Equity Issuance Methods and Dilution

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Abstract

We analyze rights offerings and public offerings when informed current shareholders strategically choose to subscribe. If all current shareholders have wealth to participate, rights offerings achieve the full information outcome and dominate public offerings. However, when some current shareholders are wealth constrained, rights offerings lead to more dilution of their stakes and lower payoffs, despite the income from selling these rights, thereby generating wealth transfers among shareholders. When firms can choose the flotation method, either all firms choose the same offer method or high and low quality firms opt for rights offerings while firms of intermediate quality select public offerings.

Keywords: Rights offerings, Public offerings, Asymmetric Information, Winner Curse, Wealth Transfer

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1 Introduction

Public companies undertake seasoned equity offerings (SEOs) to raise new equity capital from current shareholders and new investors. Broadly speaking, SEOs can be classified into two modes: public offerings and rights offerings.[1] In public offerings, firms announce the issue size, and both current shareholders and new investors can subscribe.[2] In addition, the firm may offer current shareholders some guaranteed allocation of the newly issued shares up to their fractional ownership, which we refer to as dilution protection. The public offering price must be sufficiently attractive for new investors to participate. In rights offerings, firms announce the issue size and offer short-term in-the-money call options, i.e., rights, to current shareholders on a pro-rata basis. Current shareholders receive the rights for free and decide whether to exercise them and receive new shares. Typically, rights can be sold to other investors who then exercise them. The total issue proceeds are the strike price times the number of rights (or equivalently, shares) issued.

A major friction in capital markets are the information asymmetries among the participants, which can lead to mispricing of shares. Such mispricing is a particularly important concern for shareholders and investors at the time when new shares are issued (e.g., Myers and Majluf). On the one hand, shareholders fear that their holdings get diluted due to underpricing of new shares. On the other hand, prospective investors worry that they may end up purchasing overpriced shares.

As we show, there is, however, a simple solution to the informational friction – a rights offering with a sufficiently low strike price, such that even the most pessimistic shareholders exercise their rights. If all current shareholders exercise their rights, their fractional ownership in the firm remains unchanged and no shares are issued to new investors. Accordingly, any dilution to the existing shares caused by the low strike price is exactly offset by the gains on the new shares. Consequently, all shareholders receive the full information payoff, regardless of any potential informational asymmetries among market participants or even

[1] A third way to raise equity finance are private placements in which new shares are sold to a small group of qualified investors. We are interested in equity issuance methods where the share price is determined in competitive markets and therefore do not analyse private placements. Though we briefly discuss implications of our theory for private placements in Section 5.6.

[2] In practice, issuing firms are typically assisted by underwriters who provide certification and possibly commitment to purchase all shares not taken up by investors (e.g., Eckbo and Masulis, 1992). As discussed later, we abstract from underwriters.

among shareholders. In contrast, a public offering always generates some wealth transfer among shareholders and investors because new shares are sold to investors at a premium or discount.

This suggests that rights offerings dominate public offerings in the sense that the former can avoid wealth transfers. In addition, rights offering have lower direct floating costs than public offerings (Smith, 1977; Ecbko, et al., 2007). However, empirical evidence shows that rights offerings are infrequent in the U.S. (e.g., Ecbko, et al., 2007). Outside of the U.S., rights offerings are more common but are often not the predominate issue mode (see Massa et al., 2016). This so-called rights puzzle has been explained with adverse selection problems which are mitigate in public offerings through underwriter certification (e.g., Eckbo and Masulis, 1992).[3] Nonetheless, a fundamental question remains: Why do firms bother with underwriter certification in public offerings given that rights offerings can circumvent the information problem?

In this paper, we relax one crucial condition that allows rights offerings to resolve the information problem, namely that all current shareholders have the resources needed to exercise their rights. In other words, we assume that some current shareholders cannot, for some exogenous reasons, subscribe to new shares in public offerings or exercise their rights in rights offerings. Henceforth, we refer to these shareholders as cash-poor. We study wealth transfers in public and rights offerings, and the firms' choice of issue method in a setting with information asymmetries and some wealth-constrained shareholders.

Surprisingly, cash-poor shareholders fare better in public offerings than in rights offerings, despite the fact that they obtain proceeds from selling their rights, but receive no (extra) compensation in a public offering. Intuitively, rights have a positive value only if the strike price is lower than the equilibrium price in a public offering. Such a lower strike price implies that more new shares must be issued in a rights offerings to fund the investment. However, rights are priced at a discount on average due to the winner's curse problem, similar to Rock (1986). This implies more dilution to the existing holdings, which is not fully compensated by the proceeds from selling the rights.

We begin our analysis by comparing the equilibrium outcomes of public offerings with different degree of dilution protection respectively, rights offerings with different strike prices, each solved assuming that firms can raise funding only through a given public or rights

[3]Other explanations for the choice of issue mode are discussed in the related literature.

offering. Throughout the paper, we assume that informed capital is scarce. In particular, some current shareholders know the value of the firm, specifically, the net present value of its investment opportunity, whereas other shareholders as well as new investors are uninformed. Furthermore, informed shareholders can at most purchase the shares allocated to them on a pro-rata basis, but not buy any additional shares or rights from uninformed shareholders or new investors. Otherwise, informed capital would not be scarce.

As we show, the flotation methods can be ranked according to the ex-ante (before shareholders learn the firm type/value) wealth transfer between cash-rich and cash-poor shareholders. First, cash-poor shareholders lose more to cash-rich shareholders in rights offerings. This transfer increases with lower strike prices, because more new shares have to be issued, thereby diluting the ownership stakes of cash-poor shareholders more without due compensation from the sale of the rights. Thus, cash-poor shareholders are neither indifferent about the offer method nor the chosen strike price in rights offerings. By contrast, new investors always break even on average and are therefore indifferent about offer mode and terms.

Second, the public offering with full dilution protection of current shareholders is equivalent to the rights offering with a strike price chosen such that the equilibrium rights price is zero. Intuitively, current shareholders in a public offering with full dilution protection can maintain their fractional ownership by subscribing, replicating the outcome of a rights offering. Conversely, a zero rights price implies that current shareholders who do not exercise but sell their rights receive no additional income. Hence, their payoff in this rights offering is the same as in the public offering with full dilution protection.

Finally, public offerings with less dilution protection entail less wealth transfer between cash-poor and cash-rich shareholders. With less dilution protection, informed cash-rich shareholders can purchase fewer newly issued shares, thereby reducing the adverse selection problem. In the limiting case of zero dilution protection, cash-rich shareholders receive no new shares even when they subscribe and are therefore the same as cash-poor shareholders. Hence, there is no redistribution among current shareholders. As summarized in Figure 1, we can rank all issue methods based on the wealth transfers among current shareholders.

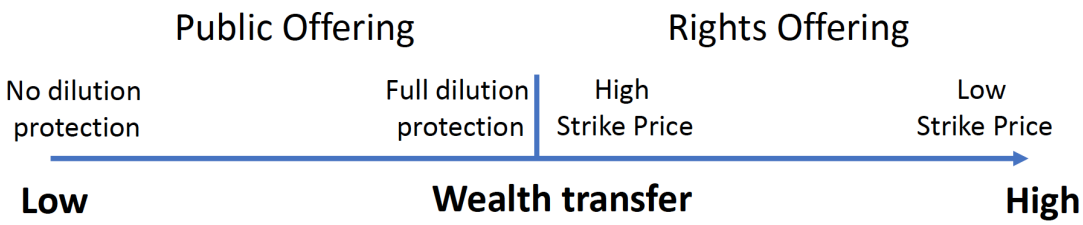

Figure 1: Rank issue modes based on wealth transfers.

These results imply that if offer mode and terms are set prior to firms knowing their type, firms which care more about equality among current shareholders select public offerings with no dilution protection. Conversely, firms which cater (more) for cash-rich shareholders opt for rights offerings with low strike prices.

In the second part of the paper, we analyze how firms choose offer mode and terms when knowing their type. The choice of flotation method may therefore serve as a signal to uninformed investors. For this signaling game, we assume that firms maximize the total payoff to all current shareholders, or equivalently, minimize the payoff to new investors. As we show, only two kinds of equilibria can exist: The first kind is a pooling equilibrium in which all firms choose the same dilution protection in a public offer, or alternatively all firms choose the same strike price in a rights offering. In the second kind of equilibrium, a single rights and a single public offering co-exist where high and low quality firms opt for the rights offering, while intermediate firm types select the public offer. Intuitively, a rights offering can only exist if the strike price is lower than the public offering price, resulting in a bigger fraction of the firm being sold. Low quality firms use rights offerings to sell a bigger fraction of their overvalued firms. On the flip side, high quality firms favor rights offering because the ability to maintain the fractional ownership is more valuable to the cash-rich shareholders when firms are more undervalued.

Literature Review

We focus our discussion on papers that - like ours - consider asymmetric information problems the primary concern when raising equity financing. We only briefly discuss other explanations for the choice of issue method and also abstract from papers that analyse private

placements or compare them with either public or rights offerings. The literature recognizes that rights offerings allow current shareholders to avoid – in principle – dilution. If all shareholders participate proportionally in a rights offering they maintain their fractional ownership. Consequently, there are neither adverse selection problems nor wealth transfers (e.g., Myers and Majluf 1984, p. 195 footnote 5; Berk and DeMarzo 2017, p. 856). However, as noted by e.g., Ursel (2006) or Wu et al. (2016), if some shareholders sell their rights to other investors, adverse selection problems arise as in the Myers and Majluf (1984) setting.[4] Our analysis shows that the ensuing adverse selection problems are aggravated in rights offerings by the winner's curse problem given some current shareholders strategically decide whether to exercise or sell their rights.

Ecbko and Masulis (1992) argue that underwriter certification and low shareholder take-up can explain why firm prefer rights offering. In their framework, underwritten offers are not direct sales as in Myers and Majluf (1984), but come with a noisy though informative certification of the firms' value. There is no such certification in uninsured rights offerings and the fraction of the issue taken up by current shareholders is exogenously given. Clearly, undervalued firms experience a wealth loss, which increases as the shareholder take-up becomes smaller. Consequently, the choice of issue mode depends upon the shareholder take-up: If it is high (low), the uninsured rights offering entails less (more) wealth transfers to investors than the underwritten issue. Our framework differs along two important dimensions. First, public offers do not feature an underwriter who plays an informational role or guarantees the offer. Second, shareholder take-up is a strategic decision rather than driven by factors outside the model.[5]

Heinkel and Schwartz (1986) also consider an extended Myers and Majluf (1984) setting to examine the choice between fully underwritten public offers and uninsured rights offerings. In their model, firms differ in the probability distribution of their terminal stock price, and the distribution depends on a parameter which is private information to the firm. All firms want to raise the same amount of equity capital, and if the realized terminal share price is less than the subscription price, the offer fails and the firm incurs a fixed cost per share. Thus, issuing

[4]If rights are non-tradeable, wealth transfers between current shareholders and investors are eliminated, though not necessarily transfers among shareholders.

[5]Eckbo and Norli (2005) add more structure to the framework of Eckbo and Masulis (1992) to prove equilibrium. They also allow for a larger menu of flotation methods. As in Eckbo and Masulis (1992), the exogenous shareholder take-up is the crucial determinant for the issue choice.

a larger number of shares – as lower quality firm must to raise the financing – makes failure more costly. Failure (costs) are avoided by using an underwriter who guarantees the offer proceeds. The failure cost of the uninsured rights offer enables high quality firms to use the subscription price to credibly reveal their types (expected terminal share price). Low quality firms prefer to sell shares at a pooling price through an uninformed underwriter, because the (expected) failure cost of an uninsured rights offering exceed underwriter fees and possible undervaluation. Heinkel and Schwartz assume that firms choosing an underwritten offer sell their issue to the underwriter at the same price they would announce in a rights offer, if they were to choose that financing method. Therefore, the extent to which shareholders participate in the rights offering plays no role.[6] By contrast, our framework features a meaningful market for rights and current shareholders who strategically decide whether to participate respectively exercise or sell their rights. Furthermore, there is no failure risk because issue respectively rights prices adjust to allow investors to break even in equilibrium.

There are several other explanation for the choice of issuance methods that are not based on informational frictions. Smith (1977) attributes the prevalence of public offerings in the US to agency conflicts among managers and shareholders. Hansen (1988) argues that shareholders face additional flotation cost in form of price concessions in rights offering which are absent in public offerings. Hence, public offerings are more attractive even though the direct flotation costs are larger. Hansen and Pinkerton (1982) propose that differences in ownership structures account for the choice of flotation method. Firms with large blockholders opt for rights offering, whereas dispersedly held firms find public underwritten offer the more cost efficient way to raise new equity financing. Ursel (2006) argues that firms in poor financial condition with low net worth use rights offerings since current shareholders have larger incentives to inject new funds to keep the firm alive than outside investors. Thus, rights issues are a (equity) financing of last resort. Wu et al. (2014) propose that the flotation choice is driven by rent-protection motives of controlling shareholders.[7] In their model, the controlling shareholder can maintain her fractional ownership in a rights offering, but her stake is

[6] In an extension Heinkel and Schwartz introduce standby rights offers as a third issue mode. The underwriter promises to purchases any not taken-up shares in exchange for a fee and also learns the firm type at some cost. In equilibrium, the highest quality firms choose the standby rights offer as they find it less expensive to reimburse the underwriter for becoming informed.

