Cheap-Stock Tunneling Around Preemptive Rights

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Abstract

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Keywords: Controlling shareholder, tunneling, preemptive rights, rights issue, rights offer, equity issuance

JEL Classifications: G14, G18, G32, G34, G38, K22

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1 Introduction

Corporate insiders may engage in tunneling—transactions to transfer value from outside shareholders to themselves. Reducing tunneling is corporate law’s most basic function, as fear of tunneling undermines entrepreneurs’ ability to raise capital from outside investors ex ante (Shleifer and Vishny, 1997). Preemptive rights are the oldest and most widely-used tool for preventing one of the main forms of tunneling, which we in this paper call “cheap-stock tunneling”: an equity issue to the insiders at a low price that economically dilutes the interest of outside shareholders. To defend against cheap-stock tunneling, preemptive rights give all shareholders the right to participate pro rata in equity offerings. In listed firms, preemptive rights are implemented via “rights issues” in which a firm distributes to all shareholders pro rata rights to buy additional shares (Holderness, 2017; Massa et al., 2016).

The conventional view is that preemptive rights are effective against cheap-stock tunneling. According to the leading comparative corporate law treatise (Kraakman et al. 2017, p. 182), “preemptive rights . . . discourage controlling shareholders from acquiring additional shares from the firm at low prices.” Similarly, La Porta et al. (1998) included preemptive rights as one of six elements in their famous anti-director rights index because “in the absence of preemptive rights, insiders may expropriate minority shareholders by offering shares to related parties, or even to themselves, at below-market prices” (Djankov et al., 2008, p. 454). Around the world, issues featuring preemptive rights (“preemptive-right issues”) are quite common because many non-U.S. jurisdictions grant preemptive rights as a default whose waiver can require super-majority shareholder approval, regulator review, or both (Kraakman et al., 2017). And although preemptive rights are no longer the default in the U.S., the logic of preemptive rights still has currency in U.S. law: U.S. courts have rejected cheap-stock tunneling claims by minority stockholders on the grounds that the controlling shareholder had voluntarily offered pro rata participation in the issue, an offer often made by controllers precisely to cut off minority remedies (Fried, 2018b). (Henceforth, we will refer to insiders with the ability to approve (and set the price of) preemptive-right issues as “controllers,” and other shareholders as “the minority.”)

This paper shows, however, that preemptive rights provide only partial protection against cheap-stock tunneling when the controller knows that the shares are cheap but the minority, with inferior

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1 See, e.g., Johnson et al. (2000); Bertrand et al. (2002); Baek et al. (2006); Cheung et al. (2006); Jiang et al. (2010). For a typology of tunneling mechanisms, see Atanasov et al. (2014).

2 Under the corporate, securities, and stock-exchange listing rules that might apply to a particular firm, an equity issue by that firm must be approved by the board of directors; shareholder approval may also be required, depending on the circumstances (Kraakman et al., 2017). In our terminology, a “controller” is a party that controls enough board seats and shares to obtain all necessary approvals for the issue at the board and shareholder levels. By contrast, it is not necessary for a “controller” to have sufficient votes to waive preemptive rights for all shareholders, as we are precisely interested in what happens when the minority does have preemptive rights but does not control the decision whether to issue new stock, and at what price.
information, believes that the shares could be either cheap or overpriced. The crux of the matter is that cheap-stock tunneling is not all the minority needs to worry about when deciding whether to exercise their preemptive rights: They also need to worry that the controller might have set the offer price high, either because the controller hopes to sell overpriced shares to others or because the controller expects to privately benefit from the issue proceeds. While preemptive rights clearly cannot solve these two other problems, our novel insight in this paper is that the mere possibility of their presence partially undermines preemptive rights even in the domain where these rights are thought to be effective: cheap-stock tunneling. If the minority cannot figure out whether the offer is cheap or overpriced, the minority is damned if it participates in the issue and damned if it does not (at least probabilistically). In equilibrium, we show that some minority shareholders will not exercise their preemptive rights when the price is in fact (and unbeknownst to them) cheap and thus cheap-stock tunneling will occur. We explain this mechanism first in an extended numerical example (section 3) and then in a general model (sections 4 and 5).

One way to understand the situation is that because the new shares represent a claim on the firm's initial assets, the issue puts on the market an asset that partly belongs to the minority. However, this market is one in which the controlling shareholder has an information advantage. As is well understood, the trader with an information advantage gains in trading, and uninformed traders will avoid trading with an informed trader (cf. Milgrom & Stokey 1982). In our setting, however, the minority shareholders have no choice: thanks to her control, the controller can put the minority's asset (i.e., the minority's share of the firm) on the market, i.e., force them to sell, whether they like it or not. Consistent with this, in our model, rational minority shareholders do not lose as buyers of the asset—the equilibrium price is equal to the asset's expected value conditional on the price—but as sellers: the price they receive is on average too low. In other words, cheap-stock tunneling occurs, preemptive rights notwithstanding. Preemptive rights are still useful to the extent the controller does not have an information advantage, i.e., at prices that everyone, even the outsiders, understand to be either good or bad. But preemptive rights do not protect the minority from being dragged into a rigged market as sellers against their will.

Our analysis provides an additional reason why regulators of listed firms supplement preemptive rights with other mechanisms to reduce cheap-stock tunneling, such as director fiduciary duties (Ventrüzzo, 2013) and supermajority voting requirements (Kraakman et al., 2017), and may even block

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3Controllers frequently serve as senior executives and directors of their firms (Broughman and Fried, 2018; Fried et al., 2018, Claessens et al., 2000), providing them easy access to inside information.

4For the avoidance of doubt, the minority's losses from cheap-stock tunneling around preemptive rights—the focus of our analysis—are above and beyond their losses from purchasing overpriced shares or the diversion of issue proceeds.

5To be precise, and using notation introduced below, \( \frac{\alpha v}{v + pq'} \) of the asset on the market “belongs to the minority,” i.e., represents the pre-issue share of the minority, where \( q' \) is the amount of stock actually issued.
preemptive-right issues, as has occurred in Hong Kong (Charltons, 2016). Similarly, our analysis can help explain why sophisticated investors in unlisted firms, such as VCs, insist on issue-veto rights alongside pro rata participation rights (Bengtsson, 2011). Finally, our analysis sheds light on minority-shareholder and controller behavior around equity issues, particularly rights issues by listed firms with controllers (see infra section 6.4).

We assume that minority shareholders and the controller are sophisticated, risk-neutral, and neither liquidity constrained nor otherwise unable to exercise their rights; all of the effects we identify require only information asymmetry. We thus abstract from the additional problems that would arise if preemptive rights were procedurally unworkable or if minority shareholders were unsophisticated, risk-averse, or liquidity constrained, in which case the controller might well maximize cheap-stock tunneling profits by setting the price so far below the value of the shares that the underpricing would be plain to any rational observer. Nor do we consider the possibility that the controller itself is liquidity constrained, which could also lead the controller to set the price obviously low, but for a different reason: namely, the controller wants the firm to raise a certain amount of capital (for business reasons or to increase private benefits) and wishes to induce the minority (or other outsiders) to contribute the needed capital.

Our paper proceeds as follows. Section 2 situates our paper in the literature. Section 3 illustrates the main idea of the present paper with an extended numerical example involving an unlisted firm. Section 4 describes and solves the basic model in general form where asymmetric information pertains to the value of the assets in an unlisted firm and the number of shares to be issued is fixed. Section 5 modifies the model such that the asymmetric information pertains to the controller’s private benefits from the issue. Section 6 discusses how the analysis would be affected by endogenizing the number of shares to be issued, combining the two sources of information asymmetry, adding concerns about voting power, or listing the firm’s shares. Section 7 concludes.

2 Related Literature

Our paper is the first to model the interactions of a controller and minority shareholders around an equity issue in which the controller has superior information, and to evaluate how preemptive rights change this interaction. The paper most closely related to ours is Atanasov et al. (2010), which derives formulas for the accounting and stock price impact of expected and actual equity tunneling and low-price freezeouts in connection with an empirical study of investor-protection reforms in the Bulgarian stock market. Atanasov et al. (2010) take as given the probabilities of, and discounts applied in, such tunneling (and freezeouts), which we model explicitly. They then show that the ex ante stock price
will be higher if there are preemptive rights or safeguards against low price freezeouts, and higher still if there are both. Although they do not focus on cheap-stock tunneling per se, they note, consistent with our results, that minority shareholders will not use preemptive rights to participate in a discounted offering if the risk of a subsequent low-price freezeout is high.

More broadly, our paper is connected to the literature on equity issues under asymmetric information. Most theoretical work in this area assumes that managers seek to maximize value for all existing shareholders (cf. Stein, 2003), and derives implications for the choice between debt and equity financing (e.g., Myers and Majluf, 1984), as well as between different methods of equity issues, in particular between rights issues and underwritten offers (e.g., Heinkel and Schwartz, 1986; Eckbo and Masulis, 1992; Burkart and Zhong 2018). By contrast, we model the conflict of interest between different groups of shareholders, while assuming that financing takes the form of a preemptive-right issue. In our model, managers are under the control of a controlling shareholder, and seek to maximize that controlling shareholder’s benefit at the expense of other investors. In this context, we ask if and to what extent preemptive rights—or equivalently, limiting a listed firm to the rights-issue method—protect the minority from cheap-stock tunneling, an issue outside the purview of models that assume managers seek to benefit all current shareholders.

Our focus on the insider-outsider conflict is more closely connected to recent empirical work on equity issues by listed firms. In particular, Holderness (2017) points out that announcement returns tend to be negative for issues that do not require shareholder approval, but positive for those that do, suggesting that agency conflicts are of first-order importance in equity issues. Importantly, Holderness (2017) finds that announcement returns tend to be negative even for rights issues that do not require shareholder approval. This finding is consistent with our main conclusion: that preemptive rights by themselves do not protect a less-informed minority from expropriation via issue mispricing (although obviously such negative announcement returns are also consistent with other forms of expropriation, such as issue proceeds being diverted or used for empire building. 

Two papers focus specifically on equity issue choices of controlled listed firms when the controller’s interests diverge from those of other stockholders. Cronqvist and Nilsson (2005) find empirically, and Wu et al. (2016) show theoretically, that controllers tend to use rights issues rather than underwritten offers to third-party investors, and suggest that this choice is due to controllers’ desire to preserve voting control and private benefits. Unlike us, however, they do not consider the possibility of expropriating

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6Our analysis assumes that the controller has enough votes to unilaterally obtain any required shareholder approval for a preemptive-right issue. From the perspective of our model, the finding in Holderness (2017) that shareholder approval affects the quality of preemptive-rights issues indicates that insiders do not always have unilateral power to effect such issues, and must sometimes obtain some minority support.
the minority through the issue.

3 A Numerical Example

Consider an unlisted firm with 100 shares and $100 of assets in place. Half the stock (50 shares) is owned by a controlling shareholder; the remaining 50 shares are owned by minority shareholders who own one share each. Thus, barring tunneling, each share is worth $1. Applicable law permits the firm to issue 100 additional shares at a price set by the board, which acts in accordance with the controller’s wishes. The proceeds of any equity issue will increase the firm’s assets by an equal amount.

3.1 Cheap-stock tunneling with full information

As an example of straightforward cheap-stock tunneling, consider an issue of 100 additional shares for price zero only to the controller (directly, or indirectly via related parties). The firm will still have assets worth only $100 because no new assets enter the firm. However, the amount of shares outstanding will now be double, namely 200, and hence each share will be worth only $0.50. The minority, who will not receive any new shares, will thus lose half the value of its pre-issue stake. By contrast, the controller, who will now own 150 shares or 75% of the firm, will see the value of her stake increase to $75. $25 of value will have changed hands.

Under full information, preemptive rights would effectively protect the minority against the foregoing cheap-stock tunneling. If each minority stockholder had the right to participate in the issue at the same price as the controller, each minority shareholder could (and would) also buy one new share at price zero, concomitantly lowering the amount of new stock that the controller can “purchase.” The amount of stock outstanding would still double to 200 shares but, as each shareholder’s holding would increase proportionally, nobody would gain or lose from the issue, which would be tantamount to a simple stock split.

3.2 Asymmetric information about the value of assets in place

Realistically, however, minority shareholders are likely to know less than the controller about the value of the firm’s assets in place, especially in an unlisted firm where there are no or minimal mandatory disclosure requirements. And the controller may wish to use its superior information about the value of firm assets to expropriate value from the minority via an issuance of shares. In our example, although the assets in place appear to be worth $100, they might well be worth less. To simplify, assume they
might actually be worth $0. Perhaps there have been hidden business losses of $100 or the controller has already tunneled out the $100 that was previously there. The controller would know if this is the case, but outsiders would not. Imagine that there is a 50-50 chance that the firm is of either type ($100 firm or $0 firm).

