

Uncertainty and Contracting in Organizations

Finance Working Paper N° 704/2020

October 2020

David L. Dicks

Baylor University

Paolo Fulghieri

University of North Carolina and ECGI

© David L. Dicks and Paolo Fulghieri 2020. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

This paper can be downloaded without charge from:
http://ssrn.com/abstract_id=3285458

www.ecgi.global/content/working-papers

ECGI Working Paper Series in Finance

Uncertainty and Contracting in Organizations

Working Paper N° 704/2020

October 2020

David L. Dicks
Paolo Fulghieri

We would like to thank Jesse Davis, David Hofmann, Yunzhi Hu, Paige Ouimet, Alejandro Rivera, Jacob Sagi, Andreas Stathopoulos, Laura Veldkamp, and seminar participants at the University of Amsterdam, Erasmus, Iowa, Tilburg, George Mason, Purdue, UNC, University of Texas at Austin, the Third Annual CEPR Spring Symposium in Financial Economics at Imperial College, the Workshop on Corporate Finance, Lancaster University, the SFS Cavalcade Asia-Pacific 2019, and the 2020 American Finance Association for very helpful comments. All errors are only our own.

© David L. Dicks and Paolo Fulghieri 2020. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Abstract

We study a multidivisional firm where headquarters are exposed to moral hazard by division managers under uncertainty (or “ambiguity”) aversion. We show the aggregation and linearity results of Holmström and Milgrom (1987) hold in an environment with IID ambiguity, as in Chen and Epstein (2002). While uncertainty creates endogenous disagreement that aggravates moral hazard, by hedging uncertainty headquarters can design incentive contracts that reduce disagreement, lower incentive provision costs, and promote effort. Because hedging uncertainty can conflict with hedging risk, optimal contracts differ from standard principal-agent models. Optimal contracts involve exposure to other divisions even when division cash flows are uncorrelated and, with sufficient uncertainty, involve equity-based pay, even when division cash-flows are positively correlated. Our model helps explain the prevalence of equity-based incentive contracts and the rarity of relative performance contracts.

Keywords: Contracting, Organizations, Hierarchy, Uncertainty Aversion, Ambiguity Aversion

JEL Classifications: D81, D84, M12

David L. Dicks*
Assistant Professor
Baylor University, Hankamer School of Business
Forster Business and Innovation
Waco, Texas 76706, United States
phone: +1 (254) 710-4771
e-mail: David_Dicks@baylor.edu

Paolo Fulghieri
Professor of Finance
University of North Carolina Kenan-Flagler Business School
300 Kenan Center Drive
Chapel Hill, NC 27599-3490, United States
phone: +1 919-962-3202
e-mail: Paolo_Fulghieri@unc.edu

*Corresponding Author

Uncertainty and Contracting in Organizations*

David L. Dicks
Hankamer School of Business
Baylor University

Paolo Fulghieri
Kenan-Flagler Business School
University of North Carolina
CEPR and ECGI

October 15, 2020

Abstract

We study a multidivisional firm where headquarters are exposed to moral hazard by division managers under uncertainty (or “ambiguity”) aversion. We show the aggregation and linearity results of Holmström and Milgrom (1987) hold in an environment with IID ambiguity, as in Chen and Epstein (2002). While uncertainty creates endogenous disagreement that aggravates moral hazard, by hedging uncertainty headquarters can design incentive contracts that reduce disagreement, lower incentive provision costs, and promote effort. Because hedging uncertainty can conflict with hedging risk, optimal contracts differ from standard principal-agent models. Optimal contracts involve exposure to other divisions even when division cash flows are uncorrelated and, with sufficient uncertainty, involve equity-based pay, even when division cash-flows are positively correlated. Our model helps explain the prevalence of equity-based incentive contracts and the rarity of relative performance contracts.

*We would like to thank Jesse Davis, David Hofmann, Yunzhi Hu, Paige Ouimet, Alejandro Rivera, Jacob Sagi, Andreas Stathopoulos, Laura Veldkamp, and seminar participants at the University of Amsterdam, Erasmus, Iowa, Tilburg, George Mason, Purdue, UNC, University of Texas at Austin, the Third Annual CEPR Spring Symposium in Financial Economics at Imperial College, the Workshop on Corporate Finance, Lancaster University, the SFS Cavalcade Asia-Pacific 2019, and the 2020 American Finance Association for very helpful comments. All errors are only our own. We can be reached at David_Dicks@baylor.edu and Paolo_Fulghieri@kenan-flagler.unc.edu

A classical theme in the theory of incentive contracts is to determine the appropriate performance measures to use as a base for incentive pay in organizations. Managerial incentive contracts often are a combination of a base-pay, which depends on narrowly defined division-specific performance measures (“pay-for-performance”), plus an additional component linked to the overall firm profitability (such as firm-wide bonuses or “equity-based” pay).¹ The distinction between equity-based contracts and division-specific pay is particularly important for lower-level managers.² The case for equity-based contracts for top managers is rather strong, as they are responsible for the performance of the overall firm. More puzzling is the use of compensation contracts that include equity-based components for division managers and rank-and-file employees who are deeper down in the organization. This is because, for lower-level employees, equity-based compensation reduces the responsiveness of pay to their actions, thus “diluting” incentives, at the cost of increasing their risk exposure. In addition, when cash-flows are positively correlated across divisions, theory suggests that, to reduce such harmful risk bearing, incentive contracts should display a “relative-performance” component, a feature that is infrequently observed in practice.

This paper studies the impact of uncertainty (or “ambiguity”) aversion on the design of incentive contracts in organizations. The key feature of our model is to acknowledge that most corporate decisions are taken without full knowledge of the probability distributions involved, a situation that is characterized as uncertainty (Knight, 1921). We model uncertainty aversion by adopting the Minimum Expected Utility approach of Gilboa and Schmeidler (1989), within the continuous time framework of Chen and Epstein (2002).

We consider a multi-division firm endowed with company headquarters (HQ) and (two) division managers. Division cash-flows, which can be (positively or negatively) correlated, depend on unobservable effort exerted by division managers, creating moral hazard. Building on Holmström and Milgrom (1987), division managers exert a continuous level of effort and consume only at the end of a finite horizon. Both HQ and division managers are uncertainty averse and, as in Chen and Epstein (2002), face stationary IID ambiguity.

HQ must design optimal incentive contracts for both division managers. Traditional principal-agent theory (see, for example, Holmström, 1979) suggests that, to limit harmful risk-exposure to

¹See Murphy (1999, 2013), Frydman and Jenter (2010) and Oyer and Schaefer (2011) for extensive surveys.

²An example of pay-for-performance compensation is a contract based on Economic Value Added, or EVA.

risk-averse division managers, incentive contracts should depend only on performance measures that are (directly or indirectly) informative on their actions. In addition, to reduce division managers' risk bearing, incentive contracts should hedge their risk by including a negative exposure to variables that are positively correlated to division cash-flow, and a positive exposure to variables that are negatively correlated. Negative exposure can be obtained through incentive contracts displaying a "relative-performance" component, and positive exposure can be obtained with "cross-pay," that is, for example, with broader equity-based incentive contracts.

These predictions change substantially in the presence of uncertainty aversion. We begin by showing that the aggregation and linearity property of Holmström and Milgrom (1987) hold in our environment with stationary uncertainty. Next, we argue that the presence of uncertainty aversion creates a divergence between division managers' and HQ beliefs. Such disagreement is endogenous, and has two adverse effects on HQ payoff. First, traditional incentive contracts, by loading primarily on division cash-flow, lead division managers to more conservative estimates of the productivity of their own division, with a negative effect on incentives to exert effort. More conservative beliefs are due to division managers' greater exposure to uncertainty on their division productivity than HQ, who instead have exposure to the overall firm. Second, the disagreement with HQ leads division managers to value compensation contracts at a discount with respect to the value attributed by the (more optimistic) HQ, increasing the cost of incentive provision. We identify this discount, due to uncertainty aversion, as the (Knightian) "disagreement discount."

We show that HQ can reduce the negative impact of disagreement in the organization by managing individual exposure to uncertainty through contracts, with a beneficial effect on the provision of incentives. The role of contracts in managing beliefs is a novel feature in the theory of contract design. This new role of contracts is a direct consequence of uncertainty aversion, and it depends on the property that the beliefs constellation in an organization, and thus the degree of internal agreement, is endogenous and it depends on each agent's exposure to uncertainty. Our paper shows that, in this situation, by proper design of incentive contracts, HQ can realign beliefs in the organization promoting a "shared view," with a positive impact on incentives.³

³The role of equity-based compensation to promote consensus in organizations is examined in Organization Behavior literature, such as Klein (1987), Pearsall, Christian, and Ellis (2010), and Blasi, Freeman, and Kruse (2016), among others. The importance of promoting a shared view in organizations is discussed in Zohar and Hofmann (2012). The impact of belief formation and the advantages and disadvantages of disagreement in organizations has been stud-

The key economic driver of our paper is that uncertainty-averse agents are (weakly) more optimistic when they are exposed to multiple sources of uncertainty, a feature known as uncertainty hedging.⁴ This happens because beliefs (in the sense of de Finetti, 1974) held by uncertainty-averse agents are not uniquely predetermined, as in a Bayesian framework but, rather, are the outcome of a minimization problem. By being exposed to multiple sources of uncertainty, uncertainty-averse agents can lower their exposure to each source of uncertainty and, thus, hold more optimistic beliefs overall. The benefits of uncertainty hedging are analogous, within uncertainty aversion, to the benefits of risk diversification within a traditional risk aversion framework. In our context, HQ can use the design of incentive contracts to affect managers' exposure to uncertainty and, in this way, to promote more favorable beliefs in the organization.

Optimal contracts depend on the level of uncertainty faced by division managers and HQ, and on the correlation between division cash-flows. We consider two leading configurations of beliefs. In the first configuration, HQ are uncertainty neutral and are confident on their assessment of the productivity of both divisions. We describe this situation as one of "visionary" leadership.⁵

With visionary HQ, optimal contracts depend on division managers' exposure to uncertainty and the sign of the correlation between division cash-flows. With low uncertainty, incentive contracts have the same qualitative features as with no uncertainty: they have a component that depends on the performance of a manager's division, the pay-for-performance part, plus a second component that depends on the cash-flow of the other division. When division are positively correlated, incentive contracts display relative performance, while when division cash-flows are negatively correlated, incentive contracts have cross-pay, that is have an equity component. The effect of uncertainty is to decrease pay-for-performance sensitivity and cross-division exposure.

When uncertainty faced by division managers is sufficiently large, uncertainty aversion creates the potential for a significant divergence between beliefs held by division managers and HQ. In this case, HQ find it desirable to offer division managers compensation contracts with greater exposure to the other division. The benefit of greater cross-division exposure is to align more closely division managers' expectations on future cash-flows with those held by the HQ. By providing cross-division

ied in several papers: Dessein and Santos (2006); Landier, Sraer, and Thesmar (2009); Bolton, Brunnermeier, and Veldkamp (2013); and Van den Steen (2005) and (2010).

⁴Uncertainty hedging is the direct effect of the "uncertainty aversion" axiom of Gilboa and Schmeidler (1989).

⁵The impact of leadership style on incentives is examined in Rotemberg and Saloner (1993) and (2000).

exposure, which hedges the division managers' exposure to uncertainty, HQ induce them to hold more favorable expectations on their divisions, with a positive impact on effort.

Improvement of division managers beliefs also lowers the disagreement discount and, thus, the cost of providing incentives. Interestingly, when division managers face large uncertainty, optimal contracts have cross-division exposure (with either cross-pay or relative performance) even in the case of uncorrelated cash-flows. This property is in sharp contrast with optimal incentive contracts in traditional principal-agent problems without uncertainty.

In the second leading configuration, HQ are uncertainty averse as well. Because in this case HQ beliefs about the two divisions' productivity are also determined endogenously, we will refer to this case as one of "pragmatic" leadership. This type of leadership is pragmatic in that HQ will adapt its beliefs and those beliefs it wishes to induce (in equilibrium) in the organization, endogenously, depending on firm characteristics.

HQ uncertainty aversion introduces an additional source of disagreement with the division managers that has the effect of making it costlier for HQ to offer incentive contracts with relative performance. This happens because compensation contracts with relative performance essentially involve division managers holding a "short" position in the other division, while HQ hold a "long" position in both divisions. This difference exacerbates the disagreement discounts, making it even costlier for HQ to offer incentive contracts with relative-performance features. The overall effect is to make contracts with cross-pay more desirable. Interestingly, we find that pure equity-based contracts are optimal when uncertainty is sufficiently large, irrespective of the correlation between divisional cash-flows.⁶ Similarly, our model suggests that compensation contracts for top executives that include relative-performance measures may not be optimal because they create disagreement between shareholders and company HQ.⁷

We argue that the relative importance of equity-based and pay-for-performance compensation depends on firms' characteristics: firms less affected by uncertainty have incentive contracts where the cross-division exposure has the opposite sign as the correlation between divisional cash-flows.

⁶DeMarzo and Kaniel (2017) argue that relative-performance compensation contracts may not be desirable when division managers have "keep-up-with-the-Joneses" features.

⁷Executive contracts without relative performance components rewards top managers for factors influenced by market forces outside their control rather than their own efforts ("pay-for-luck"). See Murphy (2013) for an extensive survey of the literature.

In contrast, firms more affected by uncertainty, such as, for example, young firms involved in innovative technologies, will be characterized by incentive contracts with positive cross-division exposure, that is with equity components. In addition, our model can explain the beneficial role of employee bonuses geared to the performance of the entire firm (or one of its larger subdivisions), rather than more narrowly defined performance measures.⁸

Our paper is linked to several streams of literature. The first one is the traditional principal-agent theory and the theory of optimal contracts design within organizations. Contract theory builds on the seminal work by Mirrlees (1975), (1999) and (1976), Holmström (1979) and Grossman and Hart (1983).⁹ One of the key results of the early stages of this literature is that optimal contracts can be thought as the solution of an inference problem where contractual compensation should be a function of all and only observable variables that are informative on the action selected by the agent. Incentive contracts more directly tailored to shareholder value, such as shareholder equity, are shown to be optimal when agents can choose their hidden action from rich sets of possible action-profiles (see, for example, Diamond, 1998, and Chassang, 2013).¹⁰

The second stream is the emerging literature on contract theory under uncertainty.¹¹ Lee and Rajan (2020) consider a model in the spirit of Innes (1990) and study the optimal incentive contract between a principal and an agent where both parties are uncertainty-averse. The source of uncertainty in the model is the exact probability distribution of the random cash-flow. The paper shows that, contrary to basic case of uncertainty-neutrality of Innes (1990), the optimal contract has equity-like components. Szydlowski (2019) considers a dynamic contracting model where an uncertainty-neutral principal designs an optimal dynamic contract for an uncertainty-averse agent, where the source of uncertainty is the agent's cost of effort. In that setting, uncertainty (specifically, the worst-case scenario) evolves over time depending on firm performance, inducing dynamic contracts with over- and under-compensation with respect to the case where the agent is

⁸For example, the compensation of mutual fund managers depends not only on the performance of their funds, but also on the performance of the entire family of funds, implying a positive cross-fund exposure. However, the majority of funds are exposed to shared macroeconomic risk, suggesting a positive correlation.

⁹See Hart and Holmström (1987) for a survey of the earlier literature on contract theory.

¹⁰See Prendergast (1999) for an extensive survey of the early literature.

¹¹A growing literature focuses on the impact of ambiguity aversion in finance, including Mukerji and Tallon (2001), Epstein and Schneider (2008), Easley and O'Hara (2009) and (2010), Caskey (2009), Bossaerts et al. (2010), Illeditsch (2011), Drechsler (2013), Jahan-Parker and Liu (2014), Mele and Sangiorgi (2015), Gallant, Jahan-Parvar and Liu (2019), Dimmock et al. (2016), Dicks and Fulghieri (2019) and (2020), and Lee and Rivera (2020). For a thorough literature review, see Epstein and Schneider (2010).

uncertainty-neutral. Miao and Rivera (2016) consider the optimal contract between uncertainty-averse principal and an uncertainty-neutral but risk-averse agent: they show that the principal's preference for robustness can cause the incentive-compatibility constraint to be lax.¹² The main feature of these papers is to consider principal-agents problems in isolation. In contrast, in our paper we consider the problem of incentive contracting within organizations, where the principal (company HQ) design contracts with multiple agents who are exposed to multiple sources of uncertainty.¹³

Closer to our paper, Carroll (2015) shows that a risk-neutral principal, who is uncertain about the set of actions available to a risk- and uncertainty-neutral agent, optimally grants the agent a linear contract that aligns their payoffs.¹⁴ The paper shows that linear (or affine) contracts are optimal robust contracts under very weak assumptions on the uncertainty characterizing the set of technologies available to the agent. Carroll and Meng (2016) provides a micro-foundation of uncertainty that results in optimal linear contracts. In the spirit of Holmström (1982), Dai and Toikka (2018) examines a moral hazard in teams problem, where a risk-neutral principal wishes to design contracts that are robust to uncertainty regarding the underlying game played by uncertainty neutral agents. The paper shows that optimal robust contracts must have the property that agents' compensation covaries positively, and provides conditions under which optimal robust contracts are linear (or affine). Finally, Walton and Carroll (2019) show that, under mild conditions, optimal contracts are linear within several possible configurations of the organization structure, again when principal are risk neutral and agents are risk- and uncertainty neutral.

Our paper differs from these in several important ways. A common theme of these papers is to show that linear (or affine) contracts emerge quite naturally as optimal robust contracts in situations where linearity would not be obtained in absence of uncertainty. In our paper we take the opposite tack, and we start from a situation similar to Holmström and Milgrom (1987), where optimal contracts are indeed linear absent uncertainty. We first show that the linearity property is preserved under stationary and IID uncertainty. We then characterize optimal linear contracts when principals are risk neutral, agents are risk averse, and they can both be uncertainty averse.

¹²Lee and Rivera (2020) consider optimal liquidity management under IID Ambiguity of Chen and Epstein (2002) without agency problems.

¹³An exception is Garlappi, Giammarino and Lazrak (2017), which shows that group decision-making by individuals with heterogeneous beliefs may generate decisions that have ambiguity-like features.