[7] Focusing exclusively on rights offerings, Fried and Spamann (2018) show that pre-emptive rights do not protect minority shareholders against expropriation through an equity issue, so-called cheap-stock tunneling.

getting diluted in a public offer which is (more) costly when private benefits are large. In a cross-country study with a sample of share issues from 41 countries during 1990-2008 McLean et al. (2013) find that the likelihood of public offerings relative to both private placements and rights offerings increase with investor protection. Finally, Holderness (2017) covers in his meta-analysis over 100 studies on equity issuances in different countries. He argues that the flotation choice is driven by the presence or absence of mandatory shareholder approval. In countries where shareholders must approve an issue, rights offerings are much more common, whereas public offers are more common in countries that allow management/boards to issue equity without shareholder approval.[8]

2 Model Setup and Benchmark

2.1 Model

Consider an economy that is populated by publicly traded firms with assets in place a and an unfunded investment opportunity which requires an outlay I and generates a payoff $I+b$. For simplicity, we assume that both the value of the assets in place a as well as the investment cost I are the same across all firms and publicly known. By contrast, the net present value (NPV) of the investment $b > 0$ varies across firms and is distributed on $[\underline{b}, \overline{b}]$ according to the density function $f(b)$, respectively its distribution function $F(b)$. As we discuss in Section 5.2, it is largely inconsequential whether the information asymmetry concerns the assets in place or the investment b. The number of existing shares is normalized to 1. Since we want to compare equity flotation methods we restrict firms to raise I by issuing new equity through either a public offering (PO) or a rights offering (RO), which we describe later. Current shareholders and competitive new investors are all risk-neutral.

The key frictions in the model are information asymmetry and the scarcity of informed capital. In particular, among current shareholders only a fraction $(1-\eta)$ know the project's NPV b while the remaining η shareholders merely know its distribution. Like the latter, new investors only know the distribution of b. The relevant information asymmetry is the one between current shareholders and investors. Except for the benchmark in Subsection

[8] He reports that shareholder-approved issues are associated with positive and higher announcement returns than managerial issuances, and that this holds across and within countries as well as for different issue methods.

2.2, we make the simplifying assumption $\eta = 0$, that is, all current shareholders know the project's NPV b. We discuss the scenario when some shareholders are uninformed ($\eta > 0$) in Subsection 5.4.

In addition and independent of information, a fraction π of shareholders have no spare wealth to participate in the equity issuance. Furthermore, these cash-poor shareholders can neither borrow nor sell (part of) their current shares to participate in an offering. We relax this assumption in Subsection 5.5. The remaining $(1-\pi)$ shareholders have financial slack to purchase additional shares. However, they cannot trade with either cash-poor shareholders or investors. That is, cash-rich current shareholders can at most purchase those newly issued shares which are allocated to them on a pro-rata basis. Without this restriction, informed capital would not be scarce and any asymmetric information friction would be eliminated.[9]

Our assumption of cash-poor shareholders can be interpreted in different ways. First, some shareholders may have exhausted their buying power, and borrowing on margin account for an extended period of time can be too expensive. Furthermore, we argue in Section 5.5 that borrowing to exercise the rights and immediately selling the underwritten shares to pay back the loan is equivalent to selling the rights directly. Second, inattentive shareholders whose rights are sold by their brokers on their behalf are equivalent to cash-poor shareholders in the model. (See Section 5.3). Finally, managers in public firms typically do not purchase significant amounts of the newly issued shares, since much of their wealth is already tied to the firm.

Throughout the paper, we consider stylized versions of public and rights offerings. In public offerings, the firm issues new shares in the public market, but shareholders may receive some dilution protection. That is, current shareholders are given priority over the some fraction $\lambda \in [0, 1]$ of the new shares on a pro-rata basis. Obviously, only the cash-rich shareholders may buy additional shares and possibly benefit from the dilution protection. Investors get to buy all $(1 - \lambda)$ non-dilution protected shares and those dilution protected shares that shareholders do not to take up.[10] Shareholders and investors simultaneously

[9]We could allow cash-rich shareholders to buy more shares than those allocated to them on a pro-rata basis, as long as new investors' participation is still necessary for the offer to succeed. If instead cash-rich shareholders could purchase all new shares, informed capital would no longer be scarce, and the information frictions would disappear.

[10]Dilution protection is very common in the UK. In countries where shares are allocated on a pro-rata basis based on the subscription, the parameter λ can be interpreted as the demand of the shareholders relative to that of the investors (similar to Rock 1986).

decide whether to subscribe. Finally, the investors' break-even condition determines the per share price P_{PO} and the number of newly issued shares such that $N_{PO} = \frac{I}{P_{PO}}$.[11]

The payoff to shareholders in a public offering depends on the offer price P_{PO}, the number of shares issued N_{PO}, and their subscription decision. After issuing new shares and investing, the firm value is equal to $I + a + b$. Given the number of shares is $N_{PO} + 1$, the share price must equal $\frac{1}{N_{PO}+1}(I+a+b)$. If a cash-rich shareholder with β shares subscribes, she receives $\lambda\beta N_{PO}$ new shares in exchange for investing an amount $\lambda\beta N_{PO}P_{PO} = \lambda\beta I$. As a result, her payoff as a function of the true firm type b is

$$\beta\left[\frac{\lambda N_{PO}+1}{N_{PO}+1}(I+a+b) - \lambda I\right]. \tag{1}$$

If a shareholder chooses not to subscribe or has no cash to do so, her payoff is

$$\beta\frac{1}{N_{PO}+1}(I+a+b). \tag{2}$$

In rights offerings with a strike price P_S, $N_{RO} = \frac{I}{P_S}$ rights are issued to shareholders on a pro-rata basis at no cost. Each right gives its owner the option to purchase a newly issued share at the strike price P_S. Cash-rich shareholders can choose between exercising the rights or selling them to new investors. Since cash-poor shareholders can neither borrow nor sell their current shares, they have no choice but to sell their rights to new investors. Doing nothing, that is, neither exercising nor selling the rights, is weakly dominated by selling the rights as long as $P_R \geq 0$. Therefore, we rule out doing nothing as an option here, but discuss it in Section5.3. The break even constraint of the competitive investors determines the rights price P_R, and we exclude negative prices.

Similar to public offerings, the post-right-offering firm value is $I + a + b$ and the number of shares is $N_{RO} + 1$. Hence, the share price is $\frac{I+a+b}{N_{RO}+1}$. If a cash-rich shareholder with β shares exercise her rights, she receives βN_{RO} new shares and invests $\beta N_{RO}P_S = \beta I$. As a result, her payoff as a function of the true firm type b is equal to

[11] Our stylized public offering is a direct share sale and resembles an At-The-Market (ATM) offerings, except that an ATM offering may split the total issuance into smaller quantities spread over some time period. In the US ATM offerings have recently become more popular and the number of ATMs in 2015 was 40 percent compared to the number of Seasoned Equity Offerings (Billett et al., 2016).

$$\frac{a+I+b}{N_{RO}+1}(N_{RO}\beta+\beta)-\beta N_{RO}P_S=\beta\left(a+b\right). \tag{3}$$

If a current shareholder sells her rights, her payoff is

$$\frac{a+I+b}{N_{RO}+1}\beta+P_R\beta N_{RO}. \tag{4}$$

Subsequently, we first solve the benchmark below and then in Section 3 the equilibrium outcomes when all firms adopt a given flotation method. We compare these equilibrium outcomes for different offer modes and terms, i.e., public offers with different λ and rights offering with different P_S. In Section 4, we let firms choose both, offer mode and terms, knowing their type (realization of b). In Section 5 we discuss empirical implications of our model and the robustness of our results with respect to the source of the information asymmetry, shareholder participation, in particular, allowing uninformed as well as cash-constrained shareholders to participate in the offerings, and possible trading of shares.

2.2 Benchmark

Here we examine the outcome of the two offering methods in the absence of cash-constrained shareholders ($\pi = 0$). The only friction is the information asymmetry. Specifically, a fraction $\eta > 0$ of shareholders only knows the distribution of the project's NPV b, as do the investors. The main result is that rights offerings can achieve the full information payoff for all shareholders, whereas public offerings necessarily create wealth transfers between shareholders and investors.

As noted in the literature, rights offering can avoid such wealth transfers if "stockholders can be compelled to exercise their rights and hold the newly issued shares" (Myers and Majluf, 1984 footnote 5). We extend this intuition by showing that rights offerings can resolve asymmetric information problems among current shareholders, ensuring that each and every shareholder receives the full information payoff $a + b$.

Proposition 1 *Given all current shareholders are cash-rich, they all receive a net payoff of $a+b$ in the unique equilibrium outcome of a rights offering. Moreover, this equilibrium exists only if $P_S \leq a + \underline{b}$, and is implemented by all current shareholders exercising their rights.*

When a shareholder exercises the rights allocated to her on a pro-rata basis, her payoff does not depend on the strike price. Indeed, exercising the rights implies that her fractional ownership stake in the firm remains unchanged. Therefore, any mispricing of the issue (strike price P_S) is fully offset by a corresponding value change of her "old" shares. However, informed shareholders of firms with low project values b may find it more profitable to sell their rights in the market. A sufficiently low strike price in combination with market beliefs that any rights sold would come from the worst firm type $\underline{b}$ make this an inferior option. As a result, informed as well as uninformed shareholders find it in their interest to exercise their rights. Consequently, they all receive a net payoff equal to $a + b$, as they would under complete information. The proof in the appendix shows that this equilibrium outcome is unique.

As shown by Myers and Majluf (1984), selling shares to investors in a public offering inevitably leads to wealth transfers. This holds true also in our setting.

Proposition 2 *Any public offering with incomplete dilution protection $\lambda < 1$ leads to wealth transfers among informed shareholders and uninformed investors in (almost) all firms.*

In public offerings without dilution protection ($\lambda = 0$), new investors purchase all new shares in a successful offering, as in Myers and Majluf (1984). Since they are uninformed, the price P_{PO} must - in equilibrium - be the same for any and all firms, irrespective of the net present value of the investment opportunity. Moreover, investors only purchase shares if the price is such that they break even on average. Consequently, there is mispricing and redistribution across firm types: For firms whose investment project has a low (high) net present value, the new shares are overpriced (underpriced), and investors make a loss (profit). Accordingly, shareholders receive a payoff which is either larger or smaller than $a + b$, their full information payoff.[12]

The asymmetric information problems are exacerbated by the dilution protection, since it adds a winner's curse problem. Informed cash-rich shareholders take up their allocated quota $\lambda(1 - \eta)$ only if the issue is underpriced. As a result, investors end up buying more shares when a firm is overpriced. Hence, dilution protection leads to additional redistribution among informed shareholders and investors. As discussed in Subsection 5.4, uninformed shareholders benefit from taking up their allocated quota.

[12]This does not hold for the one firm type whose project happens to have the value such that the price is fair, i.e., $b = P_{PO} - a$.

Comparing Propositions 1 and 2, we establish the following benchmark: Rights offerings dominate public offerings since the former but not the latter overcomes informational frictions and avoids redistribution both among shareholders and between shareholders and new investors. Hence, the widespread use of public offerings cannot be attributed exclusively to asymmetric information problems. There must be at least one other friction. Subsequently, we (re-)introduce wealth constraints of some shareholders (i.e., $\pi > 0$) and show how this may reverse the ranking of the two offer methods.

3 Offer Methods and Wealth Transfer

In this section, we characterize the equilibrium outcomes and the wealth transfers among shareholders and investors in each offer mode. In addition, we analyze how the equilibrium payoffs vary with the extent of dilution protection (λ) in public offerings, respectively with the strike price P_S in rights offerings. As a result, we can rank all issue methods based on the wealth transfers they generate.

To simplify the exposition, we assume all shareholders are informed, i.e., we set $\eta = 0$, and focus on the information asymmetry between shareholders and investors. In fact, as we show in Subsection (5.4), introducing some uninformed shareholders would not materially change the analysis.

3.1 Public Offerings

We first study the equilibrium outcome of a public offering with a given common dilution protection λ. Cash-rich shareholders subscribe to the new shares only if the payoff from subscribing (1) is higher than the payoff from abstaining (2).

