Competition for Flow and Leverage

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Abstract

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Keywords: blockholder monitoring, activist hedge funds, competition for flow, corporate governance, delegated portfolio management

JEL Classifications: G34, G23

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Abstract

We develop a dual-layered agency model to study blockholder monitoring by activist funds that compete for investor flow. Competition for flow affects the manner in which activist funds govern as blockholders. In particular, funds inflate short-term performance by increasing payouts financed by higher (net) leverage, which subsequently discourages value-creating interventions in economic downturns due to debt overhang. Our theory suggests a new channel via which asset manager incentives may foster economic fragility and links together the observed procyclicality of activist block formation with the documented effect of such funds on the leverage of their target companies.

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1 Introduction

Activist blockholders play a key role in mitigating governance problems in publicly traded corporations with dispersed owners who have limited incentives to monitor managers. The potential benefits of blockholders have been widely recognized in the theoretical literature on corporate governance since Grossman and Hart (1980) and Shleifer and Vishny (1986). In recent decades institutional investors such as hedge funds and private equity funds have taken the lead in shareholder activism (Gillan and Starks (2007)). It is important to recognize that—unlike the blockholders of classical corporate governance models—such institutional activists are delegated portfolio managers who rely on the approval of the investors who finance them. In particular, funds must compete via performance to retain and attract investor capital, commonly referred to as competition for flow.[1]

In this paper we develop a dual-layered agency model to study blockholder monitoring by activist funds who compete for flow. Funds are principals as active owners in target firms, who tackle a managerial agency problem and enhance target firm value by intervention. Simultaneously, funds are agents who manage portfolios for clients and must compete for flow. We show that such competition for flow affects how activist funds govern as blockholders. In particular, competition induces them to inflate short-term fund performance by increasing payouts, financed by higher (net) target firm leverage. This, in turn, discourages value-creating interventions in economic downturns due to debt overhang.

While our paper primarily examines the role of institutional investors in corporate governance, it also offers a new perspective on systemic risks arising from asset managers' incentives. Following the recent deleveraging in the banking sector, several commentators call for attention to be shifted to incentives and competition in the asset management industry as a potential source of financial instability (e.g., the IMF (2015)

[1] Rewards for performance have been documented across many classes of institutional investors, e.g., Chung et al. (2012) for private equity and Lim et al. (2013) for hedge funds.

Financial Stability Report). Recent academic work in response to such concerns has focussed on fire sales of financial assets triggered by redemption threat as a source of systemic risk (e.g., Morris and Shin (2016), Zheng (2017)). In contrast our model shows how competition for flow amongst activist funds can foster excessive leverage in target firms which acts as a source of fragility by exacerbating procyclicality. We thus provide a complementary perspective by demonstrating how fund manager incentives can impact leverage and investment in the real economy.

The key elements of our model can be summarized as follows. Activist funds own blocks in target firms, tackle manaagerial agency problems, and intervene to raise target firm value. Some of these activities are feasible in the short-term (e.g., releasing excess cash from target firms) while others take time and extended effort to implement (e.g., business improvements, restructuring, or merger of the target). The potential returns to longer-term activism are exposed to changes in economic conditions (e.g., takeover premia are sensitive to aggregate economic conditions). Activist funds differ in their intrinsic ability to generate returns: Good funds are able to generate higher cash flows from each form of activism than bad ones. Funding for activists is provided by their fee-paying investors to whom the funds provide periodic returns. These investors make (rational) inferences about the ability of their funds based on these returns, and then decide whether to take their money elsewhere.

While tackling managerial agency problems, the need to compete to keep investor capital, tempts funds to enhance their intrinsically generated returns. They do so by surreptitiously moving resources forward in time, i.e., by borrowing today against the target firm's future cash flows. Investors, in turn, are fully capable of detecting and nullifying such enhancement activity by incurring a verification cost, which can be arbitrarily small in our model. We impose a (small) verification cost because the financing of target firms is arguably not fully transparent (in real time) to fund investors.

We reach our main conclusion via a series of results. We first show that, no matter how small the verification cost, investors never verify in equilibrium whether payouts have been enhanced by leverage (Proposition 1). Intuitively, if investors were to verify, then funds would not enhance payouts, nullifying the investors' incentive to pay the verification cost. Yet, when the verification cost is small, pooling equilibria cannot arise (Proposition 2): If bad funds were to successfully enhance their early returns in an attempt to pool with the good, investors would prefer to verify and thereby nullifying the mimicking attempt. Thus, in any feasible equilibrium good funds borrow to enhance payouts and thus separate from bad funds. Thus, competition for flow is an essential part of equilibrium. Since investors do not verify, such separation can only be achieved by a payout high enough such that bad funds are incapable of mimicking. This, in turn, establishes a minimum level of payout to investors in any separating equilibrium (Proposition 3) and implies that leverage is essential for separation. Next, in our core result, we characterize conditions under which—even in separating equilibria with the *minimal* amount of leverage—borrowing is high enough to generate debt overhang in low aggregate states leading to a shutdown in activist effort (Proposition 4). Importantly, such debt overhang — which underpins all subsequent results — can only arise in the presence of competition for flow. As detailed in Section 4.3, absent competition, there is no need to boost intrinsic performance by borrowing, and thus no overhang. Nonetheless, even with competition for flow, activism increases target firm value (Proposition 5).

Thus, in our model activism is beneficial overall, but the incentives of activists to compete for flow limit its efficacy. Further, competition for flow amplifies the link between firm level outcomes and aggregate economic flucutations. When economic prospects are better, (i) activism is more procyclical (Proposition 6), (ii) investment in activist funds is more attractive to investors (Proposition 7), and (iii) target firms are more highly levered (Proposition 9). Further, competition for flow makes returns to investors in activist funds more sensitive to economic prospects (Proposition 8).

For expositional reasons, we present our theoretical analysis in two steps. The results summarized above are first derived in a simplified setting (developed in Section 2) in which contracts are taken as given: activists use target firm debt to boost short-term performance, and investors compensate activists via a given contract that is motivated by real world money management contracts. Subsequently, in Section 6, we endogenize these contract choices in an enriched setting featuring a single contractual friction – the nonverifiability of aggregate economic states. In this setting, we show that debt is the optimal contract for raising external financing (Proposition 12) and no compensation contract for fund managers dominates the previously assumed one (Proposition 13).

Our paper is theoretical, but it offers several points of contact with observed activism by leading classes of institutional investors. One such class is activist hedge funds. These funds have taken centre stage in activism (e.g., Gillan and Starks (2007)), generating gains to targets in terms of share prices and operating performance (see the survey by Brav et al. (2010)). Two key themes emerge from our analysis. First, since activist funds enhance payouts via increased net leverage, target firms experience increases in payout and leverage. Second, as a result of the procyclicality discussed above, investment in activist funds are higher in bull markets. Both implications resonate with the available empirical evidence on activist hedge funds, as discussed in Section 5. In that section, we also extend our baseline analysis, to deliver results that are geared toward specific findings and debates in the empirical literature on activist hedge funds.

Another class of institutional investors pertinent to our model is private equity. It is often argued that the buyout activity of private equity funds is procyclical.[2] Further, the use of extensive leverage in private equity buyouts is well known. Thus, at a qualitative level, our debt overhang story provides an explanation for the cyclical features of private equity buyout activity as well. Indeed, consistent with our results in section 4.4, Axelson et al. (2013) find that private equity buyout leverage is procyclical.[3]

Our paper belongs to the large literature on blockholder monitoring — active monitoring via “voice” or passive monitoring via “exit” — in publicly traded corporations (see Edmans and Holderness (2017) for a recent survey). This literature abstracts from the delegated nature of blockholding, a phenomenon particularly prominent in the US

[2] In a model of the optimal financing structure of private equity funds, Axelson et al. (2009) demonstrate how the procyclicality of funding implies overinvestment in booms and underinvestment in busts.

[3] Two recent theoretical papers that examine specifically the procyclicality of private equity buyout activity are Martos-Vila et al. (2013) and Malenko and Malenko (2015).

and the UK, but also relevant elsewhere. By contrast, we focus on delegated blockholders and model how competition for flow affects the degree to which they govern via voice. A few recent papers have started to explicitly consider the impact of the incentives of fund managers on blockholder monitoring. Song (2017) considers multiple blockholders who govern via voice and exit. While we show how competition for flow can generate debt overhang, he argues that such competition can be beneficial when there are multiple heterogeneous blockholders. This is because Song models fund managers as stock pickers (i.e., non-activist funds). Such fund managers are reluctant to intervene in a company in which they hold a position because the need to intervene may reveal their poor stock selection. The presence of such a blockholder who is reluctant to intervene catalyzes other blockholders (with longer horizons) to intervene. In contrast, our fund managers are specialists in activism and thus intervention per se is never a negative signal. Song's model builds on Dasgupta and Piacentino (2015) who, like us, consider the negative impact of microfounded flow motivations but, in constrast to us, focus on passive monitoring via exit. Similarly Goldman and Strobl (2013) also differ from us by examining passive monitoring, and further, also assume that delegated blockholders have short horizons. Their focus — on the interplay of horizon mismatch, stock price manipulation, and investment complexity — is very different from ours.

Last but not least, our model builds on the insights of classical corporate finance papers on debt overhang (Myers (1977)) and dividend signalling (e.g., Bhattacharya (1979), Miller and Rock (1985)).[4] As in the latter strand, payouts act as a signal; such payouts are financed via debt, leading to overhang as in the former strand. The distinctive feature of our paper is the governance problem as the incentive for signalling. In a classical governance model, if managers' short-termism leads to excessive payouts, a large shareholder should attempt to curtail these. In our setting, corporate managers are not short-termist. The source of agency problems at the firm level is a free cash flow problem. However, it is the large shareholders who are endogenously short-termist

[4]Another classical contribution to the latter strand is by Ross (1977), in which (unlike in our model) external financing is itself a signalling device.

because they are funds that compete for flow. As a result, our large shareholders can solve the firm-level free cash flow problem, but at the cost of paying out excessively and fostering debt overhang. This is driven by the dual layered nature of the agency problem: the fund is the principal with respect to the firm but simultaneously an agent with respect to their investors.

2 Model

In our model *activist funds* (AF) are involved in two agency relationships, as principals in one and as agents in the other. On the one hand, funds are active owners (principals) in *target* firms (TF) who increase firm value by tackling a managerial agency problem and by contributing their expertise. On the other hand, funds are delegated portfolio managers (agents) financed by *investors* (IN) who pay fees to them and evaluate their performance. In addition, there are competitive, deep-pocketed *financiers* (FI) who may provide financing to firms targeted by funds.

There are two periods ($t = 1, 2$), and many firms, funds, investors, and financiers. Each fund is financed by an investor and enters the first period having used the investor's capital to acquire a stake in a target firm. We assume that each fund is in de facto control of its target firm. In other words, we do not model the phenomenon by which activist funds are able to obtain decisive influence in target firms, despite holding minority stakes.[5] Accordingly, we assume for simplicity that each fund owns all shares of its target firm. Each target firm can subsequently borrow funds from a financier. All actors are risk-neutral and there is no discounting.

Activism. Activist funds come in two types $\theta \in \{G, B\}$, where $\Pr(\theta = G) = \gamma_\theta$. Regardless of type funds can engage in two forms of activism, each of which increases target firm cash flows. The first form of activism can be implemented relatively quickly while the second takes time and effort. For concreteness, we consider specific manifestations of these two types of activism. In the short run, activists ameliorate a free-cash flow

[5] An analysis of such phenomena can be found in Brav et al. (2017).

problem in the target firm. In the long run, activists add value by contributing their expertise to a range of activities that we collectively term restructuring. Furthermore, the model can be more broadly interpreted, as we discuss in Section 5.4.

