpha})$) and γ is large then $\beta \neq \beta_L(y^*(\beta))$ ($\beta \neq \beta_H(y^*(\beta))$). Then,

$$\begin{aligned}
\frac{\partial \Pi^*(\beta)}{\partial \beta} &= \frac{\partial \Pi(y^*(\beta), \beta)}{\partial y}\Big|_{y=y^*(\beta)} \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta), \beta)}{\partial \beta} \\
&= 0 \cdot \frac{\partial y^*(\beta)}{\partial \beta} + \frac{\partial \Pi(y^*(\beta), \beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta))\frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial \beta} - y^*(\beta)\frac{\partial p^*(y^*(\beta), \beta)}{\partial \beta} \\
&= (\alpha + y^*(\beta))\left[\frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial \beta} + \frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial q^*}\frac{\partial q^*(y^*(\beta), \beta)}{\partial \beta}\right] \\
&\quad - y^*(\beta)\frac{\partial v(\mathbb{E}[b], q^*(y^*(\beta), \beta))}{\partial q^*}\frac{\partial q^*(y^*(\beta), \beta)}{\partial \beta}
\end{aligned}$$

If $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then for large γ we have $q^*(y, \beta) = -\beta$. Thus,

$$\begin{aligned}
\frac{\partial v(\beta, -\beta)}{\partial \beta} &= \frac{\partial}{\partial \beta}\int_{-\beta}(\theta + \beta)\, f(q)\, dq \\
&= H(-\beta) > 0
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial p^*(y^*(\beta), \beta)}{\partial \beta} &= \frac{\partial v(\mathbb{E}[b], -\beta)}{\partial \beta} \\
&= \frac{\partial}{\partial \beta}\int_{-\beta}(\theta + \mathbb{E}[b])\, f(q)\, dq \\
&= -(\beta - \mathbb{E}[b])\, f(-\beta)
\end{aligned}$$

Thus,

$$\frac{\partial \Pi^*(\beta)}{\partial \beta} = (\alpha + y^*(\beta))\, H(-\beta) + y^*(\beta)(\beta - \mathbb{E}[b])\, f(-\beta)$$

Recall that $\lim_{\gamma\to\infty} y^*(\beta) = 0$, and thus, for large γ we have $\frac{\partial\Pi^*(\beta)}{\partial\beta} \approx \alpha H(-\beta) > 0$, that is, the first order effect is on the blockholder endowment rather than on his gains from trade.

If $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ $(G^{-1}(\frac{1-\tau}{1-\alpha}) < \beta)$ then $q^*(y^*(\beta), \beta) = \beta_L(y^*(\beta))$ $(q^*(y^*(\beta), \beta) = \beta_H(y^*(\beta)))$ does not depend on β directly, that is, $\frac{\partial q^*(y^*(\beta),\beta)}{\partial\beta} = 0$. And in this case,

$$\begin{aligned}
\frac{\partial\Pi^*(\beta)}{\partial\beta} &= (\alpha + y^*(\beta)) \frac{\partial v(\beta, q^*(y^*(\beta), \beta))}{\partial\beta} \\
&= (\alpha + y^*(\beta)) H(q^*(y^*(\beta), \beta)) \\
&> 0
\end{aligned}$$

Since $\Pi^*(\beta)$ is continuous in β when $\beta \in \left\{G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}), G^{-1}(\frac{1-\tau}{1-\alpha})\right\}$, $\Pi^*(\beta)$ increases globally in β. ■

Suppose a block of α shares is acquired by a bidder with a bias β who paid $\Pi^*(\beta) - \Delta\alpha$ where $\Delta > 0$. Parameter Δ captures the competitiveness of the block market, where smaller Δ implies more competitiveness. Recall the share price is given by $p^*(y^*(\beta)) = \gamma y^*(\beta) + v(\mathbb{E}[b], q^*(y^*(\beta), \beta))$, then the block premium is

$$P_B \equiv \Pi^*(\beta)/\alpha - \Delta - p^*(y^*(\beta), \beta) \tag{84}$$

Here, we replace $\Pi^*(\beta_R)$ in the text with $\Pi^*(\beta)$ and the discount blockholder R receives on the block price, $(1-\delta)(\Pi^*(\beta_R) - \Pi^*(\beta_I))$ with the constant Δ.

Proposition 11. *There exist $\overline{\gamma} < \infty$ and $\overline{\eta} < \infty$ such that if $\gamma > \overline{\gamma}$ and $\eta > \overline{\eta}$, then*

(i) If $\beta \le \mathbb{E}[b]$ then the equilibrium block premium is strictly negative.