Lemma 1 *In a public offering with a given λ, cash-rich shareholders subscribe to the new shares if and only if*

$$b \geq b^*_{PO} \equiv P_{PO} - a. \tag{5}$$

Cash-rich shareholders follow a simple threshold strategy and subscribe to an offer only if the sum of assets in place a and net present value of the investment b (weakly) exceed the

price P_{PO}. That is, they subscribe only if the new shares are underpriced, similar to e.g., Rock (1986). Clearly, cash-poor shareholders have no choice but to abstain from the offer.

Since investors do not know the net present value of the investment, their participation in the offer must be unconditional, that is, cannot depend on the firm type b. At the same time they anticipate that cash-poor shareholders never subscribe but that cash-rich shareholders only subscribe if the issue is not overpriced. Accordingly, the investors' collective payoff when subscribing is equal to

$$Pr\left(b|b<b^*_{PO}\right)\left[\frac{N_{PO}}{N_{PO}+1}(a+I+E[b|b<b^*_{PO}])-I\right]$$

$$+Pr\left(b|b\geq b^*_{PO}\right)\left[1-(1-\pi)\lambda\right]\left[\frac{N_{PO}}{N_{PO}+1}(a+I+E[b|b\geq b^*_{PO}])-I\right], \tag{6}$$

In the above expression, the first line represents the case when the offer is overvalued and all N_{PO} new shares are purchased by the investors. They receive a fraction $\frac{N_{PO}}{N_{PO}+1}$ of the firm any in exchange for contributing I. The second line reflects the case when the offer is undervalued and $1-\pi$ cash-rich shareholders take-up a total of $(1-\pi)\,\lambda N_{PO}$ shares. The remaining $[1-(1-\pi)\lambda]\,N_{PO}$ shares are purchased by the investors. Rearranging the terms in (6) using the fact that $N_{PO}=\frac{I}{P_{PO}}$ and factoring out $\frac{N_{PO}}{N_{PO}+1}$ yields

$$Pr\left(b|b\geq b^*_{PO}\right)\left[1-(1-\pi)\lambda\right]\left[a+E[b|b\geq b^*_{PO}]-P_{PO}\right]+Pr\left(b|b<b^*_{PO}\right)\left[a+E[b|b<b^*_{PO}]-P_{PO}\right],$$

which can be rewritten as

$$a+E(b)-P_{PO}-Pr\left(b|b\geq b^*_{PO}\right)(1-\pi)\lambda\left[a+E[b|b\geq b^*_{PO}]-P_{PO}\right]. \tag{7}$$

Given that informed capital is scarce, the zero profit condition of the competitive investors determines the equilibrium issue price P_{PO}.

Proposition 3 *For any given $\lambda\in[0,1]$, there exists an equilibrium in which all firms raise I. The equilibrium price P_{PO} is decreasing in λ, and $a+\underline{b}<P_{PO}\leq a+E(b)$, with equality holding only for $\lambda=0$.*

Since all firms issue and invest, the price P_{PO} in any equilibrium must exceed the value of firm with the lowest net present investment $(a+\underline{b})$ and can be at most equal to the

unconditional mean of firm values $(a + E(b))$. Otherwise, investors would on average either earn a profit or not break even. Furthermore, the equilibrium price P_{PO} is decreasing with the extent of the dilution protection, because it exacerbates the winner's curse problem. When shareholders enjoy better dilution protection, investors get to buy a smaller fraction $(1 - (1 - \pi)\lambda)$ of underpriced shares while still buying all overpriced shares. Consequently, they can only break even if the equilibrium price is lower. In the limiting case of no dilution protection, the information advantage of the cash-rich shareholders becomes irrelevant. They never receive any new shares, and there is no winner's curse problem. Hence, investors are willing to purchase the new shares at the unconditional average firm value, that is, $P_{PO} = a + E(b)$.

We now turn to the wealth transfers between cash-poor and cash-rich shareholders. Since investors break even on average, we can define the ex-ante (prior to knowing b) wealth transfer from cash-poor to cash-rich shareholders as the difference between the expected actual payoff and the fair expected payoff $(a + E(b))$. Using the payoff of the cash-poor shareholders (2), we can express the wealth transfer explicitly as

$$WT_{PO} \equiv a + E\left(b\right) - \frac{1}{1 + \frac{I}{P_{PO}}}\left[I + a + E\left(b\right)\right]. \tag{8}$$

We next rank all public offerings with different dilution protections according to the extent of the ex-ante wealth transfers among current shareholders.

Proposition 4 *Public offerings with a given dilution protection λ feature ex-ante wealth transfers from cash-poor shareholders to cash-rich shareholders amounting to*

$$WT_{PO} = \frac{I}{I + P_{PO}}\left[a + E(b) - P_{PO}\right] \geq 0, \tag{9}$$

These transfers decrease in the equilibrium issue price P_{PO} and increase in λ.

A lower price P_{PO} necessitates that more shares are issued. This in turn makes the cash-rich shareholders' ability to subscribe more valuable. Hence, the wealth transfer is larger when the price P_{PO} is lower. Since a better dilution protection λ grants cash-rich shareholders the option to purchase more new shares, it exacerbates the winner's curse problem. Hence, the equilibrium price P_{PO} must decrease to allow investors to break even, resulting in more wealth transfers among shareholders.

3.2 Rights Offerings

In this section, we consider the equilibrium outcome in rights offerings with a given strike price $P_S = I/N_{RO}$. Cash-rich current shareholders exercise their rights if and only if (3) is weakly greater than (4).

Lemma 2 *In a rights offering, cash-rich current shareholders exercise their rights if and only if:*

$$b \geq b^*_{RO} \equiv P_S + P_R(N_{RO} + 1) - a. \tag{10}$$

As in public offerings, cash-rich shareholders follow a simple threshold strategy. They prefer to sell their rights if the investment return falls below the cut-off value b^*_{RO}. The expression of the cutoff value b^*_{RO} is more complicated than that in a public offering because the "fair" value in a rights offering contains the bundle of strike price P_S and rights price P_R. Investors who purchase one right and exercise it have a payoff equal to

$$\frac{a + I + b}{N_{RO} + 1} - P_S - P_R.$$

Rational investors also anticipate that cash-poor shareholders always sell their rights, whereas cash-rich shareholders sell them only if the project returns are low ($b < b^*_{RO}$). Consequently, the expected payoff to investors is

$$\begin{aligned} Pr(b \geq b^*_{RO})\pi N_{RO} \left(\tfrac{a+I+E(b|b\geq b^*_{RO})}{N_{RO}+1} - P_S - P_R \right) \\ + Pr(b < b^*_{RO}) N_{RO} \left(\tfrac{a+I+E(b|b<b^*_{RO})}{N_{RO}+1} - P_S - P_R \right) \end{aligned} \tag{11}$$

Investors break even if the sum of strike and rights prices equals the conditional expected firm value (on a per share basis), taking into account when cash-rich shareholders subscribe.

$$P_S + P_R = \frac{1}{N_{RO} + 1} \left[a + I + \frac{\pi Pr(b \geq b^*_{RO}) E(b|b \geq b^*_{RO}) + Pr(b < b^*_{RO}) E(b|b < b^*_{RO})}{\pi P(b \geq b^*_{RO}) + Pr(b < b^*_{RO})} \right]. \tag{12}$$

Proposition 5 *For any given $P_S \in (0, a + b^*_{RO}]$, there exists a rights offering equilibrium in which all firms issue rights with the same strike price P_S. The cutoff type b^*_{RO} solves*

$$b^*_{RO} = \frac{(1-\pi) Pr(b < b^*_{RO}) E(b|b < b^*_{RO}) + \pi E(b)}{(1-\pi) Pr(b < b^*_{RO}) + \pi}, \tag{13}$$

lies in $(\underline{b}, E(b))$*, and is independent of* P_S*.*

In equilibrium cash-rich shareholders follow their threshold strategy and investors break even, that is, the equilibrium rights price P_R is such that (10) and (12) hold. At first glance it may seem surprising that the cutoff value b^*_{RO} does not depend on the strike price P_S. To understand this feature it is perhaps best to consider the sell/exercise decision of cash-rich shareholders in firms with low project returns ($b < b^*_{RO}$). Clearly, exercising is not attractive if the strike price is overvalued, that is, if $P_S > a+b$. Exercising at low strike prices $P_S < a+b$ is attractive, but selling the rights is even more profitable because rights are priced by the investors' beliefs about the conditional average firm value. Since cash-poor investors in all firms sell their rights, the rights price for firms with low investment returns is effectively subsidized. Regardless of the strike price, firms above or below the conditional average belief are the same, and the cutoff value is therefore not affected by the strike price.

The strike price does, however, affect the wealth transfers from cash-poor to cash-rich shareholders. Given a strike price P_S, the payoff to cash-poor shareholders in a type-b firm is equal to

$$\frac{a+I+b}{N_R+1} + P_R N_R$$

which can be rewritten as

$$a + b + \frac{I}{I+P_S}[b^*_{RO} - b].$$

(See the proof of the subsequent proposition for details.) As in public offerings, we can define the ex-ante wealth transfer among shareholders in rights offerings as the difference between the expected actual payoff and the fair expected payoff $(a + E(b))$:

$$WT_{RO} \equiv \frac{I}{I+P_S}\left[E(b) - b^*_{RO}\right]. \tag{14}$$

Proposition 6 *Rights offerings with a strike price* P_S *feature ex-ante wealth transfers from cash-poor shareholders to cash-rich shareholders amounting to*

$$\frac{I}{I+P_S}\left[E(b) - b^*_{RO}\right] > 0, \tag{15}$$

which decrease in P_S*.*

The intuition is similar to that of Proposition 4. Lower strike prices P_S require more

rights N_{RO} to be issued. Since rights are on average underpriced due to the winner's curse problem, the financial ability to exercise the rights becomes more valuable when more rights are issued. For cash-poor shareholders who cannot exercise the rights, a lower strike price implies that a larger fraction of the company is sold at a discount on average. Hence, lower strike prices and larger numbers of rights N_{RO} lead to more wealth transfers from cash-poor to cash-rich shareholders.

Absent information asymmetries, exercising the rights and buying the new shares or simply selling the rights yield the same payoff. Gains made from exercising are matched by the proceeds from the rights sale (Farinha et al. 2017). Proposition 6 implies that this does not hold in our setting with asymmetric information. Due to the winner curse problem, the rights price does not fully compensate for the dilution of existing share holdings. Therefore, the pricing of the new shares matters for cash-poor shareholders.

3.3 Comparing Public and Rights Offerings

In rights offerings, non-participating current shareholders (can) sell the rights instead of doing nothing in public offerings. In addition, new investors are exposed to a larger extent to the winner's curse problem than in public offerings. Despite these two differences, one specific public offering (with full dilution protection) is equivalent to one specific rights offering (with zero rights price).

Proposition 7 *Rights offerings with a strike price P_S such that the equilibrium rights price P_R equals 0 are equivalent to public offerings with full dilution protection ($\lambda = 1$).*

Denote by P_{PO} the equilibrium issue price in a public offering with full dilution protection. Intuitively, in a rights offering with an equal strike price $P_S = P_{PO}$ the value of the rights is zero in equilibrium ($P_R = 0$), because such a right resembles an at-the-money option at expiration. Under this conjecture, the payoffs to current shareholders are the same in both issue modes. First, if cash-rich shareholders participate in the offering, they can maintain their fractional ownership at the same price in either offering. Conversely, if shareholders cannot or choose not to subscribe, their payoffs must again be the same in either offering. Their holdings are equally diluted since the prices P_{PO} and P_S are equal and hence also the number of newly issued shares. Moreover, selling the rights does not generate any income.

Because all shareholders receive the same payoffs in either issue modes, cash-rich shareholders in the same firm types find it profitable to subscribe respectively to abstain, generating the same extent of the winner's curse problem in either offering. Finally, if one were to create in the public offering an option to buy shares at a price P_{PO}, such an option had no value, confirming that P_R is zero.

The equivalence result allows to rank the flotation methods according to the extent to which they entail wealth transfers between cash-poor and cash-rich shareholders.

Corollary 1 *Any rights offering entails more wealth transfers from cash-poor to cash-rich current shareholders than public offerings.*

On the one hand, wealth transfers increase in the dilution protection λ (Proposition 4). Therefore, a public offering with full dilution protection comes with the largest wealth transfers among all public offerings, whereas an offer without any dilution protection features no such wealth transfers. On the other hand, wealth transfers decrease with the strike price P_S in rights offerings (Proposition 6) which in turn is highest when rights have zero (resale) value. Hence, among all rights offerings the one with an equilibrium rights price P_R equal to 0 leads to the least wealth transfer. This least-wealth-transfer rights offering ($P_R = 0$) is equivalent to the aforementioned most-wealth-transfer public offering ($\lambda = 1$). Consequently, any rights offering generates more wealth transfer from cash-poor to cash-rich shareholders than any public offerings.