At price zero, it would still be safe to buy the new stock. Now imagine, however, that the controller sets a price of $0.37 per share. At that issue price, the stock would be vastly overpriced if the firm’s pre-issue assets were worthless: Assuming full subscription of the issue, each post-issue share would be worth only half the issue price, namely \( (\$0 + 100 \times \$0.37) / 200 = \$0.185 \) (initial value plus money raised, divided by post-issue share count). On the other hand, the stock would be considerably underpriced if the firm’s pre-issue assets were actually worth $100: in that case, each post-issue share would be worth almost twice the issue price, namely \( (\$100 + 100 \times \$0.37) / 200 = \$0.685 \).

Not knowing which type of firm they are dealing with, what should minority shareholders do? If they buy, they might end up vastly overpaying. If they don’t buy, they might end up getting economically diluted by cheap-stock tunneling.\(^7\) Of course, the controller would not want to overpay or get diluted either, so the controller’s participation decision would reveal the firm type and hence the right course of action – if the controller’s decision were known, which is often not the case.\(^8\) In fact, controllers have good reason to keep the minority in the dark. By doing so, they force minority shareholders to choose one of the two responses just mentioned, each of which will benefit the controller in one type of firm: minority shareholders who do not buy allow the controller of the $100 firm to buy cheap stock at the expense of those non-buying shareholders (their loss is the controller’s gain), whereas minority shareholders who do buy increase the value of the shares in the $0 firm through their overpayment (this benefits all other shareholders, including the controller).

Nevertheless, minority shareholders must choose. Whether participation is better or worse in expectation for a minority shareholder depends on the controller-set issue price and minority shareholders’ subjective probability assessments, given that price, about firm type and other minority shareholders’ responses. For example, in the above numerical example, a minority shareholder will be approximately indifferent between participating or not at a price of $0.37 per share if that shareholder expects controllers of both $100 firms and $0 firms to set that price and 16 other minority shareholders to participate, because in that case the expected value of purchasing such a share is

\[
\frac{1}{2} \left( \frac{\$100 + 100 \times \$0.37}{200} \right) + \frac{1}{2} \left( \frac{\$0 + 100 \times \$0.37}{200} \right) - \frac{1}{2} \frac{\$0 + 17 \times \$0.37}{100 + 17} \approx \$0.37. \quad (\text{This takes into account that the controller will snap up})
\]

\(^7\)Here and elsewhere, we speak of “they...getting economically diluted” (etc.) in a loose way. In our model, minority shareholders are atomistic, and individual minority shareholders might actually gain in the issue: this will happen if they, like the controller, subscribe to more than their pro rata share. See the main text, two paragraphs down.

\(^8\)No jurisdiction appears to require the controller of an unlisted firm to disclose this decision (Fried, 2018a).
every remaining share if and only if the price is low.) As we show in general form below, this price and participation decision is in fact the only equilibrium behavior in this example.

Importantly, minority shareholders in both types of firms lose even though the issue price is fair in expectation, i.e., averaged across both types of firms:

- In the $100 firm, minority shareholders lose because the value of their existing equity drops from $1 per share to $0.685 per share. Again, for such a firm the issue is underpriced at $0.37 per share, and minority shareholders lose as the controller snaps up a disproportionate number of new shares at this low price. True, participating minority shareholders also buy at the low price, and profit from each share purchased. But for minority shareholders as a group, participating shareholders' gains are more than offset by their and non-participating shareholders' losses from dilution of their existing shares. These are losses through cheap-stock tunneling that occur in spite of preemptive rights.

- In the $0 firm, it is now participating minority shareholders' turn to lose, as the shares they buy at $0.37, increasing their proportional interest, are overpriced. The controller gains at participating minority shareholders' expense. Again, non-participating minority shareholders gain as well, but for minority shareholders as a group, these gains are more than offset by the losses of the participating minority shareholders.

In either case, the ultimate intuition is simple: the controller knows when to buy and when not to buy and thus always does the profitable thing, whereas the minority is unsure and hence buys some and only some in either case. Thus the controller extracts value from some minority shareholders either way, whether it is a $100 firm (through cheap-stock tunneling) or a $0 firm (through the sale of overpriced stock).

The underlying source of minority shareholders' losses illustrates the crucial difference between our setting and the standard problem of equity issue with asymmetric information (e.g., Myers and Majluf, 1984). In both the standard equity issue and our settings, the equilibrium issue price is “fair” for buyers: in expectation, those buying shares do not profit or lose from buying the issued shares. However, only in our setting do outside shareholders also already own shares of the firm. And these existing shares, in expectation, decline in value as a result of the issue. In the $100 firm, the drop in share value is $1 − $0.685 = $0.315 (which is not offset by the much smaller gain in the value of the existing shares of the $0 firm, \( \frac{80 + 17 \times 0.37}{100 + 17} - 0 = 0.054 \)). This is precisely the type of loss that preemptive rights are supposed to protect against. But the protection fails because the minority can never be sure that this is what is going on in any given issue. As a result, minority shareholders in our example collectively
lose $50 \times 0.315 - 17 \times (0.685 - 0.37) = 10.40$ of the $100$ firm, or over 40\% of what they could have lost if they had no preemptive rights, in which case the controller could have issued stock to herself at price zero and appropriated $25$.

### 3.3 Asymmetric information about the controller’s ability to divert issue proceeds

Things get even worse for minority shareholders if the asymmetric information pertains to the controller’s ability to divert some of the issue proceeds, or to obtain non-pecuniary private benefits from the issue. For example, the issue proceeds might or might not be used to purchase overpriced assets or securities from the controller. In principle, such lopsided self-dealing transactions can occur even without a new issue of stock, but often they require what only a stock issue can deliver: fresh cash. Alternatively, the controller may simply enjoy running a larger firm. All that matters is that the controller derives a benefit from the issue that is not shared with the minority.

To be sure, a controller’s ability to benefit disproportionately from the proceeds of the issue is likely to depend on the minority’s ability to monitor the firm and applicable legal restrictions on self-dealing. But such disproportionate benefits from an issue could be substantial even if the minority is able to prevent lopsided self-dealing between the controller and the firm. Pecuniary benefits may be generated not by explicit self-dealing transactions between the firm and another party, but rather via transfers of value among different types of securities already issued by the firm (Fried, 2018b). For example, a pro rata issuance of common stock may disproportionately benefit the controller if it, but not the other shareholders, holds (or has guaranteed) a loan to the firm whose value is increased by the equity issue. And non-pecuniary benefits are and will always be beyond the reach of the law, as they are undetectable. Thus, even if the minority knows there will be no self-dealing transactions, there is likely to be asymmetric information over the extent of private benefits from the issue. In situations where lopsided self-dealing transactions may occur, the asymmetry is likely to be more severe.

We now construct an example along these lines. Imagine that it is known to all with certainty that the value of the firm’s initial assets (say, a machine) is $100$, but that there is a 50-50 chance that the controller, after having the firm issue new 100 shares, can divert all issue proceeds into her own pockets. (Equivalently, there might be a 50-50 chance that the project into which the firm will invest the issue proceeds is worthless in cash flow terms but generates a non-pecuniary psychic benefit to the controller equal to the money invested. Our analysis would be exactly the same. For concreteness and simplicity,

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\(^9\)Such transactions appear common in Hong Kong listed firms (Fong and Lam, 2014; Kim et al., 2015).
however, we will speak of diversion of the cash proceeds throughout.) The controller knows whether it will divert; minority shareholders do not.

If all knew the controller can divert these funds, no outside stockholder would be willing to buy the new stock at a price equal to the pre-issue pro-rata value of $1 per share, as the post-issue value would be only half that: $\frac{100 + 10}{200} = 0.50$. But any stockholder, new or old, would be willing to buy at the post-issue value of $0.50$: while the issue proceeds themselves would be diverted, a new share would still represent a 1/200 claim on the initial pool of assets worth $100, and hence would be worth $0.50. Of course, this issue-related diversion would be at the expense of the initial minority shareholders. Meanwhile, the controller would gain by appropriating the issue proceeds, which would more than offset her losses on her existing stock. Regardless of who buys the shares (the controller, existing minority investors, or new investors), and after subtracting any amounts paid to buy stock in the issue, the controller would own shares and diverted funds worth $75, while the minority would be left with $25.

With asymmetric information, however, the controller with the ability to divert can do even better, and controllers without the ability to divert gain by the mere possibility that some controllers can divert. The reason is similar to the previous example with asymmetric information about initial asset value. For prices between $0.50$ and $1$, minority shareholders do not know if the stock is over- or under-priced. If they buy, and if diversion ensues, they will have overpaid. But if they do not buy, and there was not going to be any diversion, the controller expropriates through cheap-stock tunneling. Whatever minority shareholders do, they will lose some of the time. Now, however, things are even worse for the minority than in the no-diversion setting discussed in section 3.2 because the controller’s purchase decision, even if known and credible, is no longer revealing about the type of firm/controller. The reason is that from the controller’s perspective, the issue cannot be overpriced in either scenario. In the no-diversion scenario, the shares are worth $1$. In the diversion scenario, the controller receives a “rebate,” so to speak, on the full price of the stock, reducing its effective price (to the controller) to zero. Thus, even if the minority were to know the controller’s purchase decision, they could not figure what to do.

As before, minority shareholders’ purchase decisions and the controller’s price choice are interdependent. We show below in general form that in our example, the only equilibrium is for both types of controllers to set a price of $\frac{2}{3}$ and for $\frac{1}{3}$ of the minority shareholders to participate, with the controller buying the rest of the stock. In our example, this means that the controller who can divert ends up with $77.78$, even more than the $75$ this controller could have obtained if minority shareholders had full information about the diversion ability: this controller is making a gain from the sale of overpriced stock on top of the gain from diverting issue proceeds. More interestingly and importantly, however,
the mere possibility that some controller may divert proceeds enables even a no-diversion controller to tunnel out $5.56 through an issue of cheap stock despite the minority having preemptive rights.

In all of the preceding examples, the problem is not that minority shareholders do not have preemptive rights or that they are worthless, but that the controller will set the price such that minority shareholders will be indifferent between exercising their rights or not. Preemptive rights will prevent the controller from doing the worst (issue at price zero), but this does not mean that cheap-stock tunneling disappears. We now explore these issues more systematically and formally.

4 Asymmetric information about value of assets in place

4.1 Model Setup

Consider an (unlisted) firm with two types of stockholders: a controlling stockholder, and a continuum of atomistic minority stockholders who do not coordinate their actions. (Qualitatively, nothing would change if we modeled the minority as a single, coordinated block.) There is initially one share divided into infinitesimally small increments. Collectively, minority stockholders initially own fraction $\alpha \in (0,1)$ of the stock. The firm is risky: with unconditional probabilities $\rho$ and $1-\rho$, the value of assets in place, $v$, is either $1-\delta_a$ or $1$, respectively, with $\delta_a \in (0,1)$. The controller observes the realization of $v$; the minority does not.

The firm now issues a quantity $q > 0$ of new stock at price $p > 0$ per share, $p,q \in \mathbb{R}_{\geq 0}$. The controller chooses $p$, whereas we treat $q$ as exogenous for now, an assumption we discuss and relax in section 6.1. Existing shareholders have preemptive rights, i.e., they are guaranteed an allotment proportional to their existing stake if they wish to subscribe to the new issue. We denote $\Delta$ as the fraction of minority shareholders’ preemptive rights that is exercised. To the extent some existing shareholders do not subscribe, we assume that others can pick up the remaining shares. In particular, the controller will want to purchase any leftover shares when the firm value is high.

To focus on the effect of interest, we assume that the firm invests the issue proceeds in a zero NPV project. That is, the social value of the firm’s use of the issue proceeds is exactly equal to $pq$. For

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10 Technically, what matters is the expected value of the assets in place that will be available for pro rata distribution to all shareholders. Anticipated future diversion depresses that expected value, including anticipated future diversion through an equity issue as modeled here or in the next section. Hence asymmetric information about the probability of future asymmetric information is enough to generate the effect we model here!

11 We do not restrict $\Delta$ to lie in $[0,1]$, but it will be seen that only $\Delta \in (0,1)$ is consistent with equilibrium (see Appendix B.1).

12 We do not explicitly include new outside investors (i.e., other than the existing shareholders) in our model, but the optimality condition for minority shareholders’ purchase decision would be the same for an outside investor; in this sense, our model thus applies fully to listed firms as well (see section 6.4 below).