(ii) If $\beta > \mathbb{E}[b]$ then there exist $\alpha^* > 0$ and $\Delta^* > 0$ such that if $\alpha \in (0, \alpha^*)$ and $\Delta \in (0, \Delta^*)$ then the equilibrium block premium is strictly positive.

Proof of Proposition 11. Notice that

$$\begin{aligned}
\lim_{\gamma\to\infty} P_B &= \lim_{\gamma\to\infty} \Pi^*(\beta)/\alpha - \Delta - \lim_{\gamma\to\infty} p^*(y^*(\beta), \beta) \\
&= v(\beta, q^*(0, \beta)) - v(\mathbb{E}[b], q^*(0, \beta)) - \lim_{\gamma\to\infty} [\gamma y^*(\beta)] - \Delta \\
&= (\beta - \mathbb{E}[b]) H(q^*(0, \beta)) - \frac{1}{2}\left[\lim_{\gamma\to\infty} MPV(y^*) + (\beta - \mathbb{E}[b]) H(q^*(0, \beta))\right] - \Delta \\
&= \frac{1}{2}\left[(\beta - \mathbb{E}[b]) H(q^*(0, \beta)) - \lim_{\gamma\to\infty} MPV(y^*)\right] - \Delta
\end{aligned}$$

Recall $\lim_{\gamma\to\infty} MPV(y^*) \ge 0$. Thus, if $\beta \le \mathbb{E}[b]$ then $\lim_{\gamma\to\infty} P_B < 0$. Suppose $\beta > E[b]$. Thus, $\frac{1}{2}(\beta - \mathbb{E}[b]) H(q^*(0, \beta)) - \Delta$ is bounded away from zero, and if $\lim_{\gamma\to\infty} MPV(y^*)$ is sufficiently small then $\lim_{\gamma\to\infty} P_B > 0$. There are three cases to consider:

1. If in addition $G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}) < \beta < G^{-1}(\frac{1-\tau}{1-\alpha})$ then $\lim_{\gamma\to\infty} MPV(y^*) = 0$.

2. If in addition $\beta < G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$ then $q^*(0,\beta) = -G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})$, and

$$\lim_{\gamma\to\infty} MPV(y^*) = \frac{\tau}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(G^{-1}(\frac{1-\alpha-\tau}{1-\alpha})-\beta)$$

Notice that $\lim_{\alpha\to 0}[\lim_{\gamma\to\infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

3. Suppose $\beta > G^{-1}(\frac{1-\tau}{1-\alpha})$. Then, $q^*(0,\beta) = -G^{-1}(\frac{1-\tau}{1-\alpha})$, and

$$\lim_{\gamma\to\infty} MPV(y^*) = \frac{1-\tau}{1-\alpha}\frac{f(-G^{-1}(\frac{1-\tau}{1-\alpha}))}{g(G^{-1}(\frac{1-\tau}{1-\alpha}))}\frac{\alpha}{1-\alpha}(\beta - G^{-1}(\frac{1-\tau}{1-\alpha})).$$

Notice that $\lim_{\alpha\to 0}[\lim_{\gamma\to\infty} MPV(y^*)] = 0$, and hence, there exists α^* as required.

■

D.1 Analysis of market for votes

In this section we extend our analysis by adding a separate market of voting rights to the baseline model. In our model, dispersed shareholders are never pivotal, and hence, would be willing to supply their votes for an arbitrarily small amount. Hereafter, we assume the price of a vote is zero, and that the blockholder will not buy any vote if he is indifferent. For simplicity, we assume that trades of votes and voting shares are simultaneous. That is, the blockholder submits an order to buy y voting shares and a fraction $\lambda \in [0,1]$ of all voting rights that are held by dispersed shareholders (through their ownership of voting shares post-trade). Then trades take place in both markets. As in the market for voting shares, we assume that the blockholder does not observe the bias of individual dispersed shareholders when trading votes, and thus, votes are sold by dispersed shareholders in proportion to their ownership of voting shares.[35]

For any given trade (y,λ), the blockholder owns a total of $\alpha + y + (1-\alpha-y)\lambda$ votes, and each share owned by dispersed shareholders has the right for $1-\lambda$ vote. Thus, the blockholder is pivotal for the vote outcome if and only if

$$\begin{aligned} s(q;y,q_e^*)(1-\lambda) &< \tau < s(q;y,q_e^*)(1-\lambda) + \alpha + y + (1-\alpha-y)\lambda \Leftrightarrow \\ \frac{\tau-\lambda}{1-\lambda} - \alpha - y &< s(q;y,q_e^*) < \frac{\tau}{1-\lambda} \end{aligned} \tag{85}$$

