Taxes, Ownership and Capital Structure

Giovanna Nicodano
University of Turin and ECGI

Luca Regis
IMT Institute for Advanced Studies Lucca

© Giovanna Nicodano and Luca Regis 2015. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

This paper can be downloaded without charge from: http://ssrn.com/abstract_id=2682570

www.ecgi.org/wp
Taxes, Ownership and Capital Structure

October 2015

Giovanna Nicodano
Luca Regis

We are grateful to Michela Altieri, Bruno Biais, Nicola Branzoli, Marco Da Rin, Jon Danielsson, Gerardo Ferrara, Paolo Fulghieri, Ulrich Hege, Florian Heider, Giulia Iori, Saqib Jafarey, Elisa Luciano, Arie Melnik, Mario Pascoa, Marti Subrahmanyam, Silvia Tiezzi, Guillaume Vuilleme

© Giovanna Nicodano and Luca Regis 2015. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.
Abstract

This paper studies the ownership connection between two units that share a common controlling entity. Our results generate diverse organizations, including the horizontal groups of US family firms and the hierarchical ownership of both multinationals and European groups. The driver of ownership is the optimal capital structure associated with the tax-bankruptcy trade-off. We also examine optimal mutations in response to dividend taxes, to caps on interest deductions and to "no bailout" rules.

Keywords: ownership structure; capital structure; dividend taxes; Thin Capitalization; groups; securitization; private equity.

JEL Classifications: G32, H32.

Giovanna Nicodano*
Professor of Financial Economics
Collegio Carlo Alberto and ESOMAS, University of Turin
Corso Unione Sovietica 218/bis
10100 Torino, Italy
e-mail: giovanna.nicodano@unito.it

Luca Regis
Assistant Professor in the Area of Economics and Institutional Change
IMT Institute for Advanced Studies Lucca and Collegio Carlo Alberto
Piazza San Francesco 19
55100 Lucca, Italy
e-mail: luca.regis@imtlucca.it

*Corresponding Author
Taxes, Ownership and Capital Structure*

Giovanna Nicodano\textsuperscript{a}, Luca Regis\textsuperscript{b}

\textsuperscript{a}Corresponding Author. Collegio Carlo Alberto and ESOMAS, Università di Torino; postal address: Corso Unione Sovietica 218/bis, 10100 Torino, Italy; e-mail:giovanna.nicodano@unito.it.
\textsuperscript{b}IMT School for Advanced Studies Lucca; postal address: Piazza San Francesco 19, 55100 Lucca, Italy; e-mail: luca.regis@imtlucca.it.

Abstract

This paper investigates the ownership connection between two firms that share a common controlling stockholder. Our model generates diverse organizations, including horizontal groups of United States family firms and the hierarchical ownership of both multinationals and European groups. The driver of ownership is the optimal capital structure associated with the tax bankruptcy trade-off. We also examine optimal ownership mutations, in response to dividend taxes, to caps on interest deductions and “no bailout” rules.

\textbf{Keywords:} ownership structure, capital structure, dividend taxes, Thin Capitalization, groups, securitization, private equity.

\textbf{JEL classification:} G32, H32.

This version: December 16, 2015

\textsuperscript{*}The authors thank Michela Altieri, Bruno Biais, Nicola Branzoli, Marco Da Rin, Jon Danielsson, Gerardo Ferrara, Paolo Fulghieri, Ulrich Hege, Florian Heider, Giulia Iori, Saqib Jafarey, Elisa Luciano, Arie Melnik, Mario Pascoa, Marti Subrahmanyam, Silvia Tiezzi, Guillaume Vuilleme and Klaus Zauner for useful suggestions, together with seminar participants at Cass Business School, City University London, the ESSFM at Gerzensee, the FMG at LSE, Surrey and Tilburg University, the 2013 SIE Meetings, the IRMC 2014 and the 13th LAGV Conferences, the XXIII International Conference on Money, Banking and Finance, the XVI Workshop in Quantitative Finance, the 3rd Workshop on Money, Banking and Finance and the GRETA 2015 Conference. Luca Regis gratefully acknowledges financial support from the Crisis Lab project funded by the Italian Ministry of Education.
1. Introduction