¹⁴In a similar spirit, Chassang (2013) shows robustness properties of linear contracts.

This approach allows us to isolate the specific effect of uncertainty aversion on optimal contracts. Our paper shows the key role played by uncertainty hedging in shaping optimal contracts, a feature that is absent in situations where agents are uncertainty neutral. Specifically, we show that, when agents are both risk- and uncertainty averse, hedging uncertainty exposure can interact in important ways with hedging risk exposure, and that the two goals can be in conflict with each other. When uncertainty is sufficiently large, the uncertainty-hedging motive can overcome the risk-hedging motive, reversing some of the properties of optimal incentive contracts absent uncertainty concerns.

Our paper is also related to the literature on the optimal employee compensation structure and, particularly, on the use of equity-based compensation. Oyer (2004) suggests that equity-based compensation (for example, through stock option plans) have the advantage of adjusting employees' compensation to their outside options, which may be correlated to firm value. Oyer and Schaefer (2005) document that broad-based stock option plans are more common at smaller and riskier firms, and argue that (in calibrations) option-based compensation provide weak incentives to middle-level managers. Rather, option plans seem to be more effective as tools to attract and retain more optimistic employees. Bergman and Jenter (2007) argue that firms adopt option-based compensation to attract (and under-pay) over-optimistic employees. Duchin et al. (2018) document that a change in industry pay in one division of a conglomerate generates spillovers on divisional managerial pay in other divisions of the same firm.

Other papers test the more traditional theory that equity-based compensation has a positive effect on incentives, resulting in better performance ex-post. For example, Hochberg and Lindsey (2010) argue that firms with option-plans that have higher implied incentives exhibit higher subsequent operating performance, a feature that is concentrated in firms with fewer employees and with higher growth opportunities. Baker, Jensen and Murphy (1988) suggests that non-executive option plans may induce cooperation among employees helping to overcome the free-rider problem implicit in large organizations, resulting in better firm performance. Our paper provides a novel explanation for the seemingly “cooperative” outcome due to equity-based compensation based on the better coordination of beliefs in the organization that is induced by such plans.

Our approach differs from theories where equity-based compensation is a way for cash constrained firms to raise “cheap” capital. Core and Guay (2001) argue that firms use stock option

compensation plans when facing greater capital requirements and stronger financing constraints. Guiso, Pistaferri and Schivardi (2013) argue that credit constrained firms offer lower initial wages, but offer a steeper wage progression over time. Kim and Ouimet (2014) argue that ESOP plans allows (financially constrained) firms to save cash by substituting wages with equity compensation, with a smaller effect on incentives (especially in larger firms) due to the free-rider problem.

Finally, our paper is related to recent literature on disagreement and heterogenous priors.¹⁵ In this paper, we develop a novel source of disagreement grounded in decision theory. We argue that the presence of uncertainty, and the aversion to it, generates differences of beliefs among agents, even in cases where agents are ex-ante identical and share the same set of “core beliefs.” Disagreement in our economy emerges endogenously as the consequence of agents’ different exposure to uncertainty, as generated by contractual relationships.¹⁶

The paper is organized as follows. Section 1 presents the model. Section 2 shows that the aggregation and linearity properties of Holmström and Milgrom (1987) hold with stationary and IID uncertainty. In Section 3 we study the impact of incentive contracts on beliefs and effort under uncertainty. Section 4 discusses the case of a visionary leadership. Section 5 examines the case of a pragmatic leadership. Section 6 presents empirical implications of our paper, and Section 7 concludes with directions for further ongoing research. All proofs are in the appendix.

1 Uncertainty and Contracting

1.1 The Basic Model

We consider a firm composed by two divisions: $d \in \{A, B\}$. Each division is run by a division manager who is supervised by HQ. At each point in time $t \in [0, 1]$ each division manager continually chooses a level of effort, $a_{d,t} \in \mathbb{R}_+$, which affects the probability distribution of divisional cash-flows. Specifically, we assume that the cash-flows of both divisions, $Y_t \equiv (Y_{A,t}, Y_{B,t})$, follow the (joint) process

$$dY_t = \mu_t dt + \Gamma dW_t, \tag{1}$$

¹⁵See, for example, Boot, Gopalan, and Thakor (2006) and (2008), Boot and Thakor (2011), and Bayer, Chemmanur, and Liu (2011).

¹⁶A similar approach was adopted in Dicks and Fulghieri (2015) in the context of corporate governance.

where $W_t = (W_{A,t}, W_{B,t}) \in \mathbb{R}^2$ is a standard bivariate Brownian motion defined on the filtered probability space, $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P^a)$, with $Y_{A,0} = Y_{B,0} = 0$. Note that (Y, W, P^a) is a weak solution to the stochastic differential equations in (1), and all processes are progressively measurable with respect to the filtration $(\mathcal{F}_t)_{t \geq 0}$. Following Chen and Epstein (2002), we will refer to P^a as representing the reference probability, which is assumed to be common for both division managers and HQ.¹⁷

We assume that division manager efforts affect only the drift of its own division with no externalities (or synergies) across divisions, and we set $\mu_t \equiv (\mu_{A,t}, \mu_{B,t})'$ with $\mu_{d,t} = a_{d,t}q_d$, where q_d represents the productivity of division $d \in \{A, B\}$ under the reference probability, P^a . We will refer to division managers' action profile as $a_t = (a_{A,t}, a_{B,t})'$. We assume that the division cash-flows are homoskedastic, with constant variance σ^2 , and that division cash-flows may be (positively or negatively) correlated, with correlation coefficient ρ . Thus, Γ is assumed to be the symmetric square root of the variance-covariance matrix, Σ . Thus,

$$\Sigma \equiv \Gamma' \Gamma = \begin{bmatrix} \sigma^2 & \rho \sigma^2 \\ \rho \sigma^2 & \sigma^2 \end{bmatrix} \quad \text{where } \Gamma \equiv \begin{bmatrix} \frac{\sigma}{2} (\sqrt{1+\rho} + \sqrt{1-\rho}) & \frac{\sigma}{2} (\sqrt{1+\rho} - \sqrt{1-\rho}) \\ \frac{\sigma}{2} (\sqrt{1+\rho} - \sqrt{1-\rho}) & \frac{\sigma}{2} (\sqrt{1+\rho} + \sqrt{1-\rho}) \end{bmatrix}. \quad (2)$$

Exerting effort is costly to division managers, and we assume that each division manager exerting effort level $a_{d,t}$ suffers an instantaneous monetary cost $c_d(a_{d,t}) dt$, where $c_d : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a continuous, increasing and convex function. For analytical tractability, we assume that $c_d(a_{d,t}) = \frac{1}{2Z_d} a_{d,t}^2$, where Z_d is a measure of division manager effort efficiency. Following Holmström and Milgrom (1987), we assume that both division managers and HQ exhibit preferences with constant absolute risk aversion (CARA), and that both division managers and HQ consume only at the end of the period, $t = 1$.¹⁸

Effort exerted by each division manager is not observable by either HQ or the other division manager, creating moral hazard. HQ promote effort by providing incentive contracts, $\{w_d\}_{d \in \{A, B\}}$, to division managers, as follows. We assume that output from each division, $Y_{d,t}$, is publicly

¹⁷In the context of robust control theory, Hansen et al. (2006) refer to the probability measure P^a as the “approximating model.”

¹⁸By restricting pay to occur only at the end, we avoid two complications: intertemporal consumption and private savings. These issues are examined, for example, in He et al. (2017) who study a dynamic agency problem but in a setting without Knightian uncertainty.

observable, and we let $h_t = \{Y_s | s \leq t\}$ represent the entire history of cash-flow from both divisions at each point in time t . HQ can condition compensation to each division manager on the entire history, that is $w_d(h_1)$. We impose the usual square-integrable condition that $E^{P^a} [w_d(h_1)]^2 < \infty$.

Given an incentive contract $w_d(h_1)$, and effort level process $a_d \equiv \{a_{d,t}\}_{t \in [0,1]}$, division manager $d \in \{A, B\}$ earns an end-of-period payoff

$$U_d(h^1) \equiv u \left(w_d(h^1) - \int_0^1 c_d(a_{d,t}) dt \right), \quad (3)$$

where $u(w) = -e^{-rw}$, and r represents the coefficient of absolute risk aversion for both divisional managers. Similarly, HQ earn end-of-period payoff equal to

$$\Pi(h^1) \equiv \pi(Y_{A,1} + Y_{B,1} - w_A(h_1) - w_B(h_1)), \quad (4)$$

where $\pi(X) = -e^{-RX}$, and R represents the coefficient of absolute risk aversion for company HQ. Because processes are in L^2 , they both have finite expectation.

The differential game unfolds as follows. At the beginning of the period, $t = 0$, HQ choose incentive contracts $w_d(h_1)$ for each division manager $d \in \{A, B\}$. We assume that HQ can commit to incentive contracts $\{w_d(h_1)\}_{d \in \{A, B\}}$, and that they are observable to both managers. After incentive contracts are offered and accepted, division managers continuously and simultaneously choose their level of effort exerted, $a_{d,t}$, after observing history h_t . At the end of the period, $t = 1$, division managers are compensated according to the realized history, h_1 , and consumption takes place.

1.2 Uncertainty aversion

We model uncertainty aversion by adopting the minimum expected utility (MEU) approach of Chen and Epstein (2002), a dynamic extension of Gilboa and Schmeidler (1989). We assume that both HQ and division managers are not sure about (i.e., they are ambiguous on) the probability measure P^a . Following Chen and Epstein (2002), we consider beliefs distortions that are mutually absolutely continuous measures with respect to P^a , allowing us to use Girsanov's Theorem. Define a density generator to be a \mathbb{R}^2 -valued \mathcal{F}_t -predictable process θ_t satisfying the Novikov condition,

$E^{P^a} \left[\exp \left(\frac{1}{2} \int_0^1 \theta_s \cdot \theta_s ds \right) \right] < \infty$, so that the process

$$z_t^\theta \equiv \exp \left\{ -\frac{1}{2} \int_0^t \theta_s \cdot \theta_s ds - \int_0^t \theta_s dW_s \right\} \quad (5)$$

is a (P^a, \mathcal{F}_t) martingale. By Girsanov's Theorem, θ_t generates an equivalent probability measure $\tilde{P}^{a,\theta}$ on (Ω, \mathcal{F}) such that

$$\frac{d\tilde{P}^{a,\theta}}{dP^a} \Big|_{\mathcal{F}_t} = z_t^\theta, \quad (6)$$

where z_t^θ is the Radon-Nikodym derivative of $\tilde{P}^{a,\theta}$ with respect to P^a when restricted to \mathcal{F}_t . Note that, from Girsanov's Theorem, the process W_t^θ defined as

$$W_t^\theta = W_t + \int_0^t \theta_s ds \quad \text{or as} \quad dW_t^\theta = dW_t + \theta_t \quad (7)$$

is a standard Brownian motion under the new measure $\tilde{P}^{a,\theta}$. Thus, under the measure $\tilde{P}^{a,\theta}$, divisional cash-flows Y^θ follow the process

$$dY^\theta = Qa_t dt + \Gamma \left(dW_t^\theta - \theta_t dt \right) = \mu^\theta(a_t) dt + \Gamma dW_t^\theta, \quad (8)$$

where

$$Q \equiv \begin{bmatrix} q_A & 0 \\ 0 & q_B \end{bmatrix} \quad \text{and} \quad \mu^\theta(a_t) \equiv Qa_t - \Gamma\theta_t, \quad (9)$$

and the new probability $\tilde{P}^{a,\theta}$ is a new measure reflecting a distortion of the cash-flow processes. The values q_d , for $d \in \{A, B\}$, represent the division productivity assessment that are implied by the reference probability P^a . Thus, from (8) and (9), the density generator process θ_t can be interpreted as describing decision makers' beliefs on the instantaneous productivity of both divisions.

Following Chen and Epstein (2002), we assume that uncertainty is IID. We allow for the possibility that HQ and division managers may be exposed to different degrees of uncertainty, as follows. For division manager $d \in \{A, B\}$, we let $\theta_{d,t} \in K_d(a_t)$, for all $t \in [0, 1]$, where $K_d \in \mathbb{R}^2$ is set-continuous (both upper- and lower-hemicontinuous) with $K_d(a_t)$ a convex set for all a_t . Similarly, for HQ we let $\theta_{HQ,t} \in K_{HQ}(a_t)$, for all $t \in [0, 1]$, where $K_{HQ} \in \mathbb{R}^2$ is also set-continuous (both upper-

and lower-hemicontinuous) with $K_{HQ}(a_t)$ a convex for all a_t . Let $\mathcal{P}_d^\Theta(a_t) = \{\tilde{P}^{a,\theta} | \theta_t \in K_d(a_t), \forall t\}$ for $d \in \{A, B\}$, and $\mathcal{P}_{HQ}^\Theta(a_t) = \{\tilde{P}^{a,\theta} | \theta_t \in K_{HQ}(a_t), \forall t\}$ be the set of admissible priors for division managers and company HQ, respectively. At times, we will assume that divisions are symmetric, that is, we set

$$(S) : \quad Z_A = Z_B \equiv Z, \quad q_A = q_B \equiv q, \quad \text{and} \quad K_A(a_t) = K_B(a_t) \equiv K(a_t). \quad (10)$$

Note that IID uncertainty can be interpreted as nature independently drawing increments $\{dW_t^{a,\theta} : t \in [0, 1]\}$ of the process $W_t^{a,\theta}$ from different urns at each point in time. These assumptions imply that a division's past cash-flow realizations are not informative on future cash-flows for either division, thus excluding learning (similar to Chen and Epstein, 2002). Importantly, they ensure that divisional managers and HQ face stationary uncertainty over time. Note that the core of beliefs is rectangular over time, as required for time consistency by Chen and Epstein (2002). This is because the set of priors considered does not vary over time. However, it is important to note that the set K may not be a rectangle (that is, a square box). Indeed, similar to Equation (3.12) of Chen and Epstein (2002), we will consider strictly convex, "round" sets K . The key feature of this approach will allow us to have time-invariant uncertainty hedging.

2 Aggregation and Linearity under Uncertainty

At the beginning of the game, $t = 0$, HQ offer to division managers a pair of incentive contracts, $w(h_1) \equiv \{w_d(h_1)\}_{d \in \{A, B\}}$, and a set of (history dependent) instructions $a \equiv \{a_d\}_{d \in \{A, B\}}$ to maximize expected payoff, that is to solve

$$\max_{\{w, a\}} \min_{\tilde{P} \in \mathcal{P}_{HQ}^\Theta(\{a_d, a_{d'}\})} E^{\tilde{P}} \pi(Y_{A,1} + Y_{B,1} - w_A(h_1) - w_B(h_1)) \quad (11)$$

subject to the constraint that each division managers choose an effort process, a_d , given the other division manager's action profile, to solve

$$\max_{\tilde{a}_d} \min_{\tilde{P} \in \mathcal{P}_d^\Theta(\{\tilde{a}_d, a_{d'}\})} E_t^{\tilde{P}} u \left(w_d(h^1) - \int_0^1 c_d(\tilde{a}_d, t) dt \right), \quad (12)$$

and satisfies their participation constraints

$$\min_{\tilde{P}_d^{\Theta}(\{a_d, a_{d'}\})} E_0^{\tilde{P}} u \left(w_d(h^1) - \int_0^1 c_d(a_{d,t}) dt \right) \geq u_0 = 0 \quad (13)$$

for $d \in \{A, B\}$, where u_0 is a division manager's reservation utility, which is normalized to zero (without loss of generality).

Definition 1 *An equilibrium is a set of contracts, $w(h_1) \equiv \{w_d(h_1)\}_{d \in \{A, B\}}$, and action processes $\{a_A, a_B\}$, such that:*

- (i) *Given incentive contracts $w(h_1)$, for every history h_t each division manager selects effort, a_d , optimally, solving (12), given the other division manager's action process, $a_{d'}$ for $d' \neq d$;*
- (ii) *HQ offer an optimal contract $w(h_1)$ that maximizes (11) subject to (12) and (13).*

The following theorem establishes that the aggregation and linearity property of Holmström and Milgrom (1987), with a single principal and agent, also holds in the case of stationary (IID) uncertainty with two division managers.

Theorem 1 *The optimal contract between HQ and division managers is linear in cash-flows, $w_d(h_1) = s_d + \beta_d Y_{d,1} + \gamma_d Y_{d',1}$, with constant s_d, β_d, γ_d . The optimal contract induces constant effort, $a_{d,t} = a_d$, and constant beliefs, $\tilde{P}^{a,\theta}$, with constant distortions, $\theta_{d,t} = \theta_d$ and $\theta_{HQ,t} = \theta_{HQ}$, for all t .*

Similar to Holmström and Milgrom (1987), the property of translation invariance displayed by CARA utility functions (which leads to absence of wealth effects) and IID uncertainty ensure that both HQ and division managers face the same instantaneous optimization problem at every point in the tree. In the optimal contract, HQ grant division manager d a constant share β_d of his own division, a constant share γ_d of the other division, and induces division manager d to implement constant effort, $a_d = a_{d,t}$ for all $t \in [0, 1]$. In addition, both division managers and HQ hold constant belief distortions, (θ_d, θ_{HQ}) , for all t .

The coefficient β_d determines the strength of the pay-for-performance component of the incentive contract (that is, its sensitivity to performance), while the coefficient $|\gamma_d|$ determines its

cross-division exposure (that is, the breadth of the performance measure at the base of the incentive contracts). A smaller value of $|\gamma_d|$ leads to compensation that is more narrowly based on the performance of a division manager's own division. A greater value of $|\gamma_d|$ broadens the base of compensation, increasing the relatedness of division managers' compensations. For $\gamma_d = 0$, division managers' compensation depends only on the performance of their own division, a feature that we denote as "pay-for-performance." For $\gamma_d > 0$ compensation depends positively also on the performance of the other division, $d' \neq d$, a feature that we denote as "cross pay." In this case, overall pay is a combination of an equity-based component and a pay-for-performance component. Specifically, if $\beta_d \leq \gamma_d$ (which will be the case in optimal contracts in our economy) we can interpret $\gamma_d(Y_{d,1} + Y_{d',1})$ as the "equity-based" component, and $(\beta_d - \gamma_d)Y_{d,1}$ as the "pay-for-performance" component of compensation. For $\gamma_d = \beta_d$, we have $w_d = \beta_d(Y_{d,1} + Y_{d',1})$, and division managers' compensation depends entirely on the performance of the whole firm.¹⁹ Finally, setting $\gamma_d < 0$ corresponds to a compensation structure with a relative-performance evaluation. In this case, compensation can be decomposed as

$$w_d = s_d + (\beta_d + \gamma_d)Y_{d,1} - \gamma_d(Y_{d,1} - Y_{d',1}), \quad (14)$$

where the first term represents the "adjusted" pay-for-performance sensitivity component, and the second term represents the additional relative-performance component.