Short-term activism (t = 1). Short-term activism addresses a free cash flow problem in the target firm. Each target firm has an amount of cash $C > 0$ and is run by an empire building manager. If left under the manager's discretion, C will be invested in wasteful projects. For simplicity, these wasteful projects are assumed to have zero return. Funds are differentially skilled in identifying potential projects as being wasteful and can thus salvage a type-dependent amount of cash x^θ. We assume that x^G is distributed uniformly on $[\Delta x, C]$ and that $x^B = x^G - \Delta x$ where $\Delta x > 0$. Any identified excess cash is disbursed to shareholders at the end of the first period. In addition, funds can increase payouts as follows: By expending an infinitesimal non-pecuniary cost, they can make the target borrow some amount $F \in R_+$ from financiers against its second period cash flows. As a result the payout at the end of the first period is $D_1 = x^\theta + F$.[6] As noted in the introduction, we show that debt is the optimal contract for raising external finance in Section 6.

Long-term activism (t = 2). Suppose that activists can, in the second period, apply their skills to restructure, generate business improvements, or sell the firm. Further, the cash flows generated by such activism are affected by a state which is exogenous to the firm. There are two possible states, $s \in \{H, L\}$, with $\Pr(s = H) = \gamma_s$. The state is publicly revealed at the beginning of the second period. Following the revelation of the state, funds can exert effort $e \in \{0, \bar{e}\}$ at private cost

$$c_e = \begin{cases} 0 & \text{if } e = 0 \\ c_{\bar{e}} > 0 & \text{otherwise} \end{cases},$$

giving rise to cash flows, $X_s^\theta > 0$ with probability e and 0 otherwise. These cash flows, net of any payments to financiers, are paid out to shareholders at the end of the second

[6] D_1 does not literally have to be paid out to fund investors, but can instead be reinvested in other targets on their behalf. Further, as discussed in Section 5.4, the model also allows for borrowing at the level of the fund.

period (D_2). We make standard monotonocity assumptions, i.e., $X_s^G > X_s^B$ for both s (good activists generate more cash flows than bad ones), and $X_H^\theta > X_L^\theta$ for both θ (effort generates higher cash flows in the high state).[7]

Information. Funds are the most informed party in the model. At the beginning of the first period funds learn their type θ and the realized values of x^B and x^G.[8] Investors only learn the realized values of x^B and x^G. At the end of the first period, investors see the payout D_1 and form beliefs $\mu_{IN}^{pre}(D_1) = \Pr(\theta = G|D_1)$. They may then, at private cost $c_v > 0$, verify ($a_{IN}^v = 1$) the amount of funding F (in which case they observe F perfectly, and thus infer θ) or choose not to do so $(a_{IN}^v = 0)$. Funds have multiple methods for increasing leverage at the level of the target firm such as bank borrowing, drawing down credit lines, lengthening trade credit terms, etc. It therefore seems plausible that investors do not *costlessly* observe the precise composition of the payout in real time.[9] Following verification, the investor's beliefs are denoted by $\mu_{IN}^{post}(a_{IN}^v)$ where $\mu_{IN}^{post}(0) = \mu_{IN}^{pre}(D_1)$ and $\mu_{IN}^{post}(1) \in \{0, 1\}$ since verification reveals the fund's type perfectly. They then decide whether to retain ($a_{IN}^r = 1$) or to fire ($a_{IN}^r = 0$) the fund. If $a_{IN}^r = 0$, the fund is shut down, and the target firm is sold to outside buyers at prices corresponding to target firm values without fund effort in the second period. Financiers do not observe the realized values of x^G, x^B, but observe F (since they are providing it). They form beliefs $\mu_{FI}(F) = \Pr(\theta = G|F)$ and set the face value K due at the end of the second period to break even, making all relevant equilibrium inferences. Financiers,

[7] These payoffs imply a perfect correlation in ability (by type) across the two forms of activism. Our qualitative results only require that this correlation is sufficiently high. For example, we could allow a small probability ϵ that bad funds get lucky and generate x^G in the first period.

[8] By assuming that funds do not initially know their type we effectively rule out signalling via compensation contracts. The lack of initial self-knowledge could be understood in a broader dynamic context where new funds are born every period and incumbent funds do not know their skills relative to these newcomers.

[9] Drastic examples of investors not being able to observe leverage in real time are the governance scandals of the early 2000s, e.g., Enron or Parmalat. For a related model in which the composition of financing is costlessly observed, see an early version of the paper (Burkart and Dasgupta (2015)).

like funds, investors, and target firms observe the state s at the beginning of the second period.

Fund fees. Motivated by standard compensation arrangements in the asset management industry, fees in our model are made up of two parts. The first part is an assets-under-management (AUM) fee, w, paid at the beginning of each period of employment. The second part is an incentive fee—a so-called "carry"—which is $\alpha \max(D_2, 0)$ for some $\alpha \in (0, 1)$. This implies that funds that are retained by their investors for the second period get a share of the liquidating cash flows to equity holders in addition to their second period AUM fee. The prospect of the carry and AUM fee implies that funds have an incentive to be retained by their investors and may be tempted to take actions to ensure retention. This is how we model competition for flow.

Abstracting from the first period carry is a simplification which—as will be clear later—*reduces* incentives for leveraging. Since our paper emphasizes the negative implications of excessive leverage induced by competition for flow, this simplification works *against* us. We also abstract from lock-up provisions. All that we require is that there is an additional payoff to a fund from being viewed as good as opposed to bad. Instead of bad funds being closed down, we could have lock-up provisions and additional inflows to those funds that are identified to be good—possibly put into a second fund run by the same manager.

Parameter restrictions. To focus on the interesting constellation of parameters, we make two assumptions. The first ensures that the free cash flow problem in the target firm is sufficiently severe by itself to make it worthwhile to engage an activist:

$$C > \triangle x + 2w. \tag{1}$$

The second assumption relates to restructuring:

$$\bar{e} X_H^B < c_{\bar{e}} \leq \alpha \bar{e} X_L^G. \tag{2}$$

The inequality on the left implies that effort exertion by the bad fund is negative NPV and thus guarantees that investors would not wish to retain a bad fund if identified.

If investors were to retain both good and bad funds, there is no competition for flow, eliminating the sole source of agency problems at the fund level in our model. The inequality on the right excludes the possibility that the good fund does not exert effort in the low state *purely* due to the high cost of activism. Violating this inequality is tantamount to hard-wiring a connection between low states and reduced activism.

Key model ingredients. We conclude the model section by highlighting the key ingredients. A signalling model with debt overhang requires both (i) asymmetric information (for signalling) and (ii) some agency cost borne by equity holders, in our model costly effort (for debt overhang). As regards (i), funds signal to their investors by first period performance, boosting such performance as necessary by moving resources forward in time (i.e., levering up). In order for such payout boosting to have any salience for signalling (i.a) investors cannot freely observe the composition of payouts (otherwise it would be pointless to boost payout) and (i.b) financiers, who must observe the amount of debt raised cannot know exactly how much the bad type needs to borrow to imitate the good type (otherwise they could recognize and not lend, removing the need for good types to signal).

3 Equilibrium

A perfect Bayesian equilibrium is given by $(F^*, e^*(\cdot), a_{IN}^{v^*}, a_{IN}^{r^*}, K^*, \mu_{IN}^{pre^*}, \mu_{IN}^{post^*}, \mu_{FI}^*)$ where (i) the verification decision $a_{IN}^{v^*}$ is optimal given beliefs $\mu_{IN}^{pre^*}$, and the retention decision $a_{IN}^{r^*}$ is optimal given beliefs $\mu_{IN}^{post^*}$; (ii) The face value K^* allows the financier to break even; (iii) Funding F^* and state-contingent effort $e^*(\cdot)$ are best responses of the fund to $(a_{IN}^{v^*}, a_{IN}^{r^*}, \mu_{IN}^{pre^*}, \mu_{IN}^{post^*})$ and (K^*, μ_{FI}^*); and (iv) The beliefs $\mu_{IN}^{pre^*}, \mu_{IN}^{post^*}, \mu_{FI}^*$ are consistent with Bayes updating along the equilibrium path and are arbitrarily chosen otherwise. In this section, we derive the perfect Bayesian equilibria of our model.

3.1 No verification in equilibrium

In the first period, the fund salvages a type-dependent amount of cash x^θ. Prior to paying investors the fund may choose to enhance the payout by using target firm leverage. Investors, in turn, can – at cost c_v – choose to verify whether and how much was borrowed by the target firm to make the payout. We show:

Proposition 1. *Investors never verify in equilibrium as long as c_v is small.*

All proofs are in the appendix. If the investor were to always verify, funds would not enhance their intrinsic cash flows x^θ and dividends would reveal types. This would render (costly) verification by the investor redundant. Thus, in equilibrium, the investors cannot verify with probability one. The next possibility is that the investor verifies with probability strictly between 0 and 1. Randomization can only arise if both types pay the same dividend and the investor obtains the same expected payoffs from verification and non-verification. Verification gives the investor the option of making the retention decision type-dependent. Due to this option, for small enough verification cost, verifying is preferred to retaining without verification because the investor saves on paying the fee w to the bad fund in the second period. The comparison of verification and an uncontingent firing strategy is more intricate. On the one hand, if verification leads to retention of the good fund—as it will in equilibrium of our game—then, for sufficiently small verification costs, verification is always preferrred to uncontingent firing. On the other hand, if even the good fund is fired after verification, then firing without verification must be preferred for any positive verification cost. In none of these cases is the investor indifferent between verification and non-verification.

These arguments have implications for the possibility of pooling equilibria in our model. From the above, we know that verification always dominates uncontingent retention. Given this, there are two possibilities. First, verification also dominates uncontinent firing. In this case, the investor will verify and thus bad funds will not wish to pay the infinitesimal cost of borrowing to pool. Second, uncontingent firing dominates verification, which in turn dominates uncontingent retention. However, this means that both types of fund will be fired in equilibrium, and thus neither will have an incentive to pay the cost of borrowing to pool. Thus:

Proposition 2. *There exists no pooling equilibrium.*

The only remaining possibilities are separating equilibria without verification. For brevity, we shall henceforth refer to these as separating equilibria. In what follows, we do not allow for the unrealistic possibility that all financiers commit to provide arbitrary but identical amounts of funding to each and every target firm. Therefore, we only consider equilibria without such commitment.[10]

3.2 Borrow to separate

We begin the analysis by making a few straightforward observations about separating equilibria. The corresponding results are formally stated and proved in the appendix. Since investors never knowingly retain bad funds such funds are always closed down at the end of the first period in any separating equilibrium. This means that in any separating equilibrium, the bad fund will not borrow (Lemma 3). Now, since the bad fund does not borrow in a separating equilibrium, the financier will rationally assume that any positive amount F is raised by a good type (Lemma 4) and therefore is willing to lend up to the (equilibrium) expected cash flows generated by the good type in the second period, which we henceforth refer to as the **pledgable income** of the good type (PI^G).

These observations sharply restrict the set of separating equilbria that can arise. Since the financier does not know x^B and x^G he cannot infer how much the good type would need to raise in equilibrium. Thus, the financier cannot detect potential deviations by the bad type which involve borrowing any amount up to PI^G. But this means that,

[10]Equilibria with commitment can formally be ruled out, for example, by imposing the requirement that financiers' beliefs are always $\mu^*_{FI}(\widehat{F}) = 1$ for all $\widehat{F} \neq F^*$. Such beliefs are compatible with the equilibria we derive below.

to separate, the good fund must pay out an amount so high that, even by borrowing the maximum amount possible, the bad type cannot imitate.

Proposition 3. *In separating equilibria,* $D_1^*(G) \geq x^B + PI^G$.

Except in the uninteresting case in which future cash flows that can be generated by the activist fund are so low that $x^B + PI^G \leq x^G$, i.e., that $PI^G < \Delta x$, separation requires the use of external finance. Thus, the good fund must raise external finance $F^*(G) = D_1^*(G) - x^G \geq PI^G - \Delta x$.