The Corollary suggests one consideration that may affect firms' choice of issuance mode — wealth transfers among shareholders. When firms care more about equality among shareholders, they opt for a public offering with no dilution protection to avoid such wealth transfers. Conversely, when firms favor their cash-rich shareholders, they use rights offerings with low strike price to create wealth transfer from the cash-poor shareholders. This result is depicted in Figure 1 in the introduction.

4 Choosing Offer Terms and Modes

The above analysis derives equilibrium outcomes for rights and public offerings for given strike prices, respectively dilution protections, and then compares these outcomes in terms of wealth transfers among shareholders. We now examine the outcomes when firms can

strategically choose offer mode and terms, knowing their type b at the time of the offering. When solving this signaling game, we assume that firms choose the issue to maximize current shareholder wealth. This objective function is less evident than it may appear at first sight since in our framework cash-rich and cash-poor shareholders typically disagree over the optimal flotation method, the extent of dilution protection, or the strike price. In view of the diverging preferences among shareholders some arbitrariness in choosing the firms' objective function is unavoidable. We opt for firms maximizing the weighted sum of their cash-poor and cash-rich shareholders because it it is equivalent to minimizing new investors' payoffs. The later seems to us the least controversial objective function in our framework.

When subscribing to an offer by type-b firm in a public offering with dilution protection λ, price P_{PO}, and number of new shares $N_{PO} = \frac{I}{P_{PO}}$, the investors realize a payoff equal to

$$\Pi_{PO}(b) = \begin{cases} [1-\lambda(1-\pi)]\, N_{PO}\left(\frac{a+I+b}{N_{PO}+1} - P_{PO}\right) & \text{if } \frac{a+I+b}{N_{PO}+1} \geq P_{PO} \\ N_{PO}\left(\frac{a+I+b}{N_{PO}+1} - P_{PO}\right) & \text{if } \frac{a+I+b}{N_{PO}+1} < P_{PO}, \end{cases} \tag{16}$$

Similarly, in a rights offering with strike price P_S, number of rights $N_{RO} = \frac{I}{P_S}$, and rights price P_R, the investors' payoff is equal to

$$\Pi_{RO}(b) = \begin{cases} \pi N_{RO}\left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R\right) & \text{if } \frac{a+I+b}{N_{RO}+1} \geq P_S + P_R \\ N_{RO}\left(\frac{a+I+b}{N_{RO}+1} - P_S - P_R\right) & \text{if } \frac{a+I+b}{N_{RO}+1} < P_S + P_R \end{cases}. \tag{17}$$

Only two kinds of equilibria can exist: a pooling equilibrium where all firms use the same issue method (Subsection 4.1), and a semi-pooling equilibrium where some firms pool on a unique public offering and others pool on a unique rights offering (Subsection 4.2).

4.1 Single Offer Mode Equilibria

When firms can strategically decide on the offer mode and terms, all firms choosing the same offer remains an equilibrium. Such pooling outcomes are supported by the beliefs that any deviating firm is perceived to be the lowest-value firm type $\underline{b}$. We first establish the pooling equilibria for public offerings. Denote by $(P_{PO}(\lambda), N_{PO}(\lambda))$ the public offering equilibrium outcome for any given dilution parameter λ, as characterized by Proposition 3.

Proposition 8 *There exist pooling equilibria in which all firms choose some common dilution protection λ in a public offering. An issue with a given λ is such an equilibrium if and only if*

$$[1-\lambda(1-\pi)]\, N_{PO}(\lambda)\left(\frac{a+I+\overline{b}}{N_{PO}(\lambda)+1}-P_{PO}(\lambda)\right)\leq \pi I\left(\frac{\overline{b}-\underline{b}}{a+I+\underline{b}}\right). \tag{18}$$

Moreover, the condition always holds for any λ sufficiently close to 1.

Proposition 3 guarantees that $P_{PO}(\lambda)$ is indeed the equilibrium issue price associated with dilution protection λ. Condition (18) rules out any deviation to any other public or rights offerings given the investors' belief that any such firm would be the lowest type $\underline{b}$.

Specifically, consider a deviation to another public offering $\hat{\lambda}$. A deviating firm would have to sell its new shares at a price $\hat{P}_{PO} = a+\underline{b}$, given the investors' off-equilibrium beliefs. Thus, any firm type (except type $\underline{b}$) deviating to $\hat{\lambda}$ would sell its shares at a discount. Clearly, this is not attractive for all low-value firms $b \in [\underline{b}, b^*_{PO}(\lambda))$ which sell overpriced shares to investors in the pooling equilibrium. High-value firms $b \in (b^*_{PO}(\lambda), \overline{b}]$ sell underpriced shares in the pooling offer or if they were to deviate, and in either case cash-rich shareholders would participate. By Proposition 3 the pooling price $P_{PO}(\lambda)$ is higher than $\hat{P}_{PO}$ (price effect), but investors can purchase more shares since $\lambda < \hat{\lambda}$ (quantity effect). Clearly, the price effect benefits the investors and hurts the shareholders, while the quantity effect has the opposite impact. Hence, undervalued firms prefer not to deviate if the price effect dominates, implying lower gains to investors from the pooling offer than from the deviating offer.

The condition for the price effect to prevail is determined by the highest-value type $\overline{b}$ since it suffers most from selling underpriced shares. The left hand side of condition (18) is the investors' equilibrium payoff when subscribing to firm $\overline{b}$. The right hand side is their payoff when firm $\overline{b}$ chooses full dilution protection $\hat{\lambda} = 1$. This is the best deviating public offer since it lets the firm sell as few underpriced shares as possible to investors. Depending on parameters, notably the support of firm types $\left[\underline{b}, \overline{b}\right]$, the condition may not hold for offerings with little dilution protection. In this case, current shareholders in firm $\overline{b}$ prefer to sell fewer shares to new investors at the more deflated price $\hat{P}_{PO}$.

The advantage of selling fewer shares becomes increasingly smaller when the dilution protection λ of the pooling offer increases. In the limit when λ approaches 1, investors buy the same number of shares in the pooling and deviating offer. Once there is only the price

effect firms strictly prefer the pooling offer. By continuity, all firm types choose pooling offers with sufficiently good dilution protection.

Such pooling offers must also dominate deviations to any rights offering $\hat{P}$. Given the investors' beliefs, a weakly positive rights price $\hat{P}_R$ must imply a strike price $\hat{P}_S \leq a + \underline{b}$. Hence, buying and exercising the rights would be (weakly) profitable. Therefore, deviating to the rights offering cannot be attractive for low-value firms $b \in [\underline{b}, b^*_{PO}(\lambda))$. They prefer to sell overpriced shares to new investors. High-valued firms $b \in (b^*_{PO}(\lambda), \overline{b}]$ would not want to switch to a rights offering with a zero strike price ($\hat{P}_S = a + \underline{b}$) since it is equivalent to a public offering with full dilution protection (Proposition 7). Furthermore, rights offerings with positive rights prices (but lower strike prices) dilute the stakes of the cash-poor shareholders more at deflated prices. Cash-rich shareholders can maintain their fractional ownership and are therefore indifferent across different combinations of strike and rights prices. Hence, high-valued firms (in fact, all firms) have no incentive to deviate to a rights offering with a lower strike price ($\hat{P}_S < a + \underline{b}$).

Next we turn our attention to rights offering. For any P_S, denote by $(P_R(P_S), N_{RO}(P_S))$ the rights offering outcome given by Lemma 2 and Proposition 5.

Proposition 9 *There exist pooling equilibria in which all firms choose some common strike price P_S in a rights offering. An issue with a given P_S is such an equilibrium if and only if*

$$\pi N_{RO}(P_S)\left(\frac{a + I + \overline{b}}{N_{RO}(P_S) + 1} - P_S - P_R(P_S)\right) \leq \pi I \left(\frac{\overline{b} - \underline{b}}{a + I + \underline{b}}\right) \tag{19}$$

Moreover, the condition always holds for any $P_S \in [a + \underline{b}, a + b^*_{RO}]$.

Similar to (18) in the pooling public offering equilibrium, condition (19) rules out any deviation to any other public or rights offerings. Consider an initial rights offering with some strike price $P_S \in [a + \underline{b}, a + b^*_{RO}]$. A firm may deviate either to another rights offering or to switch to a public offer. In either deviation the financing terms are set by the investors' belief that the firm is of type $\underline{b}$. Hence, if a firm were to choose another rights offering, the strike price would be (weakly) lower ($\hat{P}_S \leq a + \underline{b}$) and the number of new shares (weakly) larger ($I/\hat{P}_S \geq I/P_S$). Clearly, all low-value firms $b \in [\underline{b}, b^*_{RO})$ prefer the initial rights offering since they sell overpriced shares to investors and the stakes of their shareholders get less diluted. High-value firms $b \in (b^*_{RO}, \overline{b}]$ sell underpriced shares whether they adhere to the initial rights

offering or deviate. In either case cash-rich shareholders subscribe, thereby maintaining their fractional ownership. That is, their payoff is $a+b$ irrespective of the strike price, and they are indifferent. By contrast, cash-poor shareholders in high-value firms prefer the initial rights offering as it dilutes their ownership stake (weakly) less. Moreover, any possible difference in rights price $(\hat{P}_R(\hat{P}_S) - P_R(P_S))$ never fully compensates them for being more diluted. The reason is that the sum of $\hat{P}_R$ and $\hat{P}_S$ are such that investors break even for the lowest-value firm type $\underline{b}$, whereas the sum of P_R and P_S is based on all firm types (taking into account the winner's curse problem).

As in Proposition 8, the highest-value firm type $\overline{b}$ suffers most from issuing underpriced rights/shares. The left hand side of equation (19) is the investors' payoff from purchasing and exercising the rights of firm $\overline{b}$ in the initial offering. If firm $\overline{b}$ were to deviate to another rights offering, it would set $\hat{P}_S = a + \underline{b}$ which in turn implies $\hat{P}_R = 0$, since it dilutes the ownership stakes of its cash-poor shareholders the least. The right hand side of condition (19) is the corresponding payoff to the investors. Such a deviation can only be attractive to firm type $\overline{b}$ if the strike price in the initial rights offering is lower and therefore were to dilute its cash-poor shareholders more. Hence, a pooling equilibrium in which all firms choose the same strike price P_S always exists for $P_S \in [a + \underline{b}, a + b^*_{RO}]$.

As argued in the discussion of Proposition 8, the best deviating public offers for high-value firms is full dilution protection ($\hat{\lambda} = 1$), again because it lets the firm sell as few underpriced shares as possible to investors. Since this public offer is equivalent to the rights offering with $\hat{P}_R = 0$ (Proposition 7), neither the highest-value firm $\overline{b}$ nor any other undervalued type $b \in (b^*_{RO}, \overline{b})$ would want to deviate.

4.2 Coexistence of Rights and Public Offers

Propositions 8 and 9 establish that all firms choosing a public offering with the same dilution protection or a rights offering with the same strike price are equilibrium outcomes. In this subsection, we characterize all possible equilibria featuring multiple offer modes and/or terms. We begin by showing that offers of the same mode (PO or RO) with different terms cannot co-exist in equilibrium.

Lemma 3 *In any equilibrium, all public offerings have the same dilution protection λ, and all rights offerings have a common strike price P_S.*

Intuitively, multiple public or rights offerings cannot co-exist because overvalued firms would invariably deviate to the offer with the highest price. Suppose to the contrary that there are two public offerings. Any low-value firm type with non participating cash-rich shareholders prefers the offer with the higher issue price, irrespective of whether the dilution protection is greater or smaller than that of the offer with the lower price. Hence, there can only be a single issue price in equilibrium. This in turn must imply a single dilution protection since cash-rich shareholders in undervalued firms strictly prefer more to less dilution protection.

The argument why multiple rights offerings do not co-exist is similar, though slightly more involved. Suppose that there are two rights offerings with different strike prices. High-value firm types must prefer the higher strike price, since dilution is more costly for those firms and the lower strike price leads to more dilution of the cash-poor shareholders. Further, for each of the two rights offerings there must be some undervalued and some overvalued firm types. Firms which are undervalued under the offer with the lower strike price prefer to deviate to the offer with the higher strike price and get subsidized by higher-valued types, rather than subsidizing lower-valued types in the rights offer with the low price.

While Lemma 3 rules out equilibria with multiple rights offerings, respectively multiple public offerings, the two offer modes may co-exist in equilibrium. In particular, high- and low-valued firms may choose a (common) rights offering, while intermediate types opt for a public offering.