Shareholders in control of multiple activities may directly own equity in each unit. This gives rise to a horizontal group, an organization that often characterizes family firms in the U.S. (Masulis et al., 2011; Amit and Villalonga, 2009). Groups are instead hierarchical when the controlling investors own the shares of one unit through the other unit. Outsiders may own minority stakes in group affiliates, but this is not the rule. For instance, parent companies in U.S. multinational groups often fully own their subsidiaries (Lewellen and Robinson, 2013). One common trait of connected units is their reliance on debt financing (Huizinga et al., 2008; Kolasinski, 2009; Bianco and Nicodano, 2006). Sometimes one group affiliate specializes in the raising of most of the debt. Alternatively, debt may appear more evenly spread out across affiliates, albeit with larger debt in the parent company, since the parent company collects most of the dividends from the affiliates. Complex organizations, composed of connected units, represent a large share of world GDP (Altomonte and Rungi, 2013), yet scholars mostly focus on the case of one unit in isolation.

This investigation sets out to explain the ownership and capital structure in two units that share a common controlling entity. We call them “units”, instead of firms or banks, because in our model, as in Leland (2007), there is no explicit production or intermediation activity and, hence, no real synergy. The controlling entity initially owns the stochastic cash flows from its activities. It maximizes their no arbitrage value, net of corporate taxes, with respect to the level of debt in each unit. As in trade-off theory, debt provides a tax shield because interest is deductible from the corporate income tax. At the same time, higher debt increases the expected default costs. The controlling entity also chooses whether to own the second unit (that we will call the “subsidiary”, irrespective of the optimal ownership structure), either directly or indirectly through the first unit (the “parent”). The parent receives dividends in proportion to its equity ownership in the subsidiary. The parent may
also support its subsidiary, after meeting its own debt obligations (Boot et al., 1993).\footnote{Formal and informal bailout commitments are frequent in both industrial and financial groups (Bodie and Merton, 1992; Gorton and Souleles, 2006).}

Our first result rationalizes the hierarchical ownership structure of multinationals and European groups. It indicates that indirect ownership is optimal when the parent tax rate is high, relative to the subsidiary bankruptcy cost rate, inducing the parent company to lever up. Large subsidiary dividends help the parent repaying its own debt, allowing it to better exploit the tax shield. It follows that the parent company will own 100% of its subsidiary shares.

Our second result is a benchmark ownership irrelevance proposition. Subsidiary ownership is irrelevant when the ratio of the parent company tax rate, to its subsidiary default cost, is so small that the parent is optimally unlevered. The zero-leverage affiliate specializes in providing support to the levered affiliate, a role whose value increases in the subsidiary default cost rate. Support, which is conditional on a non-negative cash flow, helps the subsidiary better exploit its tax shield. More importantly, such a levered unit may display any type of ownership connection to the former, including horizontal and hierarchical groups.

This irrelevance result implies that ownership of a subsidiary may sharply react to additional taxes and regulations, which are often associated with hierarchical groups. Therefore, we proceed to investigate the mutations of ownership in response to taxes associated with intercorporate ownership, as well as the limits to either interest deductions or parent support.