3.3 Consequences of borrowing to separate

We have shown to date that competition for investor flow implies that good funds always separate in equilibrium, and that such separation implies borrowing. In this section, we explore the consequences of borrowing to separate. Before stating our formal result, we introduce some suggestive terminology. To motivate this terminology, note that since the fund receives only the second-period carry, she does not wish to borrow too much: The more she borrows, the less is this carry (by definition). So, it is reasonable to focus on the separating equilibrium that delivers separation with as little leverage as possible. In addition, since—as will be clear from our result below—borrowing to separate may (under certain conditions) shut down fund activism in low states, focussing on separating equilibria with *minimal* leverage establishes the conditions under which such reduced activism is an *essential* element of equilibrium. In the remainder of the paper, we shall refer to the equilibrium which delivers separation with as little leverage as possible as the *separating equilibrium with minimal leverage* (SEML). It follows from Proposition 3, that in a SEML the good fund borrows $F^*(G) = PI^G - \Delta x_1$.

Proposition 4. *As long as* $\gamma_s \bar{e}\left(X_H^G - X_L^G\right) > \Delta x > \frac{w}{1-\alpha}$*, the separating equilibrium with minimal leverage involves:*

i. For $c_{\bar{e}} \in \left(0, \frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+\right)$, $e^*(s) = \bar{e}$ *for all* s.

ii. For $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$, $e^*(H) = \bar{e}$ *and* $e^*(L) = 0$.

When effort costs are relatively low, the fund exerts effort in both states, but when effort costs are relatively high it does so only in the high state. This reduction of activist effort is, however, *not* due to high effort cost *alone*: Given condition (2), if the good fund were the sole claimant to the incremental cash flows generated by effort in the low state, she *would* have exerted effort in that state. She does not do so because, in equilibrium, she can*not* claim a sufficient fraction of the incremental cash flow due to leverage taken on to separate from the bad type. Thus, leverage induced by competition for flow generates debt overhang in the low state and shuts down activist effort.[11] Since this arises in the separating equilibrium with minimal leverage, for the relevant range of effort cost, such a state-contingent reduction of activist effort is an *essential* part of equilibrium.

The proof of this result is detailed in the appendix and heuristically summarized here. The incentive compatibility condition implies that the minimum face value which triggers debt overhang in the low state is $\underline{K} = X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}$. Similarly, the maximum face value which ensures effort exertion in the high state is $\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}$. In the SEML the good fund pays out just enough to separate even if the bad fund were to borrow the full pledgeable income of the good. Hence, the good fund must use the contract with the higher pledgeable income. Otherwise, the bad type could mimic the good type's SEML payout, contradicting separation. The choice between the contract that promises $\underline{K}$ and one that promises $\bar{K}$ involves the following trade-off. On the one hand, the former contract pays less conditional on success than the latter. On the other hand, creditors are paid in full more often under the former contract (with probability $\bar{e}$) than under the latter (with probability $\gamma_s \bar{e}$). This can be shown to jointly imply that the pledgeable income associated with the former contract is higher when the effort cost is low. In that case, separation involves the use of a lower face-value contract which maintains incentives to exert effort in both states. In contrast, when effort costs are relatively high, separation

[11] The reduction of activist effort due to debt overhang would arise even if effort choices were continuous. With continous effort choices, optimal effort may be higher in the high state even without leverage. Nonetheless, leverage would endogenously amplify the wedge between the effort choices.

involves the use of a higher face-value contract which destroys incentives to exert effort in the low state. This is the dichotomy captured in the result above.

The upper and lower bounds on Δx in the condition in Proposition 4 can be understood as follows. Consider the upper bound. Proposition 3 implies that good funds must borrow $PI^G - \Delta x$ to separate. Thus, in the SEML, good funds borrow exactly $PI^G - \Delta x$, and therefore leverage is decreasing in Δx. If Δx is too large, there would be insufficient borrowing to generate debt overhang in the low state. At the same time, Δx cannot be too small, because otherwise investors would not wish to retain good funds: in the SEML all but Δx of the pledgeable income is paid out to the investor in the first period, hence retaining the good fund is only attractive if investor's second-period after-fee payoff is positive.

3.4 Activism and target firm value

Having characterized the equilibrium outcome of our game, we can now discuss how the presence of activists affects the value of target firms. In our model, activism is hampered by the incentives of funds to compete for flow. Nevertheless, target firms are better off with activist funds than without.

Proposition 5. *Activism increases total cash flows generated by the target firm.*

In the absence of the activist fund, the free cash flow problem remains unresolved and no value is added in the second period. Due to our normalizations, therefore, the total cash flow generated by the target firm is zero. In contrast, activism leads to strictly positive cash flows net of fees. This is because, the resolution of the free cash flow problem results in a positive net cash flow in the first period, which is further enhanced by a leveraged payout if the fund present is of the good type. Furthermore, the good fund generates a positive second period net cash flow in the high state, and also in the low state if effort costs are small.

Governance mechanisms typically come with both costs and benefits. For example, a functioning market for corporate control discplines managers but also allows managers of cash rich firms to empire build (Burkart and Panunzi (2008)). This is also true for activism in our setting: While funds improve target firm performance through better governance, their incentives to compete for flow fosters debt overhang and thus forgoes value improvements. Further, activism amplifies the exposure of firm level variables to aggregate economic states, as discussed in the next section.

4 Activism, Competition, and Economic Prospects

We now discuss the implications of Proposition 4 for activism over the economic cycle and the role of competition for flow. To date we have cast all our results in terms of a representative {firm, fund, investor, financier} quadruple. To explore empirical implications, we now consider an economy populated by many such quadruples. Success or failure is idiosyncratic across each of these quadruples, but all are subject to the common exogenous state $s \in \{H, L\}$. The appropriate interpretation of the state s depends on the universe of quadruples included in the economy under consideration. As we subsequently link our implications to available evidence linking activism by several classes of institutional investors to macroeconomic conditions, we henceforth interpret the state s to represent aggregate economic conditions.

4.1 Procyclicality

Since we interpret the state $s \in \{H, L\}$ to represent aggregate economic conditions, Proposition 4 implies that fund activism is procyclical: it always arises when economic conditions are good but if conditions deteriorate activism ceases unless costs are low. Under this interpretation, γ_s represents economic prospects and we now proceed to characterise the impact of these prospects on activism.

Proposition 6. *Better economic prospects make fund activism more prone to procyclicality. For sufficiently good (poor) economic prospects, activism is always (never) procyclical.*

Proposition 4 states that when effort costs are relatively low, the fund exerts effort in both states, but when effort costs are relatively high it does so only in the high state. Economic prospects affect the threshold level of effort cost, $\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+$, at which activist effort becomes sensitive to economic conditions. Upon inspection of this condition, it is clear that this threshold is 0 for $\gamma_s \geq {}^{X_L^G}/_{X_H^G}$ (so that activism is procyclical for all possible cost levels), attains a maximum of $\alpha\bar{e}X_L^G$ for $\gamma_s = 0$ (so that activism is never procyclical), and is decreasing in γ_s for intermediate values.

Proposition 6, in turn, suggests that economic prospects exacerbate the fragility of activism. When economic prospects seem particularly good—say, during a bull (equity) market—even relatively low cost forms of interventions may not be immune to an economic downturn ex post. The key intuition is that higher economic prospects raise the pledgable income associated with the higher face value ($\bar{K}$) contract but do not affect the pledgable income associated with the lower face value ($\underline{K}$) contract. Accordingly, even for relatively low costs of engagement, it becomes optimal to promise to repay enough to generate debt overhang in the low state, exacerbating fragility.

4.2 Block formation

Economic prospects also have implications for activist block formation. When effort costs are high, activist funds exert effort only in the high state, and investment in activist funds thus becomes more attractive when economic prospects are better. To show this, we focus on the case in which $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$. We continue to focus on this case throughout the remainder of Section 4.

When the investor employs a fund, he is matched with a good fund with probability γ_θ since both types of funds participate. In the SEML, the good fund pays out $x^G + PI^G - \Delta x$ in the first period, and then in the second period the investor always pays w but the fund exerts effort only in the high state. Instead, with probability $1 - \gamma_\theta$ he is matched with a bad fund. The bad fund pays out x^B in the first period and is closed down, eliminating all future cash flows.

Proposition 7. *Activist fund block formation is more attractive to investors when economic prospects are better.*

At an intuitive level, two elements drive this result. First, conditional on being matched with a good fund, the investor receives a payout in the first period that is positively linked to the firm's pledgable income under the stewardship of the good fund. Since the good fund produces cash flows only in the good state, this pledgable income is increasing in the probability of the good state. Second, upon reaching the second period, matched to a good fund, the investor—as an equity holder—receives a positive cash flow only if the firm does not default, which again arises only in the good state.

4.3 The role of competition for flow

The key mechanism driving all our results is that funds compete for investor flow: it is the need to convince investors of their high ability, and thus avoid losing investment mandates, that leads good funds to lever up the target firm, generating debt overhang in the low state. To isolate the role of competition for flow, we compare our setting with a (counterfactual) environment without such competition: imagine that investors retain the fund with some arbitrary exogenous probability $\xi \in (0, 1)$ regardless of first period performance. Now funds cannot influence their retention probability, and thus do not inflate first period payouts. (Their preference for not inflating first period payouts is actually strict since they receive only a second-period carry which is reduced by borrowing in the first period.) Instead, they pay out x^θ and then (if retained by chance into the second period) the good fund exerts effort in the second period while the bad fund does not regardless of the economic state (Assumption 2). This implies that competition for flow is both necessary and sufficient for activist effort to become procyclical. Further:

Proposition 8. *Competition for flow makes returns to investors in activist funds more sensitive to economic prospects.*

The key intuition is that, by fostering debt overhang, competition for flow prevents cash flow production in the low economic state, whereas without such competition some (albeit lower) cash flows are always produced in the low state.

4.4 Target leverage

Economic prospects also affect target firm leverage. Better economic prospects imply a higher debt capacity for the target, which in turn implies that more borrowing is necessary for good type funds to separate.

Proposition 9. *When economic prospects are better, fund target firms are more highly leveraged.*

Consistent with Proposition 9, Axelson et al. (2013) report that private equity buyout leverage is procyclical. Proposition 9 also helps us interpret some anecdotal evidence with respect to the post-crisis behavior of activist hedge funds around 2010. *The Economist* writes at the time: "Activists are toning down their attempts to get companies to take on more debt. Many were burned before, and are reluctant to put their hands back in the fire."[12] Our model suggests that this may simply be a case of lower market confidence about future prospects for the economy in 2010 than in the heady days of optimism prior to the financial crisis.

5 Activist Hedge Funds

Activism by hedge funds represents a good illustration of our theory. On the one hand, the mitigation of free cash flow problems is a central goal of these funds. As Brav et al. (2010) note in their survey, hedge fund targets can be characterised as "…"cash-cows" with low growth potentials that may suffer from the agency problem of free cash flow." On the other hand, longer-term forms of activism by hedge funds often include changes

[12] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

in business strategy and the sale of target companies. Such changes, taken together, constitute almost half of 13D filings.[13]

In this section, we delve deeper into hedge fund activism. As a first step, in section 5.1 we relate our model predictions to available empirical and anecdotal evidence on activist hedge funds. In section 5.2 we provide a minor variation of our model to examine how hedge fund activism affects target firm *bond*holders. Third, in section 5.3 we study a related model variation in which our core results obtain through changes in payout policy alone while holding leverage constant. Hence, we can interpret our results more generally in terms of *net* debt.

Finally, in section 5.4, we argue that our model can be more broadly interpreted.

5.1 Interpreting the empirical evidence

Two key applied themes emerge from our analysis. First, since activist funds enhance payouts via increased net leverage, target firms experience increases in payout and leverage. Second, as a result of the procyclicality discussed above, investment in activist funds are higher in bull markets. Both implications resonate with the available empirical evidence on activist hedge funds.