Given the CARA specification and the Brownian framework, Theorem 1 shows that the solution to the dynamic model is equivalent to the solution of a corresponding static problem where HQ offer only affine contracts that depend on divisional cash-flows. The static problem that corresponds to the dynamic model can be written in certainty equivalent form, as follows. Letting $b_A \equiv (\beta_A, \gamma_A)'$, $b_B \equiv (\gamma_B, \beta_B)'$, $b_{HQ} \equiv (\mathbf{1} - b_A - b_B)'$, where $\mathbf{1} = (1, 1)'$, HQ choose a pair of incentive contracts and action profiles, $\{w_d, a_d\}_{d \in \{A, B\}}$, that maximize their (instantaneous) certainty equivalent objective function, given by

$$\pi \equiv \min_{\theta^{HQ} \in K_{HQ}(a)} b'_{HQ} \mu^{\theta^{HQ}}(a) - \frac{R}{2} b'_{HQ} \Sigma b_{HQ} - s_A - s_B, \quad (15)$$

¹⁹Note β would be like b^{II} of Edmans, Gabaix, and Landier (2009), but with the incentive base chosen by headquarters, rather than the whole firm.

subject to the constraint that division managers maximize the certainty equivalent of their objective function

$$\max_{a_d} \min_{\theta \in K_d(a)} u_d^\theta \equiv s_d + b'_d \mu^{\theta_d}(a) - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d), \quad (16)$$

and the individual rationality constraints

$$\min_{\theta_d \in K_d(a)} s_d + b'_d \mu^{\theta_d}(a) - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d) \geq 0 \quad (17)$$

for $d \in \{A, B\}$. Note that, absent uncertainty, $K_{HQ}(a) = K_d(a) = \{0\}$ and problem (15) - (17) collapses to the corresponding static problem of Holmström and Milgrom (1987). Note that $\tilde{P} \in P_d^\theta(\{a_d, a_{d'}\})$ if and only if there is a θ such that $\tilde{P} = \tilde{P}^{a, \theta}$ and $\theta_t \in K_d(a_t)$ for all t . Further, Theorem 1 shows that the optimal contract implements constant effort, a_t , and constant distortions θ_t , so is sufficient to consider $\theta \in K_d(a)$.

The main trade-offs faced by HQ in problem (15) - (17) can be seen as follows. Because of translation invariance of CARA utility, the fixed component of incentive contracts, s_d , can be chosen to make the participation constraint (17) bind in optimal contracts. Therefore, we can substitute this constraint into the objective function, obtaining

$$\pi = \mathbf{1}' \mu^{\hat{\theta}_{HQ}}(a) - \frac{R}{2} b'_{HQ} \Sigma b_{HQ} - \sum_{d \in \{A, B\}} \left[\frac{r}{2} b'_d \Sigma b_d + c_d(a_d) + b'_d \left(\mu^{\hat{\theta}_{HQ}}(a) - \mu^{\hat{\theta}_d}(a) \right) \right], \quad (18)$$

where $\hat{\theta}_{HQ}$ and $\hat{\theta}_d$ are, respectively, the worst-case scenario for HQ and division manager d , for $d \in \{A, B\}$, given any incentive contract. HQ payoff consists of four components. The first one is the expected value of the two divisions, $\mathbf{1}' \mu^{\hat{\theta}_{HQ}}(a)$, which depends on effort exerted by the two division managers; the second one is the sum of the required risk-premia for HQ and division managers, $\frac{R}{2} b'_{HQ} \Sigma b_{HQ}$ and $\frac{r}{2} b'_d \Sigma b_d$; the third component, $c_d(a_d)$, is the cost of providing effort by division managers. These components are common to the traditional problem without uncertainty aversion.

The presence of uncertainty aversion affects HQ in two ways. First, from the incentive constraint (16), effort exerted by division managers depends on their worst case scenario, $\hat{\theta}_d$. Because division

managers beliefs are endogenous, and they depend on their overall exposure to uncertainty, which is affected by the incentive contracts offered by company HQ. Thus, by appropriate choice of incentive contracts, by hedging a division manager uncertainty, HQ can affect her expectations of her division productivity, $\mu^{\hat{\theta}_d}(a)$, thus affecting the incentives to exert effort. The beneficial effect of uncertainty hedging through incentive contract design on division managers beliefs, and thus effort, is new and it represents a key driver of our paper.

The second effect of uncertainty aversion is to create a divergence between HQ and division managers on the valuation of the compensation contracts, $b'_d \left[\mu^{\hat{\theta}_{HQ}}(a) - \mu^{\hat{\theta}_d}(a) \right]$, which represented the fourth component in (18). This term reflects the fact that HQ value compensation contracts at its worst-case scenario, $\hat{\theta}_{HQ}$, while division managers value contracts at their worst case scenarios, $\hat{\theta}_d$, creating disagreement. In particular, if HQ are more optimistic than division managers on division productivity, $\mu^{\hat{\theta}_{HQ}}(a) > \mu^{\hat{\theta}_d}(a)$, division managers discount the value of their compensation contracts, relative to the HQ valuation, making it more costly to satisfy their participation constraint (17). This additional cost of incentive-based pay is new, and it represents the (Knightian) cost of disagreement.

3 Uncertainty and Incentive Contracts

The previous section established the aggregation and linearity of incentive contracts under uncertainty. In its generality, however, Theorem 1 precludes derivation of closed form expressions for the optimal contracts. We derive explicit solutions to the optimal contracting problem by assuming that HQ are risk neutral, $R = 0$, and by introducing a parametric specification of the core-beliefs sets $K_\delta(a)$, for $\delta \in \{HQ, A, B\}$. We consider two cases: first when HQ are uncertainty neutral and then when HQ are uncertainty averse.

As a benchmark, we start by characterizing the solution to the optimal contracting problem for our two-division firm without uncertainty, a setting similar to Holmström and Milgrom (1987).

3.1 The No-Uncertainty Benchmark

Absent uncertainty concerns, $K_{HQ}(a_t) = K_d(a_t) = \{0\}$, HQ and division managers share the same beliefs, and agree on the probability measure P^a , which we will denote as the “reference probability.” Optimal incentive contract with no uncertainty is characterized in the following.

Theorem 2 (*Holmström and Milgrom*) *Let HQ be risk neutral, $R = 0$. Optimal contracts are linear functions of the end-of-period cash-flows of both divisions: $w_d(h_1) = s_d + \beta_d Y_{d,1} + \gamma_d Y_{d',1}$, with*

$$\beta_d = \frac{Z_d q_d^2}{Z_d q_d^2 + r\sigma^2(1 - \rho^2)}, \quad \gamma_d = -\rho\beta_d, \quad (19)$$

and induce stationary optimal effort:

$$a_d = \beta_d Z_d q_d = \frac{Z_d^2 q_d^3}{Z_d q_d^2 + r\sigma^2(1 - \rho^2)}, \quad (20)$$

for all t and $d \in \{A, B\}$.

As in Holmström and Milgrom (1987), the optimal contract is linear in the end-of-period cash-flow of both divisions, where the coefficient β_d represents the strength of the pay-for performance incentives and γ_d represents the breadth of the performance measure. Optimal contracts $w(h_1)$ depends on the correlation between divisional cash-flows, as follows. When division cash-flows are uncorrelated, $\rho = 0$, optimal contracts for division managers are linear only in their own divisional cash-flow, $\beta_d > 0$, with no exposure to the other division performance, $\gamma_d = 0$. This feature reflects the property that it is never optimal to give risk-averse agents exposure to random variables that are not informative on their own effort (Holmström, 1979).

If division cash-flows are (positively or negatively) correlated, cash-flows from both divisions are informative on each divisional manager effort, and it is optimal for HQ to make incentive contract contingent on performance from both divisions, as follows. If division cash-flows are positively correlated, $\rho > 0$, HQ set $\gamma_d < 0$ and incentive contracts display “relative-performance” compensation. Intuitively, poor performance by both divisions, for example, is likely to signify to HQ “bad luck” rather than shirking by division managers, attenuating the negative impact on compensation, thus reducing their risk exposure. Alternatively, if division cash-flows are negatively

correlated, $\rho < 0$, HQ set $\gamma_d > 0$ and incentive contracts display “cross-pay.” Intuitively, when cash-flows are negatively correlated, cross pay allows HQ to reduce division managers’ risk exposure by “diversifying” their compensation structure over performance of both divisions. In both cases, as the correlation between cash-flows increase, HQ can increase pay-for-performance sensitivity β_d , improving efficiency.

The difference of our paper with Holmström-Milgrom (1987) is that in problem (15) - (17) HQ and both division managers perceive divisions’ productivity as uncertain. The impact of uncertainty on the structure of optimal contracts is examined next.

3.2 Incentive contracts and beliefs

A key property of the minimum expected utility (MEU) approach of Gilboa and Schmeidler (1989) is that decision makers assess future uncertain prospects by using probability measures that minimizes their expected utility, given the set of priors characterizing their level of confidence. Thus, probabilistic beliefs held by both HQ and division managers are endogenous, and depend on their overall exposure to the sources of uncertainty. The key property that we exploit in this paper is that division managers may become relatively more optimistic about the prospects of their own division if they have exposure also to the other division’s source of uncertainty, a feature that is the direct of consequence of uncertainty hedging. This means that, by optimal design of incentive contracts, HQ can affect the probability measure used by division managers to assess the productivity of their division, and thus mitigate the adverse effect of uncertainty on effort.

Our results hold when the sets $K_\delta(a_t)$, with $\delta \in \{HQ, A, B\}$, are strictly convex sets with smooth boundaries, and where beliefs distortions are in proportion to effort. For tractability, however, and to generate closed-form solutions, we assume that both HQ and division managers consider deviations, \mathcal{P}_d^θ , that are in a neighborhood of the reference probability, P^a , and we assume that

$$K_\delta(a_t) = \left\{ \theta_t \mid \sup_{t \in [0,1]} \left[-\ln \left(1 - \frac{|D\theta_{A,t} + N\theta_{B,t}|}{a_A q_A} \right) - \ln \left(1 - \frac{|N\theta_{A,t} + D\theta_{B,t}|}{a_B q_B} \right) \right] \leq \kappa_\delta \right\} \quad (21)$$

for $\delta \in \{HQ, A, B\}$, where, from (2), we have that

$$D \equiv \frac{\sigma}{2} \left(\sqrt{1+\rho} + \sqrt{1-\rho} \right), \quad N \equiv \frac{\sigma}{2} \left(\sqrt{1+\rho} - \sqrt{1-\rho} \right), \quad (22)$$

and where κ_δ measures the degree of uncertainty faced by HQ and division managers, respectively. The parameter κ_δ depends on the degree of confidence of agent δ , for $\delta \in \{HQ, A, B\}$, where $\kappa_\delta = 0$ indicates full confidence, and increasing levels of uncertainty are characterized by larger values of κ_δ . If division are symmetric, we set $\kappa_A = \kappa_B \equiv \kappa$. Note that in the special case of uncorrelated cash-flows, $\rho = 0$, (21) simplifies to

$$K_\delta(a_t) = \left\{ \theta_t \sup_{t \in [0,1]} \left[-\ln \left(1 - \frac{\sigma |\theta_{A,t}|}{a_A q_A} \right) - \ln \left(1 - \frac{\sigma |\theta_{B,t}|}{a_B q_B} \right) \right] \leq \kappa_\delta \right\}. \quad (23)$$

Note these expressions give a special case of IID uncertainty as in Chen and Epstein (2002), because they are not history dependent.²⁰

An important implication of the specification of the core beliefs set based on (21) and (22) is that decision makers behave as if divisional productivity, q_d , is uncertain. This property can be seen as follows. Recall that division managers beliefs are determined by minimizing their objective function u_d^θ , from (16), where the cash-flow process Y has drift $\mu^\theta(a_t) \equiv Qa_t - \Gamma\theta_t$. Consider the alternative representation of a division manager objective function

$$\hat{u}_d \equiv s_d + b'_d \hat{Q}^d a - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d), \quad (24)$$

with

$$\hat{Q}^d \equiv \begin{bmatrix} \hat{q}_A^d & 0 \\ 0 & \hat{q}_B^d \end{bmatrix}, \quad (25)$$

where \hat{q}_d^d represents the belief of division manager d on the productivity of division $d' = \{A, B\}$, where the drift of the cash-flow process can be expressed as $\mu^\theta(a_t) \equiv \hat{Q}^d a_t$.

²⁰See Chen and Epstein (2002), Section 3.4. An important difference from their specification is that we allow the source-dependent ambiguity aversion to be scaled by effort, a_d .

Lemma 1 *The following two problems are equivalent:*

$$\min_{\theta \in K_d(a)} u_d^\theta = \min_{\hat{q}^d \in C_d} \hat{u}_d, \quad (26)$$

where $\hat{q}^d \equiv (\hat{q}_A^d, \hat{q}_B^d)$ and

$$C_d \equiv \left\{ \vec{q} \mid \left(1 - \frac{|\hat{q}_A - q_A|}{q_A} \right) \left(1 - \frac{|\hat{q}_B - q_B|}{q_B} \right) \geq e^{-\kappa_d} \right\}. \quad (27)$$

Lemma 1 implies that problem in the LHS of (26), where the core belief set is characterized by (21), and problem in the RHS of (26), where the core belief set is characterized by (27), are equivalent. This implies that the characterization of uncertainty with density generators in (7) can be interpreted as modeling uncertainty on productivity of both divisions, as specified in (24). The advantage of focusing on the latter problem is that the form for distance of probability measures provided by C_d in (27) allows us to obtain closed form solutions for optimal contracts under uncertainty. Figure 1 (on page 45) provides a numerical example of the core belief set (27).

Division managers' beliefs of the productivity of both divisions in the firm depend on the incentive contracts offered by HQ. From (24), given incentive contract $w(h_1) = \{w_d(h_1)\}_{d \in \{A, B\}}$, division managers beliefs, $\{\hat{q}_d^d, \hat{q}_{d'}^d\}$, are obtained as

$$\begin{aligned} \hat{q}^d(a, w) &= \arg \min_{\hat{q}_d} \hat{u}_d = s_d + b'_d \hat{Q}^d a - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d), \\ \text{s.t.} \quad &\left(1 - \frac{|\hat{q}_A - q_A|}{q_A} \right) \left(1 - \frac{|\hat{q}_B - q_B|}{q_B} \right) \geq e^{-\kappa_d} q_d q_{d'} \end{aligned}$$

and are characterized in the following lemma.

Lemma 2 *Let $\beta_d > 0$ and*

$$H_d \equiv \frac{\gamma_d a_{d'} q_{d'}}{\beta_d a_d q_d}. \quad (28)$$

A division manager's assessment of the productivity of both divisions, $\hat{q}_d \equiv \{\hat{q}_d^d, \hat{q}_{d'}^d\}$, is equal to:

- i) $\hat{q}_d^d = q_d$, and $\hat{q}_{d'}^d = e^{-\kappa_d} q_{d'}$ for $H_d \geq e^{\kappa_d}$
- ii) $\hat{q}_d^d = (e^{-\kappa_d} H)^{\frac{1}{2}} q_d$ and $\hat{q}_{d'}^d = (e^{-\kappa_d} \frac{1}{H})^{\frac{1}{2}} q_{d'}$ for $H_d \in (e^{-\kappa_d}, e^{\kappa_d})$
- iii) $\hat{q}_d^d = e^{-\kappa_d} q_d$ and $\hat{q}_{d'}^d = q_{d'}$ for $H_d \in [-e^{-\kappa_d}, e^{-\kappa_d}]$
- iv) $\hat{q}_d^d = (e^{-\kappa_d} |H|)^{\frac{1}{2}} q_d$ and $\hat{q}_{d'}^d = \left[2 - \left(e^{-\kappa_d} \frac{1}{|H|} \right)^{\frac{1}{2}} \right] q_{d'}$ for $H_d \in (-e^{\kappa_d}, -e^{-\kappa_d})$
- v) $\hat{q}_d^d = q_d$ and $\hat{q}_{d'}^d = (2 - e^{-\kappa_d}) q_{d'}$ for $H_d \leq -e^{\kappa_d}$

Lemma 2 shows that the probability assessment, or beliefs, that division managers hold toward the each division productivity, \hat{q}_d , is endogenous and it depends on the relative exposure to cash-flow from each division, measured by H_d , which is determined by their incentive contract w_d . Note that every incentive contract offered by HQ must have $\beta_d > 0$, otherwise a division manager will exert no effort; in addition, note that $sign(H_d) = sign(\gamma_d)$ and that H_d is an increasing function of γ_d .

Several important features emerge from Lemma 2. First, if HQ grant to division managers pay-for-performance only, that is $\gamma_d = 0 = H_d$, or a small exposure to the other division cash-flow, as in case (iii), division managers will assess the prospects of their own division very conservatively, and set $\hat{q}_d^d = e^{-\kappa} q_d$. In this case, division managers will be rather pessimistic on the productivity of their own division, with a depressing effect on effort.

Division manager assessments of productivity of their own division, \hat{q}_d^d is however an increasing function of their exposure to the other division's, $|\gamma_d|$. Thus, incentive contracts that offer progressively increasing exposure to other division, greater $|\gamma_d|$ as in case (ii) and (iv), will induce division managers to become more optimistic on their own division, \hat{q}_d^d . Finally, if an incentive contract offers a substantial increase of the exposure to other division, large $|\gamma_d|$ as in case (i) and (v), the division manager will become very optimistic on their own division, setting $\hat{q}_d^d = q_d$. Note that the beneficial effect on division manager beliefs can be obtained by either giving a division manager cross-pay, $\gamma_d > 0$, or with relative-performance compensation, $\gamma_d < 0$.