Proposition 10 *Any co-existence equilibrium is characterized by three cutoffs* $\underline{b}^\dagger < b^\dagger < \overline{b}^\dagger$. *Low-value firms* $b \in (\underline{b}, \underline{b}^\dagger)$ *and high-value firms* $b \in (\overline{b}^\dagger, \overline{b})$ *choose rights offering and intermediate types* $b \in (\underline{b}^\dagger, \overline{b}^\dagger)$ *choose public offerings. Furthermore, in all firm types* $b > b^\dagger$, *cash-rich shareholders participate.*

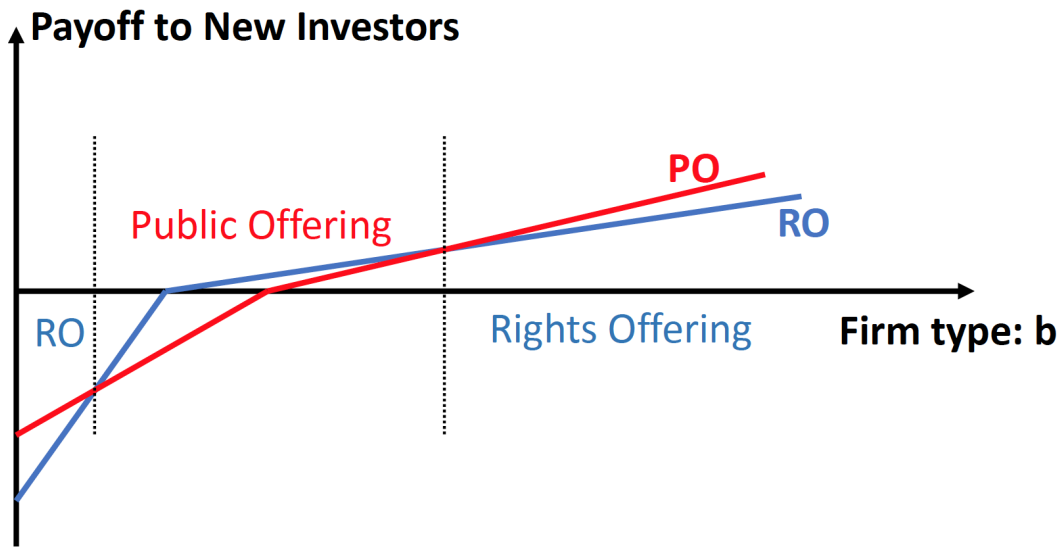

Figure 2: Payoff to new investors in a co-existence equilibrium

Figure 2 plots the investors' payoff as a function of the true firm type b, with the red curve representing the payoff from the public offering and the blue one the payoff from the rights offering. Investors buying shares from overvalued firms ($b < b^*_{PO}$ and $b < b^*_{RO}$) realize a loss, depicted by the parts of the curves below the horizontal axis. Shareholders of these firms do not purchase any new shares, leaving the entire issue to the investors. By contrast, cash-rich shareholders take-up their shares in undervalued offers ($b \geq b^*_{PO}$ or $b \geq b^*_{RO}$), and the investors can buy only a fraction of the new shares. The resulting positive payoffs are the parts of the curves above the horizontal axis. Since the slope of the curves is equal to the investors' fractional ownership, the curves have a kink at zero since cash-rich shareholder buy shares in all undervalued firms.

Firms choose the issue mode to maximize shareholder payoff which is equivalent to minimizing the payoff to investors. That is, they choose the lower contour of the respective payoff curves in Figure 2. The key feature is that the payoff curve in the negative region ($b < b^*_{RO}$) is steeper for the rights offering than for the public offering. The reason is that the strike price P_S must be smaller than the public offering price P_{PO},[13] and hence more shares are being issued and sold to investors. As a result, the lowest quality firms choose the rights offering

[13]Otherwise, all overvalued types whose cash-rich shareholders do not subscribe (to either offer) would prefer the rights offering since it would entail less dilution and (possibly) some revenues from the rights sale.

to sell more overvalued shares to investors. In a co-existence equilibrium, some high quality firms must also choose the rights offering. Since the rights offering provides full diltuion protection to cash-rich shareholders, investor can buy fewer shares of undervalued firms in the rights offering. Consequently, the payoff curve from the public offer has a steeper slope above the horizontal axis. Being able to sell fewer shares to the investors is more valueable to the highest quality firms which therefore choose rights offerings.[14]

Finally, we establish existence of such a co-existence equilibrium as described in Proposition 10 by means of an numerical example: The equilibrium payoffs depicted in Figure 3 features an investment cost $I = 1$, assets in place $a = \frac{1}{2}$, a fraction of cash-poor investor $\pi = 0.2$, four equally likely firm types $b \in \{0, 2, 6, 12\}$, a dilution protection parameter $\lambda = 0.4$ in the public offer, and a strike price $P_S = 1$ in the rights offering.

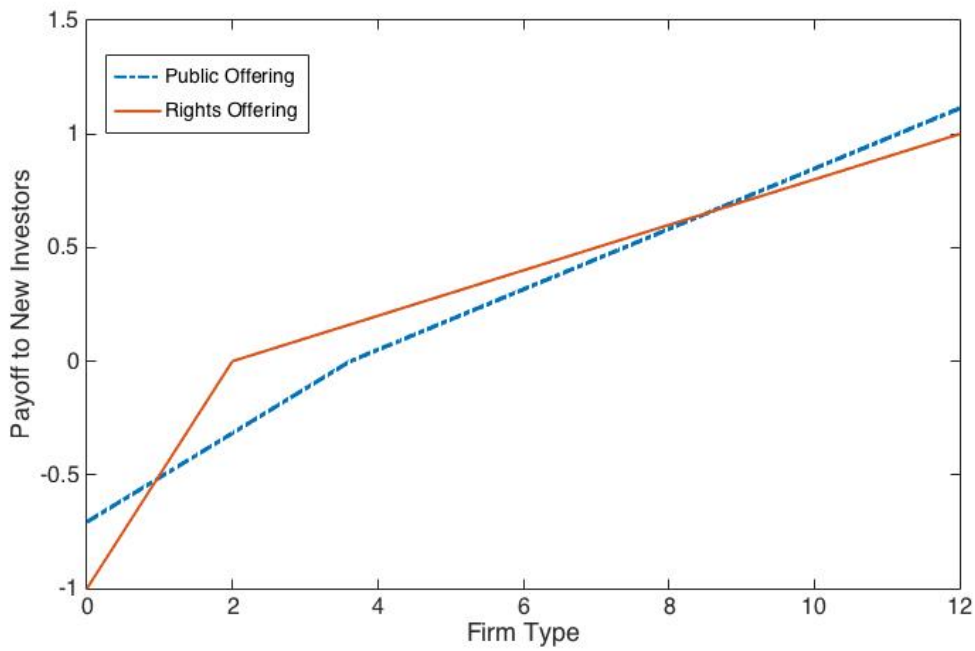

Figure 3: Numerical example for the co-existence equilibrium

[14]For undervalued firms the benefit of the public offer is the higher price while its cost is the lesser extent to which it protects shareholders from dilution. As the dilution cost increases in the firm type, the higher-valued types among all undervalued firms opt for the rights offering.

5 Empirical Predictions and Discussion

In this section, we first explore empirical implications of our model and then discuss the robustness of our results with respect to the source of the information asymmetry, shareholder participation, in particular, allowing uninformed as well as cash-constrained shareholders to participate in the offerings.

5.1 Discounts, Underpricing, and Announcement Returns

Prior to choosing the flotation method, firm types are indistinguishable to the (uninformed) market participants and therefore trade at a common initial share price P_0. The offer discount in a public offering (rights offering) can be expressed as the difference between initial price and public offering price (strike price), normalized by the initial price:

$$\frac{P_0 - P_{PO}}{P_0} \qquad \text{and} \qquad \frac{P_0 - P_S}{P_0}.$$

We choose to be agnostic about the extent to which market participants anticipate the investment and the offering mode and hence do not pin down the price P_0 at which shares initially trade. Consequently, we cannot make predictions about the size or sign of the discount in either flotation mode but merely rank the discount across the two modes. As discussed in Subsection 3.3, the strike price P_S must be smaller than the issue price P_{PO} in a public offering when all firms choose the same issue mode. Otherwise the rights price would be negative. In any equilibrium where a public offering and a rights offering co-exist (Proposition 10), the strike price must also be smaller than the issue price. Hence, our model implies that discounts are larger in rights offerings than in public offerings.

Following the IPO literature, we define underpricing as the (one-day) return on shares purchased in an issue. That is, underpricing is the difference between the post-issue share price and the public offering price P_{PO} (the strike price P_S), normalized by the public offering price P_{PO} (the strike price P_S). Since all firm types invest, the expected firm value and hence the post-issue price is equal to $E\frac{(I+a+b)}{1+I/P_{PO}}$ following a public offer and equal to $E\frac{(I+a+b)}{1+I/P_S}$ following a rights offering. Formally, the underpricing in a public offering and a

rights offering is

$$\frac{\frac{I+a+E(b|PO)}{1+I/P_{PO}} - P_{PO}}{P_{PO}} \qquad \text{and} \qquad \frac{\frac{I+a+E(b|RO)}{1+I/P_S} - P_S}{P_S}$$

which after some manipulation, can be expressed as

$$\frac{a + E\left(b|PO\right) - P_{PO}}{P_{PO} + I} \qquad \text{and} \qquad \frac{a + E\left(b|RO\right) - P_S}{P_S + I}$$

The comparison of underpricing in public and rights offerings is more involved.[15] When comparing underpricing across the pooling equilibria where all firms either opt for the same rights or for the same public offering, the conditional expectations $E\left(b|RO\right)$ and $E\left(b|PO\right)$ reduce to the unconditional mean $E\left(b\right)$. Hence, the comparison is solely driven by the rights and issue prices P_S and P_{PO}. Since, the strike price is (weakly) smaller than the issue price ($P_S \leq P_{PO}$), underpricing is more severe in rights offerings.

In the equilibrium in which a rights and a public offer coexist (Proposition 10), underpricing is determined by the prices P_{PO} and P_S as in the pooling equilibria and, in addition, by the conditional expectations $E\left(b|RO\right)$ and $E\left(b|PO\right)$ which are in general not identical. Unfortunately, we cannot analytically establish generic (qualitative) results. Though, numerical simulations suggest that firms which issue rights are the types with the higher average project NPV b. In the numerical example in Section 4.2, the average project NPV of firms issuing rights is $\frac{0+12}{2} = 6$, whereas the average is $\frac{2+6}{2} = 4$ for the firms using the public offering. Still, the price effect (P_{PO} vs P_S) clearly dominates the firm quality effect ($E\left(b|RO\right)$ vs $E\left(b|PO\right)$), and underpricing is 2.75 in the rights offering and 0.0744 in the public offering. Trusting that this example (and our other simulations) are reasonably representative, we argue that there is less underpricing in public offerings.

Finally, the announcement return is the difference between post-issue share price and the initial share price, normalized by the initial price:

$$\frac{E\frac{(I+a+b)}{1+I/P_{PO}} - P_0}{P_0} \qquad \text{and} \qquad \frac{E\frac{(I+a+b)}{1+I/P_S} - P_0}{P_0}$$

Like underpricing, the announcement returns across the two pooling equilibria are purely

[15] The underpricing in rights offering is defined for shareholders rather than investors as the formula does include the rights price P_R.

driven by the price effect since both initial price and post-issue price are the same when all firms either opt for a public or a rights offer. Hence, announcement returns are lower following rights offering since the strike price is weakly smaller than the issue price ($P_S \leq P_{PO}$). In the co-existence equilibria, there are again the opposing price and firm quality effects at work. Our numerical simulations suggest that overall the price effect dominates the firm quality effect. Hence, we are inclined to argue that announcement returns are higher in public offerings.

Our predictions receive some support in the empirical literature. Armitage (2007) studies discounts in rights offers and open offers (similar to public offerings) in the UK. Consistent with our prediction he documents that rights are often issued at a discount of 15% to 20% relative to the market price, whereas open offers are usually discounted by less than 10%.[16] International evidence provides a similar picture. Asem et al. (2016) report that the average discount in US public offerings is around 3%. In contrast, rights offerings have an average discount of 17% discount in the UK and an average of 19% in Australia.

In terms of announcement return, Slovin et al. (2000) and Barnes and Walker (2006) find that in the UK, abnormal returns are significantly more negative for rights offerings (on average -3.1% announcement return) than for private placements (3.3%). In France where rights and public offerings are both common, Gajewski and Ginglinger (2002) report significant two-day average excess returns of -1.28% for standby rights issues, -2.84% for uninsured rights issues, and an insignificant negative return for public offerings. The proportion of public offerings increases from 4.84% over the 1986-1989 period to 16.84% over the 1990–1996 period.

While we are unaware of any empirical research directly comparing the underpricing in public offerings and rights offerings, we follow Altinkilic and Hansen (2003) and decompose underpricing into the discount and the announcement return.[17] Based on the above empirical

[16]In the UK, a firm is not permitted to offer shares to the public without initially making an offer to existing qualifying shareholders (Barnes and Walker, 2006). In a rights issues, shareholders who do not wish to take-up their rights can sell them. In an open offer, the new shares are offered pro-rata to the existing shareholders, but the shareholders cannot sell their entitlements. Instead, the placees commit to take the remaining shares. To the extent that the current shareholders do not receive any compensation should they not participate in the issue, an open offer is similar to a public offering in our model.