The empirical literature suggests that activist hedge funds increase target firm leverage or payout or both (e.g. Brav et al. (2008), Klein and Zur (2009)). There is also evidence—consistent with our results—that the induced rise in leverage increases the credit risk of target firms: Target companies disproportionately experience credit downgrades (e.g., Byrd et al. (2007), Klein and Zur (2011)). Our model, of course, also suggests that the increase in leverage induced by activists potentially undermines future value creation at the level of target firms. This view receives support from prominent market participants. For instance, Larry Fink, the chairman of BlackRock, recently

[13] Our model assumes that a fund potentially engages in more than one form of activism. This is consistent with Brav et al. (2010). In their sample 52% of 13D filings declare specific goals falling into four categories but the percentages of 13D filings, when summed over specific goals, amount to nearly 85%. Thus, *on average*, hedge funds state close to two distinct activist goals per 13D declaration.

wrote to executives of all portfolio firms in the context of hedge fund activism that "Too many companies have... increased debt to boost dividends", and that such actions "can jeopardize a company's ability to generate sustainable long-term returns."[14]

There is also growing evidence that activist block formation is higher in bull markets. See, for example, Figures 1 and 2 in Brav et al. (2013) which depict the number of activist hedge funds and their engagement disclosures (e.g., 13D filings) over time in the US. These findings are echoed in the financial press. According to *The Economist*, "In America investors began only two new activist campaigns in the fourth quarter of 2008, down from 32 in the preceding nine months and 61 in 2007."[15] It is only after a "strangely quiet" period during the two years following this steep decline in activism, during which "[m]any [activist investors] scaled back or even closed shop,"[16] that activist campaigns started to re-emerge. Indeed, it is only another eighteen months later, in mid-2012, when the market had regained most of the value lost in the 2008 crisis, that – according to Peter Harkins of D.F. King, a proxy-advisor – shareholder activism is "getting back to normal after the financial crisis of 2008."[17] Further supporting evidence from more recent years can be found in Khorana et al. (2017).

It is sometimes suggested in the financial press that the procyclicality of returns from activist hedge funds is caused by the relative lack of diversification of activist portfolios.[18] Further, since one of the commonly declared objectives of activist hedge funds is the eventual sale of the target firm, it may also be tempting to attribute the procyclicality of hedge fund activism to the procyclicality of M&A markets. While these other potential channels may have a bearing on the procyclicality of activism, it is worth emphasizing that our analysis—apart from delivering a self-contained model with fully rational agents—delivers an endogenous link between the observed procyclicality

[14] *The Wall Street Journal*, 21 March 2014.

[15] *The Economist*, "Activist Investors: Flight of the Locusts", April 8, 2009.

[16] *The Economist*, "Shareholder activism: Ready, set dough", December 2, 2010.

[17] *The Economist*, "Corporate Governance in America: Heating Up," April 7, 2012.

[18] It is worth noting that an explanation based upon idiosyncratic shocks is hard to square with patterns related to the business cycle.

of activism and the documented effect of activism on the net debt of target firms.

5.2 Do activists expropriate bondholders?

There is general agreement in the literature that—as in our model—hedge fund activism produces significant positive returns to target shareholders. However, the empirical literature is not unanimous on whether (some of) these gains are at the expense of existing bondholders. At one end of the spectrum, Klein and Zur (2011) argue that hedge fund activism leads to an expropriation of existing bondholders, a conclusion shared – with caveats and qualifications – by Sunder et al. (2014). However, Brav et al. (2008) argue that expropriation of existing bondholders is unlikely to be a source of significant shareholder value because they find that returns to target shareholders are *higher* in companies which are previously *un*levered.

Our core mechanism does not turn on the interaction between existing bondholders and shareholders: Since the representative target firm is unlevered in our model, our baseline results are silent on the issue of bondholder expropriation. Nevertheless, our framework can be used to interpret the seemingly conflicting evidence in Brav et al. (2008) and Klein and Zur (2011). Reconsider the baseline model with the following modifications. Assume that the representative firm has some liquid assets of $Y_0 > 0$ in the first period. Unlike the pre-existing excess cash C, which is subject to a free cash flow problem, these liquid assets Y_0 are not under the target firm manager's discretion and thus cannot be wasted. Thus, absent hedge fund activists, this Y_0 would be retained until the second period and available to pay pre-existing creditor claims, if any. Hedge fund activists may pay out part or all of these liquid assets in the first period to enhance early returns to their investors, in addition to leveraging the target as in the baseline model. As before, investors do not directly verify the composition of the payout but infer it in equilibrium. We compare two capital structures for the target firm: Either the target firm has no pre-existing debt (as in the baseline model) or it has pre-existing debt maturing in the second period with a face value of $K_0 \in (\Delta x, Y_0)$. We slightly modify Assumption 2 to account for pre-existing debt and liquid assets Y_0 and assume $\bar{e}\left(X_H^B - K_0 + Y_0\right) < c_{\bar{e}} \leq \alpha\bar{e}\left(X_L^G - K_0 + Y_0\right)$.

Proposition 10. *For* $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$ *and* $\gamma_s\bar{e}\left(X_H^G - X_L^G\right) > \Delta x > \frac{w}{1-\alpha}$*, pre-existing target leverage may reduce shareholder returns from activism even when activism expropriates existing bondholders.*

Using arguments that parallel those of Proposition 4, we show in the appendix that competition for flow induces the good fund to pay out all available liquid assets in the first period and also to leverage the target sufficiently to generate debt overhang in the low state in the second period. This implies that activist funds reduce the cash available for existing creditors: In the absence of hedge funds, pre-existing debt is safe and creditors are paid in both states. In the presence of hedge funds, the pre-existing debt becomes risky and creditors are only paid with probability $\bar{e}$ in the high state, consistent with the findings of Klein and Zur (2011). However, comparing target firms with and without pre-existing leverage in the presence of activist funds, Proposition 10 shows that returns to shareholders are higher when the target firm is unlevered. This is because pre-existing target debt reduces the (residual) debt capacity of the target, which in turn reduces the payout necessary for separation and hence the equilibrium first period payout to target firm shareholders. The second period payout is unaffected because activist funds borrow all but Δx of the target's debt capacity. Hence, in the presence of activist funds, returns are lower to the target firm shareholders when there is pre-existing leverage, consistent with the findings of Brav et al. (2008). Thus, our model provides a simple, stylized framework that helps to resolve some of the seemingly contradictory empirical evidence in Brav et al. (2008) and Klein and Zur (2011).

5.3 Excessive payout

The enriched framework introduced in section 5.2 delivers a further benefit: It enables us to show that our results hold if we restrict hedge funds to changing payout policy *only*, i.e., preclude them from issuing *new* target debt. Consequently, our results can be interpreted more broadly in terms of increases in *net* debt – i.e., debt minus cash – thus linking them more directly to evidence on increased payout (Brav et al. (2008), Klein and Zur (2009)).

Our results are indeed robust to payout policy changes provided that target firms have both pre-existing debt and liquid assets: For targets with pre-existing debt, a reduction in liquid assets increases net debt. Competition for flow can deliver sufficiently high net debt to foster debt overhang in the low state. We consider the same variation of the model as in section 5.2 except that new borrowing is precluded. Activist hedge funds salvage excess cash of x^θ and pay it out at the end of the first period. They may augment the payment by tapping into liquid assets Y_0. In the absence of a hedge fund activist, the liquid assets Y_0 would be retained until the second period and available to pay pre-existing creditor claims.

Proposition 11. *Competition for flow leads to high payouts which in turn may cause debt overhang even without new target firm borrowing.*

The intuition is that – as before – good funds must pay high enough dividends at the end of the first period to prevent mimicking by bad funds. Since either fund can tap into the liquid assets, the good fund must pay out at least $x^B + Y_0$ to separate, i.e., can retain only $\triangle x$ liquid assets. But, then, for target firms with sufficient pre-existing leverage, debt overhang arises in the low state.

5.4 Broader Model Interpretations

In our model there are two periods and aggregate economic variation arises only in the second one. Needless to say, one can interpret the state of the economy in the second period as being *relative* to its state in the first. We can then view our current first period analysis as being conditional on a realised first-period state. Given any such state in the first period, the economy may improve or decline in the second. This means that, in principle, returns from both first- and second-period activism could be made state dependent without altering our qualitiative results. This paves the way for a broader intepretation of our two forms of activism. This is because the remaining difference across the two forms of activism—namely, the effort required to undertake them—can also be relaxed.

Our formal analysis assumes, purely for simplicity, that there is no effort cost associated with the first form of activism, which we have interpreted as free cash flow mitigation. Nothing would change if free cash flow mitigation requires effort and funds learn their types in the first period as effort is exerted. It would still remain the case, that in equilibrium the good funds would lever up to an extent that bad funds are unable to match the enhanced dividend. Since, therefore, both forms of activism can be costly and generate state-dependent returns, neither the sequence nor the labels given to the two forms of activism are critical for the core mechanism. The assumed sequence of free cash flow mitigation and restructuring can be reversed. For example, restructuring via potential spin-offs of non-core assets could occur in the first period with costly capital structure adjustments occuring later. Activism would still be procyclical, since leverage generated in an attempt to boost restructuring returns in the first period would interfere with capital stucture adjustments in the second.

Indeed, it is not even necessary that the activist fund potentially intervenes in two different ways in the same target firm, as in the model. Consider instead a setting in which each fund has a portfolio of target firms, intervening (in one way or the other) only once per firm, in different periods for different firms. Procyclicality would still emerge in such a setting if leverage is undertaken at the fund level rather than at the target firm level. Competition for flow would still tempt funds into enhancing early returns to investors by levering up. Under qualitatively similar conditions, endogenously generated leverage would be sufficient to discourage activists funds from exerting effort in *any* portfolio firm that subsquently required costly intervention if aggregate economic conditions decline. Note that since borrowing at the hedge fund level is also not fully transparent, it is reasonable to assume that it is at least somewhat costly for investors to verify the source of returns, as in the baseline model, giving rise to endogenous opacity as before.

6 Financing and compensation contracts

In our analysis to date, we have imposed two key contracting restrictions: (i) activists use target debt to inflate early payouts and (ii) are compensated via an assets under management fee (w) and a carry (α). While both of these contracts are well justified on empirical grounds, we now show that a single contracting friction can simultaneously rationalize both of them.

The friction we introduce is that contracts can only be made contingent on project success or failure, not on aggregate economic states ($s \in \{H, L\}$). That is, while aggregate states are publicly observable at the beginning of period 2 they are not verifiable. In practice, agents can relatively easily contract on aggregate indices (e.g., S&P 500) which reflect future economic prospects.[19] However, it is difficult to contract in real time on current aggregate states due to measurement difficulties (Shiller (1998)).[20] For example, GDP — a key measure of macroeconomic states — is often revised with substantial delay. As Orphanides (2001) points out with respect to real-time Taylor rules: "... as is well known, the actual variables required for implementation of such a rule — potential output, nominal output, and real output — are not known with any accuracy until much later."

Given the non-contractibility of aggregate states, we show that debt is the optimal form of financing at the level of the target firm and there is no loss of generality in restricting the compensation contract to an AUM fee with a second-period carry.

It is well known that with binary outcomes, where the failure cash flow is zero, debt and equity are indistinguishable (Tirole (2006), p. 119). Therefore, we enrich the set of cash flows generated in the second period, while keeping the rest of the model

[19] In our model, contracts are indeed de facto contingent on economic prospects, γ_s, which – as discussed in Section 4 – is our proxy for an aggregate market index. This is because the pledgable income PI^G varies with γ_s.

[20] Shiller (1998), p. 2: "These economic causes of changes in standards of living that should be insurable without moral hazard because they are beyond individual control are still not insurable today because they are not so objective or easy to verify as fires or disabilities."

unchanged. We assume that effort level e gives rise to cash flow, $\bar{X}_s^\theta$ with probability e and $\underline{X}_s^\theta > 0$ with probability $1-e$. As before, we impose standard monotonocity assumptions: $\bar{X}_s^\theta > \underline{X}_s^\theta$ for all θ, s (fund effort increases cash flow), $\bar{X}_s^G > \bar{X}_s^B$ for all s (good funds are better than bad ones), and $\bar{X}_H^\theta > \bar{X}_L^\theta$ for all θ (effort generates higher cash flows in the high state).