13 Allowing for investment at a loss or profit would enhance the realism of the model but not the economic insight. This reflects a key difference to the well-known model of Myers and Majluf (1984), which also involves asymmetric information.
example, the firm might just invest the receipts in treasury securities or some diversified portfolio. Note that with zero NPV investments, the following holds:

**Lemma 1.** *In the absence of private benefits diverted from the issue, participating in the issue is profitable if and only if the issue price is less than the realized per-share value of the assets in place \( p < v \), and breaks even when they are equal \( p = v \).*

*Proof.* See Appendix.

The lemma would be self-evident except that for existing shareholders, there is an attenuating effect to overpaying or underpaying for new shares: to the extent investors overpay (underpay), the value of their existing shares goes up (down). But the latter effect is always smaller than the former because the latter effect is shared with all other existing shareholders, while the former effect is borne by the investor alone. Buying into a firm at a price above (below) pro rata value is a losing (winning) proposition, even taking into account the technical complication just mentioned.

**Timeline of the model.** The timeline of the model is as follows, with two variants of period 2:

1. The controller privately observes realization of the value of assets in place and then announces \( p \).
   (Equivalently, the controller might observe the realization of the investment opportunities of the firm – nothing would change in substance.)

2. Both minority shareholders and the controller announce their participation decision, i.e., whether they will buy their allotted shares:
   (a) variant 1: the controller announces first;
   (b) variant 2: the controller announces second or contemporaneously.

3. The firm is liquidated and proceeds distributed pro rata according to share ownership.

We now describe the perfect Bayesian equilibria (PBE) for each variant.

**4.2 Variant 1: controller announces first**

If the controller has to announce her (binding) participation decision up front, all parties receive zero net payoffs (i.e., relative to the pre-issue status quo) in all PBE of the model. The reason is that all parties about the value of the existing assets in the firm. In Myers and Majluf, the high type firm, whose stock will be underpriced in equilibrium, would not conduct the equity offering unless it is compensated for the underpricing by a sufficiently large positive NPV from the growth opportunity to be financed by the offering. By contrast, in our model, the controller of the high type firm profits from cheap-stock tunneling, and hence does not require a positive NPV project. Focusing on zero NPV projects helps to emphasize this important difference.
have a strategy that ensures they obtain at least zero, and the game is zero sum. The minority can guarantee a zero payoff by mimicking the controller, while the controller is guaranteed zero by setting $p = v$.\textsuperscript{14}

### 4.3 Variant 2: controller does not announce first

If the controller need not announce first, then there is a unique equilibrium price equal to a weighted average of 1 and $1 - \delta_a$ and some minority shareholders buy and some do not (a possibly stochastic decision at the individual level) because they are unable to tell if true firm value is low and hence the price too high, or if the true firm value is high and hence the price a bargain. Obviously, the controller only participates if it is a bargain. In expectation, the controller gains and the minority loses. Ex post, the controller always gains and at least some minority lose.

Concretely, we have

**Proposition 1.** If there is asymmetric information about the value of the assets in the firm and the controller does not need to announce first, then there exist only pooling perfect Bayesian equilibria in which the controller always sets the price $p^* \equiv 1 - \frac{\delta_a \rho (1 + q)}{1 + \rho q + \alpha q (1 - \rho)/r} \in (1 - \delta_a, 1)$ and buys if and only if $v = 1$, and a fraction $\Delta^* \equiv \frac{1 - \rho}{1 + \rho q} \in (0, 1)$ of the minority rights are exercised; the purchase decisions of individual members of the minority as well as minority shareholders’ off-equilibrium beliefs and purchases are not unique.\textsuperscript{15}

**Proof.** See Appendix.

The intuition for the proof is that by pooling on an intermediate price in the range $(1 - \delta_a, 1)$, the two types of controllers can force minority shareholders into at least one of two decisions that are bad for them: buying overpriced stock if the value of assets in place is low, or allowing the controller to cheap-stock-tunnel if the value of assets in place is high. The closer the price is to 1, the higher the losses from buying overpriced stock; and vice versa for prices closer to $1 - \delta_a$. In equilibrium, minority

\textsuperscript{14}Even if the price is high (above post-issue share value) and the minority follows the controller and buys, the minority would not lose anything because the controller equally overpays (and hence firm value grows proportionally). If the price is low and the controller does not buy, the minority misses an opportunity by not buying, but at least it does not lose anything.

Thus, there is an infinity of PBE in which the controller announces some price-participation combination and the minority mimics. The only price-participation combinations that cannot be part of these equilibria are those involving $(p < v_l, \text{not buy})$ and $(p > v_h, \text{buy})$. The only non-mimicking that can occur in equilibrium is when the controller sets $p = v_l$ and does not buy, or when the controller sets $p = v_h$ and does buy; the controller would do so in equilibrium only when these are actually the realized values. In any event, the controller and the minority both receive zero payoffs in any of these equilibria.

\textsuperscript{15}These formulas reflect (a) the normalization of the initial outstanding stock to 1, such that the value per share and the value of the firm coincide and $q$ is both the number of new shares issued and the ratio of new stock to old stock, and (b) the normalization of the high asset value to 1. If either the initial number of shares or the high asset value differ from 1, the right hand side of the price formula must be divided by the former and multiplied by the latter to obtain the correct price per share. In either case, $q$ must be input as a ratio.
shareholders balance the two risks and buy intermediate amounts that decline with the price. This allows both types of controllers to earn a positive profit.

By contrast, in a separating equilibrium, both types of controllers would earn zero profit because type revelation would allow the minority to buy if and only if the price is advantageous for them. But no off-equilibrium minority reaction can sustain such a zero-profit equilibrium: for any $\Delta(p)$ at any $p \in (1 - \delta_a, 1)$, at least one of the controllers earns a positive profit and hence would have an incentive to deviate from the candidate equilibrium. Ultimately, the reason that the controllers inevitably gain in any equilibrium is that the two controller types have inverse preferences over minority participation $\Delta$, and hence what hurts one helps the other, making it impossible for the minority to “fend off” both.

How does this situation compare to one without preemptive rights? In that latter case, the worst that can happen to the minority is that the controller issues $q$ shares at price zero to herself. This would increase the controller’s stake to $\frac{1 - \alpha + q}{1 + q}$ while not affecting the value of the firm. The minority’s stock, worth $\alpha v$ before the issue, would be worth $\frac{\alpha}{1 + q} v$ after the issue, for a loss of $L_{NPR}^{\rho} (v) \equiv \alpha v \frac{q}{1 + q}$. (On the other hand, with preemptive rights and no asymmetric information and no diversion of issue proceeds, the minority loses nothing from an issue.)

The most interesting case with asymmetric information is “the high type” ($v = 1$), as here actual cheap-stock tunneling occurs despite preemptive rights. Some tedious algebra shows that the minority loses (and the controller gains) $L_{h}^{PR} = \frac{\delta_a (1 + q)^2 \rho^2}{(1 + \rho q)^2 + \alpha q (1 - \rho)^2} L_{NPR}^{PR} (1)$ from participating less than pro rata in the issue of what turns out to be cheap stock. To repeat the explanation given above, the reason for the minority not to participate fully is that they are concerned about falling into the opposite trap, which is to buy overpriced stock. This intuition can be seen in the comparative statics for $\rho$. As the fraction of low value firms $\rho$ increases, the probability of buying overpriced stock increases as well, and hence minority shareholders are more reluctant to participate – and lose more from non-participation in the (rarer) case that the firm is, in fact valuable. In the limit as $\rho \to 1$ or $q \to \infty$, the minority loses a fraction $\delta_a$ of what it would have lost without preemptive rights. In that limiting case, when $\delta_a = 1$ (minority shareholders fear that the firm is worthless) minority shareholders lose as much as if they did not have preemptive rights (intuitively, nobody would want to buy worthless stock, so preemptive rights are irrelevant in this limiting case). At the other extreme, the minority’s losses in the “high type” firm tend to zero as $\rho \to 0$ (or, obviously, as $q \to 0$): when minority shareholders are virtually sure that the firm is valuable ($\rho \approx 0$), they risk little by exercising their preemptive rights, avoiding cheap-stock tunneling.

\footnote{In our simple model with zero NPV investments, minority losses from the issue (if any) are equal to the controller’s gain from the issue.}
On the other hand, more tedious algebra shows that minority shareholders of the low value firm \((v = 1 - \delta_a)\) lose \(L_{pr}^l \equiv \frac{\delta_a \alpha q (1-\alpha)(1-\rho)^2}{(1+\rho q)^2+\alpha q(1-\rho)^2}\). The source of these losses is not cheap-stock tunneling but the purchase of overpriced stock. In expectation—i.e., before the firm’s type is revealed—this (at that point, merely probabilistic) loss is exactly offset by the (then still possible) gain of purchasing underpriced stock in the valuable firm.

Naturally, in both high and low value firms, the minority’s loss, and the controller’s gain, increases in the value difference \(\delta_a\), which measures the value relevance of the information asymmetry in this model. The minority’s total expected losses from stock issues, \(\rho L_{pr}^l + (1 - \rho) L_{pr}^h\), also tend to be larger when the information asymmetry itself—as measured by the entropy—is larger, i.e., when \(\rho\) takes on intermediate values, and tend to zero if \(\rho \to 0\) or \(\rho \to 1\).

5 Asymmetric information about controller’s private benefits from the issue

We now consider asymmetric information about the controller’s private benefits from the issue. These issue-related private benefits should not be confused with private benefits that have arisen or will arise regardless of the size of the new issue (i.e., even if \(qp\) were zero). The case of asymmetric information about issue-unrelated private benefits is merely a variant of the previous model (asymmetric information about the value of assets in place), and the mimicking defense would continue to work: we can reinterpret \(\delta_a\) as the fraction of the initial assets possibly diverted by the controller, either before or after the issue.\(^{17}\)

By contrast, the issue-related benefits on which we focus in this section derive specifically from the amount of issue proceeds. They have different implications that we now analyze in more detail. In theory and practice, the first and second type of private benefits could be combined, such as when private benefits scale proportionally with firm size. To emphasize the conceptual difference, however, we first build a model only with the second type, and defer consideration of the combination of types of private benefits to section 6.2 below.

As before, we consider an issue \(q > 0\) of new stock at price \(p > 0\) set by the controller, where atomistic minority stockholders initially own fraction \(\alpha \in (0, 1)\) of the one initial share outstanding and have preemptive rights. However, we now fix the value of the firm’s initial assets at \(v = 1\), and instead introduce asymmetric information about whether some of the issue proceeds will accrue to the controller.

\(^{17}\)Cf. footnote 10 above.
in private benefits rather than to the corporation to be shared among all shareholders: with probability \( \rho \), a fraction \( \delta_i \in (0, 1) \) of the issue proceeds benefits only the controller. This can be thought of as describing different kinds of controllers: those that can obtain private benefits, who comprise a fraction \( \rho \) of the population of controllers, and those who cannot, who comprise a fraction \( 1 - \rho \). Alternatively, it can be thought of as describing the same individual controller who may or may not have an opportunity to divert issue proceeds. In either case, the parameters \( \rho \) and \( \delta_i \) are common knowledge, but only the controller herself knows her type.

As we already said in motivating our example in section 3.3 above, the private benefits \( \delta_i \) can take various forms and, importantly, do not need to be pecuniary.\(^{18}\) What matters is that the controller and the minority shareholders derive different payoffs from the issue. This could be because the controller diverts some of the cash, as in having the corporation use the proceeds to make a payment to the controller for an overpriced asset. But it could also be because the firm will invest the issue proceeds \( pq \) in a way that yields less than \( pq \) in present value of cash flows to the firm but an offsetting private benefit to the controller, such as the ability to run a larger firm or schmooze with the stars at an event sponsored by the firm. In either case, we continue to assume that the social value (i.e., including the controller’s private benefits) of the investment of the issue proceeds is exactly \( pq \), even though less than \( pq \) accrue to the firm. From now on, we will refer to the private benefits as diversion for simplicity, but the reader should keep in mind the broader interpretation.

We begin by formalizing an important observation about differential reservation prices that we mentioned informally in the introduction.

**Lemma 2.** If the controller diverts a fraction \( \delta_i \) of the issue proceeds into her own pockets, participating in the issue is strictly profitable for the controller if and only if \( p < \frac{1}{1-\delta_i} \), i.e., even for prices above the pro-rata value of the assets in place before the issue (which equals 1). By contrast, other shareholders, regardless of whether they owned any stock before the new issue, find participation profitable only for prices below the pro-rata value of the assets in place before the issue: Given that a total \( q'' \in [0, q] \) new shares will be subscribed to, atomistic shareholders find participation strictly profitable if and only if \( p < \overline{p}(q'') \equiv \frac{1}{1+\delta_i q''} \leq 1 \); non-atomistic shareholders (other than the controller) have even lower break-even prices. If the offer is fully subscribed, atomistic minority shareholders’ reservation price is thus \( \overline{p} \equiv \overline{p}(q) = \frac{1}{1+\delta_i q'} < 1 \).