[17]For details see equations (1) and (2) in Altinkilica and Hansen (2003). Follow their notation, denote by p_{-1}, p_0, and p_1 the pre-issue share price, the offering price (i.e., P_S in a rights offering or P_{PO} in a public offering), and the post issue price. Then, underpricing $\equiv \log \frac{p_1}{p_0} = \log \frac{p_1}{p_{-1}} + \log \frac{p_{-1}}{p_0} \equiv$ announcement return+ discount.

evidence, the discount is approximately 10% larger in rights offerings and the announcement return is approximately 4% lower. Hence, we may conclude that rights offerings feature higher underpricing, broadly consistent with our theoretical predictions.

5.2 Source of Information Asymmetry

In the main model investors and shareholders are asymmetrically informed about the NPV of the project b, whereas the value of the asset in place a is common knowledge. Here we consider the reverse case when shareholders have private information about a, while b is common knowledge. The first observation is that, as long as all firms issue shares, the pre-money total firm value $a+b$ is the crucial term incorporating the informational friction. Consequently, it is irrelevant whether the information asymmetry is about the NPV of the project or the value of the assets in place. In fact, the term $a+b$ jointly appears in all payoff expression throughout the analysis. Hence, one merely needs to replace all as with bs (and vice versa) in the lemmas and propositions, and all the results carry over to the case where the information asymmetry is about a instead of b.

The more interesting observation is that asymmetric information about a - though not about b - can lead some firms to abstain from investing as in Myers-Majluf (1984). When firms can choose issue mode as in Section 4 as well as whether to issue at all, the firm types with the most valuable assets in place may prefer to forgo the investment because issuing shares to investors may dilute the stake of the shareholders too much. Formally, there are the cutoff types $\underline{a}^\dagger$, $a^\dagger$, and $\overline{a}^\dagger$ parallel to those in terms of project NPV in Proposition 10. When the support of a is large enough, there exists an additional cutoff type $\hat{a} \in \left(\overline{a}^\dagger, \overline{a}\right)$ which satisfies

$$\pi N_{RO}\left(\frac{I+\hat{a}+b}{1+N_{RO}} - P_{RO} - P_S\right) = b. \tag{20}$$

Since $\hat{a} > \overline{a}^\dagger > a^\dagger$, type $\hat{a}$ firm uses rights offering if it issues new equity, and the cash-rich shareholders exercise their rights. The left-hand side of condition (20) is the payoff to the investors from exercise the rights sold by the cash-poor shareholders. Condition (20) implies that when type $\hat{a}$-firm conducts a rights offering, the investors extract the entire NPV b, leaving the shareholders indifferent about investing or not. Hence, all firm types below $\hat{a}$ issue and those above do not.

5.3 Shareholder Participation in Rights Offerings

Our model assumes a functioning rights market where shareholders and investors trade without frictions other than the adverse selection problem.[18] Therefore, all unexercised rights are in equilibrium sold to investors, and rights offerings do not face a subscription risk. This requires that rights are in fact tradeable which holds true in most countries (Holderness and Pontiff, 2016). Furthermore, many countries offer protection to shareholders who do not to respond to a rights offering by either having brokers sell the rights on their behalf (e.g. Italy or Sweden) or by having an investment bank sell all unexercised rights and credit the proceeds to the non-participating shareholders (e.g., Australia). In some countries – most notably – the US, firms can choose whether or not to make rights transferable.[19] In the study of Holderness and Pontiff (2016) about 50 percent of the firms in their US sample opt to have transferable rights while in the international sample of Massa et al. (2016) more than 60 percent of the rights offering have tradeable rights in countries which do not make transferability mandatory. Thus, shareholders do indeed either exercise or sell their rights (or have them sold on their behalf) in many countries as our model assumes. Still, rights do in practice at times lapse due to inattention, wealth constraints, or restricted transferability. When valuable rights expire those shareholders who hold these rights lose out even more than the cash-poor shareholders in our analysis. Thus, introducing some passive or inattentive shareholders into our framework would reinforce our result that there are more wealth transfers in rights offerings than in public offerings.

5.4 Uninformed Current Shareholders

With the exception of the benchmark case in Subsection (2.2), all current shareholders are informed about the true quality of the project b, but only a fraction π has the cash to buy new shares, respectively exercise their rights. One can conceive a more general setting with four types of shareholders; cash-rich informed and uninformed ones and cash-poor informed and uninformed ones. Our (qualitative) results carry over to such an extended setting for

[18] In their international study Massa et al. (2016) find that rights are typically less liquid than the underlying shares and often undervalued. The latter feature is consistent with a winner's curse problem in the rights market.

[19] In the UK, Singapore, and Hong Kong offers without tradeable rights are called open offers and are separately regulated (Massa et al., 2016).

two reasons. First, it is immaterial whether cash-poor shareholders are informed or not since they can – by assumption – not act strategically. Either they do nothing in public offerings or mechanically sell their rights. Hence, it is without loss of generality to assume that all cash-poor shareholders are informed. Second, uninformed cash-rich shareholders can never purchase more new shares than those allocated to them on a pro-rata basis, that is, $(1-\pi)\eta\lambda N_{PO}$ in a public offer and $(1-\pi)\eta N_{RO}$ in a rights offer. Consequently, they are not directly exposed to the winner's curse problem. Given that the equilibrium prices are set such that investors break even, uninformed cash-rich shareholders make – on average – a gain from participating and therefore always subscribe to new shares, respectively exercise their rights. In equilibrium, these shareholders therefore always take up the same fixed fraction of shares $(1-\pi)\eta\lambda$ in public offerings, respectively $(1-\pi)\eta$ in rights offerings. Hence, one can abstract from these and apply the analysis of the main model to the remaining shares $[1-(1-\pi)\eta\lambda]N_{PO}$, respectively $[1-(1-\pi)\eta]N_{RO}$, generating qualitatively the same results.

5.5 Margin Borrowing and Trading of Shares

Relaxing the assumption that cash-constrained shareholders can neither borrow nor trade their shares to participate in an offer seems an obvious extension. Here we discuss how the equilibrium outcomes may be affected in such an extended model settings. In a public offering, the strategy of selling existing shares to buy newly issued ones is futile given both shares are traded simultaneously at the same market clearing price. Still, when shareholders can trade their existing shares, investors are confronted with a winner's curse problem in both the primary and secondary market. While this exacerbates the extent of the adverse selection, it does not qualitatively change the nature of the winner's curse problem.

In rights offerings, cash-poor shareholders can sell part of their shares to have the funds to exercise the remaining rights. A complete analysis requires a fully specified trading environment, e.g. whether only rights or also shares can be traded separately. Regardless of the chosen model setup, the cash-poor shareholders always need to sell some shares or rights or both to participate in the rights offering, which again changes the extent but not the qualitative nature of the winner's curse problem.

Instead of selling some of their shares, cash-poor shareholders could borrow to participate in an offer. For example, they could borrow on their margin accounts and exercise underval-

ued rights. However, if the newly acquired shares are sold immediately to cover the margin loan, the payoff is the same as if the rights were sold instead.[20] Exercising the rights and getting the full information payoff (3) requires shareholders to hold the shares sufficiently long until the true project value b is realized. In practice, this may take a long time, making borrowing on margin accounts prohibitively expensive or even infeasible.

5.6 Private Placement

In a private placement, the issuing firm negotiates a share sale to a small group of qualified investors who may be current shareholders or new investors. Since most - if not all - shareholders do not qualify, private placements can be viewed as being similar to public offerings with zero dilution protection in our model. The key difference is the pricing mechanism. The public offering price P_{PO} is the market clearing price set by competitive investors, whereas the issue price in a private placement is the outcome of the bargaining between firm and qualified investors. In practice, private placements are sold at a discount relative to the current share price (Eckbo et al., 2007). The discount may be a reflection of the qualified investors' strong bargaining position or it may be compensation for costs of investigating the firm or for valuable monitoring. In either case, there is a wealth transfer between current shareholders and qualified investors. Current shareholders are treated equally and typically have no available action to take. In this sense, they are similar to the cash-poor shareholders in our model.

6 Conclusion

We analyze seasoned equity offerings where some shareholders are informed and can strategically choose to participate. When all shareholders have wealth to participate in the issue, right offerings achieve the full information outcome and therefore dominate public offerings which necessarily generate wealth transfers. We show that this ranking may be reversed when some of the existing shareholders are wealth-constrained. In rights offerings, investors must purchase the rights to buy the underlying shares, rather than only buying these shares

[20] A simple example illustrates this point. Suppose the market price after a rights offering is £10 and the strike price is £4. If a shareholder exercises her right and immediately sell the share at the market price, the payoff is £10 − £4, which is exactly the price the right commands in the market.

as in a public offering. Hence, a positive right price implies a discount in the strike price relative to the public offering price. Therefore, cash-poor shareholders become more diluted in a rights offering, and lower strike prices increase the wealth transfer from them to informed cash-rich shareholders. More generally, cash-poor and cash-rich shareholders have diverging preference over flotation methods and terms.

When firms choose the flotation mode and terms to maximize the total payoff to all shareholders, there are only two kinds of equilibria. On the one hand, there exist pooling equilibria in which all firms choose the same public offering, or alternatively all firms choose the same rights offering. On the other hand, there exist equilibria with a single rights and a single public offering. In such an equilibrium, high and low quality firms opt for the rights offering, while intermediate firm types choose the public offer. Low quality firms prefer a rights offering to sell a larger fraction of their overvalued firms. High quality firms favor a rights offering because it allows cash-rich shareholders to maintain their fractional ownership, thereby selling fewer undervalued shares to investors.

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A Proofs

Proof of Proposition 1: In any equilibrium, the informed shareholders can secure a net payoff of at least $a+b$ by exercising the rights. Similarly, the uninformed shareholders must receive at least a net payoff $a+E(b)$. New investors must on average at least break even. Because the total firm value net of investment I is $a+b$, the above payoffs are exactly the equilibrium payoffs for investors, informed and uninformed shareholders.

Next, we show that uninformed shareholders receive exactly $a+b$ net of investment in equilibrium as well. Suppose otherwise, then some uninformed shareholders can earn a net payoff strictly larger than $a+b$. The informed shareholders in the same firm would deviate to this strategy to earn a strictly larger payoff, a contradiction. Thus, all shareholders receive exactly $a+b$, which can be implemented by exercising the rights.

We now prove that no equilibrium exists for strike prices $P_S > a+\underline{b}$. Consider the informed shareholders of a firm type $b \in [\underline{b}, P_S - a)$, which implies $P_S > a+b$. If they choose not to exercise, their payoff is

$$\frac{I+a+b}{1+\frac{I}{P_S}} > \frac{I+a+b}{1+\frac{I}{a+b}} = a+b.$$

Thus, these informed shareholders' equilibrium payoff must be strictly higher than $a+b$. A contradiction and therefore, no such equilibrium exists.

To complete the proof, we show that when the strike price $P_S \leq a+\underline{b}$, all shareholders have an incentive to exercise their rights. The equilibrium is supported by the investors' belief that any sold rights come from the worst firm type $\underline{b}$. The price of the rights is therefore

$$P_R = \frac{I+a+\underline{b}}{1+\frac{I}{P_S}} - P_S.$$

The payoff to shareholders of a type b firm should they choose to sell the rights is

$$\frac{I+a+b}{1+\frac{I}{P_S}} + P_R N_{RO}.$$

Since $P_R \leq \frac{I+a+b}{1+\frac{I}{P_S}} - P_S$ for any $b \geq \underline{b}$, the above payoff is bounded by

$$\frac{I+a+b}{1+\frac{I}{P_S}} + \left(\frac{I+a+b}{1+\frac{I}{P_S}} - P_S\right)\frac{I}{P_S} = a+b,$$

which can be achieved by exercising the rights. Therefore, all shareholders have an incentive to exercise their rights. ■

Proof of Proposition 2: Given $\lambda < 1$, investors must purchase some shares in equilibrium. Since their purchase decision cannot depend on b, their break-even condition implies a unique P_{PO}. The per-share payoff to investors is

$$\frac{I+a+b}{1+N_{PO}} - P_{PO},$$

which is linear in b and has a unique root at $b = P_{PO} - a$. Hence, the investors´ payoff is non-zero for any firm type $b \neq P_{PO} - a$, implying wealth transfers.■

Proof of Lemma 1: Cash-rich shareholders subscribe if and only if

$$(\lambda N_{PO}+1)(I+a+b) - \lambda I(N_{PO}+1) \geq (I+a+b),$$

which is equivalent to

$$\lambda N_{PO}(I+a+b) \geq \lambda I(N_{PO}+1),$$

which is in turn is equivalent to

$$N_{PO}(a+b) \geq I.$$

Together with the fact that $P_{PO}N_{PO} = I$, we have condition (5). ■

Proof of Proposition 3: When $P_{PO} = a + \underline{b}$, condition (5) always holds, and cash-rich shareholders subscribe. In this case, $Pr(b|b \geq b^*_{PO}) = 1$ and $E(b|b \geq b^*_{PO}) = E(b)$. Therefore, investor payoff (7) becomes

$$[1-(1-\pi)\lambda]\,[a+E(b)-P_{PO}]\,,$$

which is strictly positive.