Since contracts can only be made contingent on project success or failure, the differences between cash flows across aggregate states in the event of either success ($\bar{X}_H^G - \bar{X}_L^G$) or failure ($\underline{X}_H^G - \underline{X}_L^G$) are non-verifiable and hence divertible. Following the corporate governance literature (Shleifer and Vishny (1997)), we assume that divertible cash flows accrue to the controlling party, which here is the activist fund (see the discussion in Section 2).[21]

We also have to take a stand on whether the divertible component of cash flows is higher in the event of project success or failure. We assume the former:[22]

$$\bar{X}_H^G - \bar{X}_L^G > \underline{X}_H^G - \underline{X}_L^G. \tag{3}$$

This assumption is *not* sufficient for our results, because—as we point out below—without (endogenous) leverage generated by competition for flow, activist effort and investor returns would not be procyclical. Finally, our Assumption 2 has to be adjusted to the richer payoff structure as follows:

$$\bar{e}\left(\bar{X}_H^B - \underline{X}_H^B\right) < c_{\bar{e}} \leq \alpha \bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right). \tag{4}$$

To avoid a plethora of cases, we make a minor simplification by setting $\bar{X}_H^B = \underline{X}_H^B$. Since Assumption 4 already precludes effort by the bad fund in the high state, this simplification is entirely without loss of generality.

We first note that Propositions 1, 2, and 3 go through unmodified in our richer setting. The equivalent formal results are stated and proved in the appendix. This means that

[21]Had we explicitly modeled target firm managers, such divertible cash flows could be shared between firms and funds, without qualitatively affecting the incentives of the fund.

[22]In the reverse case, the divertibility of cash flows would make failure more attractive, undermining effort provision regardless of the aggregate state.

investors never verify in equilibrium, there are no pooling equilibria, and—in separating equilibria—the good fund must pay out an amount so high that, even by raising the maximum amount of external financing possible, the bad type cannot imitate.

We now solve for the optimal contract for external financing. At the time of investing F, financiers set the repayments $R\left(\bar{X}_s^\theta\right)$ and $R\left(\underline{X}_s^\theta\right)$ due at the end of the second period to break even, making all relevant equilibrium inferences.

Proposition 12. *Debt is the optimal contract for raising external funding F.*

Since project success/failure is verifiable but the state is not, promised repayments can take on at most two possible values, say $\bar{R}$ and $\underline{R}$. Since, conditional on separation (which eliminates the bad fund in the first period) the future cash flows are increasing in the good fund's effort, we look for $\bar{R}$ and $\underline{R}$ which maximize the good fund's incentives to exert effort. While effort is costly for the fund, it allows it to obtain an $\alpha-$share of a larger cash flow with probability $\bar{e}$. In addition, the fund—having de facto control—can appropriate all nonverifiable cash flows (like the borrower/entrepreneur in e.g., Bolton and Scharfstein (1990)). In particular, if the project succeeds, the fund can appropriate $\bar{X}_H^G - \bar{X}_L^G$ whereas if it fails it can appropriate $\underline{X}_H^G - \underline{X}_L^G$. Since effort increases the probability of success from 0 to $\bar{e}$, in the high state effort also generates an additional payoff of $\bar{e}\left(\left(\bar{X}_H^G - \bar{X}_L^G\right) - \left(\underline{X}_H^G - \underline{X}_L^G\right)\right)$ to the fund. Thus, as the proof in the appendix shows, the incentive compatibility constraints of the good fund are:

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) \geq c_{\bar{e}} \text{ in state } s = L\text{, and}$$

$$\alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \geq c_{\bar{e}} \text{ in state } s = H.$$

For arbitrarily chosen parameters, these two constraints are clearly most slack if $\bar{R} - \underline{R}$ is minimized. Imposing monotonicity, as is standard in this literature following Innes (1990), leads to two possible optimal financing arrangements: If the fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$.[23]

[23]Needless to say, absent the contracting friction that underlies all results in this section, state

We now re-derive our core result, Proposition 4, in this richer setting, labelling it Proposition 4′ for ease of reference. For technical reasons, the subsequent analysis needs to be split into two cases:

$$\text{Case A: } \left(\bar{X}_H^G - \underline{X}_H^G\right) \geq (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right) \tag{5}$$

and

$$\text{Case B: } \left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right). \tag{6}$$

Since α is typically on the order of 0.2 for funds, Case B is restrictive. Accordingly, we only discuss Case A in the body of the paper and relegate Case B to the appendix, where we show that the economic content of our results is essentially identical across the cases.

Proposition 4′. *As long as* $\gamma_s(1-\gamma_s)\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) > \Delta x_1 > \frac{w}{1-\alpha}$, *the separating equilibrium with minimal leverage involves:*

i. For $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$, $e^*(s) = \bar{e}$ *for all* s.

ii. For $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$, $e^*(H) = \bar{e}$ *and* $e^*(L) = 0$.

The full set of our applied implications stated above which follow from Proposition 4, follow similarly in this richer setting from Proposition 4′. We do not therefore restate these implications. Instead, we conclude this section by considering whether debt overhang in the low state could be avoided by using a more sophisticated compensation contract for the fund.

Proposition 13. *There is no fund compensation contract that simultaneously*

i. Prevents mimicking by bad funds, and

ii. Generates effort by good funds in both states for the full range of effort costs that—absent leverage—would induce effort in the low state.

Given the non-verifiable component of returns in the high state, the bad fund always has an incentive to try to survive into the second period: Irrespective of the second

contingent debt would be the optimal contract which, by virtue of being state contingent, would rule out debt overhang.

period contractual payments, survival enables the bad fund to effortlessly earn at least an expected payoff of $\gamma_s \left(\underline{X}_H^B - \underline{X}_L^B \right) > 0$. As a result, she tries to mimic the good fund. In turn, the good fund levers up to separate. Whenever she levers, there is *some* cost range for which she subsequently does not work in the low state while she would have done so unlevered. Thus, the non-contractibility of economic states implies that there is no contract between investors and funds that can preclude debt overhang in the low state.

7 Conclusions

We propose a model of activism by asset managers in the presence of competition for flows. Our self-contained theory highlights how agency frictions arising out of the delegation of portfolio management can affect the nature of blockholder monitoring and, more broadly, may help to enrich our understanding of corporate governance issues. In addition, our model suggests a new channel by which the incentives of asset managers can amplify booms and busts and foster economic fragility. In addition to these broader implications, our paper sheds light on the observed procyclicality of hedge fund activism and reconciles it with the documented effect of activist hedge funds on the net leverage of their target firms. Finally, we generate some testable implications which resolve some contradictory empirical evidence on the wealth effects of hedge fund activism on different stakeholders in target firms.

8 Appendix

Proof of Proposition 1: The result follows immediately from the following two lemmata.

Lemma 1. *There is no equilibrium in which the investor verifies with probability 1.*

Proof of Lemma: If the investor verifies for sure, he identifies the type and thus there is no benefit to borrowing while there is an infinitesimal cost. Thus, $F(\theta) = 0$ for all θ. But, if $F(\theta) = 0$ for both θ then it is a best response for the investor not to verify, since $c_v > 0$.■

Lemma 2. *As long as c_v is small, there is no equilibrium in which the investor verifies with interior probability.*

Proof of Lemma: For the investor to verify with interior probability, the only equilibria to consider are those in which $D_1^G = D_1^B := D_1^P$. Let the gross (of verification cost) expected payoff to verification be Π_v. Following verification, investors can retain or fire the fund in a type dependent manner, and therefore their payoff will be:

$$\Pi_v = \gamma_\theta max\left(\Pi^G(D_1), 0\right) + (1 - \gamma_\theta)(0),$$

where $\Pi^G(D_1)$ denotes the investor's expected second period net payoff from retaining a good fund given D_1 and 0 is the price at which the firm can be sold. Since the bad type never makes an effort in the second period, she is always fired and the firm is sold. The good type may or may not be retained, depending on whether $\Pi^G(D_1)$ is larger or smaller than 0. Without verification the investor may always retain (with expected payoff Π_1) or always fire (with expected payoff Π_0). We have:

$$\Pi_1 = \gamma_\theta \Pi^G(D_1) + (1 - \gamma_\theta)(-w),$$

and

$$\Pi_0 = 0.$$

Randomization requires that $\Pi_v - c_v = \max(\Pi_1, \Pi_0)$.

Since $max\left(\Pi^G(D_1), 0\right) \geq \Pi^G(D_1)$ and $0 > -w$, it follows that that $\Pi_1 < \Pi_v$. Then, for small enough c_v, $\Pi_v - c_v > \Pi_1$. Further, inspection yields that $\Pi_v \geq \Pi_0$. This is because verification does not preclude the option of always firing both types. If $\Pi_v > \Pi_0$, for small enough c_v, $\Pi_v - c_v > \Pi_0$ and randomization cannot be supported in equilibrium. Alternatively, if $\Pi_v = \Pi_0$, for any $c_v > 0$ there is no verification, and thus again randomization cannot be supported.■

Proof of Proposition 2: From the proof of Proposition 1, we have that $\Pi_v \geq \Pi_0$ and $\Pi_v > \Pi_1$. Hence there are two possibilities. Either $\Pi_v > \Pi_0 \geq \Pi_1$, and for small c_v, $\Pi_v - c_v > \Pi_0$. As a result, $F(B) = 0$ and $D_1(G) \neq D_1(B)$. Or, we have $\Pi_v = \Pi_0 > \Pi_1$, and both types are fired following non-verification. Consequently $F(G) = F(B) = 0$ and $D_1(G) \neq D_1(B)$. Finally, a pooling equilibrium where investors never verify but randomise between retaining and firing cannot arise because for $\Pi_0 = \Pi_1$ it must be the case that $\Pi_v > \Pi_0 = \Pi_1$.■

Lemma 3. *If $D_1^*(G) \neq D_1^*(B)$, then $F^*(B) = 0$.*

Proof: If $D_1^*(G) \neq D_1^*(B)$, then $\mu_{IN}^{pre^*}(D_1^*(B)) = 0$. Assumption (2) implies that bad funds does not exert effort. Thus, investors fire the bad fund, because by doing so they save the fee, w, to be paid in the second period. Thus, $a_{IN}^{r^*}(D_1^*(B)) = 0$, and $F^*(B) = 0$ since choosing $F > 0$ creates an infinitesimal cost for the fund.■

Lemma 4. *If $D_1^*(G) \neq D_1^*(B)$, then $\mu_{FI}^*(F) = 1$ for $F \in \left(0, PI^G\right]$.*

Proof: The equilibrium payout $D_1^*(G)$ can be represented as a map $f : \left(x^G, x^B\right) \to \mathbb{R}_+$. The required borrowing is therefore $F^*(G) = f\left(x^G, x^B\right) - x^G$. Except in the special case in which $f\left(x^G, x^B\right) - x^G = k$ for some $k \in \mathbb{R}$ – which by definition can only arise in equilibria in which financiers commit/coordinate to lend only specific amounts and are thus ruled out in our analysis – financiers cannot compute $F^*(G)$ before the funding request is made because they do not know x^G . However, since $F^*(B) = 0$ (Lemma 3), any requested amount $F \in \left(0, PI^G\right]$ is consistent with $\mu_{FI}^*(F) = 1$.■

Proof of Proposition 3: Since in a separating equilibrium $\mu_{FI}^*(F) = 1$ for $F \in \left(0, PI^G\right]$, financiers are willing to lend up to PI^G. Suppose that $D_1^*(G) < x_1^B + PI^G$. Then, type B can deviate and raise $D_1^*(G) - x^B < PI^G$ and successfully imitate type G violating $D_1^*(G) \neq D_1^*(B)$.■

Proof of Proposition 4: The derivation proceeds in three steps.