The impact of $|\gamma_d|$ on a division manager's assessment of the productivity of the other division depends on the sign of γ_d . If the incentive contract includes cross-pay, $\gamma_d > 0$, increasing exposure to the other division progressively worsens the assessment of that division productivity, as in cases (ii) and (i). If the incentive contract includes relative performance, $\gamma_d < 0$, increasing exposure to the other division (that is, a lower γ_d) progressively improves the assessment of its productivity,

as in cases (iv) and (v), where in both cases $\hat{q}_d^d > q_{d'}^d$. The more optimistic assessment reflects the fact that, when $\gamma_d < 0$, better performance in the other division reduces a division manager's compensation, which is a concern to the division manager. In these cases, division managers will have an overly optimistic assessment of the other division, relative to theirs, generating "envy" in the organization, a possibly undesirable feature.

Lemma 2 shows a new key benefit of granting cross-pay or relative performance that is due to uncertainty aversion: it makes division managers more confident about their division's prospects, with a beneficial effect on effort. This property is the consequence of uncertainty hedging, and it reflects the fact that increasing the exposure to the other division uncertainty will make division managers less concerned about the uncertainty in their own division and, thus, more "optimistic" about it. Thus, HQ can use incentive contracts to affect beliefs in the organization, with a beneficial effect on effort and overall economic efficiency.

3.3 Incentive contracts and effort

We now determine equilibrium effort exerted by division managers. Given division managers' beliefs formation, characterized in Lemma (2), optimal division manager's effort is determined by

$$a_d(w) = \arg \max_{a_d} \hat{u}_d(a, \hat{q}_d^d(a, w)) = s_d + b'_d \hat{Q}^d a - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d), \quad (29)$$

for $d \in \{A, B\}$. By direct inspection of (29), it can immediately be seen that optimal action by division managers satisfies

$$a_d = Z_d \beta_d \hat{q}_d^d. \quad (30)$$

This means that division managers exert more effort when they are more optimistic about their division (greater \hat{q}_d^d). In addition, exposure to the other division payoff, $|\gamma_d| > 0$, makes effort exerted by a division manager, a_d , an increasing function of effort exerted by the other division manager, $a_{d'}$, creating a (beneficial) strategic complementarity between division managers. This complementarity reflects the fact that greater effort from the other division decreases the relative exposure of a division manager to uncertainty on her own division, leading to more favorable beliefs and, thus, greater effort.

Given incentive contracts $\{w_d = (\beta_d, \gamma_d)\}_{d \in \{A, B\}}$ for the two division managers, the Nash equilibrium of the dynamic effort selection by the two managers is characterized in the following.

Lemma 3 *Given a pair of incentive contracts, $\{w_d = (\beta_d, \gamma_d)\}_{d \in \{A, B\}}$, there is a unique Nash equilibrium level of effort exerted by each division manager, $\{a_d^*\}_{d \in \{A, B\}}$. The equilibrium effort level, a_d^* , is increasing in the pay-performance sensitivity, β_d , the exposure to the other division, $|\gamma_d|$, the efficiency of effort, Z_d , and decreasing in uncertainty κ_d . Furthermore, if $H_d \in (e^{-\kappa_d}, e^{\kappa_d})$, equilibrium effort a_d^* is also increasing in the other division's contract $\{\beta_{d'}, |\gamma_{d'}|\}$ and efficiency, $Z_{d'}$, and decreasing in $\kappa_{d'}$.*

Lemma 3 provides one of the key drivers of our paper. It implies that incentive contracts affect a division manager effort through two distinct channels. The first one is the traditional effect of inducing effort by rewarding division managers on the basis of (appropriate) measures of performance. The second channel is through the impact of incentive contracts on managerial assessment of the success probability of their projects. Specifically, incentive contracts can be used by HQ to lead uncertainty-averse division managers to hold more favorable assessment of the productivity of their division, thus generating a positive effect on effort. This is a new channel and it provides the key driver of our paper.

If division managers are uncertainty neutral, their optimal level of effort is (20): a division manager's effort, a_d , is an increasing function of his own division-based pay, β_d , but is affected by neither their cross-division pay, γ_d , nor the action of the other division manager, $a_{d'}$. If division cash-flows are correlated, the only benefit of linking the compensation of one division manager to the performance of the other division (through either cross-pay, $\gamma_d > 0$, when cash-flows are negatively correlated, or relative-performance measures, $\gamma_d < 0$, when cash-flows are positively correlated) is to decrease risk exposure, thus reducing the required risk premium. The reduction of risk-premia allows HQ to increase the incentive pay component, β_d , and thus to improve incentives.

The presence of uncertainty aversion introduces a link across division managers' optimal effort levels, driven by beliefs. This happens because beliefs held by uncertainty-averse division managers on the productivity of their division are endogenous, as characterized in Lemma 2. A division manager is more confident about his division if he is also granted pay that depends on the payoff

from the other division's, through cross-pay. The presence of cross pay makes a division manager's beliefs an increasing function of both the effort level and the productivity of the other division. This means that effort levels by division managers are strategic complements.

4 Visionary Leadership

We consider the ex-ante problem faced by HQ in selecting the optimal incentive pay for the two divisions. We start with the case in which HQ are uncertainty neutral and holds a belief $\hat{q}_d^{HQ} = q_d$ for both divisions. In contrast to Theorem 2, division managers are uncertainty averse. Because HQ hold firm beliefs $q = (q_A, q_B)$ on the productivity of the two divisions, and which are not affected by uncertainty, we will denote this case as one of "visionary leadership."

Given Lemma 1, with uncertainty-neutral (and risk neutral) HQ and uncertainty-averse division managers, problem (11) - (13) become

$$\max_{\{w_d, a_d\}_{d \in \{A, B\}}} (\mathbf{1} - b_A - b_B)' Qa - s_A - s_B \quad (31)$$

subject to the incentive and participation constraints

$$\max_{a_d} \min_{\hat{q}_d} \hat{u}_d = s_d + b'_d \hat{Q}^d a - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d), \quad (32)$$

$$\min_{\hat{q}_d} s_d + b'_d \hat{Q}^d a - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d) \geq 0 \quad (33)$$

for the division managers $d \in \{A, B\}$. The optimal contract is characterized in the following theorem.

Theorem 3 *There is a critical threshold $\bar{\kappa}_d(r, \rho)$, defined in Appendix, such that:*

(i) *If $\kappa_d \leq \bar{\kappa}_d$, HQ offer contracts with exposures to both divisions, setting*

$$\beta_d = \frac{e^{-\kappa} Z_d q_d^2}{[(2 - e^{-\kappa}) e^{-\kappa} Z_d q_d^2 + r \sigma^2 (1 - \rho^2)]} > 0 \quad \text{and} \quad \gamma_d = -\rho \beta_d, \quad (34)$$

inducing division managers beliefs $\hat{q}_d^d = e^{-\kappa} q_d$ and $\hat{q}_d^d = q_d$, and a Nash equilibrium effort levels

$$a_d = \beta_d Z_d e^{-\kappa} q_d = \frac{e^{-2\kappa} Z_d^2 q_d^3}{[(2 - e^{-\kappa}) e^{-\kappa} Z_d q_d^2 + r \sigma^2 (1 - \rho^2)]}, \quad (35)$$

for $d \in \{A, B\}$. Pay-for-performance sensitivity, β_d , and effort, a_d , are both decreasing in uncertainty, κ_d . In addition $\bar{\kappa}_d(r, \rho)$ is increasing in r and in $|\rho|$.

(ii) If $\kappa_d > \bar{\kappa}_d$, HQ offer contracts with exposures to both divisions: $\beta_d > 0$ and $|\gamma_d| > 0$. If divisions are symmetric, and condition (S) holds, optimal incentive contract is

$$\begin{aligned}\beta_d &= \frac{Ze^{-\frac{\kappa}{2}}q^2}{Ze^{-\frac{\kappa}{2}}q^2\left(4 - 3e^{-\frac{\kappa}{2}}\right) + 2r\sigma^2(1 - |\rho|)}, \\ |\gamma_d| &= \beta, \text{ with } \text{sign}(\gamma_d) = -\text{sign}(\rho),\end{aligned}$$

for $d \in \{A, B\}$. Optimal incentive contracts induce division managers beliefs on their own division $\hat{q}_d^d = e^{-\frac{\kappa}{2}}q_d < q_d$ and on the other division

$$\hat{q}_{d'}^d = e^{-\frac{\kappa}{2}}q_{d'} < q_{d'} \text{ for } \gamma > 0 \text{ and } \hat{q}_{d'}^d = \left[2 - e^{-\frac{\kappa}{2}}\right]q_{d'} > q_{d'} \text{ for } \gamma < 0,$$

and a Nash Equilibrium effort levels

$$a_d = \frac{Z^2e^{-\kappa}q^3}{Ze^{-\frac{\kappa}{2}}q^2\left(4 - 3e^{-\frac{\kappa}{2}}\right) + 2r\sigma^2(1 - |\rho|)},$$

for $d \in \{A, B\}$. When $\rho = 0$, HQ are indifferent between setting $\gamma = \pm\beta$. In addition, pay-performance sensitivity, β_d , and effort, a_d , are both decreasing in uncertainty, κ .

When division managers are affected by low levels of uncertainty, $\kappa_d \leq \bar{\kappa}_d$, uncertainty aversion does not significantly affect their beliefs and incentives to exert effort. The effect of uncertainty is to make division managers more pessimistic than HQ on the productivity of their own division, with $\hat{q}_d^d = e^{-\kappa}q_d < q_d$ (corresponding to case (iii) in Lemma 2). At these lower levels of uncertainty, however, the disagreement between division managers and HQ is relatively small, and HQ respond by decreasing pay-for-performance sensitivity and offer a contract with lower β_d . This happens because the presence of uncertainty has two adverse effects on division managers. First, by reducing the assessment of the productivity of their division, uncertainty has a detrimental effect on their incentives to exert effort. Second, more conservative beliefs reduce the value of an incentive contract w_d as assessed by division managers, relative to the value assessed by HQ, increasing the

“disagreement discount.” The combined effect is to make it costlier for HQ to induce effort with incentive pay, leading to lower pay-for-performance compensation, β_d , and thus lower effort, a_d . The optimal cross-division exposure $|\gamma_d|$ is still proportional to $|\rho|$, and is set to limit a division manager’s overall risk exposure, reducing the corresponding required risk-premium compensation.

Overall, when uncertainty is sufficiently low, optimal incentive contract mirrors those in Theorem 2, with the difference that HQ allow some (endogenous) disagreement to persist in the organization. Interestingly, the optimal contract has the property that HQ are (endogenously) more optimistic than division managers. In other words, HQ appear in the organization as “visionary.”

When division managers are sufficiently concerned about uncertainty on division productivity, $\kappa_d > \bar{\kappa}_d$, and divisions are symmetric, HQ find it optimal to increase cross-pay. Note that, in this case, HQ grant division managers a sufficient share of the other division to induce division managers to hold beliefs that are more closely aligned with theirs, with $\hat{q}_d^d = e^{-\frac{\kappa}{2}} > e^{-\kappa}$ (corresponding to case (ii) and (iv) in Lemma 2). In other words, when division managers are exposed to great levels of uncertainty, the presence of such uncertainty, if left unchallenged, would depress the effort they exert. In this case, by granting greater cross-division exposure, $|\gamma| = \beta > |\rho|\beta_d$, HQ can limit the pessimism held by division managers, promoting effort. This means that, when uncertainty is sufficiently large, with a detrimental effect on division managers beliefs and, thus, effort, HQ respond by setting the cross-division exposure $|\gamma|$ to a level that is greater than the one offered absent uncertainty.

Note that when divisions are symmetric, optimal incentive contracts have $|\gamma| = \beta$, even when divisions are not correlated, $\rho = 0$. This means that the presence of sufficiently large Knightian uncertainty leads to incentive contracts that would not be optimal under traditional risk aversion alone, namely exposing a risk-averse division managers to the outcome of an uncorrelated division. Setting $|\gamma| = \beta$ has two opposing effects on division managers. First, by exposing a risk-averse division manager to the outcome of an uncorrelated division is costly to HQ because it increases the risk premium required to meet the division manager’s participation constraint. Such exposure, however, has two benefits for company HQ. First, it makes division managers more confident about their own division, increasing effort. The second effect is that the presence of the cross-division exposure $|\gamma|$ affects the belief system in the organization, realigning the division managers’ beliefs

and bringing them to be closer to the beliefs held by company HQ. This induces division manager to assess the value of incentive contracts, which is based on \hat{q}_d , closer to the value assessed by HQ, which is based on q_d , reducing the uncertainty discount. Note that HQ are indifferent between granting cross pay, $\gamma_d > 0$, or relative-performance evaluation, $\gamma_d < 0$, as the optimal contracts depends only on the size of the cross-division exposure, $|\gamma|$, and not on its sign. This property is the direct outcome of the symmetric features of our model.

5 Pragmatic Leadership

We now consider the optimal incentive contract for division managers when HQ are uncertainty averse as well. Different from the case of visionary leadership, where HQ held firm beliefs $q = \{q_A, q_B\}$, for an uncertainty-averse HQ beliefs are not fixed but, rather, they will be determined endogenously as well. Specifically, uncertainty-averse HQ will pragmatically adapt their beliefs to the company characteristics. Thus, we will identify this type of leadership as the “pragmatic leadership.”

In this case, both HQ and division managers are uncertainty averse and are characterized by a level of uncertainty given by κ_{HQ} , and κ_d , for $d \in \{A, B\}$, respectively. Since the properties of Lemma 1 applies to HQ as well, their beliefs $\{\hat{q}_{HQ}^A, \hat{q}_{HQ}^B\}$ are determined endogenously by solving

$$\min_{\{\hat{q}_{HQ}^A, \hat{q}_{HQ}^B\} \in C_{HQ}} \hat{\pi} \equiv \sum_{d \in \{A, B\}} b_{HQ}^d \hat{q}_{HQ}^d a_d - s_A - s_B \quad (36)$$

where $b_{HQ}^d \equiv 1 - \beta_d - \gamma_{d'} > 0$, for $d, d' \in \{A, B\}$, $d \neq d'$ and

$$C_{HQ} \equiv \left\{ \vec{q} \mid \left(1 - \frac{|\hat{q}_A - q_A|}{q_A}\right) \left(1 - \frac{|\hat{q}_B - q_B|}{q_B}\right) \geq e^{-\kappa_{HQ}} \right\}. \quad (37)$$

HQ beliefs are characterized in the following lemma.

Lemma 4 *Let $b_{HQ}^d > 0$, for $d \in \{A, B\}$ and*

$$H_{HQ}^d \equiv \frac{b_{HQ}^{d'} a_{d'} q_{d'}}{b_{HQ}^d a_d q_d}.$$

Headquarters assessment of both divisions, $(\hat{q}_A^{HQ}, \hat{q}_B^{HQ})$, is equal to:

- i) $\hat{q}_d^{HQ} = q_d$ and $\hat{q}_{d'}^{HQ} = e^{-\kappa_{HQ}} q_{d'}$ for $H_{HQ}^d > e^{\kappa_{HQ}}$
- ii) $\hat{q}_d^{HQ} = [e^{-\kappa_{HQ}} H_d]^{\frac{1}{2}} q_d$, for $d \neq d'$ for $H_{HQ}^d \in [e^{-\kappa_{HQ}}, e^{\kappa_{HQ}}]$
- iii) $\hat{q}_d^{HQ} = e^{-\kappa_{HQ}} q_d$ and $\hat{q}_{d'}^{HQ} = q_{d'}$ for $H_{HQ}^d < e^{-\kappa_{HQ}}$

Similar to division managers beliefs in Lemma 2, company HQ beliefs depends on their relative exposure to the two divisions. When HQ have moderate exposure to both divisions, in case (ii) with $H_{HQ} \in [e^{-\kappa_{HQ}}, e^{\kappa_{HQ}}]$, HQ have conservative beliefs toward both divisions, $\hat{q}_d^{HQ} < q_d$, and they become more pessimistic toward any division when relative exposure to that division increases. When HQ have a sufficiently large exposure to a division $d \in \{A, B\}$, as in cases (i) and (iii) with $H_{HQ} > e^{\kappa_{HQ}}$ or $H_{HQ} < e^{-\kappa_{HQ}}$, they will be very pessimistic toward that division, $\hat{q}_d^{HQ} = e^{-\kappa} q_d$, and correspondingly very optimistic on the other division, $\hat{q}_{d'}^{HQ} = q_{d'}$.

In the previous section, when HQ considered offering a contract to a division manager, they considered not only the direct effect of that contract on incentives, but also the effect that the contract would have on the beliefs of the manager. Now, the HQ also consider the impact of the contract on their beliefs as well. Beliefs for the division managers are still given in Lemma 2, and effort is as described in Lemma 3.

The optimal contract is characterized in the following theorem.

Theorem 4 *Let divisions be symmetric, and condition (S) holds. There are thresholds $(\hat{\kappa}^{HQ}, \hat{\kappa})$, defined in the appendix, such that optimal incentive contract are:*

1. If $\kappa_d \leq \hat{\kappa}$ and $\kappa_{HQ} \leq \hat{\kappa}^{HQ}$, optimal contract has

$$\beta_d = \frac{e^{-\frac{\kappa_{HQ}}{2}} e^{-\kappa} Z q^2}{\left(2M + 2(1 - M) e^{-\frac{\kappa_{HQ}}{2}} - e^{-\kappa}\right) e^{-\kappa} Z q^2 + r\sigma^2 (1 - 2\rho M + M^2)} \quad \text{and } \gamma_d = -M\beta_d, \quad (38)$$

for $d \in \{A, B\}$, where $M = \max \left\{ \min \left\{ \hat{M}, -e^{-\kappa} \right\}, e^{-\kappa} \right\}$ and $\hat{M} \equiv \rho - \bar{\rho}$, where $\bar{\rho} \equiv \frac{e^{-\kappa} q^2 Z}{r\sigma^2} \left(1 - e^{-\frac{\kappa_{HQ}}{2}}\right)$, inducing beliefs for division managers equal to $\hat{q}_{d'}^d = q > \hat{q}_d^d = e^{-\kappa} q$, and for HQ equal to $\hat{q}_d^{HQ} = e^{-\frac{\kappa_{HQ}}{2}} q < q$.