For any $P_{PO} \geq a + E(b)$, it follows by definition that $E(b|b \geq P_{PO} - a) \geq P_{PO} - a$, with strict inequality for $P_{PO} < a + \overline{b}$. Investor payoff (7) for such prices is strictly negative. By continuity, investor payoff (7) as a function of P_{PO} has a root and all roots lie in $(a + \underline{b}, a + E(b))$.

Finally, we show that P_{PO} is decreasing in λ. Suppose $\lambda_1 < \lambda_2$. Denote by $P_{PO,i}$ the corresponding solution to (7) for λ_i $(i = 1, 2)$. Also denote by $Pr_i(b|b \geq b^*_{PO,i})$ and $E_i(b|b \geq b^*_{PO,i})$ the corresponding values for λ_i. Since $P_{PO,i} < a + E(b) < a + E_i(b|b \geq b^*_{PO,i})$, we have

$$\begin{aligned} & a + E(b) - P_{PO,1} - Pr_1(b|b \geq b^*_{PO,1})(1-\pi)\lambda_2 \left[E_1(b|b \geq b^*_{PO,1})\right] \\ < \; & a + E(b) - Pr_1(b|b \geq b^*_{PO,1})(1-\pi)\lambda_1 \left[E_1(b|b \geq b^*_{PO,1})\right] \\ = \; & 0. \end{aligned}$$

Since (7) is positive for $P_{PO} = a + \underline{b}$, there must exist a $P_{PO,2} \in (a + \underline{b}, P_{PO,1})$, completing the proof. ■

Proof of Proposition 4: From (8), we have

$$\begin{aligned} WT_{PO} &= [a + E(b)]\left(1 - \frac{P_{PO}}{I + P_{PO}}\right) - \frac{P_{PO}}{I + P_{PO}} I \\ &= [a + E(b)]\frac{I}{I + P_{PO}} - \frac{I}{I + P_{PO}} P_{PO} \\ &= [a + E(b) - P_{PO}]\frac{I}{I + P_{PO}}, \end{aligned}$$

which establishes (9). Since $P_{PO} \leq a + E(b)$ from Proposition 3, the wealth transfer $WT_{PO} \geq 0$, which clearly is decreasing in P_{PO}. Furthermore, the issue price P_{PO} is decreasing in λ(Proposition 3). Hence, WT_{PO} is increasing in λ.■

Proof of Lemma 2: Cash-rich shareholders exercise their rights if

$$a + b \geq \frac{I + a + b}{N_{RO} + 1} + P_R N_{RO},$$

which implies

$$N_{RO}(a+b) \geq I + P_R N_{RO}(N_{RO}+1),$$

which in turn implies

$$a+b \geq \frac{I}{N_{RO}} + P_R(N_{RO}+1).$$

Using the fact that $\frac{I}{N_{RO}} = P_S$, condition (10) follows immediately. ■

Proof of Proposition 5: Rewriting (10) as

$$P_R = \frac{1}{N_{RO}+1}\left[I + \frac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO}) + \pi E(b)}{(1-\pi)P(b<b^*_{RO}) + \pi} + a\right] - P_S$$

and using to substitute P_R in (12), we have

$$\begin{aligned} b^*_{RO} &= P_S + \left[\tfrac{1}{(N_{RO}+1)}\left(I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a\right) - P_S\right](N_{RO}+1) - a \\ &= P_S + \left(I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a\right) - P_S(N_{RO}+1) - a \\ &= P_S + I + \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi} + a - I - P_S - a \\ &= \tfrac{(1-\pi)P(b<b^*_{RO})E(b|b<b^*_{RO})+\pi E(b)}{(1-\pi)P(b<b^*_{RO})+\pi}, \end{aligned}$$

which is the expression in the statement.

Next, we show that $b^*_{RO} \in (\underline{b}, E(b))$. At $b^*_{RO} = \underline{b}$, the right hand side of (13) is $E(b) > \underline{b}$. Since $E(b|b < b^*_{RO}) < E(b)$ whenever $b^*_{RO} < \bar{b}$, the right hand side of (13) is in turn dominated by $E(b)$. Therefore, b^*_{RO} must exist and lie in $(\underline{b}, E(b))$.■

Proof of Proposition 6: The payoff to cash-poor shareholders and to shareholders in firms of type $b < b^*_{RO}$ is

$$\frac{I+a+b}{N_R+1} + P_R N_{RO}$$

Using (10) and the definition of b^*_{RO} (Proposition 5) this payoff can be rewritten as

$$\begin{aligned} & \tfrac{I+a+b}{N_{RO}+1} + \tfrac{N_{RO}}{N_{RO}+1}\left(I + a + b^*_{RO}\right) + N_{RO}P_S \\ = \; & \tfrac{a+b+N_{RO}b^*_{RO}+N_{RO}a}{N_{RO}+1} \\ = \; & a + b + \tfrac{N_{RO}}{N_{RO}+1}(b^*_{RO} - b) \\ = \; & a + b + \tfrac{I}{I+P_S}(b^*_{RO} - b) \end{aligned}$$

Since investors break even, the ex ante wealth transfer among shareholders is therefore

$$a + E(b) - \left\{a + E(b) + E\left[\frac{I}{I+P_S}(b^*_{RO} - b)\right]\right\} = \frac{I}{I+P_S}\left[E(b) - b^*_{RO}\right].$$

■

Proof of Proposition 7: For $\lambda = 1$ equation (7) and $b^*_{PO} = P_{PO} - a$ imply the following condition for b^*_{PO}:

$$E(b) - b^*_{PO} - P(b \geq b^*_{PO})(1-\pi)\left[E(b|b \geq b^*_{PO}) - b^*_{PO}\right] = 0.$$

Solving for b^*_{PO}, we have

$$b^*_{PO} = \frac{E(b) - (1-\pi)P(b \geq b^*_{PO})E(b|b \geq b^*_{PO})}{1 - P(b \geq b^*_{PO})(1-\pi)}. \tag{21}$$

Together with the fact that

$$E(b) = P(b < b^*_{PO})E(b|b < b^*_{PO}) + P(b \geq b^*_{PO})E(b|b \geq b^*_{PO}),$$

condition (21) is equivalent to the condition for b^*_{RO} in a rights offering (13). Therefore, $b^*_{PO} = b^*_{RO}$ which implies that $P_{PO} = P_S$ and $N_{PO} = N_{RO}$ as well. In both types of offerings, existing shareholders receive $a + b$ if they participate (subscribe or exercise the rights) and the same payoff $\frac{I+a+b}{N_{RO}+1}$ if they do not participate. Overall, everyone's payoff in a public offering is exactly the same as in a rights offering. ■

Proof of Proposition 8: Consider a public offering equilibrium with issue price $P_{PO}(\lambda)$

and number of new shares $N_{PO}(\lambda)$, as characterized in Proposition 3. Suppose further that any firm which deviates is perceived by the investors as being of type $\underline{b}$. Hence, if a firm deviates to another public offering with $\hat{\lambda} \neq \lambda$, it has to sell its shares at $\hat{P}_{PO} = a + \underline{b}$ with $\hat{N}_{PO} = \frac{I}{\hat{P}_{PO}}$. The payoff (for investors and cash-rich shareholders) from buying these shares is

$$\frac{I + a + b}{\hat{N}_{PO} + 1} - \hat{P}_{PO} = \frac{b - \underline{b}}{\hat{N}_{PO} + 1} \geq 0.$$

As a result, no firm of type $b \leq b^*_{PO}$ wants to deviate to $\hat{\lambda}$ because $\Pi_{PO}(b|\lambda) \leq 0$. Firms of type $b > b^*_{PO}$ do not deviate either if

$$[1 - \lambda(1 - \pi)]\, N_{PO} \left(\frac{I + a + b}{N_{PO} + 1} - P_{PO} \right) \leq \left[1 - \hat{\lambda}(1 - \pi) \right] \hat{N}_{PO} \left(\frac{I + a + b}{\hat{N}_{PO} + 1} - \hat{P}_{PO} \right) \quad (22)$$

holds. Since both sides are linear in b, it is sufficient to show that the inequality is satisfied at the endpoints b^*_{PO} and $\overline{b}$. By definition of b^*_{PO} (Lemma 2), the left hand side equals 0 for $b = b^*_{PO}$, while the right hand side is

$$\left[1 - \hat{\lambda}(1 - \pi) \right] \hat{N}_{PO} \left(\frac{b^*_{PO} - \underline{b}}{\hat{N}_{PO} + 1} \right) > 0$$

For $b = \overline{b}$, the left-hand side of (22) is the left-hand side of (18). The right-hand side of (22) can be rewritten as

$$\left[1 - \hat{\lambda}(1 - \pi) \right] \left(\frac{I + a + \overline{b}}{1 + \hat{P}_{PO}/I} - I \right) = \left[1 - \hat{\lambda}(1 - \pi) \right] I \left(\frac{\overline{b} - \underline{b}}{I + a + \underline{b}} \right)$$

which is (weakly) larger than the right-hand side of (18) since $\hat{\lambda} \leq 1$. Hence, condition (18) implies that (22) holds for $b = \overline{b}$.

If a firm deviates to a rights offering with $\hat{P}_S$, the associated rights price $\hat{P}_R$ is given by

$$\hat{P}_R = \frac{I + a + \underline{b}}{\hat{N}_{RO} + 1} - \hat{P}_S. \quad (23)$$

For $\hat{P}_R \geq 0$, it must be that $\hat{P}_S \leq a + \underline{b}$. Condition (23) and Lemma 2 imply that

$$b \geq \hat{P}_S + \hat{P}_R(\hat{N}_{RO} + 1) - a = \underline{b}.$$

Hence, the payoff from exercising (and buying) rights is (weakly) positive. Consequently, no firm of type $b \leq b^*_{PO}$ wants to deviate to a rights offering since $\Pi_{PO}(b|\lambda) \leq 0$. Firms of type $b > b^*_{PO}$ do not deviate either if

$$[1 - \lambda(1-\pi)]\, N_{PO} \left(\frac{I + a + b}{N_{PO} + 1} - P_{PO} \right) \leq \pi \hat{N}_{RO} \left(\frac{I + a + b}{\hat{N}_{RO} + 1} - \hat{P}_S - \hat{P}_R \right). \tag{24}$$

As above, both sides are linear in a, and it is sufficient to show that the inequality is satisfied at the endpoints. For $b = b^*_{PO}$, the left hand side equals 0, while the right hand side is

$$\pi \hat{N}_{PO} \left(\frac{b^*_{PO} - \underline{b}}{\hat{N}_{PO} + 1} \right) > 0.$$

For $b = \overline{b}$, the right-hand side of (24) is equal to

$$\pi \hat{N}_{RO} \left(\frac{\overline{b} - \underline{b}}{\hat{N}_{PO} + 1} \right) = \pi I \left(\frac{\overline{b} - \underline{b}}{I + \hat{P}_S} \right).$$

Since $\hat{P}_S \leq a + \underline{b}$, it must be that

$$\pi I \left(\frac{\overline{b} - \underline{b}}{I + \hat{P}_S} \right) \geq \pi I \left(\frac{\overline{b} - \underline{b}}{I + a + \underline{b}} \right)$$

holds. Hence, condition (18) implies that (24) holds for $a = \overline{a}$.