Step 1: Debt Overhang thresholds

For a given face value of debt K debt overhang arises in state $s = L$ only if

$$\alpha\bar{e}\left(X_L^G - K\right) < c_{\bar{e}}.$$

Thus, the maximum face value of debt associated with effort exertion in state $s = L$ is

$$\underline{K} = X_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

Conversely, for a given face value K, there is no debt overhang in state $s = H$ if

$$\alpha\bar{e}\left(X_L^G - K\right) \geq c_{\bar{e}}.$$

Thus, the maximum face value of debt associated with effort exertion in state $s = H$ is

$$\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

Step 2: Pledgeable Income PI^G

We compare the maximum pledgable income with debt $\underline{K}$ and the one with debt $\bar{K}$. Without debt overhang in state $s = L$ pledgeable income is equal to $PI_{\underline{K}}^G = \bar{e}\underline{K}$. With debt overhang in state $s = L$ pledgable income is equal to $PI_{\bar{K}}^G = \gamma_s \bar{e}\bar{K}$. Then $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ is equivalent to

$$c_{\bar{e}} \geq \frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+.$$

Step 3(a): Equilibrium borrowing and retention given that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$

Separation requires borrowing of

$$PI_{\bar{K}}^G - \Delta x = \gamma_s \bar{e}\bar{K} - \Delta x,$$

and the corresponding face value K^* is:

$$K^* = \frac{\gamma_s \bar{e}\bar{K} - \Delta x}{\gamma_s \bar{e}} = \frac{\gamma_s \bar{e}\left(X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x}{\gamma_s \bar{e}} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s \bar{e}}.$$

For consistency we need $K^* > \underline{K}$, i.e.,

$$\frac{\gamma_s \bar{e}\bar{K} - \Delta x}{\gamma_s \bar{e}} > \underline{K},$$

i.e.,

$$\gamma_s \bar{e}\left(X_H^G - X_L^G\right) \geq \Delta x.$$

It remains to check that it is in the investor's interest to retain a good fund. Retention results in a payoff equal to

$$(1-\alpha)\,\gamma_s \bar{e}\left(X_H^G - K^*\right) - w,$$

Liquidating the fund/firm results in a payoff of 0. Retention requires:

$$(1-\alpha)\,\gamma_s \bar{e}\left(X_H^G - K^*\right) \geq w$$

which, upon substituting in the value of K^* is equivalent to:

$$(1-\alpha)\,\gamma_s \bar{e}\left(X_H^G - X_H^G + \frac{c_{\bar{e}}}{\alpha \bar{e}} + \frac{\Delta x}{\gamma_s \bar{e}}\right) \geq w.$$

Now, using the lower bound (for debt overhang) of effort costs yields:

$$\Delta x \geq \frac{w}{1-\alpha} - \frac{\gamma_s \bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+.$$

Step 3(b): Equilibrium borrowing and retention given that $PI_{\bar{K}}^G < PI_{\underline{K}}^G$

Proposition 2 implies that separation requires borrowing of

$$PI_{\underline{K}}^G - \Delta x = \bar{e}\left(X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}\right) - \Delta x,$$

and the corresponding face value is

$$K^{**} = \frac{\bar{e}\left(X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}\right) - \Delta x}{\bar{e}} = X_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}} - \frac{\Delta x}{\bar{e}}.$$

It remains to check that it is in the investor's interest to retain a good fund. Retaining the good fund generates a continuation payoff equal to

$$(1-\alpha)\,\bar{e}\left(X_L^G - K^{**} + \gamma_s\left(X_H^G - X_L^G\right)\right) - w,$$

which must be compared to a payoff of 0 for firing. Simplifying, retention requires that:

$$\Delta x \geq \frac{w}{1-\alpha} - \frac{c_{\bar{e}}}{\alpha} - \bar{e}\gamma_s\left(X_H^G - X_L^G\right).$$

This concludes the proof of the proposition.■

Proof of Proposition 5: For $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+, \alpha\bar{e}X_L^G\right)$, the investor's expected cash flows are:

$$\gamma_\theta E\left(x^G\right) + (1-\gamma_\theta)\,E\left(x^B\right) - w + \gamma_\theta\left(PI^G - \Delta x + (1-\alpha)\,\gamma_s\bar{e}\left(X_H^G - K^*\right) - w\right),$$

i.e., the sum of net payoffs from free cash flow mitigation and restructuring. Using from the proof of Proposition 4 the facts that

$$PI_{\bar{K}}^G - \Delta x = \gamma_s\bar{e}\bar{K} - \Delta x,$$

$$\bar{K} = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}},$$

and

$$K^* = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s\bar{e}},$$

the investor's expected cash flows can be simplied to:

$$\gamma_\theta E\left(x^G\right) + (1-\gamma_\theta)\,E\left(x^B\right) - w + \gamma_\theta\left(\gamma_s\left(\bar{e}X_H^G - c_{\bar{e}}\right) - \alpha\Delta x - w\right).$$

The former is positive because $E\left(x^G\right) > E\left(x^B\right) = \frac{C-\triangle x}{2} > w$ by Assumption 1. Inserting the highest effort cost, the latter can be further rearranged as follows:

$$\gamma_s\alpha\bar{e}\left(X_H^G - X_L^G - \Delta x\right) + (1-\alpha)\left(\bar{e}X_H^G - \frac{w}{1-\alpha}\right),$$

where both terms in the parentheses are positive under the conditions of Proposition 4. The investor's expected cash flow in equilibrium is clearly higher in the case of smaller effort costs.■

Proof of Proposition 6: The proposition follows from the fact that $\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G\right)^+$ is (i) decreasing in γ_s for $\gamma_s \in [0, X_L^G/X_H^G)$, (ii) 0 for $\gamma_s \geq X_L^G/X_H^G$, and (iii) $\alpha\bar{e}X_L^G$ for $\gamma_s = 0$. ■

Proof of Proposition 7: From the proof of Proposition 5, the investors expected payoff is

$$\gamma_\theta E\left(x^G\right) + (1-\gamma_\theta)\,E\left(x^B\right) - w + \gamma_\theta\left(\gamma_s\left(\bar{e}X_H^G - c_{\bar{e}}\right) - \alpha\Delta x - w\right), \tag{7}$$

which is clearly increasing in γ_s since $\bar{e}X_H^G - c_{\bar{e}} > 0$ by (2).■

Proof of Proposition 8: Investors' payoffs without competition for flow are:

$$\gamma_\theta E\left(x^G\right) + \left(1-\gamma_\theta\right) E\left(x^B\right) + \xi\gamma_\theta\left(1-\alpha\right)\bar{e}\left(\gamma_s X_H^G + \left(1-\gamma_s\right) X_L^G\right) - \xi w. \tag{8}$$

Without competition for flow, the investor's payoff is most sensitive to economic prospects when $\xi = 1$, where it is given by $\gamma_\theta\left(1-\alpha\right)\bar{e}\left(X_H^G - X_L^G\right)$. This is smaller than $\gamma_\theta\left(\bar{e}X_H^G - c_{\bar{e}}\right)$, which is the derivative of (7) with respect to γ_s. ■

Proof of Proposition 9: The amount of borrowing in the SEML is $PI_{\bar{K}}^G - \Delta x = \gamma_s\bar{e}\left(X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x$, while the face value of the debt is $K^* = X_H^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s\bar{e}}$. Both quantities are increasing in γ_s. In addition, end of the first period ratio of the market value of debt to the market value of the firm is $\frac{PI_{\bar{K}}^G - \Delta x}{PI_{\bar{K}}^G} = 1 - \frac{\Delta x}{PI_{\bar{K}}^G}$ is also increasing in γ_s.■

Proof of Proposition 10: To separate, the good fund must pay out enough to prevent mimicking by the bad fund. The good fund always prefers to pay out liquid assets Y_0 in the first period (that would anyway go to creditors in the second period) because, holding fixed the separation payout, replacing the paying out of Y_0 with additional borrowing is costly: For each dollar borrowed the good fund must pay back either $1/\gamma_s\bar{e}$ (if debt overhang arises) or $1/\bar{e}$ (otherwise) in the second period. Both are costly to the hedge fund's payoff, as it receives a second period carry. This establishes that Y_0 is fully paid out in any separating equilibrium. The remaining steps mirror those of the proof of Proposition 4, and are thus stated in brief.

Given pre-existing debt K_0 and all liquid assets Y_0 paid out, there is debt overhang in $s = L$ if the face value of debt satisfies $K > \underline{K}_{K_0} \equiv X_L^G - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}}$, and no debt overhang in $s = H$ if $K < \bar{K}_{K_0} = X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}}$. For

$$c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X_L^G - \gamma_s X_H^G - \left(1-\gamma_s\right)K_0\right)^+, \alpha\bar{e}\left(X_L^G - K_0 + Y_0\right)\right]$$

it is easy to check that $PI_{\bar{K}_{K_0}}^G \geq PI_{\underline{K}_{K_0}}^G$. Thus, separation requires an amount of borrowing equal to $PI_{\bar{K}_{K_0}}^G - \Delta x = \gamma_s\bar{e}\left(X_H^G - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x$, with corresponding face value $K^*_{K_0} = X^G_H - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x}{\gamma_s \bar{e}}$. For consistency we need $K^*_{K_0} > \underline{K}_{K_0}$, which is always satisfied as long as $\gamma_s \bar{e}\left(X^G_H - X^G_L\right) > \Delta x$, as in the baseline model.

Next we check that the investor wants to retain a good hedge fund. Since w paid at $t = 1$ is sunk and the investor has already received $D^*_1 = x^G + Y_0 + \gamma_s \bar{e}\left(X^G_H - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x$, the investor retains the good fund if $(1-\alpha)\gamma_s \bar{e}\left(X^G_H - K_0 - K^*_{K_0}\right) \geq w$, which is guaranteed if i.e., if $\Delta x_1 > \frac{w}{1-\alpha}$ as in the baseline model.

For $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X^G_L - \gamma_s X^G_H\right)^+, \alpha\bar{e}X^G_L\right]$, the analysis of the baseline model implies that debt overhang arises in the low state in the SEML in the unleveraged target firm (since Y_0 is paid out in the first period). Further, since $\frac{\alpha\bar{e}}{1-\gamma_s}\left(X^G_L - \gamma_s X^G_H\right)^+ \geq \frac{\alpha\bar{e}}{1-\gamma_s}\left(X^G_L - \gamma_s X^G_H - (1-\gamma_s) K_0\right)^+$ and $\alpha\bar{e}X^G_L < \alpha\bar{e}\left(X^G_L - K_0 + Y_0\right)$, we can conclude that for $c_{\bar{e}} \in \left[\frac{\alpha\bar{e}}{1-\gamma_s}\left(X^G_L - \gamma_s X^G_H\right)^+, \alpha\bar{e}X^G_L\right]$, debt overhang arises in the low state in the SEML in levered and unlevered target firms.

Finally, we can compare (i) the payoffs to equity holders in firms with and without pre-existing debt in the presence of hedge fund activists and (ii) the payoffs to pre-existing creditors in levered target firms in the presence and absence of hedge fund activists.

(i) **Payoffs to equity holders:** With pre-existing leverage of K_0, target shareholders receive an expected payoff of

$$\gamma_\theta \left(E\left(x^G\right) + Y_0 + \gamma_s \bar{e}\left(X^G_H - K_0 - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x\right) + (1-\gamma_\theta) E\left(x^B_1\right)$$

in the first period and $\gamma_\theta\left(\gamma_s \frac{c_{\bar{e}}}{\alpha} + \Delta x\right)$ in the second period. Without leverage, target shareholders receive an expected payoff of

$$\gamma_\theta \left(E\left(x^G\right) + Y_0 + \gamma_s \bar{e}\left(X^G_H - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) - \Delta x\right) + (1-\gamma_\theta) E\left(x^B_1\right)$$

in the first period and $\gamma_\theta\left(\gamma_s \frac{c_{\bar{e}}}{\alpha} + \Delta x\right)$ in the second period. Thus, pre-existing leverage reduces first period payoffs to target shareholders without affecting second period payoffs.