2. If $\rho \leq 0$ and $\kappa_d > \hat{\kappa}$ or $\kappa_{HQ} > \hat{\kappa}^{HQ}$, optimal contract has

$$\beta_d = \gamma_d = \hat{\beta} \equiv \frac{Zq^2 e^{-\frac{\kappa + \kappa_{HQ}}{2}}}{Zq^2 \left(4e^{-\frac{\kappa_{HQ} + \kappa}{2}} - 3e^{-\kappa} \right) + 2r\sigma^2 (1 + \rho)}, \quad (39)$$

for $d \in \{A, B\}$, inducing beliefs for division managers equal to $\hat{q}_d^d = \hat{q}_{d'}^d = e^{-\frac{\kappa}{2}}q$, and for HQ equal to $\hat{q}_d^{HQ} = e^{-\frac{\kappa_{HQ}}{2}}q$.

3. If $\rho > 0$, and if (a) $\kappa > \hat{\kappa}$ and $\kappa_{HQ} \leq \hat{\kappa}_1^{HQ}$, optimal contract has

$$\beta_d = \frac{e^{-\frac{\kappa_{HQ} + \kappa}{2}} m^{\frac{1}{2}} Zq^2}{\left[3 + \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) m \right] e^{-\frac{\kappa_{HQ} + \kappa}{2}} m^{\frac{1}{2}} Zq^2 - 3e^{-\kappa} m q^2 Z + 2r\sigma^2 (1 - \rho m)} \quad \text{and} \quad \gamma_d = -m\beta_d < 0, \quad (40)$$

for $d \in \{A, B\}$, where m is defined in the appendix, inducing beliefs for division managers equal to $\hat{q}_{d'}^d > q$ and $\hat{q}_d^d = e^{-\frac{\kappa}{2}} m^{\frac{1}{2}} q$ and for HQ equal to $\hat{q}_d^{HQ} = e^{-\frac{\kappa_{HQ}}{2}} q < q$; if (b) $\kappa > \hat{\kappa}$ and $\kappa_{HQ} > \hat{\kappa}_2^{HQ}$, optimal contract has

$$\beta_d = \gamma_d = \hat{\beta},$$

inducing beliefs for division managers equal to $\hat{q}_{d'}^d = \hat{q}_d^d = e^{-\frac{\kappa}{2}}q$ and for HQ equal to $\hat{q}_d^{HQ} = e^{-\frac{\kappa_{HQ}}{2}}q < q$.

When both HQ and division managers are uncertainty averse, optimal incentive contracts depend on their relative exposure to uncertainty and the correlation between divisional cash-flows. When overall exposure to uncertainty is sufficiently low, Case 1, optimal contracts mirror again those absent uncertainty of Theorem 4. Company headquarters offer incentive contracts with a cross-division exposure to reduce division managers risk-premia, where again $sign(\gamma) = sign(\rho)$. The effect of uncertainty is again to reduce the pay-for-performance sensitivity, β_d . Interestingly, note however that, if $0 \leq \rho < \bar{\rho}$, we have that $\gamma > 0$ and, different from the basic case, optimal contracts include cross-pay (and, thus, an equity component). Finally, in this case, HQ and division managers are both pessimistic on divisions, and their assessment of division productivity depends on the degree of their uncertainty aversion, with $\hat{q}_d^d \geq \hat{q}_d^{HQ}$ as $\frac{\kappa_{HQ}}{2} \geq \kappa$.

When uncertainty faced by HQ or division managers is sufficiently large, Case 2, optimal incentive contracts depend on correlation between divisions. When division cash-flows are negatively

correlated, $\rho \leq 0$, optimal incentive contracts are fully equity-based, with $\beta_d = \gamma_d$. Furthermore, in this case, division managers have the same beliefs on division productivity, with $\hat{q}_d^d = \hat{q}_{d'}^d = e^{-\frac{\kappa}{2}} q$, and again $\hat{q}_d^d \geq \hat{q}_d^{HQ}$ as $\kappa_{HQ} \geq \kappa$. Finally, if $\kappa_{HQ} = \kappa$ HQ and division managers share the same vision in the firm, reaching consensus in the organization.

When division cash-flows are positively correlated, $\rho > 0$, the structure of optimal incentive contracts depends critically on the degree of uncertainty affecting HQ. When HQ are exposed to low levels of uncertainty (while division managers are exposed to large uncertainty), Case 3a, optimal incentive contracts have a relative-performance component, with $\gamma < 0$.

In contrast, when both HQ and division managers are exposed to sufficiently large uncertainty, as in Case 3b, optimal incentive contracts are again equity-based contracts with $\beta_d = \gamma_d$, with no relative performance. The reason is that, when HQ are very uncertainty averse and they hold a long position in both divisions, from Lemma 4, HQ are more pessimistic than the reference probability, $\hat{q}_{HQ}^d < q_d$. This means that, if the optimal contract offers relative compensation, $\gamma_d < 0$, from Lemma 2, division managers will be more optimistic on the other division than the reference probability, $\hat{q}_{d'}^d > q_d$, exacerbating the disagreement between HQ and divisional managers. Greater disagreement with division managers is not desirable to HQ because it will increase the disagreement discount in (18), further increasing the cost of inducing effort. Thus, in this case, HQ prefer to forego the benefit of risk-reduction of relative-performance compensation and, rather, offer an equity-only incentive contracts that realigns division managers beliefs with theirs. This case represents an important reversal from traditional predictions of the standard optimal contracting problem with no uncertainty of Theorem 2.

6 Empirical Implications

Our paper has several empirical implications that can help explaining some otherwise puzzling features of the compensation policies adopted by corporations.

1. *Firms characterized by high uncertainty, such as young firms, prefer compensation contracts with an equity component rather than relative performance.* A puzzling feature of the compensation structure of many young firms is the widespread use of equity-based compensation throughout the

organization. While equity-based compensation appears to be justified for members of the top management, such as the CEO, it is less clear why lower-level managers should receive equity-based compensation. This is because equity-based compensation reduces the sensitivity of managerial pay to their action, and thus reduces its effectiveness as an incentive. In other words, equity-based compensation makes divisional managers lose line of sight between their actions and their impact on compensation. This practice is even less justifiable for low-level employees, where the connection between an employee's actions and equity value is even more tenuous.

Our paper provides an explanation for the common occurrence of equity-based compensation and the infrequent use of relative-performance assessments. Our paper suggests that equity-based compensation plays two important roles. The first one is to better align the beliefs of members of the organization with the one held by the top management. In particular, absent the equity component in pay, individuals would hold more conservative beliefs than the top management on the expected performance of their unit. Inclusion of the equity-based compensation has the benefit of better aligning their expectations with the ones held by the top management, improving the overall disposition of the organization. The second benefit is that, because of the improvement of expectations, employees will exert greater effort, improving firm value.

2. Relative performance and pay-for-luck. It is often suggested that lack of relative-performance component in executive pay results in rewarding top managers for performance influenced by market forces outside their control rather than their own efforts (“pay-for-luck”).²¹ Our paper suggests an advantage of equity-based compensation over relative-performance. Relative-performance creates a divergence between shareholders, who typically hold “long” positions in their portfolios, and top executives, that would hold “short” positions in the benchmarks adopted as a basis for their relative performance. The presence of such divergent positions has the consequence of creating potentially harmful disagreement between shareholders and top management. Equity-based compensation, in contrast, has the benefit of aligning shareholders and top executives exposure to uncertainty, preserving agreement.

3. Mature firms adopt compensation contracts primarily based on pay-for-performance measures with relative-performance components. As firms mature, the level of uncertainty surrounding their

²¹Gopalan, Milbourn and Song (2010) argues that this feature is an optimal response to strategic uncertainty surrounding firms.

business activities decreases, reducing (or even eliminating) the need for equity-based compensation. For these firms, effort levels in the organization is better elicited by the use of pay-for-performance incentive contract, making equity-based compensation redundant. This means that firms should first start, when they are young, with incentive contracts heavily skewed toward equity-based compensation, and then move toward pay-for-performance based contracts as they mature.

4. *Optimal compensation in business groups.* Our paper has also implications for the compensation structure in business groups. Consider an executive manager in a subsidiary of a business (or family group). Traditional theory would suggest that in these cases compensation should depend only on the performance of their subsidiary or business unit. In contrast, compensation for such managers is often tied to the performance of the entire business group. For example, Ma, Tang, and Gomez (2019) study the compensation structure for the mutual funds industry and find that in about half of their sample, managers' bonuses are directly linked to the overall profitability of the advisor. A similar practice is common in the investment bank industry, where individual bonuses depend also on the overall performance of the intermediary. Such features, which would be difficult to be justified on the basis of risk-aversion only, are consistent with the findings of our paper.

5. *Managerial (over)optimism.* Our model predicts that managers in the upper echelon of corporate ladders tend to be more optimistic about their firm's future performance. This implies that, rank-and-file managers perceive members of the top management team of a firm (such as CEOs and CFOs) as overconfident and unrealistically optimistic. The role of managerial overconfidence in corporations has been extensively documented (see, for example, Heaton, 2002, and Malmendier and Tate, 2005, among others). Goel and Thakor (2008) suggest that managerial optimism can be the outcome of the managerial selection process, whereby lucky and overconfident managers are more likely to rise to the top positions of companies. Our paper suggests that top managers' optimism is the consequence of uncertainty hedging, and not necessarily the sign of a negative behavioral bias.

6. *Entrepreneur CEOs and family wealth.* Entrepreneurship is commonly associated with family wealth (Hurst and Lusardi, 2004), and access to family wealth is a primary determinant of entrepreneurship (Levine and Rubinstein, 2017). There are several reasons why family wealth may be associated with greater incentives to become entrepreneurs and, thus, CEOs. These include

relaxation of financial constraints and greater diversification opportunities (lower cost of capital). Note that traditional risk-diversification rationales would imply the wealthy families invest in industries with low (or negative) correlation with the bulk of family money. Our paper adds a novel rationale for the association between family wealth and entrepreneurship. Individuals in wealthy families, by virtue of their broad portfolio, benefit more from uncertainty hedging, giving them a comparative advantage in investing in business surrounded by greater uncertainty. As a consequence, owners/CEOs belonging to wealthy families would (endogenously) be characterized by more optimistic views of their companies. These are new and testable implications.

7 Conclusions and Future Research

In this paper we have examined the impact of uncertainty aversion on the design of optimal incentive contracts in an organization. We have studied the problem faced by a multidivisional firm, for simplicity with two divisions, where agents may be uncertainty averse. Divisional managers exert unobservable effort that affects the success probability of their division, creating moral hazard. The contracting problem is further complicated by the fact that division managers are uncertainty averse, which makes them unduly conservative (in the eyes of the company HQ) on the success probability of the investments in their divisions. We showed that the structure of optimal incentive contracts depends on the level of uncertainty that affects the firms. For firms with low uncertainty, incentive contracts exhibit pay-for-performance compensation, when division cash-flows are negatively correlated, and relative-performance compensation, when division cash-flows are positively correlated. For firms characterized by high levels of uncertainty, optimal incentive contracts are more likely to have a cross pay compensation or straight-equity contracts. Our paper can explain how young firms award equity compensation to their employees and then switch to pay-for-performance compensation as they mature.

Our paper can be extended in several ways. First, our results can be extended to cases on multitasking, as discussed in Holmström and Milgrom (1991). Our paper suggests an important aspect of multitasking: uncertainty hedging and its impact on task assignment and optimal compensation. Our paper is also essentially a partial equilibrium model. An important question is to examine the

impact of labor market forces in a process where heterogeneous agents are matched with heterogeneous firms. We leave these important questions for future research.

References

- [1] Baker, G., M. Jensen and K. Murphy (1988) “Compensation and Incentives: Practice vs. Theory,” *Journal of Finance*, **43**(3): 593–616.
- [2] Bayar, O., Chemmanur, T., and Liu, M. (2011) “A Theory of Equity Carve-Outs and Negative Stub Values under Heterogeneous Beliefs,” *Journal of Financial Economics*, **100**(3): 616–638.
- [3] Bergman, N. and D. Jenter (2007) “Employee Sentiment and Stock Option Compensation,” *Journal of Financial Economics*, **84**(3): 667–712.
- [4] Blasi, J., R. Freeman, and D. Kruse (2016) “Do Broad-based Employee Ownership, Profit Sharing and Stock Options Help the Best Firms Do Even Better?” *British Journal of Industrial Relations*, **54**(1): 55–82
- [5] Bolton, P., M. Brunnermeier, and L. Veldkamp (2013) “Leadership, Coordination, and Corporate Culture,” *Review of Economic Studies*, **80**(2): 512–537.
- [6] Boot, A., Gopalan, R. and Thakor, A. (2006) “The Entrepreneur’s Choice between Private and Public Ownership,” *Journal of Finance*, **61**(2): 803–836.
- [7] Boot, A., Gopalan, R. and Thakor, A. (2008) “Market Liquidity, Investor Participation and Managerial Autonomy: Why do Firms Go Private?” *Journal of Finance*, **63**(4): 2013–2059.
- [8] Bossaerts, P., P. Ghirardato, S. Guarnaschelli, and W. Zame (2010) “Ambiguity in Asset Markets: Theory and Experiment,” *Review of Financial Studies*, **23**(4): 1325–1359.
- [9] Carroll, G. (2015) “Robustness and Linear Contracts,” *American Economic Review*, **105**(2): 536–563.
- [10] Carroll, G. and D. Meng (2016) “Robust Contracting with Additive Noise,” *Journal of Economic Theory*, **166**: 586–604.
- [11] Caskey, J. (2009) “Information in Equity Markets with Ambiguity-Averse Investors,” *Review of Financial Studies*, **22**(9): 3595–3627.
- [12] Chassang, S. (2013) “Calibrated Incentive Contracts,” *Econometrica*, **81**(5): 1935–1971.
- [13] Chen, Z. and L. Epstein (2002) “Ambiguity, Risk, and Asset Returns in Continuous Time,” *Econometrica*, **70**(4): 1403–1443.
- [14] Core, J. and W. Guay (2001) “Stock Option Plans for Non-Executive Employees” *Journal of Financial Economics*, **61**(2): 253–287.
- [15] Dai, T. and J. Toikka (2018) “Robust Incentives for Teams,” University of Pennsylvania Working Paper.

- [16] de Finetti, B. (1974) *Theory of Probability: A Critical Introductory Treatment*. John Wiley & Sons Ltd.
- [17] DeMarzo, P. and R. Kaniel (2017) “Relative Pay for Non-Relative Performance: Keeping Up with the Joneses with Optimal Contracts” CEPR Discussion Paper No. DP11538.
- [18] Dessein, W. and T. Santos (2006) “Adaptive Organizations,” *Journal of Political Economy*, **114**(5): 956-995.
- [19] Diamond, P. (1998) “Managerial Incentives: On the Near Linearity of Optimal Compensation,” *Journal of Political Economy*, **106**(5): 931-957.
- [20] Dicks, D. and P. Fulghieri (2015) “Ambiguity, Disagreement, and Allocation of Control in Firms,” SSRN Working Paper 2357599.
- [21] Dicks, D. and P. Fulghieri (2019) “Uncertainty Aversion and Systemic Risk,” *Journal of Political Economy*, **127**(3): 1118-1155.
- [22] Dicks, D. and P. Fulghieri (2020) “Uncertainty, Investor Sentiment, and Innovation,” *Review of Financial Studies*, forthcoming.
- [23] Dimmock, S., R. Kouwenberg, O. Mitchell, and K. Peijnenburg (2016) “Ambiguity Aversion and Household Portfolio Choice Puzzles: Empirical Evidence,” *Journal of Financial Economics*, **119**(3): 559-577.
- [24] Drechsler, I. (2013) “Uncertainty, Time-Varying Fear, and Asset Prices,” *Journal of Finance*, **68**(5): 1843-1889.
- [25] Duchin, R., A. Goldberg, and D. Sosyura (2017) “Spillovers inside Conglomerates: Incentives and Capital,” *Review of Financial Studies*, **30**(5): 1696-1743.
- [26] Easley, D. and M. O’Hara (2009) “Ambiguity and Nonparticipation: The Role of Regulation,” *Review of Financial Studies*, **22**(5): 1817-1843.
- [27] Easley, D. and M. O’Hara (2010) “Microstructure and Ambiguity,” *Journal of Finance*, **65**(5): 1817-1846.
- [28] Edmans, A., X. Gabaix and A. Landier (2009) “A Multiplicative Model of Optimal CEO Incentives in Market Equilibrium,” *Review of Financial Studies*, **22**(12): 4881–4917.
- [29] Epstein, L. and M. Schneider (2008) “Ambiguity, Information Quality and Asset Pricing,” *Journal of Finance*, **63**(1): 197-228.
- [30] Epstein, L. and M. Schneider (2010) “Ambiguity and Asset Markets,” *Annual Review of Financial Economics*, **2**: 315-346.
- [31] Frydman, C. and D. Jenter (2010) “CEO Compensation,” *Annual Review of Financial Economics*, **2**: 75-102.
- [32] Gallant, A. R., M.R. Jahan-Parvar, and H. Liu, (2019) “Does Smooth Ambiguity Matter for Asset Pricing?,” *Review of Financial Studies*, **32**(9): 3617-3666.