**Proof.** See Appendix. □

\(^{18}\)Nor do the private benefits need to accrue to the controller instantaneously. Of particular interest, the private benefits could consist of the possibility to extract value through a follow-on equity issue in the future.
Lemma 2 formalizes the intuition that the controller’s and outside shareholder’s valuations of the new stock diverge when the controller can divert some of the issue proceeds. For outside shareholders, the value of a new share is only the post-issue pro-rata value of the firm net of the funds diverted by the controller. By contrast, for the controller, the value of a new share is the value to outsiders plus the private benefits obtained from the issue. Without private benefits, a price equal to the pre-issue pro-rata value of the firm’s assets would be break-even for both the controller and outsiders, as the issue proceeds would proportionally increase the value of the firm, such that the pro-rata value of the firm would be the same pre-issue and post-issue. With private benefits, outsiders do worse while the controller does better, because some of the issue proceeds flow only to the controller. The last point holds even if the controller is the only buyer of the new stock because the controller appropriates some of the value of the minority’s pre-issue stock: as long as \( p < \frac{1}{1 - \delta_i} \), the issue proceeds net of private benefits do not increase the value of the firm proportionally to the amount of new stock issued. That said, the controller obviously prefers paying a lower price for her stock if she is the only buyer, and would like it even better if the minority bought the stock at a higher price.

As a consequence of lemma 2, the mimicking defense no longer works when the controller derives private benefits from the issue. Since the controller and outside stockholders no longer have the same valuation for the stock, it is not safe for outside stockholders to buy when the controller does – outside stockholders might be overpaying, while the controller is not. Thus, even if the controller publicly and credibly pre-commits to participate in the issue, outside stockholders will not know if the price is high or low from their perspective. Outside stockholders are thus caught between a rock and a hard place. They can participate at the risk of overpaying if the controller is able to divert some of the proceeds, or they can decline to participate at the risk of letting the controller snap up stock on the cheap. Both types of controllers benefit from the outside stockholders’ dilemma. Controllers with private benefits exploit the minority’s fear of cheap stock tunneling to trick the minority into buying stock at a price above the value of the stock to the minority. Controllers without private benefits exploit the minority’s fear of the latter to trick the minority into not buying at a price below pro rata firm value. As a result, the minority loses money in expectation. Proposition 2 formalizes and quantifies this intuition.

**Proposition 2.** If there is asymmetric information about the controller’s ability to divert a fraction \( \delta_i \) of the issue proceeds into her own pockets, then regardless of whether the controller announces first, there exist only pooling perfect Bayesian equilibria in which the controller always sets the price \( p^{**} = \frac{1}{1 + \rho_i q} \in (p, 1) \) and buys as much stock as she can, and a fraction \( \Delta^{**} = \frac{1 - \rho_i}{1 + \rho_i q} \in (0, 1) \) of the minority rights are exercised; the purchase decisions of individual members of the minority as well as minority
shareholders’ off-equilibrium beliefs and purchases are not unique.

Proof. See Appendix.

The proof of proposition 2 is analogous to that of proposition 1. The main difference is that the controller who can divert will derive a positive profit even with symmetric information and hence with separation. However, that controller does better still by pooling with the no-benefit controller because such pooling lures enough minority shareholders into buying shares above value to offset the fact that the controller, too, has to overpay in that case, relative to a price of $p$.

To compare this situation to one with symmetric information, we first need to establish an appropriate benchmark. Even with complete information, the controller’s ability to divert issue proceeds obviously harms the minority. Specifically, we have

**Lemma 3.** If minority shareholders have preemptive rights and complete information but the controller can divert a fraction $\delta_i$ of the issue proceeds, the controller optimally sets $p = \bar{p}$; the offer will be fully subscribed by any combination of controller, minority shareholders, or new investors; and the controller appropriates $L_{PB,info}^{PR} = \alpha \delta_i \bar{p} q = \frac{\alpha \delta_i q}{1 + \delta_i q}$.

Proof. By lemma 2, the controller will buy any unsubscribed stock at any $p < \frac{1}{1 - \delta_i}$, whereas (1) minority shareholders exercise at any $p < \bar{p}$, such that the controller gets the same amount of shares for any such $p < \bar{p}$ but increases her private benefits (net of her own contribution) $\alpha \delta_i \bar{p} q$ by setting $p$ higher; (2) minority shareholders do not exercise at any $p > \bar{p}$, such that the only effect of increasing $p$ is to increase the controller’s payment, of which $1 - \delta_i$ will not flow back to the controller as private benefits and hence be shared with minority shareholders; and (3) the offer is “fairly priced” at $p = \bar{p}$ in the sense that everyone is indifferent between exercising or not (in the controller’s case, conditional on the offer being fully subscribed), which establishes (a) any combination of subscriptions is possible and (b) the controller’s profit function conditional on a particular pattern of participation must be continuous at $p = \bar{p}$ and hence, together with (1) and (2), $\bar{p}$ is the uniquely optimal choice for the controller, whose gain is simply $\alpha \delta_i \bar{p} q$.

The controller can appropriate, and the minority loses, even more, however, if the minority does not know if the controller is able to divert proceeds. In that case, tedious algebra shows the controller’s gain and the minority’s loss to be $L_{PB,no}^{PR} = \frac{\alpha \delta_i q (1 + \rho q)}{(1 + \rho \delta_i q)}$. If all controllers are able to divert, i.e., $\rho = 1$, then there is no asymmetric information and the expression collapses to $L_{PB,info}^{PR}$. But as $\rho$ decreases, $L_{PB,no}^{PR}$ increases, up to $\alpha \delta_i q = (1 + \delta_i q) L_{PB,info}^{PR}$ as $\rho \downarrow 0$. The reason is that uninformed minority shareholders now have to contend with the possibility that the stock is actually cheap, and thus some
minority shareholders will buy at a price that they would reject if they knew for sure that the controller can divert. This gives the controller a profit from selling overpriced stock on top of the profit from diverting the issue proceeds.

Minority shareholders’ need to balance these risks is the reason why the controller without the ability to divert still has the ability to cheap-stock-tunnel, pocketing a gain of \( L_{NPB, no}^{PR} = \frac{\alpha \delta_i^2 q^2}{(1+\rho \delta_i q)^2} \) from the issue, as can be verified by tedious algebra. It is worth emphasizing that in the limit as \( q \to \infty \), \( L_{NPB, no}^{PR} \) approaches \( L_{NPR}^{PR} (1) \). The fascinating upshot is that the mere possibility that some controllers are able to divert issue proceeds may enable even those who cannot to cheap-stock tunnel essentially as much as if the minority had no preemptive rights.

The crucial role of information asymmetry can be seen by inspecting the total expected losses of the minority, net of the losses they would incur with symmetric information. These are \( \rho \left( L_{PB, no}^{PR} - L_{PB, info}^{PR} \right) + (1-\rho) L_{NPB, no}^{PR} = \frac{\alpha \delta_i^2 q^2 (1-\rho)}{(1+\rho \delta_i q)(1+\delta_i q)}. \) They are increasing in the value relevance of the information, \( \delta_i \), and tend to be larger when the information asymmetry itself—as measured by the entropy—is larger, i.e., when \( \rho \) takes on intermediate values, tending to zero as \( \rho \to 0 \) or \( \rho \to 1 \).

### 6 Discussion: Real-world complications and extensions

For clarity of exposition of the main effect, the model above contained certain simplifications. We now comment on how additional realism would affect the outcomes. In general, added realism only aggravates the minority’s problem.

#### 6.1 Endogenous \( q \) (number of shares to be issued)

The discussion thus far has shown that both controller types’ profits are increasing in \( q \). Both controllers would therefore choose \( q \) as large as possible if they could do so costlessly. In the model thus far, the only potential cost from changing \( q \) is that which could arise from a resulting change in \( \Delta \). However, since a change in \( \Delta \) in either direction is strictly beneficial for one of the two controller types, this change alone could not constrain both controller types from increasing \( q \) from any candidate pooling equilibrium \((p', q')\), and as before and for the same reasons, only pooling equilibria are possible. Thus, the only equilibria without an exogenous cost of higher \( q \) involve both controllers pooling on the same highest possible \( q \) (and choosing \( p \) as before, conditional on \( q \)).

It is not entirely implausible to assume that issuing stock is costless (or to be more exact, that

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19 While the infinite limit is theoretical, real world controllers of listed firm have engaged (or attempted to engage in) very large stock issues. For example, certain listed firms in Hong Kong sought to increase outstanding stock by a factor of 20, only to be blocked by a securities regulator (Charltons, 2016). On endogenous choice of \( q \), see infra section 6.1.
this cost is not increasing in \( q \) and that there exists an upper bound on possible \( q \), especially when the controller must move fast to fully exploit her informational advantage. Stock exchange rules (e.g., NYSE Listed Company Manual section 312.03(c)) and authorized-share limitations in the corporate charter (e.g., Delaware General Corporation Law sections 161, 242(b)) limit the amount of new stock that may be issued absent compliance with often time-consuming voting and disclosure requirements. But once the company incurs the fixed cost of conducting an issue, the cost of including an additional share (up to these regulatory limits) should be close to zero. That said, the dilution of the controller’s voting power (for the low type that does not participate), the controller’s liquidity needs (for the high type that does participate), expected regulatory-scrutiny costs, and potential reputational costs are likely all increasing in \( q \). We therefore also prove the following:

**Proposition 3.** If the controller can also choose \( q \) at a twice continuously differentiable cost \( c(q) \), with \( c(0) = c'(0) = 0 \) and \( c''(q) > 0 \), then in the only possible equilibria, both controllers pool on one \( q^* > 0 \) (in the case of asymmetric information about the value of the assets) or \( q^{**} > 0 \) (in the case of asymmetric information about private benefits), as the case may be, and the equilibrium prices, participation rates, and minority losses established above are unaffected (with the exogenous \( q \) replaced by \( q^* \) or \( q^{**} \), as the case may be). Equilibrium issue sizes \( q^* \) and \( q^{**} \) are increasing in \( \delta_a \) and \( \delta_i \), respectively.

**Proof.** See Appendix.

Nothing hinges on the cost function \( c(\cdot) \) being identical for both types of controllers; we choose this specification merely for simplicity. That the equilibrium issue is increasing in \( \delta_a \) and \( \delta_i \), respectively, is a consequence of the fact that the controllers’ gross profits from the issue are increasing in these quantities, such that they are willing to incur larger issue costs \( c(q) \).

### 6.2 Double-Asymmetry Scenario

In our setting, minority shareholders’ losses from cheap-stock tunneling (and buying overpriced stock) are caused by information asymmetry regarding either (a) the value of assets in place (pre-issue) or (b) the extent of issue-related private benefits. For clarity, we considered each information-asymmetry scenario separately and independently. In each, the minority loses, with losses increasing in the degree of information asymmetry.

But in the real world, there may well be asymmetric information about both (a) and (b), creating more information asymmetry about the value of the issued shares than in each of the considered scenarios.

\[^{20}\text{These assumptions for } c(\cdot) \text{ are a little more restrictive than necessary, as will be seen in the proof, but we make them because they are standard and simpler than the strictly necessary version.}\]
separately. We conjecture that the expected losses to minority shareholders will, accordingly, be larger. Proving this conjecture in general form would require numerical solutions because the model can no longer be solved in closed form. However, for a restricted parameter range $\delta_a \leq \delta_i$, we can prove the following:

**Proposition 4.** If there is asymmetric information about both the value of the assets in the firm, which may be reduced by $\delta_a$, and the controller’s ability to divert a fraction $\delta_i \geq \delta_a$ of the issue proceeds into her own pockets (i.e., both or neither may be true), then regardless of whether the controller announces first, there exist only pooling perfect Bayesian equilibria in which the controller always sets the price $p^{***} = \frac{1-\delta_a}{1+\rho \delta_i \eta}$, and a fraction $\Delta^{**} = \frac{1-\rho \delta_i}{1+\rho \delta_i \eta} \in (0,1)$ of the minority rights are exercised; the purchase decisions of individual members of the minority as well as minority shareholders’ off-equilibrium beliefs and purchases are not unique. Losses to the minority are increasing in $\delta_a$ and $\delta_i$.

**Proof.** See Appendix. \qed

To see the intuition, start with the scenario where asymmetric information pertains only to issue-related private benefits, and its equilibrium issue price. If the information asymmetry about asset value is added, the expected loss from buying overpriced stock at that initial issue price will now rise as the second cause of possible loss is added to the first. To maintain equilibrium, the issue price must decline so as to “rebalance” the expected loss from buying overpriced stock and the expected gain from buying cheap stock, which will thus both be larger than before. As a result, assuming constant participation rates, the controller would thus extract more from both cheap-stock tunneling and overpriced issues than under either of the two considered scenarios.