(ii) **Payoffs to pre-existing creditors:** In the absence of the hedge fund activists, creditors would have expected to receive K_0 in the second period in either state (since $Y_0 > K_0$). In the presence of hedge fund activists, the same creditors can expect to receive K_0 in the second period in the high state with probability $\bar{e}$ but nothing otherwise. Thus, the presence of activist hedge funds expropriates pre-existing creditors.■

Proof of Proposition 11: To separate, the good type has to pay out $D_1^*(G) = x^B + Y_0$ and can therefore retain at most $x^G + Y_0 - \left(x^B + Y_0\right) = \Delta x$ liquid assets. For

$$K_0 \in \left(X_L^G + \triangle x - \frac{c_{\bar{e}}}{\alpha \bar{e}}, X_H^G + \triangle x - \frac{c_{\bar{e}}}{\alpha \bar{e}}\right)$$

the incentive compatibility constraint in state $s = L$

$$\alpha \bar{e}(X_L^G - K_0 + \Delta x) \geq c_{\bar{e}}$$

is violated but that for state $s = H$

$$\alpha \bar{e}(X_H^G - K_0 + \Delta x) \geq c_{\bar{e}}$$

is satisfied.■

8.1 Proofs for Section 6

The statements of Proposition 1 and Lemmas 1 and 2, as well as the proofs of the first to results, are entirely unchanged in this richer setting. The only change is to the proof of Lemma 2, which is stated below:

Proof of Lemma 2 for enriched payoffs: As before, the only equilibria to consider are those in which $D_1^G = D_1^B := D_1^P$. Let the gross (of verification cost) expected payoff to verification be Π_v. Following verification, the investor's payoff will be:

$$\Pi_v = \gamma_\theta max \left(\Pi^G \left(D_1\right), P^G\right) + \left(1 - \gamma_\theta\right) P^B,$$

where $\Pi^G \left(D_1\right)$ denotes the investor's expected second period net payoff from retaining a good fund given D_1 and $P^\theta = max \left(\underline{X}_L^\theta - R\left(\underline{X}_L^\theta\right), 0\right)$ is the type-dependent price at which the target can be sold. (The earlier proof of Lemma 2 shows that this proof holds for a type-uncontingent sales price as well.) Since the bad type never makes an effort in the second period, she is always fired and the firm is sold. The good type may or may not be retained, depending on whether the investors receive enough from the additional cash flows generated by the good type's effort. Without verification the investor may always retain (with expected payoff Π_1) or always fire (with expected payoff Π_0). We have:

$$\Pi_1 = \gamma_\theta \Pi^G (D_1) + (1 - \gamma_\theta) \left(max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) (1 - \alpha) - w \right),$$

and

$$\Pi_0 = \gamma_\theta P^G + (1 - \gamma_\theta) P^B.$$

Randomization requires that $\Pi_v - c_v = \max(\Pi_1, \Pi_0)$.

Since $max \left(\Pi^G (D_1), P^G \right) \geq \Pi^G (D_1)$ and

$$P^B = max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) > max \left(\underline{X}_L^B - R \left(\underline{X}_L^B \right), 0 \right) (1 - \alpha) - w,$$

it follows that that $\Pi_1 < \Pi_v$. Then, for small enough c_v, $\Pi_v - c_v > \Pi_1$. Since verification does not preclude the option of always firing both types, we have that $\Pi_v \geq \Pi_0$. This means that there are two possibilities: If $\Pi_v > \Pi_0$, for small enough c_v, $\Pi_v - c_v > \Pi_0$ and randomization cannot be supported in equilibrium. Alternatively, if $\Pi_v = \Pi_0$, for any $c_v > 0$ there is no verification, and thus again randomization cannot be supported.■

The statements and proofs for Propositions 2 and 3 remain unchanged.

Proof of Proposition 12: Since there are four possible cash flows generated by the good type (two states crossed with project success or failure), the repayment function $R(\cdot)$ takes four possible values: $R \left(\bar{X}_L^G \right)$, $R \left(\bar{X}_H^G \right)$, $R \left(\underline{X}_L^G \right)$, and $R \left(\underline{X}_H^G \right)$ respectively. The verifiability of project success coupled with the non-verifiability of realized cash flows implies that

$$R \left(\bar{X}_L^G \right) = R \left(\bar{X}_H^G \right) := \bar{R} \text{ and } R \left(\underline{X}_L^G \right) = R \left(\underline{X}_H^G \right) := \underline{R}.$$

It also implies that in state H the fund captures the incremental cash flows $\bar{X}_H^G - \bar{X}_L^G$ and $\underline{X}_H^G - \underline{X}_L^G$ conditional on success and failure respectively, since investors cannot verify whether $s = H$ or L.

Effort exertion in state $s = L$ requires that

$$\begin{aligned} \alpha\left(\bar{e}\left(\bar{X}_L^G - \bar{R}\right) + (1-\bar{e})\left(\underline{X}_L^G - \underline{R}\right)\right) - c_{\bar{e}} &\geq \alpha\left(\underline{X}_L^G - \underline{R}\right), \\ \text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) &\geq c_{\bar{e}}. \end{aligned} \tag{9}$$

Effort exertion in state $s = H$ requires that

$$\left(\begin{array}{c} \alpha\bar{e}\left(\bar{X}_L^G - \bar{R}\right) + \bar{e}\left(\bar{X}_H^G - \bar{X}_L^G\right) + \\ \alpha(1-\bar{e})\left(\underline{X}_L^G - \underline{R}\right) + (1-\bar{e})\left(\underline{X}_H^G - \underline{X}_L^G\right) \end{array}\right) - c_{\bar{e}} \geq \alpha\left(\underline{X}_L^G - \underline{R}\right) + \left(\underline{X}_H^G - \underline{X}_L^G\right),$$

$$\text{i.e., } \alpha\bar{e}\left(\left(\bar{X}_L^G - \underline{X}_L^G\right) - \left(\bar{R} - \underline{R}\right)\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \geq c_{\bar{e}}. \tag{10}$$

For arbitrarily chosen parameters, (9) and (10) are clearly most slack if $\bar{R} - \underline{R}$ is minimized. Imposing monotonicity implies $\bar{R} \geq \underline{R}$. Hence, if the fund raises less than $\underline{X}_L^G$, we have safe debt with repayment $\bar{R} = \underline{R} < \underline{X}_L^G$. Otherwise, optimal external financing is achieved via defaultable debt with $\bar{R} > \underline{R} = \underline{X}_L^G$, i.e., the face value of debt must be $K \geq \underline{X}_L^G$. The maximum (fulfillable) face value of debt is given by $K \leq \bar{X}_L^G$.■

Proof of Proposition 4′:

Step 1: Debt Overhang thresholds

For a given face value of debt K debt overhang arises in state $s = L$ only if

$$\alpha\left[\bar{e}\left(\bar{X}_L^G - K\right) - \bar{e}\left(\underline{X}_L^G - \min(K, \underline{X}_L^G)\right)\right] < c_{\bar{e}}.$$

For $K < \underline{X}_L^G$ the above reduces to $\alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) \leq c_{\bar{e}}$, which violates assumption (4). Thus, $K > \underline{X}_L^G$, and the maximum face value of debt associated with effort exertion in state $s = L$ is

$$\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}.$$

For a given face value K, there is no debt overhang in state $s = H$ if

$$\left(\begin{array}{c} \alpha\left[\bar{e}\left(\bar{X}_L^G - K\right) + (1-\bar{e})\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)\right] \\ +\bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) \end{array}\right) - c_{\bar{e}} \geq \alpha\left(\underline{X}_L^G - \min(\underline{X}_L^G, K)\right)$$

Since we look for debt levels that induce debt overhang in state $s = L$, $K > \underline{K} > \underline{X}_L^G$ so that the expression above simplifies to:

$$\alpha\bar{e}\left(\bar{X}_L^G - K\right) + \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) - c_{\bar{e}} \geq 0,$$

which gives us

$$K \leq \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right).$$

If

$$c_{\bar{e}} \leq \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$$

then the relevant constraint for K is

$$K \leq \bar{X}_L^G,$$

because of the non-verifiability of economic states. Assumption (4) guarantees that

$$c_{\bar{e}} \leq \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right).$$

Thus, if

$$\begin{aligned} \alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right) &< \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right), \\ \text{i.e., } \left(\bar{X}_H^G - \underline{X}_H^G\right) &\geq (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right), \end{aligned}$$

then, under Assumption (2) the relevant constraint for K is always

$$K \leq \bar{X}_L^G.$$

and

$$\bar{K} = \bar{X}_L^G.$$

Step 2: Pledgeable Income PI^G

To derive the conditions under which pledgable income is higher, we compare the maximum pledgable income with debt $\underline{K}$ and the one with debt $\bar{K}$. Without debt overhang in state $s = L$ pledgeable income is equal to

$$\bar{e}\underline{K} + (1-\bar{e})\,\underline{X}_L^G.$$

Inserting $\underline{K} = \bar{X}_L^G - c_{\bar{e}}/\alpha\bar{e}$ yields the maximum pledgeable income $PI_{\underline{K}}^G$:

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G.$$

With debt overhang in state $s = L$ pledgable income is equal to

$$\gamma_s\bar{e}\bar{K} + (1-\gamma_s\bar{e})\,\underline{X}_L^G.$$

Inserting the expression for $\bar{K} = \bar{X}_L^G$ yields the maximum pledgeable income $PI_{\bar{K}}^G$:

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G.$$

Then $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ is equivalent to

$$c_{\bar{e}} \geq (1-\gamma_s)\,\alpha\bar{e}\left(\bar{X}_L^G - \underline{X}_L^G\right).$$

Thus, for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ (Case A.1), while for $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, the maximum pledgeable income is $PI_{\bar{K}}^G$ (Case A.2).

Case A.1: $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$

Step 3 for A.1: Funding amount for $PI_{\bar{K}}^G < PI_{\underline{K}}^G$

Proposition 2 implies that separation requires borrowing of

$$PI_{\underline{K}}^G - \Delta x_1 = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1,$$

and the corresponding face value K^{**} solves

$$\bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G - \Delta x_1 = \bar{e}K^{**} + (1-\bar{e})\min(K^{**}, \underline{X}_L^G). \tag{11}$$

Suppose $K^{**} > \underline{X}_L^G$, then $\min(K^{**}, \underline{X}_L^G) = \underline{X}_L^G$, in which case (11) gives:

$$K^{**} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x_1}{\bar{e}},$$

which is clearly smaller than $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ so that there is indeed no debt overhang in state $s = L$. Furthermore, the condition $\bar{X}_L^G > \underline{X}_L^G + \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}$ in Proposition 4 ensures that $K^{**} > \underline{X}_L^G$. Indeed, a sufficient condition for $K^{**} > \underline{X}_L^G$ for all $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right)$ is that

$$\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{\Delta x_1}{\bar{e}} > \underline{X}_L^G$$

for $c_{\bar{e}} = (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$. This in turn, is equivalent to:

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s \bar{e}} \tag{12}$$

which always holds since $\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}} > \frac{\Delta x_1}{\gamma_s \bar{e}}$.

It remains to check that it is in the investor's interest to retain a good fund. Retaining the good fund generates a continuation payoff equal to

$$(1-\alpha)\,\bar{e}\left(\bar{X}_L^G - K^{**}\right) - w,$$

which does not depend on the aggregate state due to a combination of (i) no debt overhang and (ii) non verifiability of the state. Liquidating the fund/firm results in a payoff of $\max\left(\underline{X}_L^G - K^{**}, 0\right) = 0$. Thus retention requires:

$$(1-\alpha)\left(\frac{c_{\bar{e}}}{\alpha} + \Delta x_1\right) - w \geq 0 \tag{13}$$

which is clearly always satisfied given $\Delta x_1 > \frac{w}{1-\alpha}$. This conclude the proof of the proposition for constellation A.1.