Finally, as $\lambda \to 1$, the left-hand side of condition (18) becomes

$$\pi N_{PO}(1) \left(\frac{I + a + \overline{b}}{N_{PO}(1) + 1} - P_{PO}(1) \right) = \pi \left(\frac{I + a + \overline{b}}{1 + \frac{P_{PO}(1)}{I}} - I \right).$$

Since $P_{PO}(1) > a + \underline{b}$ (Proposition 3),

$$\pi \left(\frac{I + a + \overline{b}}{1 + \frac{P_{PO}(1)}{I}} - I \right) < \pi \left(\frac{I + a + \overline{b}}{1 + \frac{a + \overline{b}}{I}} - I \right) = \pi I \left(\frac{\overline{b} - \underline{b}}{I + a + \underline{b}} \right).$$

Thus, condition (18) is satisfied in the limit and by continuity also holds when λ is sufficiently close to 1. ■

Proof of Proposition 9: Consider a rights offering equilibrium with strike price P_S, number of new shares N_{RO}, and associated rights price P_R, as characterized in Proposition 5. Furthermore, any firm which deviates from this equilibrium rights offering is perceived by the investors as being of type $\underline{b}$. Parallel to the proof of Proposition 8, it suffices to establishes that

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq \pi\hat{N}_{RO}\left(\frac{I+a+b}{\hat{N}_{RO}+1}-\hat{P}_S-\hat{P}_R\right) \tag{25}$$

for any deviating rights offering with $\hat{P}_S$ and that

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq \left[1-\hat{\lambda}(1-\pi)\right]\hat{N}_{PO}\left(\frac{I+a+b}{\hat{N}_{PO}+1}-\hat{P}_{PO}\right) \tag{26}$$

for any deviating public offering with $\hat{\lambda}$. The right-hand sides of (25) and (26) are the same as those in (24) and (22), which are both weakly positive. Since by Lemma 2

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S-P_R(P_S)\right)\leq 0,$$

for any firm type $b\leq b^*_{RO}$, conditions (25) and (26) hold for these types. As for the firm types $b>b^*_{RO}$, it suffices to establishes that (25) and (26) hold for $b=\overline{b}$ due to the linearity in b. For $b=\overline{b}$ the left-hand sides of (25) and (26) are the left-hand side of (19). As shown in the proof of Proposition 8, the right-hand sides of (25) and (26) are weekly larger than $\pi I\left(\frac{\overline{b}-\underline{b}}{I+a+\underline{b}}\right)$, the right-hand side of (19). Hence, condition (19) implies that (25) and (26) hold for $b=\overline{b}$.

Given $P_R(P_S)\geq 0$, the left-hand side of (19) is bounded by

$$\pi N_{RO}(P_S)\left(\frac{I+a+\overline{b}}{N_{RO}(P_S)+1}-P_S\right)=\pi\left(\frac{I+a+\overline{b}}{1+\frac{P_S}{I}}-I\right).$$

Since $P_S\geq a+\underline{b}$,

$$\pi\left(\frac{I+a+\overline{b}}{1+\frac{P_S}{I}}-I\right)\leq \pi\left(\frac{I+a+\overline{b}}{1+\frac{a+\underline{b}}{I}}-I\right)=\pi I\left(\frac{\overline{a}-\underline{a}}{I+a+\underline{b}}\right).$$

Thus, condition (19) holds for any $P_S \in [a + \underline{b}, a + b^*_{RO}]$. ■

Proof of Lemma 3: Suppose there were two public offerings with corresponding $\lambda_2 > \lambda_1$ both adopted by some firms. Denote by $P_{PO,i}$ and $b^*_{PO,i}$ $(i = 1, 2)$ the corresponding issue price and cutoff type in each offering. Lemma 1 and Proposition 3 imply that some firms in each offering must weakly lie below the respective cutoff type $b^*_{PO,i}$. Let $b_i \leq b^*_{PO,i}$ $(i = 1, 2)$ be two such firms. For these firms all N_i new shares are issued to the investors. From (16), each firm's optimal choice of issue terms implies

$$N_i \left[\frac{I + a + b_i}{N_i + 1} - P_{PO,i} \right] \leq N_{-i} \left[\frac{I + a + b_i}{N_{-i} + 1} - P_{PO,-i} \right].$$

Since $N_i P_{PO,i} = I$, we have

$$N_i \frac{I + a + b_i}{N_i + 1} \leq N_{-i} \frac{I + a + b_i}{N_{-i} + 1}.$$

Hence, $\frac{N_i}{N_i+1} \leq \frac{N_{-i}}{N_{-i}+1}$ for $i = 1, 2$, which implies $N_1 = N_2$ and as a result, $P_{PO,1} = P_{PO,2}$ and $b^*_{PO,1} = b^*_{PO,2}$. Finally, consider a different pair of firms in each issue mode, with their firm types above the respective cutoff types: $b_i > b^*_{PO,i}$. Using the fact that $b^*_{PO,i} = P_{PO,i} - a$, we have

$$\frac{I + a + b_i}{N_i + 1} > \frac{I + a + b^*_{PO,i}}{\frac{I}{P_{PO,i}} + 1} = \frac{I + P_{PO,i}}{\frac{I}{P_{PO,i}} + 1} = P_{PO,i}.$$

In equilibrium, cash-rich shareholders subscribe, and new investors receive $[1 - \lambda_i(1 - \pi)] N_i$ new shares. The optimal choice of issue terms implies

$$[1 - \lambda_i(1 - \pi)] N_i \left[\frac{I + a + b_i}{N_i + 1} - P_{PO,i} \right] \leq [1 - \lambda_{-i}(1 - \pi)] N_{-i} \left[\frac{I + a + b_i}{N_{-i} + 1} - P_{PO,-i} \right].$$

Using the fact that N_i and $P_{PO,i}$ are the same across $i = 1, 2$, the above expression simplifies to

$$1 - \lambda_i(1 - \pi) \leq 1 - \lambda_{-i}(1 - \pi),$$

for $i = 1, 2$. Hence, $\lambda_1 = \lambda_2$. There is at most one public offering in equilibrium.

Suppose there were two rights offerings with strike prices $P_{S,2} > P_{S,1}$, both adopted by some firms. Denote the corresponding rights price by $P_{R,i}$ $(i = 1, 2)$. By Proposition 5 some

cash-rich shareholders must choose to (or not to) exercise the rights in each offer. Denote by $b_{i,e}$ ($b_{i,ne}$) $i = 1, 2$ the type of firms that issue rights with strike price $P_{S,i}$, and the cash-rich shareholders exercise (do not exercise) their rights. The optimal choice of offer terms states

$$\pi N_i \left[\frac{I + a + b_{i,e}}{N_i + 1} - P_{S,i} - P_{R,i} \right] \leq \pi N_{-i} \left[\frac{I + a + b_{i,e}}{N_{-i} + 1} - P_{S,-i} - P_{R,-i} \right],$$

which, combined with the fact that $P_{S,i} N_i = I$, implies

$$N_i \left[\frac{I + a + b_{i,e}}{N_i + 1} - P_{R,i} \right] \leq N_{-i} \left[\frac{I + a + b_{i,e}}{N_{-i} + 1} - P_{R,-i} \right].$$

Since $N_1 = \frac{I}{P_{S,1}} > \frac{I}{P_{S,2}} = N_2$, the above condition implies

$$b_{1,e} \leq b^{cf} \equiv \frac{P_{R,1} N_1 - P_{R,2} N_2}{\frac{N_1}{N_1+1} - \frac{N_2}{N_2+1}} - (I + a),$$

and

$$b_{2,e} \geq b^{cf} \geq b_{1,e}.$$

On the other hand, the optimal offer term choice for the $b_{i,ne}$ firm implies

$$N_i \left[\frac{I + a + b_{i,ne}}{N_i + 1} - P_{R,i} \right] \leq N_{-i} \left[\frac{I + a + b_{i,ne}}{N_{-i} + 1} - P_{R,-i} \right],$$

which following the same logic, implies $b_{1,ne} \leq b_{2,ne}$, $b_{1,ne} \leq b^{cf}$, and $b_{2,ne} \geq b^{cf}$. Hence, combined with Lemma 2, we have

$$b_{1,ne} < b_{1,e} \leq b^{cf} \leq b_{2,ne} < b_{2,e}.$$

However, this relation cannot hold in equilibrium, because $a_{1,e}$ firm has an incentive to deviate to the rights offering with strike price $P_{S,2}$. With $P_{S,1}$, investors in the $b_{1,e}$ firm collectively receive

$$\pi N_1 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,1} - P_{R,1} \right],$$

which is weakly positive because of Lemma 2 and the fact that the cash-rich shareholders exercise their rights, i.e. $b_{1,e} \geq b^*_{RO,1}$. On the other hand, with $P_{S,2}$, investors in the $a_{1,e}$ firm

collectively receive

$$N_2 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,2} - P_{R,2} \right] < N_2 \left[\frac{I + a + b_{2,ne}}{N_i + 1} - P_{S,2} - P_{R,2} \right],$$

which is in turn weakly negative because the cash-rich shareholders in $b_{2,ne}$ firms choose to sell the rights, i.e. $b_{2,ne} \leq b^*_{RO,2}$. Consequently,

$$\pi N_1 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,1} - P_{R,1} \right] \geq 0 > N_2 \left[\frac{I + a + b_{1,e}}{N_i + 1} - P_{S,2} - P_{R,2} \right].$$

Hence, the $a_{1,e}$ firm has an incentive to deviate to using a rights offering with strike price $P_{S,2}$. Contradiction! Concluding the proof. ■

Proof of Proposition (10): We begin with the following lemma that establishes the existence of $b^\dagger$.

Lemma 4 *Suppose b_1 (b_2) is any firm type where the cash-rich shareholders (do not) subscribe to the new shares. Then $b_1 > b_2$.*

Proof of Lemma 4: Suppose instead that $b_1 < b_2$. By Lemma 1 and 2 relatively better firms see shareholders participate. Therefore, b_1 and b_2 firms must have different offering modes. Without loss of generality assume b_1 adopts a public offering with dilution protection λ and b_2 adopts a rights offering with strike price P_S and rights price P_R. Because current shareholders participate, Lemma 1 states that

$$b_1 > P_{PO} - a.$$

Hence, the investors' payoff in firm b_1 is

$$[1 - \lambda(1 - \pi)] \, N_{PO} \left[\frac{I + a + b_1}{N_{PO} + 1} - P_{PO} \right] > 0.$$

Similarly, Lemma 2 implies the investors' payoff in firm b_2 is

$$N_{RO} \left[\frac{I + a + b_2}{N_{RO} + 1} - P_S - P_R \right] < 0.$$

However, b_1 firm then has an incentive to deviate to the rights offerings because

$$N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1}-P_S-P_R\right]<N_{RO}\left[\frac{I+a+b_2}{N_{RO}+1}-P_S-P_R\right]<0.$$

The contradiction establishes the lemma. ■

The next lemma establishes the existence of $\underline{b}^\dagger$.

Lemma 5 *There exist $\underline{b}^\dagger < b^\dagger$ such that all firms with $b \in (\underline{b}^\dagger, b^\dagger)$ (resp. $b < b^\dagger$) choose public (resp. rights) offerings.*

Proof of Lemma 5: By Lemma 4, the entire set of firm types $(\underline{b}, b^\dagger)$ can be partitioned into two subsets B_{RO} and B_{PO}, where RO and PO denote right offering and public offering. Lemma 1 and Lemma 4 imply that both sets are non-empty. For any firm type $b_1 \in B_{PO}$, the IC condition suggests

$$N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1}-P_S-P_R\right]\geq N_{PO}\left[\frac{I+a+b_1}{N_{PO}+1}-P_{PO}\right]. \tag{27}$$

Proposition 7 states that a rights offering with $P_R = 0$ is equivalent to a public offering with $\lambda = 1$, which in turn implies two public offerings cannot coexist. Thus, it must be that $P_R > 0$. Using the fact $N_{RO}P_S = N_{PO}P_{PO} = I$ and $P_R > 0$, condition (27) implies

$$N_{RO}\frac{I+a+b_1}{N_{RO}+1}>N_{RO}\left[\frac{I+a+b_1}{N_{RO}+1}-P_R\right]\geq N_{PO}\frac{I+a+b_1}{N_{PO}+1}.$$

Hence, $N_{RO} > N_{PO}$. This condition in turn implies

$$b_1>\frac{N_{RO}P_R}{\frac{N_{RO}}{N_{RO}+1}-\frac{N_{PO}}{N_{PO}+1}}\equiv\underline{b}^\dagger.$$

Completing the proof of the lemma. ■

Finally, we are ready to establish the existence of $\overline{b}^\dagger$. From Lemma 5, (16), (17), and the issue mode being chosen optimally, we have

$$\begin{cases}\Pi_{PO}(b)>\Pi_{RO}(b) & \text{for } b<\underline{b}^\dagger\\ \Pi_{PO}(b)<\Pi_{RO}(b) & \text{for } \underline{b}^\dagger<b<b^\dagger\end{cases}.$$

Clearly, both Π_{PO} and Π_{RO} are increasing, concave, and piece-wise linear functions. The only kink is the unique root of each respective function. Therefore, the graph of Π_{PO} and Π_{RO} must intersect exactly once at $\underline{b}^{\dagger}$ when $\Pi < 0$ and once when $\Pi > 0$. Denote the intersection by $\overline{b}^{\dagger}$. If such an $\overline{b}^{\dagger}$ does not exist, only one issue mode exists for $b > b^{\dagger}$, when shareholders subscribe, a contradiction.

Since Π_{RO} is steeper than Π_{PO} when $\Pi < 0$ ($N_{RO} > N_{PO}$), the reverse must be true when $\Pi > 0$ to generate an intersection. Hence, we must have

$$\begin{cases} \Pi_{PO}(b) > \Pi_{RO}(b) & \text{for } b > \overline{b}^{\dagger} \\ \Pi_{PO}(b) < \Pi_{RO}(b) & \text{for } b^{\dagger} < b < \overline{b}^{\dagger} \end{cases}.$$

This establishes the proposition. ■

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