### 6.3 Voting Rights

In our analysis, we set aside the possibility that the controller or the minority might desire to maintain a certain fraction of the voting rights for control or blocking purposes, respectively. For example, if certain transactions require more than 80% approval, the minority might want to preserve or obtain a 20% voting interest, and, conversely, the controller might want to preserve or obtain an interest exceeding 80%. Ignoring voting rights is proper if the current issue involves non-voting shares, or if there is otherwise no possibility that the current issue can meaningfully alter control rights, in particular because the controller’s and minority’s holdings are not close to any relevant thresholds. Otherwise, voting rights would need to be taken into account (cf. Wu et al., 2016).

In some circumstances, the controller’s need to remain above a certain voting threshold will protect the minority. This will be the case if (1) the controller is sufficiently close to the relevant threshold that...
not participating would push the controller below that threshold, (2) the controller values being above the threshold after the issue, and (3) the only threat to the minority emanates from the possibility that the controller may overprice the issue and not participate (as in our first model). Under conditions (1) and (2), the threat (3) is not credible. Importantly, however, the mere proximity of the controller to the voting threshold (condition 1) is not enough to protect the minority. First, the controller may not care about the threshold after the issue (i.e., condition 2 may fail). In particular, the controller may have short-term plans for the firm such as a post-issue liquidation or sale that do not require it remaining above the threshold after the issue. Second, the threat to the minority may not involve controller non-participation (i.e., condition 3 may fail). In particular, if the minority’s problem emanates exclusively from the controller’s private benefits from the issue proceeds (section 5), then the controller always participates anyway.

In other circumstances, the controller’s possibility to climb above a certain voting threshold will aggravate the minority’s problem. Since the minority never fully participates out of fear of overpaying, the participating controller will increase her percentage of the shares in the underpricing case. This may be valuable to the controller and a loss to the minority. In fact, the controller may set the price higher to induce lower minority participation if she does participate, and to make the minority overpay more when she does not participate.

How the minority would react to this additional threat, or to control considerations generally, would depend on the composition of the shareholder base. If minority shareholders are highly dispersed (in the limit, atomistic as in our model), then individual shareholders will not take into account the effect of their purchase decisions on the overall voting power of the minority. By contrast, large minority shareholders might strategically buy more aggressively to prevent the loss of certain blocking rights. Even they, however, would need to balance such aggressive buying against the risk of overpaying; they will not defend their blocking rights at all cost.\footnote{For a recent example of a large insider of U.S. listed firm deliberately setting the offer price high to discourage outsider participation and thereby enable the insider to increase its equity voting power, see Fried (2018a).}

### 6.4 Listing the Firm’s Shares

Thus far, we have analyzed the minority’s dilemma around equity issues in an unlisted firm. We now discuss how our analysis would be affected by listing the firm’s stock, which would subject the firm to enhanced disclosure requirements (Kraakman et al., 2017) and enable continuous (and potentially anonymous) trading of its stock. In this setting, preemptive rights are typically implemented by a rights issue in which the rights are sometimes tradable (Massa et al., 2016). Our bottom line is that listing may...
alleviate the asymmetric-information problems we have been discussing, but will certainly not fix them
(and might even exacerbate them). Indeed, evidence from listed firms is broadly consistent with what
our model would predict: in rights issues by controlled firms, minority shareholders tend to purchase
less than their pro rata share (Fong and Lam, 2014); controllers sometimes increase their percentage
ownership and other times decrease it (Fong and Lam, 2014; Larrain and Urzua I., 2013); and controllers
reduce their percentage ownership when the stock is overpriced (Larrain and Urzua I., 2013).

6.4.1 Enhanced Disclosure

Enhanced disclosure has an unambiguously positive effect on the minority’s position by reducing in-
formation asymmetry. As our model shows, the less the information asymmetry, the less the minority
loses from cheap-stock tunneling and the purchase of overpriced shares.22 But enhanced disclosure will
not solve the minority’s problem, as no disclosure regime can fully eliminate information asymmetry.
Even in the U.S., where disclosure requirements for listed firms are relatively stringent (Kraakman et
al., 2017), insiders know more than outsiders, as evidenced by the returns of executives trading directly
or indirectly in their own firms’ shares (e.g., Cohen et al., 2012; Baker and Wurgler, 2002).23

6.4.2 Trading

As a preliminary matter, we note an indirect effect of trading in the firm’s stock: to the extent trading
reveals and aggregates information, it may reduce information asymmetry, and hence minority losses,
just like increased disclosure. However, we now turn to the direct effects of trading.

Minority’s ability to trade The minority’s ability to trade would not affect the minority’s position
directly if third-party buyers are sophisticated, as sophisticated third parties would buy the stock only
at a discount reflecting the anticipated losses from the issue. If these buyers are unsophisticated and
pay above value, the old minority might lose less or even gain from trading, but only by shifting loss to
the new minority. Similarly, if the minority receives tradable rights, buyers of those rights would face
the same dilemma as the sellers.24

Controller’s ability to trade Trading by the controller has various and mostly ambiguous effects

22The expressions for the minority’s expected losses from the issue are increasing in the information asymmetry regarding
the value of the assets in the firm or the controller’s ability to obtain private benefits from the issue (δa and δi, respectively).
23The asymmetry is likely to be particularly acute in a firm with a controller, which has the power to operate the firm
in ways designed to obscure its value, as did the controller of Dole Food Corporation before freezing out public investors
(Potter Anderson Corroon, 2015).
24In fact, rights are often not easily tradable, and when there is a market for such rights it is often highly illiquid and
characterized by severe underpricing (Massa et al., 2016).
mispriced issue. However, an important caveat is that the controller may face trading constraints as a result of insider trading and similar laws (Fried, 2014).

**Trading as a substitute** If the controller conducts an issue solely to exploit its superior information about the value of assets in place, the controller might use open market trades as a substitute for a stock issue: the controller could directly buy shares when the market underestimates the firm’s prospects and otherwise sell. This would obviate the need for the issue. However, insider-trading restrictions imposed on the controller trading directly in the market tend to be more onerous than those imposed on the controller trading with the minority indirectly via the firm (Fried, 2014). The controller may therefore choose to rely exclusively on a stock issue to circumvent tighter legal restrictions on direct insider trading, or to conduct a stock issue and engage in only limited open market trading. Of course, when the issue would be conducted at least in part to exploit information asymmetry over the controller’s private benefits from the issue, trading cannot substitute for the issue.

**Trading as a complement** Trading can also be a complement for the controller exploiting its superior information in a stock issue, potentially exacerbating the problem we identify. To begin, the controller’s ability to trade could undermine the effectiveness of any ex ante disclosure of the controller’s participation decision (by enabling the controller to offset that participation via hedging or sales), and hence the minority’s ability to protect itself by mimicking the controller’s decision when the information asymmetry pertains (at least in part) to the value of assets in place (cf. supra section 4.2). In addition, when the asymmetric information relates (at least in part) to private benefits from the issue, the issue itself increases information asymmetry about the value of the firm’s shares, potentially boosting the controller’s ability to profit from informed trading directly in the market. This extra profit, in turn, could provide an additional incentive to undertake such an issue. Of course the controller would still need to reckon with restrictions on trading by informed insiders.

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25. To be effective in a listed firm, the controller’s disclosure must cover not only its participation in the issue but also a commitment to abstain from offsetting market transactions (Fried, 2018a). However, we know of no jurisdiction that requires disclosure of such a commitment and only one jurisdiction (the PRC) that imposes even a simple participation-disclosure requirement directly on controllers of listed firms, at least for certain types of issues: the PRC (Chen and Huang, 2017). In other major markets—including the U.S., the U.K., Hong Kong, Japan, Singapore, and Germany—there is no obligation on the controller itself to disclose its intended participation in an equity issue by a listed firm. However, a firm is typically required to reveal any underwriting arrangement in connection with the rights issue, including an arrangement with the controller.
7 Conclusion

In this paper, we have presented a model that shows how preemptive rights, widely used to prevent cheap-stock tunneling by controlling shareholders, can be substantially undermined by asymmetric information. In particular, asymmetric information about either the value of assets in place or the extent to which the issue disproportionately benefits the controller enables the latter to cheap-stock tunnel around preemptive rights. Importantly, these disproportionate benefits could be purely non-pecuniary. Moreover, the mere possibility that the controller might get such disproportionate benefits, either financial or non-pecuniary, enables cheap-stock tunneling even by a controller who knows it will not.

Our analysis can explain why, in unlisted firms, sophisticated investors such as VCs typically negotiate blocking rights on subsequent equity issues alongside the right to participate pro rata in future issues. It can also help explain why regulators of listed firms do not rely exclusively on preemptive rights to protect minority investors, but rather impose a range of other measures on issues (such as fiduciary duties for directors and, in some jurisdictions, a regulator veto on issues).

As we have explained, some amount of asymmetric information and disproportionate benefit is inevitable even in the most robust disclosure regimes, such as that of the U.S. However, forcing the controller of unlisted and listed firms to disclose her participation decision up front, as proposed by Fried (2018a) and already required by the PRC for certain issues by listed firms (Chen and Huang, 2017), would go at least some way towards alleviating the problem we discuss.\footnote{For listed firms, the controller should also be required to commit to refrain from offsetting market transactions, cf. supra note 25.} Requiring majority-of-minority (MoM) approval would (in our model) totally eliminate the problem, but of course come with its own costs.\footnote{The PRC has mandated minority veto rights over certain equity issues (Chen et al., 2013). For evidence that mandatory minority veto rights can curb controller tunneling, see Fried et al. (2018).} A less effective (but less costly) approach would be to require majority or super-majority shareholder approval, as this would give the minority some protection when insiders lack sufficient voting power to ensure approval.\footnote{For evidence on the effects of such shareholder-approval requirements, see Holderness (2017).} Future work should consider the trade-offs involved in these various approaches, which presumably differ as a function of the firm’s anticipated capital needs. Such work might also consider if better protective mechanisms could be designed. In the meantime, courts and others should be cognizant that the ability to participate pro rata in an equity issue does not suffice to protect the minority from cheap-stock tunneling.
Appendix

A Proof of Lemma 1

If a shareholder owning $\gamma < 1$ of the existing stock purchases $q'$ of the new stock and others purchase $q''$, the value of the shareholder’s investment, net of the purchase price, is

$$ W \equiv (\gamma + q') v_s - pq', $$

where

$$ v_s \equiv v + \frac{(q' + q'') p}{1 + q' + q''} $$

is the value of a share after the issue. Now

$$ \frac{dW}{dq'} = \frac{v - p}{1 + q' + q''} \left( 1 - \frac{\gamma + q'}{(1 + q' + q'')} \right). $$

It follows that $\frac{dW}{dq'} \leq 0$ if and only if $v - p \leq 0 \iff v \leq p$ (because for $\gamma \in (0,1)$ and $q', q'' \geq 0$, $1 - \frac{\gamma + q'}{1 + q' + q''} > 0$).

B Proof of Proposition 1

The proof proceeds in three steps:

1. The only possible equilibria are pooling equilibria with $(p, \Delta) \in (1 - \delta_a, 1) \times (0, 1)$.

2. If such pooling equilibria exist, the unique equilibrium price and participation are $p^*$ and $\Delta^*$.

3. Such pooling equilibria do exist, which we show by example.

B.1 The only possible equilibria are pooling equilibria with $(p, \Delta) \in (1 - \delta_a, 1) \times (0, 1)$

As a preliminary matter, note the following corollaries of lemma 1:

**Corollary 1.** For prices $p < 1 - \delta_a$ ($p > 1$), the only rational choice for minority shareholders is to exercise (not to exercise), such that both controller types earn zero profits at such prices.
Corollary 2. For \( p = 1 - \delta_a \) \( (p = 1) \), the only rational choice for minority shareholders is to exercise (not exercise) unless they assign zero probability to the possibility of facing the high (low) type controller; consequently, if such price is chosen in equilibrium (where the minority’s probability beliefs have to coincide with objective probabilities), the controller earns zero profit at that price (either because all minority shareholders exercise (do not exercise), or because only the low (high) type chooses this price in equilibrium).

Corollary 3. For prices \( p \in (1 - \delta_a, 1) \) that fully reveal the controller’s type, the only rational choice for minority shareholders is to exercise (not exercise) when facing the high type (low type), such that both controller types earn zero profits at such revealing prices.