Case A.2: $c_{\bar{e}} \in \left[(1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$

Step 3 for A.2: Funding amount given that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$

Separation requires borrowing of

$$PI_{\bar{K}}^G - \Delta x_1 = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1,$$

and the corresponding face value K^* is obtained by setting

$$\gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G - \Delta x_1 = \gamma_s\bar{e}K^* + (1-\gamma_s\bar{e})\,\underline{X}_L^G,$$

giving

$$K^* = \frac{\gamma_s \bar{e} \bar{X}_L^G - \Delta x_1}{\gamma_s \bar{e}} = \bar{X}_L^G - \frac{\Delta x_1}{\gamma_s \bar{e}}.$$

For consistency we need $K^* > \underline{K}$, i.e.,

$$\bar{X}_L^G - \frac{\Delta x_1}{\gamma_s \bar{e}} > \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}},$$

i.e.,

$$\Delta x_1 < \frac{\gamma_s}{\alpha} c_{\bar{e}}$$

Since $c_{\bar{e}} \geq (1-\gamma_s)\alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]$, the constraint above is always satisfied given

$$\bar{X}_L^G - \underline{X}_L^G > \frac{\Delta x_1}{\gamma_s(1-\gamma_s)\bar{e}}. \tag{14}$$

It remains to check that it is in the investor's interest to retain a good fund. Retaining the fund results in a payoff equal of

$$(1-\alpha)\left(\gamma_s\left(\bar{e}\left(\bar{X}_L^G - K^*\right) + (1-\bar{e})\max\left(\underline{X}_L^G - K^*, 0\right)\right) + (1-\gamma_s)\max\left(\underline{X}_L^G - K^*, 0\right)\right) - w,$$

Liquidating the fund/firm results in a payoff of

$$\max\left(\underline{X}_L^G - K^*, 0\right).$$

Since $K^* = \bar{X}_L^G - \frac{\Delta x_1}{\gamma_s \bar{e}} > \underline{K} > \underline{X}_L^G$, the investor retains the good fund if:

$$(1-\alpha)\gamma_s\bar{e}\left(\bar{X}_L^G - \bar{X}_L^G + \frac{\Delta x_1}{\gamma_s \bar{e}}\right) - w \geq 0 \tag{15}$$

which is clearly satisfied given $\Delta x_1 > \frac{w}{(1-\alpha)}$. This concludes the proof of the proposition for case A.2.■

Case B: $\left(\bar{\mathbf{X}}_\mathbf{L}^\mathbf{G} - \underline{\mathbf{X}}_\mathbf{L}^\mathbf{G}\right) < \left(\bar{\mathbf{X}}_\mathbf{H}^\mathbf{G} - \underline{\mathbf{X}}_\mathbf{H}^\mathbf{G}\right) < (\mathbf{1}+\alpha)\left(\bar{\mathbf{X}}_\mathbf{L}^\mathbf{G} - \underline{\mathbf{X}}_\mathbf{L}^\mathbf{G}\right)$

When $\left(\bar{X}_L^G - \underline{X}_L^G\right) < \left(\bar{X}_H^G - \underline{X}_H^G\right) < (1+\alpha)\left(\bar{X}_L^G - \underline{X}_L^G\right)$, there are two possibilities: For $c_{\bar{e}} \leq \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G$, while for $c_{\bar{e}} > \bar{e}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$, $\bar{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$.

For $c_{\bar{e}} \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, $\bar{K} = \bar{X}_L^G$ while $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ as before. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G$$

As in case **A1)**, the condition for $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ is

$$c_{\bar{e}} \geq (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]$$

Since $c_{\bar{e}} \leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition can only be satisfied if

$$\begin{aligned}(1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G] &\leq \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] \\ \gamma_s &\geq 1 - \frac{1}{\alpha}\left[\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} - 1\right] := \tilde{\gamma}_s.\end{aligned}$$

Note that $\tilde{\gamma}_s \to 0$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1+\alpha$ and $\tilde{\gamma}_s \to 1$ as $\frac{\bar{X}_H^G - \underline{X}_H^G}{\bar{X}_L^G - \underline{X}_L^G} \to 1$ so $\gamma_s \in [0,1]$. Thus, for $\gamma_s < \tilde{\gamma}_s$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for all $c_{\bar{e}} \in \left(0, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. For $\gamma_s \geq \tilde{\gamma}_s$, the maximum pledgeable income is $PI_{\underline{K}}^G$ for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]\right)$ and $PI_{\bar{K}}^G$ for $c_{\bar{e}} \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G], \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x_1$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x_1 < \frac{\gamma_s}{\alpha}c_{\bar{e}}$ which is again guaranteed by (14).

For $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}\left[\bar{X}_L^G - \underline{X}_L^G\right]\right]$, $\underline{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}$ as before and $\bar{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$. Consequently,

$$PI_{\underline{K}}^G = \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

and

$$PI_{\bar{K}}^G = \gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G$$

Hence, $PI_{\bar{K}}^G \geq PI_{\underline{K}}^G$ holds if

$$\gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\begin{pmatrix}(\bar{X}_H^G - \underline{X}_H^G) \\ -(\bar{X}_L^G - \underline{X}_L^G)\end{pmatrix}\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G \geq \bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$$

i.e.,

$$\gamma_s \geq \frac{\bar{X}_L^G - \underline{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}}{\frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) + \left(\bar{X}_L^G - \underline{X}_L^G - \frac{c_{\bar{e}}}{\alpha \bar{e}}\right)} := \hat{\gamma}_s \in (0,1)\,.$$

Thus, in the range $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ the maximum pledgeable income is $PI_{\underline{K}}^G$ for $\gamma_s < \hat{\gamma}_s$ and $PI_{\bar{K}}^G$ for $\gamma_s \geq \hat{\gamma}_s$. To ensure debt overhang in the latter case, the face value associated with raising $F = PI_{\bar{K}}^G - \Delta x_1$ has to be larger than $\underline{K}$. As shown in case A.2 (step 4) above, this holds for $\Delta x_1 < \frac{\gamma_s}{\alpha} c_{\bar{e}}$ which is again guaranteed by (14).

We now establish that $\tilde{\gamma}_s \geq \hat{\gamma}_s$. Suppose the reverse were true, i.e., $\tilde{\gamma}_s < \hat{\gamma}_s$ and consider $\gamma_s \in (\tilde{\gamma}_s, \hat{\gamma}_s)$ and effort costs immediately to the left and right of the threshold $\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$. Since $\gamma_s > \tilde{\gamma}_s$, for $c_{\bar{e}} = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon$ for some small $\epsilon > 0$, $PI_{\bar{K}}^G > PI_{\underline{K}}^G$. Yet, since $\gamma_s < \hat{\gamma}_s$, for $c_{\bar{e}} = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon$, $PI_{\bar{K}}^G < PI_{\underline{K}}^G$. Note that $PI_{\underline{K}}^G$ is given by $\bar{e}\left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}}\right) + (1-\bar{e})\,\underline{X}_L^G$ for all $c_{\bar{e}}$ and decreases in $c_{\bar{e}}$ at the rate $1/\alpha$.

In contrast, for $c_{\bar{e}} \in \left[\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$, $PI_{\bar{K}}^G$ is given by $\gamma_s \bar{e} \bar{X}_L^G + (1-\gamma_s\bar{e})\,\underline{X}_L^G$ which is invariant with $c_{\bar{e}}$. For $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon\right]$, $PI_{\bar{K}}^G$ is given by $\gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)\right] + (1-\gamma_s\bar{e})\,\underline{X}_L^G$ which decreases in $c_{\bar{e}}$ at the rate γ_s/α, i.e., more slowly than $PI_{\underline{K}}^G$ in the same interval. Thus if $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_{\bar{e}} = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] - \epsilon$ it must also be true that $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ for $c_{\bar{e}} = \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right] + \epsilon$, a contradiction.

To summarize our findings, we have three regions in terms of γ_s:

1. If $\gamma_s < \hat{\gamma}_s$, then $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ for the full relevant range of $c_{\bar{e}}$ and there is no debt overhang.

2. If $\hat{\gamma}_s \leq \gamma_s < \tilde{\gamma}_s$, then for $c_{\bar{e}} \in \left(0, \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]\right)$ we have $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ and no debt overhang, while for $c_{\bar{e}} \in \left(\bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ we have $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ and debt overhang.

3. If $\tilde{\gamma}_s \leq \gamma_s$, then for $c_{\bar{e}} \in \left(0, (1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G]\right)$ we have $PI_{\bar{K}}^G < PI_{\underline{K}}^G$ and no debt overhang, while for $c_{\bar{e}} \in \left((1-\gamma_s)\alpha\bar{e}[\bar{X}_L^G - \underline{X}_L^G], \alpha\bar{e}(\bar{X}_L^G - \underline{X}_L^G)\right)$ we have $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ and debt overhang.

It remains to check that it is in the investor's interest to retain a good fund. In all three regions of γ_s where $PI_{\underline{K}}^G > PI_{\bar{K}}^G$ the analysis of the retention decision is identical to case A.1 (step 4). In the regions $\gamma_s < \hat{\gamma}_s$ and $\gamma_s \geq \tilde{\gamma}_s$ where $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ the constraint $\bar{K} = \bar{X}_L^G$ binds, and the analysis of the retention decision is identical to case A.2. (step 4). In the region $\gamma_s \in [\hat{\gamma}_s, \tilde{\gamma}_s)$ where $PI_{\bar{K}}^G > PI_{\underline{K}}^G$ the constraint $\bar{K} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)$ binds. The corresponding face value of debt K^{***} is obtained by setting

$$\begin{array}{c} \gamma_s\bar{e}\left[\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right)\right] \\ + \left(1-\gamma_s\bar{e}\right)\underline{X}_L^G - \Delta x_1 \end{array} = \gamma_s\bar{e}K^{***} + \left(1-\gamma_s\bar{e}\right)\underline{X}_L^G,$$

giving

$$K^{***} = \bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) - \frac{\Delta x_1}{\gamma_s\bar{e}}.$$

Hence, the investor's payoff from retaining the fund is

$$(1-\alpha)\left[\bar{X}_L^G - \left(\bar{X}_L^G - \frac{c_{\bar{e}}}{\alpha\bar{e}} + \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) - \frac{\Delta x_1}{\gamma_s\bar{e}}\right)\right] - w$$

and retention is in the investor's interest if

$$\left[\frac{c_{\bar{e}}}{\alpha\bar{e}} - \frac{1}{\alpha}\left(\left(\bar{X}_H^G - \underline{X}_H^G\right) - \left(\bar{X}_L^G - \underline{X}_L^G\right)\right) + \frac{\Delta x_1}{\gamma_s\bar{e}}\right] \geq \frac{w}{(1-\alpha)}$$

Since $c_{\bar{e}} > \bar{e}\left[(\bar{X}_H^G - \underline{X}_H^G) - (\bar{X}_L^G - \underline{X}_L^G)\right]$, this condition is satisfied given $\Delta x_1 > \frac{w}{(1-\alpha)}$. This concludes the analysis for case B.■

Proof of Proposition 13: Consider any arbitrary contract with non-negative payments $w_1, \alpha_1(D_1), w_2(D_1), \alpha_2(D_1, X)$ where $X \in \left\{\underline{X}_L^B, \bar{X}_L^B, \underline{X}_L^G, \bar{X}_L^G\right\}$. First note that, for any $w_2(D_1), \alpha_2(D_1, X)$ the investor fires the bad fund if identified. Since the bad fund does not exert effort at $t = 2$, this is immediate for $max\left(w_2(D_1), \alpha_2(D_1, X)\right) > 0$.

Even if $max\left(w_2\left(D_1\right), \alpha_2\left(D_1, X\right)\right) = 0$, the investor is strictly indifferent, and so (by our tie-breaking assumption) fires. Since the investor fires if the bad fund is identified, it is easy to see that the bad fund wishes to mimick for any $w_2\left(D_1\right), \alpha_2\left(D_1, X\right)$. For $max\left(w_2\left(D_1\right), \alpha_2\left(D_1, X\right)\right) > 0$, this is immediate, but even if $max\left(w_2\left(D_1\right), \alpha_2\left(D_1, X\right)\right) = 0$, survival enables the bad fund to effortlessly earn at least an expected payoff of $\gamma_s\left(\underline{X}_H^B - \underline{X}_L^B\right) > 0$. Finally, given that the bad fund has the incentive to mimic, the good fund has to lever to separate. Whenever she levers, there is *some* cost range for which she does not work in the low state with leverage when she would have without.$\blacksquare$

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