Notice that the RHS increases in λ and LHS decreases in λ. Since $s(q;y,q_e^*)$ is an increasing function of q, we have $\beta_l(y,q_e^*,\lambda) \equiv -s^{-1}\left(\frac{\tau}{1-\lambda};y,q_e^*\right)$ decreases in λ and $\beta_h(y,q_e^*,\lambda) \equiv -s^{-1}\left(\frac{\tau-\lambda}{1-\lambda}-\alpha-y;y,q_e^*\right)$ increases in λ. Let $\beta_L(y,\lambda)$ and $\beta_H(y,\lambda)$ be the solutions of $\beta_L =$

[35] Enabling the blockholder to discriminate and buy votes only from shareholders with a certain bias would further increase the blockholder's ability to influence the identity of the marginal voter. However, such discrimination may not be feasible when biases are unobserved.

$\beta_l(y, -\beta_L, \lambda)$ and $\beta_H = \beta_h(y, -\beta_H, \lambda)$, respectively, then the marginal voter is given by

$$-q^*(y,\lambda) = \begin{cases} \beta_L(y,\lambda) & \text{if } \beta < \beta_L(y,\lambda) \\ \beta & \text{if } \beta_L(y) < \beta < \beta_H(y,\lambda) \\ \beta_H(y,\lambda) & \text{if } \beta_H(y,\lambda) < \beta. \end{cases} \tag{86}$$

Notice that $\beta_L(y,\lambda)$ decreases in λ and $\beta_H(y,\lambda)$ increases in λ. Intuitively, the blockholder's access to the market for votes further increases his ability to influence the identity of the marginal voter. In particular, there is a wider region in which the blockholder is the marginal voter (and pivotal). Indeed, for any $y \in [-\alpha, 1-\alpha]$, $\beta_L(y,1) = -\bar{b}$ and $\beta_H(y,1) = \bar{b}$. Thus, without other constraints on vote-trading, the blockholder can use the market for votes to ensure he is the marginal voter.

Proof of Proposition ??. Suppose in equilibrium the blockholder obtains his ideal marginal voter, that is, $-q^*(y^*,\lambda^*) = -q_B^*(y^*)$, where $-q^*(y^*,\lambda^*)$ is defined by (86) and $-q_B^*(y^*)$ is defined by (42). Notice that if y^* satisfies $q^*(y^*,\lambda^*) = q_B^*(y^*)$ then $MPV(y^*) = 0$. Therefore, the FOC implies $MPC(y^*) = 0$, that is, $y^* = z(-q_B^*(y^*))$ where

$$z(-q^*) \equiv \frac{1}{2\gamma+\eta}(\beta - \mathbb{E}[b])\, H(q^*). \tag{87}$$

Let q_B^{**} be the solution of

$$\begin{aligned} -q^* &= \beta + \frac{z(-q^*)}{\alpha}(\beta - \mathbb{E}[b]) \Leftrightarrow \\ -q^* &= \beta + \frac{1}{\alpha}\frac{1}{2\gamma+\eta}H(q^*)(\beta - \mathbb{E}[b])^2. \end{aligned}$$

Then, it must be $-q^*(y^*,\lambda^*) = -q_B^{**}$. But notice that $-q_B^{**} > \beta$. According to (86), $-q^*(y^*,\lambda^*) > \beta$ implies $-q^*(y^*,\lambda^*) = \beta_L(y^*,\lambda^*)$. Therefore, $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda^*)$. Notice that for every y, $\beta_L(y,\lambda)$ spans $[\beta, \beta_L(y,0)]$. Therefore, the ideal marginal voter is obtained in equilibrium as long as there exists $\lambda \in [0,1]$ such $-q_B^{**} = \beta_L(z(-q_B^{**}),\lambda)$. Since $\beta_L(y,\lambda)$ is a decreasing function of λ, the ideal marginal voter is obtained in equilibrium if and only if $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$. Notice that $\lim_{\gamma\to\infty} -q_B^{**} = \beta$ and $\lim_{\gamma\to\infty}\beta_L(z(-q_B^{**}),0) = G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$. Therefore, $-q_B^{**} < \beta_L(z(-q_B^{**}),0)$ holds for large γ if and only if $\beta < G^{-1}\left(\frac{1-\alpha-\tau}{1-\alpha}\right)$, as required. ■

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities.*

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (www.ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html