Corollary 4. For prices \( p \in (1 - \delta_a, 1) \), at least one of the controllers’ payoffs is strictly greater than zero regardless of \( \Delta (p) \), and both are strictly positive unless \( \Delta (p) \in \{0, 1\} \), since for \( p \in (1 - \delta_a, 1) \), the only rational choice for the high (low) type controller is to exercise (not to exercise) (except for indifference at the boundary \( (p = 1 \text{ for the high type and } p = 1 - \delta_a \text{ for the low type}) \)), and their respective profits are thus\(^{29}\)

\[
\pi_h (p) = \frac{(1 - \min \{\Delta (p), 1\}) (1 - p)}{1 + q} \geq 0
\]

\[
\pi_l (p) = \alpha (1 - \alpha) q \frac{\max \{0, \Delta (p)\} (p - 1 + \delta_a)}{1 + \alpha q \max \{0, \Delta (p)\}} \geq 0.
\]

With a little more work, lemma 1 also leads to

Lemma 4. If \( p \in (1 - \delta_a, 1) \) is an equilibrium price, then \( \Delta (p) \in [0, 1] \) unless the part of the order exceeding 1 or below 0 is never filled (because there are no shares available, or no buyer, respectively).

Proof. We will show that at least some individual minority shareholders would have to make a suboptimal equilibrium participation decision if for some equilibrium price \( p \in (1 - \delta_a, 1), \Delta (p) \notin [0, 1], \) and

\(^{29}\)By lemma 1, the high type will snap up not only her allotted \((1 - \alpha) q\) shares but also the \((1 - \min \{\Delta (p), 1\}) \alpha q\) shares allotted to the minority that minority shareholders do not buy, such that the value of the stock following the issue will be \(\frac{1 + \alpha p q}{1 + q}\). The controller makes a trading gain on her \((1 - \alpha) q + (1 - \min \{\Delta (p), 1\}) \alpha q = (1 - \alpha \min \{\Delta (p), 1\}) q\) purchased shares, partially offset by a loss on the value of her \((1 - \alpha)\) existing shares:

\[
(1 - \alpha \min \{\Delta (p), 1\}) q \left( \frac{1 + \alpha p q}{1 + q} - p \right) - (1 - \alpha) \left( 1 - \frac{1 + \alpha p q}{1 + q} \right) = \frac{1 - p}{1 + q} [1 - \min \{\Delta (p), 1\}] \alpha q = \pi_h (p).
\]

The low type will not purchase any shares, and hence the only change in her position will be the increase in the value of her existing shares. The increase happens because a fraction \(\max \{0, \Delta (p)\}\) of the minority exercises in ignorance of the overpricing, increasing the value of a share from \(1 - \delta_a\) to \(\frac{1 - \delta_a + \max \{0, \Delta (p)\} \alpha p q}{1 + \max \{0, \Delta (p)\} \alpha q}\). The controller’s gain is thus

\[
(1 - \alpha) \left( 1 - \delta_a + \max \{0, \Delta (p)\} \alpha p q \right) = \frac{1 - \alpha}{1 + \max \{0, \Delta (p)\} \alpha q} \max \{0, \Delta (p)\} \alpha q (p - 1 + \delta_a) = \pi_l (p).
\]
the order is actually filled at least some of the time.\textsuperscript{30} We will focus on the case of $\Delta(p) > 1$; the proof for $\Delta(p) < 0$ is parallel.\textsuperscript{31} Minority orders aggregating to net minority purchases above the minority’s pro rata share ($\Delta > 1$) can be filled only if the controller does not exercise her rights, since the shares are in fixed supply. By lemma 1, the controller will exercise if and only if $p < v$ (we do not need to consider equality because we are only dealing with $p \in (1 - \delta_a, 1)$ whereas $v \in \{1 - \delta_a, 1\}$). Consequently, $\Delta(p) > 1$ can only be filled when $p > v$, such that, by lemma 1, any purchasing minority shareholder is losing money from the purchase. Now there are three cases to distinguish. First, the only controller type who sets $p$ is one with $v > p$; in this case, the part of the minority order exceeding 1 will not be filled because the controller is exercising her rights. Second, the only controller type who sets $p$ is one with $v < p$; in this case, the minority order $\Delta(p) > 1$ will be filled in full (because the controller will not exercise and will in fact be happy to sell to the minority) but all participating minority shareholders lose and would be better off not participating. Third, both types of controllers set $p$, such that the aggregate minority order $\Delta(p) > 1$ will sometimes be filled and sometimes not be filled, and we need to distinguish by composition of the aggregate minority order: (i) If $\Delta(p) > 1$ because all minority shareholders buy at least their pro rata share of the offer and some buy more, then each minority shareholder buying more could do better by instead limiting her order to her pro rata share of the issue: this deviation would change nothing when $p < v$ (higher orders are not filled anyway, since every other shareholder exercises), but avoid purchasing additional shares when $p > v$; (ii) If the aggregate order $\Delta(p) > 1$ results from some individual shareholders buying less and some buying more than their pro rata share, then at least one of them must be able to do better by changing their order since the marginal impact on their expected payoffs is different (and hence cannot be zero for both): for both of them, a change in the order changes their allotment one-to-one when $p > v$ (because the controller is then happy to sell as many shares as desired) but changes it one-to-one when $p < v$ only for the minority shareholder not yet purchasing her pro rata share because any orders above the pro rata share are filled only at a ratio of (unsubscribed shares / subscriptions above pro rata share), which is less than one because the controller always exercises and the minority in the aggregate orders more than their pro rata share.\textsuperscript{32}
With these ingredients, it is now straightforward to show that the only possible equilibrium is a pooling equilibrium with equilibrium price and participation rate \((p', \Delta (p')) \in (1 - \delta_a, 1) \times (0, 1)\):

1. In any equilibrium, both controller types must at least partially pool on at least one price \(p' \in (1 - \delta_a, 1)\) because if they were to pool only on other prices or only choose separating prices, then both would earn zero profits by corollaries 1-2 and 3, respectively, such that by corollary 4 at least one type would have a profitable deviation to some \(p'' \in (1 - \delta_a, 1)\).

2. For any such pooling equilibrium price \(p' \in (1 - \delta_a, 1)\), \(\Delta (p') \in [0, 1]\) by lemma 4.

3. However, \(\Delta (p') \notin \{0, 1\}\) because, by corollary 4, otherwise one of the two controller types would earn zero equilibrium profit but would have a profitable deviation to some \(p'' \in (1 - \delta_a, 1)\) unless \(\Delta (p) \geq 1\forall p \in (1 - \delta_a, 1)\) or \(\Delta (p) \leq 0\forall p \in (1 - \delta_a, 1)\), as the case may be; which, by inspection of the profit functions, would in turn give the other type a profitable deviation to a price \(p''\) closer to 1 (low type) or \(1 - \delta_a\) (high type), as the case may be, unless \(p'\) is already the maximum or minimum, respectively, of \((1 - \delta_a, 1)\); which is in turn impossible for two reasons: First, technically, there is no extreme \(p'\) in \((1 - \delta_a, 1)\) because the interval is open and prices are chosen from the real numbers. Second, and more economically meaningfully, at a price close to the interval’s boundary, the minority’s reaction (buying at \(p' \approx 1\) or not buying at \(p' \approx 1 - \delta_a\)) would not be rational for the minority even if both controller types fully pooled on that price, let alone if the (at that price) “harmless” controller type only partially pooled on it.\(^{34}\)

4. Finally, this pooling equilibrium must be a full pooling equilibrium because at any \((p', \Delta (p')) \in (1 - \delta_a, 1) \times (0, 1)\) both controller types earn positive profits, whereas by corollary 3 they would earn zero profit at any partially separating price and hence would never choose that price.

\(^{33}\)The direct effect of such change in \(p\) on that type’s profits is strictly positive, while the indirect effect of any induced change in \(\Delta (p)\) would be weakly positive.

\(^{34}\)Technically, complete non-exercise by the minority (\(\Delta = 0\)) at a partial pooling price \(p\) chosen with probability \(\beta\) by the low type is optimal for the minority only if

\[
\frac{\beta p}{\beta p + 1 - p} (1 - \delta_a) + \frac{1 - p}{\beta p + 1 - p} \left(\frac{1 + qp}{1 + q} - p\right) \leq 0.
\]

For \(p > 1 - \delta_a\), the left-hand side is decreasing in \(\beta\), so the condition is most likely to be fulfilled if \(\beta = 1\). Even in that case, however, the condition implies

\[
p \geq 1 - \delta_a + \delta_a \frac{1 - \rho}{1 + \rho q},
\]

i.e., the minimal price compatible with complete minority abstention is \(\delta_a \frac{1 + \rho}{1 + \rho q}\) above the lower boundary of the interval. A parallel argument shows that complete exercise by the minority (\(\Delta = 1\)) at a partial pooling price \(p\) is possible only if

\[
p \leq 1 - \delta_a \frac{\rho (1 + q)}{1 + \rho q + \alpha q (1 - \rho)}.
\]
B.2 Unique equilibrium price and participation \(p^*\) and \(\Delta^*\)

If there is an equilibrium in which both types of controllers pool on one or more price \(p^* \in (1 - \delta_a, 1)\)
with minority participation rate \(\Delta^* \in (0, 1)\) (the only possible equilibrium, as per B.1), then in any such
equilibrium the unique equilibrium price/participation pair is \((p^*, \Delta^*)\) as defined in proposition 1. This
follows from the following two observations that must hold at any such equilibrium pair \((p^*, \Delta^*)\):

1. Optimality of the price choice \(p^*\) for both types of controllers requires that their isoprofit curves
in \((p, \Delta)\) space be tangent at \((p^*, \Delta^*)\), which implies (cf. the specification of the profit functions
in corollary 4)

\[
p^* = 1 - \delta_a \frac{1 - \Delta^*}{1 + aq\Delta^*}. \tag{1}
\]

The tangency requirement follows from the fact that the two controller types’ isoprofit curves are
smooth in the neighborhood of any \((p^*, \Delta^*) \in (1 - \delta_a, 1) \times (0, 1)\), yet they must not cross because
for prices \(p \in [1 - \delta_a, 1]\), the high type’s profits decrease and the low type’s profits increase in both
\(\Delta\) and \(p\) (strictly for \(\Delta \in (0, 1)\)).\(^{35}\)

2. Minority shareholders’ indifference requires:

\[
\rho \frac{1 - \delta_a + \alpha \Delta (p^*) q p^*}{1 + \alpha \Delta (p^*) q} + (1 - \rho) \frac{1 + q p^*}{1 + q} - p^* = 0. \tag{2}
\]

The minority shareholders must be indifferent between participating or not since some minority

\(^{35}\)More formally, the proof that the isoprofit curves must be tangent at \((p^*, \Delta^*) \in (1 - \delta_a, 1) \times (0, 1)\) is:

(a) For any candidate equilibrium point \((p^*, \Delta^*)\), the following must hold for all \(p\):

\[
\pi_h (p, \Delta (p)) \leq \pi_h (p^*, \Delta^*) \equiv \pi_h^* \quad \pi_l (p, \Delta (p)) \leq \pi_l (p^*, \Delta^*) \equiv \pi_l^*.
\]

(b) By the implicit function theorem, the isoprofit relationships \(\pi_h (p, \Delta) = \pi_h^*\) and \(\pi_l (p, \Delta) = \pi_l^*\)
uniquely define continuously differentiable functions \(\Delta_h (p)\) and \(\Delta_l (p)\) in a neighborhood of \(p^*\) because the profit functions are
continuously differentiable and strictly monotonic in \(p\) and \(\Delta\) for \((p, \Delta) \in (1 - \delta_a, 1) \times (0, 1)\) (cf. the profit functions’
specification in corollary 4). (All following statements regarding these implicit functions are restricted to this
neighborhood, which is sufficient for the proof.) By definition, \(\Delta_h (p^*) = \Delta_l (p^*) = \Delta^*\).

(c) Since \(\pi_h\) (\(\pi_l\)) is strictly decreasing (increasing) in \(p\) and \(\Delta\), (a) and (b) imply \(\Delta_h (p) \leq \Delta (p) \leq \Delta_l (p)\) \(\forall p\).

(d) It follows that \(\frac{\Delta_h (p) - \Delta_h (p^*)}{p - p^*} \leq \frac{\Delta_l (p) - \Delta_l (p^*)}{p - p^*}\) \(\forall p > p^*\) and \(\frac{\Delta_h (p^*) - \Delta_h (p)}{p^* - p} \geq \frac{\Delta_l (p^*) - \Delta_l (p)}{p^* - p}\) \(\forall p < p^*\).