- [33] Garlappi, L., R. Giammarino, and A. Lazrak, (2017) “Ambiguity and the Corporation: Group Disagreement and Underinvestment,” *Journal of Financial Economics*, **125**(3): 417-433.
- [34] Gilboa, I. and D. Schmeidler (1989) “Maxmin Expected Utility with Non-Unique Prior,” *Journal of Mathematical Economics*, **18**(2): 141-153.
- [35] Goel, A. and A. Thakor (2008) “Overconfidence, CEO Selection, and Corporate Governance,” *Journal of Finance* **63**(6): 2737-2784.
- [36] Gopalan, R., T. Milbourn and F. Song (2010) “Strategic Flexibility and the Optimality of Pay for Sector Performance”, *Review of Financial Studies*, **23**(5): 2060-2098.
- [37] Grossman, S. and O. Hart, (1983) “An Analysis of the Principal-Agent Problem,” *Econometrica*, **51**(1): 7-45.
- [38] Guiso, L., L. Pistaferri, and F. Schivardi (2013) “Credit within the Firm,” *Review of Economic Studies*, **80**(1): 211–247.
- [39] Hansen, L., T. Sargent, G. Turmuhambetova, and N. Williams (2006) “Robust Control and Model Misspecification,” *Journal of Economic Theory*, **128**(1): 45–90.
- [40] Hart, O. and B. Holmström (1987) “The Theory of Contracts,” in: T. Bewley, ed., *Advances in Economic Theory*, Fifth World Congress (Cambridge University Press, Cambridge, UK).
- [41] He, Z., B. Wei, J. Yu, and F. Gao (2017) “Optimal Long-Term Contracting with Learning,” *Review of Financial Studies*, **30**(6): 2006-2065.
- [42] Heaton, J. (2002) “Managerial Optimism and Corporate Finance,” *Financial Management*, **31**(2): 33-45
- [43] Hochberg, Y. and L. Lindsey (2010) “Incentives, Targeting, and Firm Performance: An Analysis of Non-executive Stock Options,” *Review of Financial Studies*, **23**(11): 4148–4186.
- [44] Holmström, B. (1979) “Moral Hazard and Observability,” *Bell Journal of Economics*, **10**(1): 74-91.
- [45] Holmström, B. (1982) “Moral Hazard in Teams,” *Bell Journal of Economics*, **13**(2): 324-340.
- [46] Holmström, B. and P. Milgrom (1987) “Aggregation and Linearity in the Provision of Intertemporal Incentives,” *Econometrica*, **55**(2): 303-328.
- [47] Holmström, B. and P. Milgrom (1991) “Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design,” *Journal of Law, Economics, & Organization*, **7**: 24-52.
- [48] Hurst, E. and A. Lusardi (2004) “Liquidity Constraints, Household Wealth, and Entrepreneurship,” *Journal of Political Economy*, **112**(2): 319-347.
- [49] Illeditsch, P. (2011) “Ambiguous Information, Portfolio Inertia, and Excess Volatility,” *Journal of Finance*, **66**(6): 2213-2247.
- [50] Innes, R. (1990) “Limited Liability and Incentive Contracting with Ex-Ante Action Choices,” *Journal of Economic Theory*, **52**(1): 45–67.

- [51] Jahan-Parvar, M. R., and H. Liu (2014) “Ambiguity Aversion and Asset Prices in Production Economies,” *Review of Financial Studies*, **27**(10): 3060-3097.
- [52] Kim, H. and P. Ouimet (2014) “Broad-Based Employee Stock Ownership: Motives and Outcomes” *Journal of Finance*, **69**(3): 1273-1319.
- [53] Klein, K. (1987), “Employee Stock Ownership and Employee Attitudes: A Test of Three Models,” *Journal of Applied Psychology*, **72**(2): 319-332.
- [54] Knight, F. (1921) *Risk, Uncertainty, and Profit*. Boston: Houghton Mifflin.
- [55] Landier, A., D. Sraer, and D. Thesmar (2009) “Optimal Dissent in Organizations,” *Review of Economic Studies*, **76**(2): 761–794.
- [56] Lee, S. and U. Rajan (2020) “Robust Security Design,” SSRN Working Paper 2898462.
- [57] Lee, S. and A. Rivera (2020) “Extrapolation Bias and Robust Dynamic Liquidity Management,” *Management Science*, forthcoming.
- [58] Levine, R. and Y. Rubinstein (2017) “Smart and Illicit: Who Becomes an Entrepreneur and Do They Earn More?” *Quarterly Journal of Economics*, **132**(2): 963-1018.
- [59] Ma, L., Y. Tang, and J. Gomez (2019) “Portfolio Manager Compensation and in the U.S. Mutual Fund Industry,” *Journal of Finance*, **74**(2): 587-638.
- [60] Malmendier, U. and G. Tate (2005) “CEO Overconfidence and Corporate Investment,” *Journal of Finance*, **60**(6): 2661-2700.
- [61] Mele, A. and F. Sangiorgi (2015) “Uncertainty, Information Acquisition, and Price Swings in Asset Market,” *Review of Economic Studies*, **82**(4): 1533-1567.
- [62] Miao, J. and A. Rivera (2016) “Robust Contracts in Continuous Time,” *Econometrica*, **84**(4): 1405-1440.
- [63] Mirrlees, J. (1975) “The Theory of Moral Hazard and Unobservable Behaviour” mimeo, Nuffield College, Oxford.
- [64] Mirrlees, J. (1976) “The Optimal Structure of Incentives and Authority Within an Organization,” *Bell Journal of Economics*, **7**(1): 105–131.
- [65] Mirrlees, J (1999) “The Theory of Moral Hazard and Unobservable Behaviour: Part 1,” *Review of Economic Studies*, **66**(1): 3–21.
- [66] Mukerji, S. and J.-M. Tallon (2001), “Ambiguity Aversion and Incompleteness of Financial Markets,” *Review of Economic Studies*, **68**(4): 883-904.
- [67] Murphy, K. (1999) “Executive Compensation,” *Handbook of Labor Economics*, Vol. 3, Elsevier Science, 2485-2563.
- [68] Murphy, K. (2013) “Executive Compensation: Where We Are, and How We Got There” *Handbook of Economics and Finance*, Vol. 2, Part A, Ch. 4, 211-356.

- [69] Oyer, P. (2004) “Why Do Firms Use Incentives That Have No Incentive Effects?” *Journal of Finance*, **59**(4): 1619-1650.
- [70] Oyer, P. and S. Schaefer (2005) “Why Do Some Firms Give Stock Options to All Employees?: An Empirical Examination of Alternative Theories” *Journal of Financial Economics*, **76**(1): 99-133.
- [71] Oyer, P. and S. Schaefer (2011) “Personnel economics: Hiring and incentives,” in *Handbook of Labor Economics*, O. Ashenfelter and D. Card (ed.), vol. 4, pt. B, chap. 20, 1769–1823. Amsterdam: Elsevier.
- [72] Pearsall, M., M. Christian, and A. Ellis (2010) “Motivating Interdependent Teams: Individual Rewards, Shared Rewards, or Something in Between?,” *Journal of Applied Psychology*, **95**(1):183-91.
- [73] Prendergast, C. (1999) “The Provision of Incentives in Firms,” *Journal of Economic Literature*, **37**(1): 7-63.
- [74] Rotemberg, J. and G. Saloner (1993) “Leadership Styles and Incentives,” *Management Science*, **39**(11): 1299-1318.
- [75] Rotemberg, J. and G. Saloner (2000) “Visionaries, Managers, and Strategic Direction,” *RAND Journal of Economics*, **31**(4): 693-716.
- [76] Szydlowski, M. (2019) “Ambiguity in Dynamic Contracts,” SSRN Working Paper 1986186
- [77] Walton, D. and G. Carroll (2019) “When are Robust Contracts Linear,” Working Paper, Stanford University.
- [78] Van den Steen, E. (2005) “Organizational Beliefs and Managerial Vision,” *Journal of Law, Economics, and Organization*, **21**(1): 256-283
- [79] Van den Steen, E. (2010), “On the origin of shared beliefs (and corporate culture),” *The RAND Journal of Economics*, **41**(4): 617-648.
- [80] Zohar, D. and D. Hofmann (2012) “Organizational Culture and Climate,” in *Oxford Handbook of Organizational Psychology*, S. Kozlowski (ed.) , Vol. 1, Ch. 20: 643-666, Oxford University Press, Oxford, UK.

A Appendix: Proofs

Proof of Theorem 1. From (12), each division manager selects a_t to maximize U_t , where

$$U_{d,t} = \min_{\tilde{P} \in \mathcal{P}_d^\ominus(\{\tilde{a}_d, a_{d'}\})} E_t^{\tilde{P}} u \left(w_d(h^1) - \int_0^1 c_d(\tilde{a}_{d,t}) dt \right)$$

Given worst-case scenario process, θ_d^* , $U_{d,t}$ is a martingale adapted to Brownian Motion Y^θ (Law of Iterated Expectations). By the martingale representation theorem, $U_{d,t}$ is an Ito Process adapted to Y^θ with zero drift. Define $\tilde{w}_{d,t}$ as the certainty equivalent pay, given history: $u \left(\tilde{w}_{d,t} - \int_0^1 c_d(a_{d,t}^*) dt \right) = U_{d,t}$. Note $\tilde{w}_{d,t}$ is a twice-continuously differentiable function of an Ito Process, so it is an Ito Process adapted to Y^θ . Thus, we can express

$$d\tilde{w}_{d,t} = A_{d,t}dt + B'_{d,t}dY^\theta$$

For some process $A_{d,t} \in \mathbb{R}$, $B_{d,t} \in \mathbb{R}^2$. Because $U_d(t) = u\left(\tilde{w}_{d,t} - \int_0^1 c_d(a_{d,t}^*) dt\right)$, where $u = -e^{-rw}$, note $u_t = 0$, $u_w = re^{-rw^\tau}$, and $u_{ww} = -r^2e^{-rw^\tau}$. Thus, by Ito's Lemma,

$$dU_d = re^{-r\tilde{w}_{d,t}} \left[A_{d,t} + B'_{d,t} (Qa_t - \Gamma\theta_t) - \frac{r}{2} B'_{d,t} \Sigma B_{d,t} \right] dt + re^{-r\tilde{w}_{d,t}} B'_{d,t} \Gamma dW_t^\theta.$$

Note this is the evolution of expected utility along the equilibrium path. Suppose the optimal effort process is $a_{d,t}^*$, and the worst-case scenario process is $\theta_{d,t}^*$. Because U_d is a martingale, the drift is zero, so

$$A_{d,t} = \frac{r}{2} B'_{d,t} \Sigma B_{d,t} - B'_{d,t} (Qa_t^* - \Gamma\theta_t^*).$$

Off-equilibrium, the agent could deviate from optimal effort $a_{d,t}^*$ to $\tilde{a}_{d,t}$ from time t to $t + \Delta$. This would give them utility

$$\hat{U}_d = E_{\tilde{a}_{d,t}} \left[u \left(w_d(h^1) - \int_0^1 c_d(\tilde{a}_{d,t}) dt \right) \right]$$

Because u is CARA, we can express this as

$$\hat{U}_d = U_d C_d$$

where U_d is the equilibrium utility and

$$C_d = \exp \left[r \int_t^{t+\Delta} c_d(\tilde{a}_{d,t}) - c_d(a_{d,t}^*) dt \right].$$

By the product rule, $d\hat{U}_d = dU_d \cdot C_d + U_d \cdot dC_d$. Note $\frac{dC_d}{C_d} = r(c_d(\tilde{a}_{d,t}) - c_d(a_{d,t}^*))$. Substituting in,

$$\begin{aligned} d\hat{U}_d &= re^{-r\tilde{w}_{d,t}} C_d \left[A_{d,t} + B'_{d,t} (Q\tilde{a}_t - \Gamma\theta^*(\tilde{a}_t)) - \frac{r}{2} B'_{d,t} \Sigma B_{d,t} \right] dt + re^{-r\tilde{w}_{d,t}} C_d B'_{d,t} \Gamma dW_t^\theta \\ &\quad - re^{-r\tilde{w}_{d,t}} C_d (c_d(\tilde{a}_{d,t}) - c_d(a_{d,t}^*)) \end{aligned}$$

$$d\hat{U}_d = re^{-r\tilde{w}_{d,t}} C_d \left[A_{d,t} + B'_{d,t} (Q\tilde{a}_t - \Gamma\theta^*(\tilde{a}_t)) - \frac{r}{2} B'_{d,t} \Sigma B_{d,t} - c_d(\tilde{a}_{d,t}) + c_d(a_{d,t}^*) \right] dt + re^{-r\tilde{w}_{d,t}} C_d B'_{d,t} \Gamma dW_t^\theta.$$

Note minimizing expected utility is definitionally minimizing the drift of the process expected utility, and maximizing expected utility is maximizing the drift. Because effort and beliefs do not affect $A_{d,t}$ or $\frac{r}{2} B'_{d,t} \Sigma B_{d,t}$, effort and beliefs solve

$$\max_a \min_\theta B'_{d,t} (Q\tilde{a} - \Gamma\theta) - c_{d,t}(\tilde{a}_t).$$

Define the monetary payoff to the principal as

$$X = Y_A + Y_B - w_A - w_B.$$

Substituting in w_d ,

$$dX = [(1 - B_{A,t} - B_{B,t})' (Qa - \Gamma\theta) - A_{A,t} - A_{B,t}] dt + (1 - B_{A,t} - B_{B,t})' \Gamma dW_t^\theta.$$

Note $\Pi(t) = \min_{\tilde{P}\theta \in \mathcal{P}_{HQ}^\ominus} E_t \Pi(1)$, so $\Pi(t)$ is an Ito Process. Because HQ utility is CARA, $\pi = -e^{-RX}$, so $\pi_t = 0$, $\pi_x = R e^{-RX}$, and $\pi_{xx} = -R^2 e^{-RX}$. Thus, by Ito's Lemma,

$$\begin{aligned} d\Pi &= \left[R e^{-RX} \left\{ (1 - B_{A,t} - B_{B,t})' (Qa - \Gamma\theta) - A_{A,t} - A_{B,t} \right\} - \frac{R^2}{2} e^{-RX} (1 - B_{A,t} - B_{B,t})' \Sigma (1 - B_{A,t} - B_{B,t}) \right] dt \\ &\quad + R e^{-RX} (1 - B_{A,t} - B_{B,t})' \Gamma dW_t^\theta \end{aligned}$$

Substituting in for $A_{d,t}$ and rearranging, this implies

$$d\Pi = R e^{-RX} P dt + R e^{-RX} (1 - B_{A,t} - B_{B,\tau})' \Gamma dW_t^\theta$$

where

$$P = (1 - B_{A,t} - B_{B,t})' (Qa^* - \Gamma\theta^{HQ*}) + B'_{A,t} (Qa^* - \Gamma\theta^{A*}) + B'_{B,t} (Qa^* - \Gamma\theta^{B*}) - \frac{r}{2} B'_{A,t} \Sigma B_{A,t} - \frac{r}{2} B'_{B,t} \Sigma B_{B,t} - \frac{R}{2} (1 - B_{A,t} - B_{B,\tau})' \Sigma (1 - B_{A,t} - B_{B,\tau})$$

Again, optimizing expected utility is the same as optimizing the drift. Therefore, the HQ solve at every point in time,

$$\max_B \min_{\theta \in K_{HQ}} P$$

while each division manager solves, at every point in time,

$$\max_a \min_{\theta \in K_d} B'_{d,t} (Qa - \Gamma\theta) - c_d(\tilde{a}_t).$$

Neither of these depend on t , w , or X . Therefore, the optimal $B_{d,t} = B_d$, $a_{d,t} = a_d$, $\theta_{d,t} = \theta_d$, $\theta_{HQ,t} = \theta_{HQ}$ for all $t \in [0, 1]$. Therefore, linear contracts are optimal. ■

Proof of Theorem 2. Linearity follows from Theorem 1, with $K_A = K_B = K_{HQ} = \{0\}$. Thus, compensation to DM_d is

$$w_d = s_d + \beta_d Y_{d,1} + \gamma_d Y_{d',1}$$

Note (13) binds, so substituting in for μ^θ and Σ ,

$$u_d = s_d + \beta_d q_d a_d + \gamma_d q_{d'} a_{d'} - \frac{r\sigma^2}{2} (\beta_d^2 + 2\rho\beta_d\gamma_d + \gamma_d^2) - c_d(a_d) = 0$$

Substituting into headquarter's objective, (11),

$$\pi = q_A a_A + q_B a_B - \frac{r\sigma^2}{2} (\beta_A^2 + 2\rho\beta_A\gamma_A + \gamma_A^2) - \frac{r\sigma^2}{2} (\beta_B^2 + 2\rho\beta_B\gamma_B + \gamma_B^2) - c_A(a_A) - c_B(a_B).$$

Note $\frac{du_d}{da_d} = \beta_d q_d - \frac{a_d}{Z_d}$, so $a_d = \beta_d Z_d q_d$. Because $\frac{da_d}{d\gamma_d} = 0$, $\frac{d\pi}{d\gamma_d} = -r\sigma^2 (\rho\beta_d + \gamma_d)$, so it is optimal to set $\gamma_d = -\rho\beta_d$. Substituting in, HQ maximize

$$\pi = \beta_A Z_A q_A^2 + \beta_B Z_B q_B^2 - \frac{r\sigma^2}{2} (1 - \rho^2) (\beta_A^2 + \beta_B^2) - \frac{1}{2} \beta_A^2 Z_A q_A^2 - \frac{1}{2} \beta_B^2 Z_B q_B^2.$$

Thus, $\frac{\partial \pi}{\partial \beta_d} = Z_d q_d^2 - r\sigma^2 (1 - \rho^2) \beta_d - \beta_d Z_d q_d^2$, so it is optimal to set $\beta_d = \frac{Z_d q_d^2}{Z_d q_d^2 + r\sigma^2 (1 - \rho^2)}$. ■

Proof of Lemma 1. Consider deviations $\theta_t \in K_d(a) \equiv \{\theta | \sup_t f(\theta_t) \leq \kappa_\delta\}$ and $f(\theta_t) = -\ln\left(1 - \frac{|D\theta_{A,t} + N\theta_{B,t}|}{a_A q_A}\right) - \ln\left(1 - \frac{|N\theta_{A,t} + D\theta_{B,t}|}{a_B q_B}\right)$. By Girsanov's Theorem, deviation θ_t sets drift $\mu = Qa - \Gamma\theta$, where $\Gamma = \begin{bmatrix} D & N \\ N & D \end{bmatrix}$, $D = \frac{\sigma}{2} (\sqrt{1+\rho} + \sqrt{1-\rho})$ and $N = \frac{\sigma}{2} (\sqrt{1+\rho} - \sqrt{1-\rho})$. Thus, $\mu_{A,t} = q_A a_{A,t} - D\theta_{A,t} - N\theta_{B,t} = \hat{q}_{A,t} a_{A,t}$ where $q_A - \hat{q}_{A,t} = \frac{1}{a_{A,t}} (D\theta_{A,t} + N\theta_{B,t})$. Similarly, $\mu_{B,t} = \hat{q}_{B,t} a_{B,t}$, where $q_B - \hat{q}_{B,t} = \frac{1}{a_{B,t}} (N\theta_{A,t} + D\theta_{B,t})$. Thus, $|D\theta_{A,t} + N\theta_{B,t}| = a_{A,t} |\hat{q}_{A,t} - q_A|$ and $|N\theta_{A,t} + D\theta_{B,t}| = a_{B,t} |\hat{q}_{B,t} - q_B|$. Substituting into, $f = -\ln\left(1 - \frac{|\hat{q}_{A,t} - q_A|}{q_A}\right) - \ln\left(1 - \frac{|\hat{q}_{B,t} - q_B|}{q_B}\right)$. Rearranging, note $f \leq \kappa_\delta$ iff (27) holds. By Theorem 1, optimal effort and beliefs are constant over time, allowing us to suppress time subscripts. ■

Proof of Lemma 2. By Lemma 1, the division manager's objective is $U_d = \min_{\hat{q} \in C_d} u_d$, where $\hat{q} \in C_d$ iff $g \equiv \left(1 - \frac{|\hat{q}_A - q_A|}{q_A}\right) \left(1 - \frac{|\hat{q}_B - q_B|}{q_B}\right) \geq e^{-\kappa_\delta}$, and

$$u_d = s_d + \beta_d \hat{q}_d a_d + \gamma_d \hat{q}_{d'} a_{d'} - \frac{r}{2} b'_d \Sigma b_d - c_d(a_d),$$

Let l be the Lagrangian for the minimization problem, and ϕ be the multiplier for the constraint. Thus,

$$l = -u_d + \phi [g - e^{-\kappa_d}]$$

Note $\frac{\partial l}{\partial \hat{q}_d} = -\beta_d a_d + \phi \frac{\partial g}{\partial \hat{q}_d}$ and $\frac{\partial l}{\partial \hat{q}_{d'}} = -\gamma_d a_{d'} + \phi \frac{\partial g}{\partial \hat{q}_{d'}}$. Note $\beta_d > 0$ so that $a_d > 0$, so $\frac{\partial l}{\partial \hat{q}_d} = 0$ requires that $\frac{\partial g}{\partial \hat{q}_d} > 0$, so $\hat{q}_d^d \leq q_d$. Consider first $\gamma_d \geq 0$: $\frac{\partial g}{\partial \hat{q}_{d'}} > 0$, so $\hat{q}_{d'}^d \leq q_{d'}$. Thus, $g = \frac{\hat{q}_d \hat{q}_{d'}}{q_d q_{d'}}$, so $\frac{\partial g}{\partial \hat{q}_d} = \frac{1}{q_d} \frac{\hat{q}_{d'}}{q_{d'}}$ and $\frac{\partial g}{\partial \hat{q}_{d'}} = \frac{\hat{q}_d}{q_d} \frac{1}{q_{d'}}$. First, the minimum is achieved at $\hat{q}_d^d = q_d$ and $\hat{q}_{d'}^d = e^{-\kappa_d} q_{d'}$ iff $\frac{\partial l}{\partial \hat{q}_d} \geq 0 \geq \frac{\partial l}{\partial \hat{q}_{d'}}$ iff $\gamma_d a_d q_{d'} \geq \phi \geq \beta_d a_d q_d e^{\kappa_d}$, or equivalently, if $H_d \geq e^{\kappa_d}$. An interior minimum is achieved iff $\frac{\partial l}{\partial \hat{q}_d} = \frac{\partial l}{\partial \hat{q}_{d'}} = 0$ iff $\frac{\beta_d a_d}{\hat{q}_d} = \frac{\phi}{q_d q_{d'}} = \frac{\gamma_d a_{d'}}{\hat{q}_{d'}}$ iff $\beta_d a_d \hat{q}_d = \gamma_d a_{d'} \hat{q}_{d'}$. Substituting this into $g = e^{-\kappa_d}$, $\hat{q}_d^d = \left[e^{-\kappa_d} \frac{\gamma_d a_{d'}}{\beta_d a_d} q_d q_{d'} \right]^{\frac{1}{2}} = [e^{-\kappa_d} H]^{\frac{1}{2}} q_d$. Similarly, $\hat{q}_{d'}^d = [e^{-\kappa_d} \frac{1}{H}]^{\frac{1}{2}} q_{d'}$. Finally, the minimum is achieved at $\hat{q}_d^d = e^{-\kappa_d} q_d$ and $\hat{q}_{d'}^d = q_{d'}$ iff $\frac{\partial l}{\partial \hat{q}_d} \leq 0 \leq \frac{\partial l}{\partial \hat{q}_{d'}}$ iff $\beta_d a_d q_d \geq \phi \geq \gamma_d a_{d'} q_{d'} e^{\kappa_d}$, or equivalently, if $H_d \in [0, e^{-\kappa_d}]$.

Now, consider $\gamma_d < 0$: $\frac{\partial g}{\partial \hat{q}_{d'}} < 0$, so $\hat{q}_{d'}^d \geq q_{d'}$. Thus, $g = \frac{\hat{q}_d}{q_d} \left(2 - \frac{\hat{q}_{d'}}{q_{d'}} \right)$, so $\frac{\partial g}{\partial \hat{q}_d} = \frac{1}{q_d} \left(2 - \frac{\hat{q}_{d'}}{q_{d'}} \right)$, while $\frac{\partial g}{\partial \hat{q}_{d'}} = -\frac{\hat{q}_d}{q_d} \frac{1}{q_{d'}}$. First, the minimum is achieved at $\hat{q}_d^d = e^{-\kappa_d} q_d$ and $\hat{q}_{d'}^d = q_{d'}$ iff $\frac{\partial l}{\partial \hat{q}_d} < 0$ and $\frac{\partial l}{\partial \hat{q}_{d'}} < 0$, or equivalently, $\beta_d a_d q_d \geq \phi \geq (-\gamma_d) a_{d'} q_{d'} e^{\kappa_d}$, which holds iff $H \in [-e^{-\kappa_d}, 0)$. An interior solution is achieved iff $\frac{\partial l}{\partial \hat{q}_d} = \frac{\partial l}{\partial \hat{q}_{d'}} = 0$ iff $\frac{\beta_d a_d q_d}{\left(2 - \frac{\hat{q}_{d'}}{q_{d'}} \right)} = \phi = \frac{(-\gamma_d) a_{d'} q_{d'}}{\frac{\hat{q}_d}{q_d}}$, so $\left(2 - \frac{\hat{q}_{d'}}{q_{d'}} \right) = \frac{\beta_d a_d q_d}{(-\gamma_d) a_{d'} q_{d'}} \frac{\hat{q}_d}{q_d}$. Because the constraint binds, this implies $\hat{q}_d^d = [e^{-\kappa_d} |H|]^{\frac{1}{2}} q_d$ and $\hat{q}_{d'}^d = 2q_{d'} - [e^{-\kappa_d} \frac{1}{|H|}]^{\frac{1}{2}} q_{d'}$. Finally, for a corner, $\hat{q}_d^d = q_d$, $\hat{q}_{d'}^d = (2 - e^{-\kappa_d}) q_{d'}$, it must be that $\frac{\partial l}{\partial \hat{q}_d} \geq 0 \geq \frac{\partial l}{\partial \hat{q}_{d'}}$. Thus, $(-\gamma_d) a_{d'} q_{d'} \geq \phi \geq \beta_d a_d q_d e^{\kappa_d}$ iff $H < -e^{\kappa_d}$. ■

Proof of Lemma 3. The division manager chooses a_d to maximize $U_d = \min_{\hat{q} \in C} \hat{u}_d$. Thus, $\frac{dU_d}{da_d} = \frac{\partial \hat{u}_d}{\partial a_d} + \frac{\partial \hat{u}_d}{\partial \hat{q}_d^d} \frac{\partial \hat{q}_d^d}{\partial a_d} + \frac{\partial \hat{u}_d}{\partial \hat{q}_{d'}^d} \frac{\partial \hat{q}_{d'}^d}{\partial a_d}$. Applying the minimax theorem, for interior solutions, from the proof of Lemma 2, $\frac{\partial \hat{u}_d}{\partial \hat{q}_d^d} = \phi \frac{\partial g}{\partial \hat{q}_d}$ and $\frac{\partial \hat{u}_d}{\partial \hat{q}_{d'}^d} = \phi \frac{\partial g}{\partial \hat{q}_{d'}}$, so $\frac{\partial \hat{u}_d}{\partial \hat{q}_d^d} \frac{\partial \hat{q}_d^d}{\partial a_d} + \frac{\partial \hat{u}_d}{\partial \hat{q}_{d'}^d} \frac{\partial \hat{q}_{d'}^d}{\partial a_d} = \phi \frac{dg}{da_d}$. However, $g = e^{-\kappa_d}$, so $\frac{dg}{da_d} = 0$. For corner solutions, $\frac{\partial \hat{q}_d^d}{\partial a_d} = \frac{\partial \hat{q}_{d'}^d}{\partial a_d} = 0$. Therefore, $\frac{dU_d}{da_d} = \frac{\partial \hat{u}_d}{\partial a_d} = \beta_d \hat{q}_d^d - \frac{a_d}{Z_d}$, so $a_d = Z_d \beta_d \hat{q}_d^d$, where \hat{q}_d^d is from Lemma 2. If $|H_d| < e^{-\kappa_d}$, $a_d = Z_d \beta_d e^{-\kappa_d} q_d$, and if $|H_d| > e^{\kappa_d}$, $a_d = Z_d \beta_d q_d$. Finally, if $|H_d| \in (e^{-\kappa_d}, e^{\kappa_d})$, $a_d = Z_d \beta_d [e^{-\kappa_d} |H_d|]^{\frac{1}{2}} q_d = Z_d e^{-\frac{\kappa_d}{2}} |\gamma_d|^{\frac{1}{2}} a^{\frac{1}{2}} \beta_d^{\frac{1}{2}} a^{-\frac{1}{2}} q_d^{\frac{1}{2}} q_{d'}^{\frac{1}{2}}$, or equivalently, $a_d = Z_d^{\frac{2}{3}} e^{-\frac{\kappa_d}{3}} |\gamma_d|^{\frac{1}{3}} a^{\frac{1}{3}} \beta_d^{\frac{1}{3}} q_d^{\frac{1}{3}} q_{d'}^{\frac{1}{3}}$. If the other division has $|H_{d'}| \leq e^{-\kappa_{d'}}$, then $a_{d'} = Z_{d'} \beta_{d'} e^{-\kappa_{d'}} q_{d'}$, so $a_d = Z_d^{\frac{2}{3}} Z_{d'}^{\frac{1}{3}} e^{-\frac{\kappa_d + \kappa_{d'}}{3}} |\gamma_d|^{\frac{1}{3}} \beta_{d'}^{\frac{1}{3}} \beta_d^{\frac{1}{3}} q_{d'}^{\frac{1}{3}} q_d^{\frac{2}{3}}$. If the other division has $|H_{d'}| \geq e^{\kappa_{d'}}$, then $a_{d'} = Z_{d'} \beta_{d'} q_{d'}$, so $a_d = Z_d^{\frac{2}{3}} Z_{d'}^{\frac{1}{3}} e^{-\frac{\kappa_d}{3}} |\gamma_d|^{\frac{1}{3}} \beta_{d'}^{\frac{1}{3}} \beta_d^{\frac{1}{3}} q_{d'}^{\frac{1}{3}} q_d^{\frac{2}{3}}$. Finally, if $|H_{d'}| \in (e^{-\kappa_{d'}}, e^{\kappa_{d'}})$, then $a_{d'} = Z_{d'}^{\frac{2}{3}} e^{-\frac{\kappa_{d'}}{3}} |\gamma_{d'}|^{\frac{1}{3}} a^{\frac{1}{3}} \beta_{d'}^{\frac{1}{3}} q_{d'}^{\frac{1}{3}} q_{d'}^{\frac{1}{3}}$ as well, so $a_d = Z_d^{\frac{3}{4}} Z_{d'}^{\frac{1}{4}} e^{-\frac{3\kappa_d + \kappa_{d'}}{8}} \beta_{d'}^{\frac{3}{4}} |\gamma_{d'}|^{\frac{3}{8}} \beta_d^{\frac{1}{8}} |\gamma_d|^{\frac{1}{8}} q_d^{\frac{1}{2}} q_{d'}^{\frac{1}{2}}$. Note effort is increasing not only in this divisions characteristics, $Z_d, q_d, \beta_d, |\gamma_d|, (-\kappa_d)$, but also in the other divisions characteristics. ■

Proof of Theorem 3. There are three regions to consider: $|H_d| < e^{-\kappa}$, $H_d \in (e^{-\kappa}, e^{\kappa})$, and $H_d \in (-e^{\kappa}, -e^{-\kappa})$. Finally, we will show that $H_d > e^{\kappa}$ is suboptimal when divisions are symmetric. First, consider when $|H_d| < e^{-\kappa}$, so $\hat{q}_d^d = e^{-\kappa_d} q_d$ and $\hat{q}_{d'}^d = q_{d'}$. Because the participation constraint binds, headquarter's objective is

$$\begin{aligned} \pi &= (1 - \beta_A) q_A a_A + (1 - \beta_B) q_B a_B + \beta_A e^{-\kappa_A} q_A a_A + \beta_B e^{-\kappa_B} q_B a_B \\ &\quad - \frac{r\sigma^2}{2} (\beta_A^2 + 2\rho\beta_A\gamma_A + \gamma_A^2) - \frac{a_A^2}{2Z_A} - \frac{r\sigma^2}{2} (\beta_B^2 + 2\rho\beta_B\gamma_B + \gamma_B^2) - \frac{a_B^2}{2Z_B} \end{aligned}$$

where $a_d = \beta_d Z_d e^{-\kappa_d} q_d$, by Lemma 3. Because $\frac{\partial U_d}{\partial \gamma_d} = -r\sigma^2 (\rho\beta_d + \gamma_d)$, $\gamma_d = -\rho\beta_d$.²² This implies

$$\begin{aligned} \pi &= (1 - \beta_A) \beta_A Z_A e^{-\kappa_A} q_A^2 + (1 - \beta_B) \beta_B Z_B e^{-\kappa_B} q_B^2 + \frac{1}{2} \beta_A^2 Z_A e^{-2\kappa_A} q_A^2 + \frac{1}{2} \beta_B^2 Z_B e^{-2\kappa_B} q_B^2 \\ &\quad - \frac{r\sigma^2}{2} (1 - \rho^2) (\beta_A^2 + \beta_B^2) \end{aligned}$$

²²If $|\rho|$ is high enough or divisions are sufficiently asymmetric, $\bar{\kappa}_d = 0$.

so

$$\frac{\partial \pi}{\partial \beta_d} = Z_d e^{-\kappa_d} q_d^2 - \beta_d Z_d e^{-\kappa_d} (2 - e^{-\kappa_d}) q_d^2 - r\sigma^2 (1 - \rho^2)$$

Therefore, $\beta_d = \frac{e^{-\kappa_d} Z_d q_d^2}{(2 - e^{-\kappa_d}) e^{-\kappa_d} Z_d q_d^2 + r\sigma^2 (1 - \rho^2)}$. Optimal actions follow by substitution.

Next, suppose divisions are symmetric. Consider $H_d \in (e^{-\kappa}, e^\kappa)$. Substituting in beliefs from Lemma 2 and equilibrium effort from Lemma 3, because the participation constraint binds,

$$\pi = 2(1 - \beta - \gamma) Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} q^2 - r\sigma^2 (\beta^2 + 2\rho\beta\gamma + \gamma^2) + 3Z [e^{-\kappa} \beta \gamma] q^2$$

Note

$$\frac{\partial \pi}{\partial \beta} = -2Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} q^2 + (1 - \beta - \gamma) Z [e^{-\kappa} \gamma]^{\frac{1}{2}} q^2 \beta^{-\frac{1}{2}} - 2r\sigma^2 (\beta + \rho\gamma) + 3Z [e^{-\kappa} \gamma] q^2$$

$$\frac{\partial \pi}{\partial \gamma} = -2Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} q^2 + (1 - \beta - \gamma) Z [e^{-\kappa} \beta_d]^{\frac{1}{2}} \gamma^{-\frac{1}{2}} q^2 - 2r\sigma^2 (\rho\beta + \gamma) + 3Z e^{-\kappa} \beta q^2$$

Rearranging, $\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial \gamma} = 0$ iff

$$0 = \left[(1 - \beta - \gamma) Z e^{-\frac{\kappa}{2}} \beta_d^{-\frac{1}{2}} \gamma^{-\frac{1}{2}} q^2 + 3Z \beta q^2 + 2r\sigma^2 (1 - \rho) \right] (\beta - \gamma)$$

Because the first term is strictly positive, $\beta = \gamma$. Plugging back into $\frac{\partial \pi}{\partial \beta} = 0$, $\beta = \frac{Z e^{-\frac{\kappa}{2}} q^2}{Z e^{-\frac{\kappa}{2}} (4 - 3e^{-\frac{\kappa}{2}}) q^2 + 2r\sigma^2 (1 + \rho)}$, giving

$$\text{HQ payoff } \pi = \frac{Z^2 e^{-\kappa} q^4}{2r\sigma^2 (1 + \rho) + Z e^{-\frac{\kappa}{2}} q^2 (4 - 3e^{-\frac{\kappa}{2}})}.$$

Consider $H_d \in (-e^\kappa, -e^{-\kappa})$, so $\beta > 0 > \gamma$. Substituting in beliefs from Lemma 2 and equilibrium effort from Lemma 3,

$$\pi = 2(1 - \beta + \gamma) Z e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} q^2 - r\sigma^2 (\beta^2 + 2\rho\beta\gamma + \gamma^2) + 3e^{-\kappa} \beta (-\gamma) q^2 Z$$

so

$$\frac{\partial \pi}{\partial \beta} = -2Z e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} q^2 + (1 - \beta + \gamma) Z e^{-\frac{\kappa}{2}} (-\gamma_d)^{\frac{1}{2}} q^2 \beta^{-\frac{1}{2}} - 2r\sigma^2 (\beta + \rho\gamma) + 3e^{-\kappa} (-\gamma) q^2 Z$$

$$\frac{\partial \pi}{\partial \gamma} = 2Z e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} q^2 - (1 - \beta + \gamma) Z e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} q^2 (-\gamma_d)^{-\frac{1}{2}} - 2r\sigma^2 (\rho\beta + \gamma) - 3e^{-\kappa} \beta q^2 Z$$

Rearranging, $\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial \gamma} = 0$ implies

$$0 = \left[(1 - \beta + \gamma) Z e^{-\frac{\kappa}{2}} \beta_d^{-\frac{1}{2}} q^2 (-\gamma_d)^{-\frac{1}{2}} + 3e^{-\kappa} q^2 Z + 2r\sigma^2 (1 + \rho) \right] (\beta + \gamma),$$

so $\gamma = -\beta$. Plugging back into $\frac{\partial \pi}{\partial \beta} = 0$, $\beta = \frac{Z e^{-\frac{\kappa}{2}} q^2}{2r\sigma^2 (1 - \rho) + Z e^{-\frac{\kappa}{2}} q^2 (4 - 3e^{-\frac{\kappa}{2}})}$, so $\pi = \frac{Z^2 e^{-\kappa} q^4}{2r\sigma^2 (1 - \rho) + Z e^{-\frac{\kappa}{2}} q^2 (4 - 3e^{-\frac{\kappa}{2}})}$.