(e) (b) and (d) imply

\[
\frac{d\Delta_h}{dp} (p^*) = \lim_{p \downarrow p^*} \frac{\Delta_h (p) - \Delta_h (p^*)}{p - p^*} \leq \lim_{p \downarrow p^*} \frac{\Delta_l (p) - \Delta_l (p^*)}{p - p^*} = \frac{d\Delta_l}{dp} (p^*)
\]

and

\[
\frac{d\Delta_h}{dp} (p^*) = \lim_{p \uparrow p^*} \frac{\Delta_h (p) - \Delta_h (p^*)}{p - p^*} \geq \lim_{p \uparrow p^*} \frac{\Delta_l (p) - \Delta_l (p^*)}{p - p^*} = \frac{d\Delta_l}{dp} (p^*)
\]

(the existence of the two-sided limit follows from (b)), from which it follows that \(\frac{d\Delta_h}{dp} (p^*) = \frac{d\Delta_l}{dp} (p^*),\) i.e., the
isoprofit curves are tangent at \((p^*, \Delta^*)\).
shareholders buy and some do not if $\Delta^* \in (0, 1)$. They will only be indifferent if the expected marginal profit from purchasing a new share is zero. This is the condition expressed in 2, where the probability weights $\rho$ and $1 - \rho$ follow from the fact that in a pooling equilibrium, the conditional probability of facing either type of controller given $p^*$ equals the unconditional probability (unless the two types were to pool on more than one price in unequal proportions, which is impossible because equation 1 would require that minority participation is higher at the higher pooling price than at the lower pooling price, whereas the high type controller would choose both prices only if minority participation is lower at the higher price).

Equations 1 and 2 set up a system of two equations in two unknowns that has a unique solution for $\Delta > 0$, namely $\Delta^* = \frac{1 - \rho}{1 + \rho q}$ and thus $p^* = 1 - \frac{\delta_{\rho}(1 + q)}{1 + \rho q + \rho q(1 - \rho)^2/(1 + \rho q)}$.

### B.3 Existence

There exist off-equilibrium minority participation rates $\Delta(p)$ and minority beliefs $\theta(p)$ (about the probability of facing a low type) that sustain the type of equilibrium identified above (controller pooling on $p^*$ and equilibrium minority participation $\Delta^*$). One class of examples is:

- $\Delta^*(p)$, defined as the tangency line (to both controller types’ isoprofit curves) through (their tangency point) $(p^*, \Delta^*)$ for $(p, \Delta(p)) \in (1 - \delta_a, 1) \times (0, 1)$, switching to $\Delta = 1$ to the left and

---

36 It is not a necessary feature of the model that minority shareholders are indifferent. Alternatively, we could model minority shareholders with heterogeneous priors and thus heterogeneous posteriors $\theta_i(p) \in [0, 1]$ (distributed according to some cumulative distribution function $G(\cdot | p)$) that the firm is the low type, given the announced price $p$. Then only sufficiently optimistic shareholders would participate in the offer. Specifically, given price $p$, a shareholder $i$ would participate only if $\theta_i(p) \geq 1 - \frac{\delta_{\rho}(1 + q)}{1 + \rho q + \rho q(1 - \rho)^2/(1 + \rho q)}$.

37 The plural conveys that the collective $\Delta^*(p)$ masks an infinite number of individual strategies, including a common individual participation probability or a partition of minority shareholders into a fraction $\Delta^*(p)$ that always participates given $p$, and a remainder $1 - \Delta^*(p)$ that never does given $p$. 

38 The plural conveys that the collective $\Delta^*(p)$ masks an infinite number of individual strategies, including a common individual participation probability or a partition of minority shareholders into a fraction $\Delta^*(p)$ that always participates given $p$, and a remainder $1 - \Delta^*(p)$ that never does given $p$.

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Electronic copy available at: https://ssrn.com/abstract=3185860
above of that region and $\Delta = 0$ to the right and below.\footnote{Formally, these (off-equilibrium) aggregate minority participation rates are

$$\Delta^\ast (p) \equiv \begin{cases}
1 & \text{if } p \leq 1 - \delta_a \\
0 & \text{if } p \geq 1 \\
\max \left\{ 0, \min \left\{ 1, \Delta^\ast (p) \right\} \right\} & \text{if } p \in (1 - \delta_a, 1),
\end{cases}$$

where $\Delta^\ast (p) \equiv \Delta^\ast + (p^\ast - p) \frac{1 - \Delta^\ast}{1 - p^\ast}$.

} Given $\Delta^\ast (p)$, both controllers’ unique optimal choice is $p^\ast$ because their profits are positive at $p^\ast$ (corollary 4) and strictly lower elsewhere: (1) along the tangency line, this follows from strict convexity of the controllers’ preferred sets (cf. the specification of the profit functions in corollary 4) or, equivalently, strict concavity of one controller’s isoprofit curve and strict convexity of the other’s; (2) for $p \notin (1 - \delta_a, 1)$, profits are zero for both controllers given $\Delta^\ast (p)$; and (3) for $\{ p \in (1 - \delta_a, 1) \mid \Delta^\ast (p) \in \{0, 1\} \}$, one of the controller’s profits is zero whereas the other’s is at least lower than at $p^\ast$: by point (1), the respective type’s profit is already lower at the intersection of the tangency line with the lower ($\Delta = 0$) or upper ($\Delta = 1$) boundary of the region $(p, \Delta (p)) \in (1 - \delta_a, 1) \times (0, 1)$, and only decreases further as $p$ increases or decreases, respectively.

- $\theta^\ast (p)$, defined as the belief at which individual minority shareholders are just indifferent between participating or not participating given $\Delta^\ast (p)$ for $p \in (1 - \delta_a, 1)$, and the certain belief of facing the worse controller, i.e., $\theta = 1$ or $\theta = 0$ for $p \leq 1 - \delta_a$ and $p \geq 1$, respectively.\footnote{Formally, such off-equilibrium beliefs are

$$\theta^\ast (p) \equiv \begin{cases}
1 & \text{if } p \leq 1 - \delta_a \\
0 & \text{if } p \geq 1 \\
\frac{1}{1 + \alpha q \Delta^\ast (p)(1 - p) + (1 + q)(1 - p) - \delta_a} & \text{if } p \in (1 - \delta_a, 1)
\end{cases}.$$}

$\theta^\ast (p)$ sustains the PBE because (1) given $\theta^\ast (p)$, $\Delta^\ast (p)$ is an optimal participation rate for minority shareholders even individually, and (2) $\theta^\ast (p)$ coincides with the Bayesian objective probability $\rho$ at $p^\ast$ by construction and by the derivation of the unique equilibrium point above, and always exists (i.e., lies between zero and one) even for $p \in (1 - \delta_a, 1)$ because participation is always profitable (unprofitable) in this range if the probability of facing the low type (high type) is low enough.

C Proof of Lemma 2

If the controller purchases $q'$ of the new stock and others (minority shareholders and/or third parties) purchase $q''$, the value of the controller’s investment, net of the purchase price and inclusive of private benefits, is

$$W^PB_C (q'; q'', p) \equiv (1 - \alpha + q') v^PB_s - pq' + \delta_i p (q' + q''),$$

\begin{align*}
\Delta^\ast (p) & \equiv \begin{cases}
1 & \text{if } p \leq 1 - \delta_a \\
0 & \text{if } p \geq 1 \\
\max \left\{ 0, \min \left\{ 1, \Delta^\ast (p) \right\} \right\} & \text{if } p \in (1 - \delta_a, 1),
\end{cases} \\
\theta^\ast (p) & \equiv \begin{cases}
1 & \text{if } p \leq 1 - \delta_a \\
0 & \text{if } p \geq 1 \\
\frac{1 + \alpha q \Delta^\ast (p)(1 - p) + (1 + q)(1 - p) - \delta_a}{(1 + \alpha q \Delta^\ast (p)(1 - p) + (1 + q)(1 - p) - \delta_a)} & \text{if } p \in (1 - \delta_a, 1)
\end{cases}.
\end{align*}
where

\[ v_s^{PB} \equiv \frac{1 + (1 - \delta_i) (q' + q'') p}{1 + q' + q''} \]

is the value of a share to any shareholder after the issue and diversion of fraction \( \delta_i \) of the issue proceeds to the controller. Now

\[ \frac{dW_{PB}^C}{dq'} = [1 - (1 - \delta_i) p] \frac{\alpha + q''}{(1 + q' + q'')^2}, \]

which entails \( \frac{dW_{PB}^C}{dq'} \leq 0 \) if and only if \( 1 - (1 - \delta_i) p \leq 0 \iff \frac{1}{1 - \delta_i} \leq p \) (because \( \alpha, q'' \geq 0 \) and \( \delta_i < 1 \)).

A non-controlling shareholder, new or old, values the purchase differently because that shareholder does not obtain the private benefits \( \delta_i \). Denoting any prior stake of the non-controlling shareholder \( \gamma \geq 0 \), the value of the non-controlling shareholder’s investment, net of the purchase price and given others’ purchases \( q'' \), is

\[ W_{NC}^{PB} \equiv (\gamma + q') v_s^{PB} - pq'. \]

Now

\[ \frac{dW_{NC}^{PB}}{dq'} = \frac{(1 - p) (1 - \gamma + q'') - p\delta_i (\gamma + q' + (q' + q'') (q' + q'' + 1))}{1 + q' + q''}, \]

which entails \( \frac{dW_{NC}^{PB}}{dq'} \leq 0 \) if and only if \( p \geq \frac{1 - \gamma + q''}{1 - \gamma + q'' + \delta_i (\gamma + q' + (q' + q'') (q' + q'' + 1))} \). The latter expression is decreasing in \( \gamma \) and \( q' \) – outside shareholders’ reservation price \( \bar{p} (\gamma, q', q'') \) declines with their holdings and issue purchases. The most willing to buy is an atomistic shareholder, for whom the expression converges to

\[ \bar{p} (q'') = \lim_{\gamma, q' \to 0} \bar{p} (\gamma, q', q'') = \frac{1}{1 + \delta_i q''}. \]

**D Proof of Proposition 2**

The proof of proposition 2 is very similar to the proof of proposition 1, and we merely note the differences to that proof. One difference is, of course, that we now need to show that the equilibrium is independent of the sequencing of participation announcements. At this point, we merely note that existence is unaffected by the sequencing because both types of controllers now make the same participation decisions, such that the controller’s equilibrium participation decision now contains no information about controller type. We will explain in the uniqueness parts why uniqueness is also unaffected by the sequencing.
D.1 No other types

If the controller announces her participation second or simultaneously, the first part of the proof of proposition 2 is virtually identical to that of proposition 1, with \( \overline{p} \) replacing \( 1 - \delta_a \). The only other difference comes from the fact that the profit function of the low type controller (i.e., the controller with private benefits) now has a different form. For prices \( p \in \left[ \overline{p}, \frac{1}{1 - \delta_a} \right] \) (at which, by lemma 2, the controller snaps up any left-over new shares but, by lemma 1, does not purchase any old shares), these profits are

\[
\pi^{PB} (p) \equiv W^{PB}_C (q (1 - \alpha \max \{0, \Delta (p)\}) ; q \alpha \max \{0, \Delta (p)\}, p) - (1 - \alpha)
\]

\[
= \frac{\alpha q}{1 + q} \left\{ 1 - (1 - \delta_i) p + \left( \frac{p}{\overline{p}} - 1 \right) \max \{0, \Delta (p)\} \right\},
\]

which is strictly positive regardless of \( \Delta \), implying that this controller will earn positive profits even with symmetric information, which are then maximal at \( \overline{p} \) and equal to \( \overline{\pi} \equiv \frac{\alpha q}{1 + q} \) (since, by lemma 2, \( \Delta (p) = 0 \forall p > \overline{p} \) when minority shareholders have symmetric information). By lemma 3, \( \overline{p} \) is also this controller’s globally optimal choice when minority shareholders have symmetric information. Corollaries 1-3 are thus modified to the effect that the controller with private benefits will earn \( \overline{\pi} \) or less at such prices, and corollary 4 is modified to the effect that for prices \( p \in (\overline{p}, 1) \), either the controller with private benefits earns more than \( \overline{\pi} \) or the other controller earns more than zero, or both.\(^{40}\) Lemma 4 holds unaltered (substituting \( \overline{p} \) for \( 1 - \delta_a \)).\(^{41}\) The four enumerated steps at the end of section B.1 then go through analogously, except for a simplification of step 3 at the lower boundary: if \( \Delta (p) < (1 - \delta_i) \overline{p} \forall p \in (\overline{p}, 1) \), then the low type controller herself has a profitable deviation to \( \overline{p} \) from any putative equilibrium pooling price \( p' \in (\overline{p}, 1) \).