Therefore, $\gamma = -\beta$ is more profitable than $\gamma = \beta$ iff $\rho > 0$.

Back to the optimal contract with $|H_d| < e^{-\kappa}$. If divisions are symmetric, that contract gives HQ payoff $\pi = \frac{[e^{-\kappa} Z q^2]^2}{[(2 - e^{-\kappa}) e^{-\kappa} Z q^2 + r\sigma^2 (1 - \rho^2)]}$. It is better to induce division managers to interior beliefs iff

$$\frac{Z^2 e^{-\kappa} q^4}{2r\sigma^2 (1 - |\rho|) + Z e^{-\frac{\kappa}{2}} q^2 (4 - 3e^{-\frac{\kappa}{2}})} \geq \frac{[e^{-\kappa} Z q^2]^2}{[(2 - e^{-\kappa}) e^{-\kappa} Z q^2 + r\sigma^2 (1 - \rho^2)]}.$$

iff $f \geq 0$, where

$$f(\kappa) = 2 \left(1 - 2e^{-\frac{\kappa}{2}} + e^{-\kappa} \right) Z q^2 + r\sigma^2 (1 - |\rho|) [e^\kappa (1 + |\rho|) - 2]$$

Note $f(0) = -r\sigma^2 (1 - |\rho|)^2 < 0$. Also, $f'(\kappa) = 2 \left(1 - e^{-\frac{\kappa}{2}} \right) e^{-\frac{\kappa}{2}} Z q^2 + r\sigma^2 e^\kappa (1 - \rho^2) > 0$ and $\lim_{\kappa \rightarrow \infty} f(\kappa) = +\infty$, so $\bar{\kappa}$ exists and is unique. Also, for $\rho \neq 0$, define $\kappa_\rho = -\ln(|\rho|)$. Note $f(\kappa_\rho) = 2 \left(1 - \sqrt{|\rho|} \right)^2 Z q^2 + r\sigma^2 \frac{(1 - |\rho|)^2}{|\rho|} > 0$, so $\bar{\kappa} < \kappa_\rho$. Note we can express $f = 2 \left(1 - e^{-\frac{\kappa}{2}} \right)^2 Z q^2 + r\sigma^2 [e^\kappa (1 - |\rho|^2) - 2(1 - |\rho|)]$. Because the first term is

always positive, the second term must be negative when $f = 0$. Therefore, $\frac{\partial f}{\partial r} = \sigma^2 [e^\kappa (1 - \rho^2) - 2(1 - |\rho|)] < 0$. Because $f' > 0$, $\bar{\kappa}$ is increasing in r .

Finally, note it is suboptimal to select $|H_d| > e^\kappa$, because it exposes division managers to greater risk, without affecting beliefs, relative to setting $|H_d| = e^\kappa$. Without symmetry, the only way it would be optimal to set $|H_d| > e^\kappa$ would be if $\gamma = -\rho\beta$, but symmetry precludes this because in that case $|H_d| = |\rho| \leq 1 < e^\kappa$. ■

Proof of Lemma 4. Proof is isomorphic to proof for Lemma 2 and is omitted. ■

Proof of Theorem 4. We are assuming symmetry throughout. Lemma 4 implies that $\hat{q}_d^{HQ} = \hat{q}_{d'}^{HQ} = e^{-\frac{\kappa HQ}{2}} q$. Let us first consider the optimal contract with corner beliefs: $\hat{q}_d^d = e^{-\kappa} q$ but $\hat{q}_{d'}^d = q$. From Lemma 3, $a_d = e^{-\kappa} \beta_d Z_d q_d$. Because the participation constraint binds, $s_d = \frac{r\sigma^2}{2} (\beta^2 + 2\rho\beta\gamma + \gamma^2) - \frac{1}{2} e^{-2\kappa} q^2 \beta^2 Z - \gamma q e^{-\kappa} \beta Z q$. Plugging into the objective,

$$\pi = 2e^{-\frac{\kappa HQ}{2}} (1 - \beta) e^{-\kappa} \beta Z q^2 - r\sigma^2 (\beta^2 + 2\rho\beta\gamma + \gamma^2) + e^{-2\kappa} q^2 \beta^2 Z + 2 \left(1 - e^{-\frac{\kappa HQ}{2}}\right) \gamma e^{-\kappa} \beta Z q^2$$

Thus, $\frac{\partial \pi}{\partial \gamma} = 2 \left(1 - e^{-\frac{\kappa HQ}{2}}\right) e^{-\kappa} \beta Z q^2 - 2r\sigma^2 (\rho\beta + \gamma)$, so $\frac{\partial \pi}{\partial \gamma} = 0$ iff $\gamma = -\hat{M}\beta$, where $\hat{M} = \rho - \frac{e^{-\kappa} Z q^2}{r\sigma^2} \left(1 - e^{-\frac{\kappa HQ}{2}}\right)$. To be on this region, it must be that $\frac{|\gamma|}{\beta} \leq e^{-\kappa}$, so $\gamma = -M\beta$, where $M = \min \left\{ \max \left\{ \hat{M}, -e^{-\kappa} \right\}, e^\kappa \right\}$. Substituting in,

$$\pi = 2e^{-\frac{\kappa HQ}{2}} (1 - \beta) e^{-\kappa} \beta Z q^2 - r\sigma^2 \beta^2 (1 - 2\rho M + M^2) + e^{-2\kappa} q^2 \beta^2 Z - 2 \left(1 - e^{-\frac{\kappa HQ}{2}}\right) M e^{-\kappa} \beta^2 Z q^2$$

$$\frac{\partial \pi}{\partial \beta} = 2e^{-\frac{\kappa HQ}{2}} (1 - 2\beta) e^{-\kappa} Z q^2 - 2r\sigma^2 \beta (1 - 2\rho M + M^2) + 2e^{-2\kappa} q^2 \beta Z - 4 \left(1 - e^{-\frac{\kappa HQ}{2}}\right) M e^{-\kappa} \beta Z q^2$$

Thus, $\frac{\partial \pi}{\partial \beta} = 0$ iff

$$\beta = \frac{e^{-\frac{\kappa HQ}{2}} e^{-\kappa} Z q^2}{\left(2M + 2(1 - M) e^{-\frac{\kappa HQ}{2}} - e^{-\kappa}\right) e^{-\kappa} Z q^2 + r\sigma^2 (1 - 2\rho M + M^2)}$$

This implies that

$$\pi = \frac{e^{-(\kappa HQ + 2\kappa)} Z^2 q^4}{\left(2M + 2(1 - M) e^{-\frac{\kappa HQ}{2}} - e^{-\kappa}\right) e^{-\kappa} Z q^2 + r\sigma^2 (1 - 2\rho M + M^2)}$$

As κ_{HQ} gets big, note that \hat{M} decreases, $M = -e^{-\kappa}$ when κ_{HQ} is big enough. By the same logic as the proof of Theorem 3, this will be optimal when κ and κ_{HQ} are small.

Consider now when headquarters induces division managers to have interior beliefs with positive exposure to both divisions. From Lemma 3, $a = Z e^{-\frac{\kappa}{2}} \gamma^{\frac{1}{2}} \beta^{\frac{1}{2}} q$. Because the participation constraint binds,

$$\pi = 2e^{-\frac{\kappa HQ}{2}} (1 - \beta - \gamma) Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} q^2 - r\sigma^2 (\beta^2 + 2\rho\beta\gamma + \gamma^2) + 3Z [e^{-\kappa} \beta \gamma] q^2$$

Note

$$\frac{\partial \pi}{\partial \beta} = -2e^{-\frac{\kappa HQ}{2}} q^2 Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} + e^{-\frac{\kappa HQ}{2}} (1 - \beta - \gamma) q^2 Z [e^{-\kappa} \gamma]^{\frac{1}{2}} \beta^{-\frac{1}{2}} - 2r\sigma^2 (\beta + \rho\gamma) + 3Z e^{-\kappa} \gamma q^2$$

$$\frac{\partial \pi}{\partial \gamma} = -2e^{-\frac{\kappa HQ}{2}} Z [e^{-\kappa} \beta \gamma]^{\frac{1}{2}} q^2 + e^{-\frac{\kappa HQ}{2}} (1 - \beta - \gamma) q^2 Z [e^{-\kappa} \beta]^{\frac{1}{2}} \gamma^{-\frac{1}{2}} - 2r\sigma^2 (\rho\beta + \gamma) + 3Z e^{-\kappa} \beta q^2$$

Note $\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial \gamma} = 0$ iff

$$\left[e^{-\frac{\kappa HQ}{2}} (1 - \beta - \gamma) q^2 Z e^{-\frac{\kappa}{2}} \beta^{-\frac{1}{2}} \gamma^{-\frac{1}{2}} + 2r\sigma^2 (1 - \rho) + 3Z e^{-\kappa} q^2 \right] (\beta - \gamma) = 0$$

Thus, $\beta = \gamma$, which implies $\frac{\partial \pi}{\partial \beta} = 0$ iff $\beta = \hat{\beta} \equiv \frac{Z q^2 e^{-\frac{\kappa + \kappa_{HQ}}{2}}}{Z q^2 (4e^{-(\kappa_{HQ} + \kappa)} - 3e^{-\kappa}) + 2r\sigma^2 (1 + \rho)}$. This gives HQ payoff

$$\pi = \frac{Z^2 q^4 e^{-(\kappa_{HQ} + \kappa)}}{Z q^2 (4e^{-(\kappa_{HQ} + \kappa)} - 3e^{-\kappa}) + 2r\sigma^2 (1 + \rho)}$$

Finally, consider the optimal contract with interior beliefs, but with negative exposure to the other division: $\beta > 0 > \gamma$. By Lemma 3, $a_d = Ze^{-\frac{\kappa}{2}} |\gamma|^{\frac{1}{2}} \beta^{\frac{1}{2}} q$. The participation constraint binds, so $s = \frac{r\sigma^2}{2} (\beta^2 + 2\rho\beta\gamma + \gamma^2) + 2(-\gamma)^{\frac{3}{2}} Ze^{-\frac{\kappa}{2}} \beta^{\frac{1}{2}} q^2 - \frac{3}{2} e^{-\kappa} \beta_d (-\gamma_d) q_d^2 Z$. Substituting into the objective,

$$\pi = 2e^{-\frac{\kappa_{HQ}}{2}} \left[1 - \beta + \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) \gamma \right] e^{-\frac{\kappa}{2}} \beta^{\frac{1}{2}} (-\gamma)^{\frac{1}{2}} Zq^2 + 3e^{-\kappa} \beta (-\gamma) q^2 Z - r\sigma^2 (\beta^2 + 2\rho\beta\gamma + \gamma^2).$$

Note

$$\begin{aligned} \frac{\partial \pi}{\partial \beta} &= -2e^{-\frac{\kappa_{HQ}}{2}} e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} Zq^2 + e^{-\frac{\kappa_{HQ}}{2}} \left[1 - \beta + \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) \gamma \right] e^{-\frac{\kappa}{2}} \beta_d^{-\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} Zq^2 \\ &\quad + 3e^{-\kappa} (-\gamma_d) q^2 Z - 2r\sigma^2 (\beta + \rho\gamma) \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi}{\partial \gamma} &= 2e^{-\frac{\kappa_{HQ}}{2}} \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{\frac{1}{2}} Zq^2 - e^{-\frac{\kappa_{HQ}}{2}} \left[1 - \beta + \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) \gamma \right] e^{-\frac{\kappa}{2}} \beta_d^{\frac{1}{2}} (-\gamma_d)^{-\frac{1}{2}} Zq^2 \\ &\quad - 3e^{-\kappa} \beta_d q^2 Z - 2r\sigma^2 (\rho\beta + \gamma) \end{aligned}$$

Defining $f(m) = 2 \left(e^{\frac{\kappa_{HQ}}{2}} - 1 \right) e^{-\frac{\kappa_{HQ} + \kappa}{2}} Zq^2 - \frac{r\sigma^2}{m^{\frac{1}{2}}} (1 - m^2)$, $\frac{\partial \pi}{\partial \beta} = \frac{\partial \pi}{\partial \gamma} = 0$ requires $\gamma = -m\beta$ where $m < 1$ solves

$f(m) = 0$. Further, $\frac{\partial \pi}{\partial \beta} = 0$ iff $\beta = \frac{e^{-\frac{\kappa_{HQ} + \kappa}{2}} m^{\frac{1}{2}} Zq^2}{\left[3 + \left(2e^{\frac{\kappa_{HQ}}{2}} - 1 \right) m \right] e^{-\frac{\kappa_{HQ} + \kappa}{2}} m^{\frac{1}{2}} Zq^2 - 3e^{-\kappa} m q^2 Z + 2r\sigma^2 (1 - \rho m)}$. Note this contract is

feasible only if $m > e^{-\kappa}$, or equivalently, if κ_{HQ} is not too big. ■

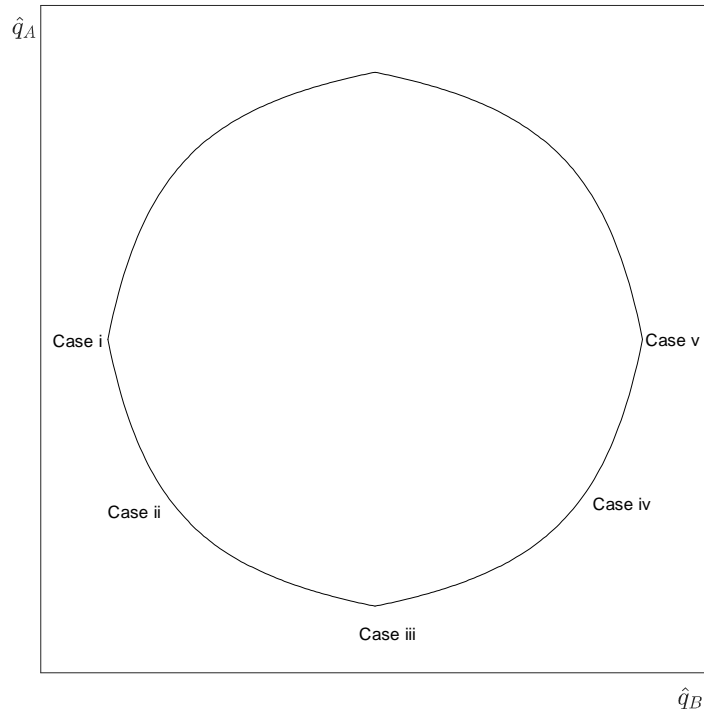


Figure 1: Core of Beliefs

The figure displays the core-belief set from Equation (27) for the following parameter values: $q_A = q_B = 100$ and $\kappa_A = \ln(5)$. It also reports the 5 cases presented in Lemma 1 for division manager A , as follows. Case (i): If the division manager has a large positive exposure to the other division, $H_A > e^{\kappa_A}$, she holds reference belief, q_A , toward her own division but pessimistic toward the other division. Case (ii): if the division manager has relatively balanced positive exposure to the other division, $H_A \in (e^{-\kappa_A}, e^{\kappa_A})$, she is pessimistic toward both divisions. Case (iii): If the division manager has little to no exposure to the other division, $H_A \in (-e^{-\kappa_A}, e^{\kappa_A})$, she is pessimistic toward her division, but she holds the reference belief, q_B , toward the other division. Case (iv): If the division manager has a balanced negative exposure to the other division, $H_A \in (-e^{\kappa_A}, -e^{-\kappa_A})$, she is pessimistic toward her division but optimistic toward the other division. Case (v): If the division manager has a large negative exposure toward the other division, $H_A < -e^{\kappa_A}$, she holds the reference belief, q_A , toward her own division but optimistic toward the other division.

about ECGI

The European Corporate Governance Institute has been established to improve *corporate governance through fostering independent scientific research and related activities*.

The ECGI will produce and disseminate high quality research while remaining close to the concerns and interests of corporate, financial and public policy makers. It will draw on the expertise of scholars from numerous countries and bring together a critical mass of expertise and interest to bear on this important subject.

The views expressed in this working paper are those of the authors, not those of the ECGI or its members.

ECGI Working Paper Series in Finance

Editorial Board

Editor	Mike Burkart, Professor of Finance, London School of Economics and Political Science
Consulting Editors	Franklin Allen, Nippon Life Professor of Finance, Professor of Economics, The Wharton School of the University of Pennsylvania Julian Franks, Professor of Finance, London Business School Marco Pagano, Professor of Economics, Facoltà di Economia Università di Napoli Federico II Xavier Vives, Professor of Economics and Financial Management, IESE Business School, University of Navarra Luigi Zingales, Robert C. McCormack Professor of Entrepreneurship and Finance, University of Chicago, Booth School of Business
Editorial Assistant	Úna Daly, Working Paper Series Manager

Electronic Access to the Working Paper Series

The full set of ECGI working papers can be accessed through the Institute's Web-site (www.ecgi.global/content/working-papers) or SSRN:

Finance Paper Series	http://www.ssrn.com/link/ECGI-Fin.html
-----------------------------	---

Law Paper Series	http://www.ssrn.com/link/ECGI-Law.html
-------------------------	---