If the controller announces her binding participation decision \( q' \) before minority shareholders make theirs, then \( \overline{p} \) must be replaced by \( \overline{p}(q') \) in the modified corollaries 1-4, corollary 4 holds (only) for \( q' > 0 \),\(^{42}\) and the profit earned by the high and low type controller in the situations described in the modified corollaries 1-3 may be even lower than zero and \( \overline{\pi} \), respectively. For the rest, however, the modified corollaries 1-4 and lemma 4 continue to hold; in particular, nothing in corollaries 1-3 hinges on the controller’s participation decision, and the proof of lemma 4 in this situation can be completed by pointing out that if no minority shareholder made a suboptimal decision when \( \Delta (p) \notin [0, 1] \), then

---

\(^{40}\)These modified versions of the corollaries follow from the combination of Lemmata 1 and 2; in particular, lemma 2 implies that the controller with private benefits will snap up any leftover stock at any \( p < \frac{1}{1 - \delta_a} \), such that the relevant cutoff price for minority participation is \( \overline{p}(q) = \overline{p} \).

\(^{41}\)Regarding lemma 4, note that while the “low type” controller (i.e., the controller with private benefits) is now willing to purchase shares in the offer even if \( p \in (\overline{p}, 1) \) (by lemma 2), this does not extend to purchases outside the offer, which do not generate private benefits, such that \( \Delta (p) < 0 \) still will be filled only when \( p > \overline{p} \). To the extent the controller with private benefits buys shares at \( p > \overline{p} \), it may prevent orders \( \Delta (p) > 0 \) being filled, but this is consistent with lemma 4.

\(^{42}\)That corollary 4 continues to hold for both controllers is the difference to the situation without private benefits, where corollary 4 holds for the high type for \( q > 1 \) and for the low type for \( q < 1 \), giving rise to separation.
the controller would.

D.2 Uniqueness of equilibrium price \( p^{**} \) and participation \( \Delta^{**} \)

Unlike in the proof of proposition 1, there is an entire sub-region, namely with participation rates \( \Delta \leq (1 - \delta_i) \bar{p} \), that cannot contain an equilibrium pooling point. In this sub-region, both \( \pi^P \) (weakly) and \( \pi_h \) are decreasing in \( p \), while \( \pi^P \) is increasing and \( \pi_h \) is decreasing in \( \Delta \), such that a deviation from a candidate equilibrium \( p' \) with \( \Delta(p') \leq (1 - \delta_i) \bar{p} \) to a lower \( p \) must be profitable for at least one of the two controller types no matter the induced change in \( \Delta \). We thus limit our search for a possible equilibrium point to \( (p, \Delta) \in (\bar{p}, 1) \times ((1 - \delta_i) \bar{p}, 1) \).

From here on, the proof of the uniqueness of the equilibrium price and participation rates is exactly analogous to that in proposition 1. Concretely, the tangency condition for the two controller types’ isoprofit functions now implies

\[
p^{**} = \frac{1 + q\Delta(p^{**})}{1 + q}.
\]

(3)

The minority indifference condition (a/k/a zero marginal profit condition for share purchases) is now

\[
0 = \rho \frac{1 + (1 - \delta_i) qp^{**}}{1 + q} + (1 - \rho) \frac{1 + qp^{**}}{1 + q} - p^{**}
\]

\[
\Leftrightarrow p^{**} = \frac{1}{1 + \rho \delta_i q}
\]

(4)

Equations (3) and (4) uniquely determine

\[
\Delta^{**} \equiv \Delta(p^{**}) = \frac{1 - \rho \delta_i}{1 + \rho \delta_i q},
\]

which also satisfies \( \Delta^{**} > (1 - \delta_i) \bar{p} \) (cf. the discussion at the beginning of this subsection).

Finally, if the controller announces her participation decision \( q' \) first, the only equilibrium announcement is full participation including purchase of leftover minority allotments. The reason is that the partial derivative of both controller types’ profits with respect to \( q' \) is positive, such that the direct effect of announcing higher \( q' \) is positive for both types, whereas one type’s profits are increasing and the other’s decreasing in \( \Delta \) in the region of possible equilibria \( (p, \Delta) \in (\bar{p}, 1) \times ((1 - \delta_i) \bar{p}, 1) \), such that any change in \( \Delta \) triggered by a change in \( q' \) would offset the direct effect of a higher \( q' \) at most for one of the two controller types.

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D.3 Existence

Off-equilibrium minority purchase fractions \( \Delta^{**}(p) \) and beliefs \( \theta^{**}(p) \) can be specified analogously to proposition 1 using the isoprofit tangent through the equilibrium point and the minority indifference condition for \( \Delta \) and \( \theta \), respectively, on \( (p, \Delta) \in (\bar{p}, 1) \times ((1 - \delta_i) \bar{p}, 1) \), and the same corners outside that region.\(^{43}\) The only slight difference in completing the proof is that the profits of the controller with private benefits are positive even for \( p \not\in (\bar{p}, 1) \) (provided \( p < \frac{1}{1 - \delta_i} \)), and for \( (p, \Delta) \in (\bar{p}, 1) \times \{0\} \).

However, this controller type will not deviate to a \( p \) such that \( \Delta^{**}(p) \) is off of the tangency line: for lower \( p \), this would imply \( \Delta^{**}(p) = 1 \) whereas \( \frac{\partial \pi_{PB}}{\partial \Delta} |_{\Delta = 1} > 0 \); for higher \( p \), this would imply \( \Delta^{**}(p) = 0 \) whereas \( \frac{\partial \pi_{PB}}{\partial \Delta} |_{\Delta = 0} < 0 \).\(^{44}\) Along the tangency line itself, \( (p^{**}, \Delta^{**}) \) is again optimal for the controller by construction of the line and strict convexity of the controller’s preferred set.

E Proof of Proposition 3

That the only possible equilibria remain pooling equilibria in the ranges specified for \( p \) in the proofs of propositions 1 and 2, respectively, and \( q > 0 \) follows immediately from the arguments established in sections B.1 and D.1 of this Appendix, which go through unaffected when the choice is extended from a choice of the singleton \( p \) to a choice of the pair \( (p, q) \), noting that all controllers earn zero profits if \( q = 0 \).

Uniqueness of \( q^* \) and \( q^{**} \), as the case may be, follows from an added tangency condition with respect to \( q \) for the controller isoprofit curves at \( (p^*, q^*, \Delta^*)/(p^{**}, q^{**}, \Delta^{**}) \), which together with the tangency conditions for \( p \) and the minority indifference condition, which both remain unaffected from the proofs of proposition 1 and 2, set up a system of three equations in three unknowns which still has a unique solution:

- That tangency is required also in the \( q \)-dimension follows from the following argument.

\[^{43}\text{Specifically, the analogues to proposition 1 are:} \]

\[
\Delta^{**}(p) = \begin{cases} 
1 & \text{if } p \leq \bar{p} \\
0 & \text{if } p \geq 1 \\
\max \{0, \min \{1, \Delta^{**}(p)\}\} & \text{if } p \in (\bar{p}, 1),
\end{cases}
\]

where \( \Delta^{**}(p) \equiv \Delta^{**} + (p^{**} - p)^{\frac{1 - \Delta^{**}}{1 - \bar{p}}} \), and

\[
\theta^{**}(p) = \begin{cases} 
1 & \text{if } p \leq \bar{p} \\
0 & \text{if } p \geq 1 \\
\frac{1}{\Delta^{**}} & \text{if } p \in (\bar{p}, 1),
\end{cases}
\]

\[^{44}\text{If there were a discontinuous drop of } \Delta^{**}(p) \text{ at } p = \bar{p}, \text{ this would be without consequence because } p = \bar{p} \Rightarrow \frac{\partial \pi_{PB}}{\partial \Delta} = 0. \text{ Similarly, if there were a discontinuous drop of } \Delta^{**}(p) \text{ at } p = 1, \text{ this would only further push the controller towards a lower price because } p = 1 \Rightarrow \frac{\partial \pi_{PB}}{\partial \Delta} > 0. \]

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Note first that at any pooling point, diminishing returns must have set in in the sense that profits are decreasing in \(q\) for one of the two controller types or (a knife-edge case) both controllers have reached indifference to changes in \(q\). If this were not the case and the or one of the controllers whose profits are still increasing in \(q\) were to set a higher \(q\) (while keeping \(p\) at the equilibrium level), then one of two things would have to happen, both of which are incompatible with equilibrium: either \(\Delta\) does not change, in which case the deviating controller increases her payoff, or \(\Delta\) does change in the direction that reduces that first controller’s profits, in which case the other controller would benefit from choosing that higher \(q\), since, as shown above, her profits move in the opposite direction with \(\Delta\).

Now we can distinguish two cases. (1) If both controllers are indifferent to infinitesimal changes in \(q\) at the equilibrium point, then their indifference curves are tangent in the \(q\)-dimension by definition. (2) If one controller’s profits are increasing in \(q\) and the other’s decreasing, then the argument for tangency given above for \(p\) (see Appendix section B.2, footnote 35 and accompanying text) applies analogously to \(q\), given the smoothness assumptions we have made for \(c(\cdot)\).

- That the solution is unique is not guaranteed a priori because the three equations (minority indifference, and tangency with respect to \(q\) and \(p\)) are not linear. However, we already established above that the equilibrium price and minority participation rate are unique for a given \(q\), and it turns out that the solution for \(q\) is unique as well. Specifically, the solution is characterized by:

  - In the case of asymmetric information about asset value:

    \[
    c'(q^*) = \frac{\delta a \alpha (1 - \alpha) \rho (1 - \rho)}{(1 + \rho q^*)^2 - \alpha^2 q^* \frac{(1 - \rho)^2}{1 + \rho q^*} + \alpha (1 - \rho) (q^* - 2 \rho q^* - 1)}
    \]

  - In the case of asymmetric information about private benefits:

    \[
    c'(q^{**}) = \frac{\delta_i \alpha \rho}{(1 + \rho \delta_i q^{**})^2}
    \]

In each case, the solution exists and is unique if the left-hand side of the equation \((c'(q))\) is positive, continuous and non-decreasing in \(q\), and its minimal value is below the maximal value of the right-hand side (which the right hand side takes at \(q = 0\)) because the right-hand side is continuous and decreasing in \(q\); these conditions are guaranteed by our somewhat stronger than necessary assumptions for \(c(\cdot)\). Under these same conditions, inspections of the characterizing
equation reveals that the equilibrium \( q \) is increasing in \( \delta_a \) or \( \delta_i \), as the case may be.

F Proof of Proposition 4

The proof is a simple extension of the proof of proposition 2. Relative to proposition 2, the addition of the asset value reduction \( \delta_a \) is a mere shrinking towards zero of the relevant cutoffs by factor \( 1 - \delta_a \): The low type’s wealth is now increasing (decreasing) in her own purchases of newly issued shares if and only if \( p < \frac{1 - \delta_i}{1 - \delta_i} \) (instead of \( p \leq \frac{1 - \delta_i}{1 - \delta_i} \)), whereas given aggregate offer uptake \( q' \leq q \), atomistic minority shareholders are willing to buy up to \( \tilde{p}(q') \equiv \frac{1 - \delta_a}{1 + \delta_i q'} \) (replacing \( p(q') = \frac{1 - \delta_a}{1 - \delta_i q} \)).

Consequently, nothing changes at all from the logic of proposition 2 (given our assumption that \( \delta_a \leq \delta_i \)); only the parametrization of the low type’s profit function and the minority’s loss function and hence of the tangency and indifference conditions change, now yielding the lower equilibrium price \( p^{***} = \frac{1 - \rho \delta_a}{1 + \rho \delta_i q} < \frac{1}{1 + \rho \delta_i q} = p^{**} \) but still the same minority participation rate \( \Delta^{**} \). Off-equilibrium participation rates and beliefs can be specified analogously to proposition 2. From constant minority participation notwithstanding the lower price, it immediately follows that the minority’s loss from dilution will be higher when facing the high type than in the scenario of proposition 2 (\( \delta_a = 0 \)), and indeed will be increasing in \( \delta_a \) (since the price is decreasing in \( \delta_a \)); since this loss is proportional to \((1 - p)(1 - \Delta)\), it also continues to be increasing in \( \delta_i \). When facing the low type, the minority’s gross loss \( \frac{\alpha \delta_i q}{(1 + \rho \delta_i q)} \{1 + \rho^2 \delta_i q - \rho \delta_a (2 + \rho \delta_i q - \rho \delta_a)\} \) relative to the no-issue baseline is decreasing in \( \delta_a \) (since \( \rho \delta_a < 2 \)), but not the net loss \( \frac{\alpha \delta_i q}{(1 + \rho \delta_i q)^2} (1 - \rho)^2 \frac{\delta_i q + \delta_a}{1 + \rho \delta_i q} \) relative to the baseline of an issue under symmetric information (where the minority would in equilibrium lose \( \alpha \delta_i q \frac{1 - \delta_a}{1 + \delta_i q} \) by extension of lemma 3); both are increasing in \( \delta_i \).
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