Ownership adjusts to binding caps on interest deductions for guaranteed units, that are known as Thin Capitalization (TC) rules. Major OECD countries impose these TC rules, with the aim of limiting interest deductions.\footnote{See, for instance, Her Majesty Revenue and Customs (INTM541010): “Thin capitalization commonly arises where a company is funded...by a third party...but with guarantees...provided to the lender by another group company... The effect of funding a .. company.. with excessive ... guaranteed debt is...excessive interest deductions. It is the possibility of excessive deductions for interest which the ..legislation on thin capitalization seeks to counteract.” We also refer the reader to Webber (2010), for a survey of TC rules around the world.} If enforced in each and every connected unit,
binding TC rules make full intercorporate ownership optimal in our model; this is also the case when the original situation was one of ownership irrelevance. The formerly unlevered parent company becomes levered so as to exploit the tax shield, in order to counterbalance the binding cap on the subsidiary interest deductions. Such a cap, in turn, preserves subsidiary dividends that help support its levered parent. Our numerical analysis illustrates the extent of these effects in calibrations that, following Leland (2007), replicate BBB-rated firms when they are not connected, with a tax rate and proportional bankruptcy costs respectively set to 20% and 23%. A cap on the subsidiary debt, requiring it not to exceed the optimal debt for an unconnected unit ($57), increases the total tax burden to $34.69, up from $25.40 for every $100 of expected cash flow. This is the result of a 37% reduction in the overall optimal leverage, even if the optimal parent debt jumps from $0 to $81. Note, however, that TC rules are effective only if they apply to each and every group affiliate. They are unable to curb interest deductions if, in the case of ownership irrelevance, limits concern proper subsidiaries of hierarchical groups only, as it often happens in practice. In such a case, proper subsidiaries transform into directly owned firms, so as to preserve their tax shield and value.

Both horizontal, as well as pyramidal, groups may become value maximizing organizations in conjunction with taxes on dividends distributed to the parent, the so-called Intercorporate Dividend Taxes (IDT), because optimal indirect ownership decreases in the IDT tax rate. In the numerical calibration with TC rules, when the dividend tax rate increases from 0% to 1%, parent ownership in the subsidiary drops from 100% to 87%. Leverage is also 5% to 20%.

---

3 The use of non-debt tax shelters by the parent (De Angelo and Masulis, 1980; Graham and Tucker, 2006) may increase these tax gains. Additionally, multinationals may exploit the different tax jurisdictions of their subsidiaries (Desai et al., 2004; Huizinga et al., 2008). Our model assumes equal tax rates, so as to focus on an additional tax arbitrage.

4 Blouin et al. (2014) finds that affiliates’ leverage responds to the introduction of TC rules in US multinationals while consolidated leverage does not. Their finding is consistent with debt shifting onto parent companies.

5 See Morck and Yeung (2005) for a historical perspective on dividend taxes and for a discussion of their rationale and their effects on corporate governance.
lower than in the absence of the IDT, because of the reduced dividend support to the parent company. A pyramid emerges if the controlling entity sells the remaining 13% equity in the subsidiary to outsiders, instead of holding it directly. Zero parent ownership is the optimal arrangement, when the IDT rate exceeds the cut-off level of 2%. We also show that an arbitrarily small IDT tax rate makes direct ownership optimal, when optimal ownership in the subsidiary is initially irrelevant.

Finally, a fully integrated hierarchical group is value maximizing when regulators impose a “no bailout” provision. Such a provision, which is, for instance, present in the Volcker rule, bans bailouts of financial conduits by sponsor banks. In such a “no bailout” case, both optimal debt and the tax shield in the subsidiary fall, while the parent-sponsor levers up.

In summary, the indirect ownership of hierarchical groups optimizes the tax bankruptcy trade-off, allowing for support to levered units. This is the only value maximizing arrangement when the tax rate of the parent is initially large, relative to the bankruptcy costs in the subsidiary. Partial indirect ownership or direct ownership are, in turn, optimal when they reduce the burden of the double taxation of dividends. The direct ownership of horizontal groups also avoids caps on interest deductions imposed on proper subsidiaries.

This paper contributes to the theory of corporate ownership. Chemmanur and John (1996) study the joint design of debt and ownership of two projects with the same controlling entity. Their model accommodates a wholly owned group when the entrepreneur has limited wealth: separate incorporation and debt financing of the subsidiary allow for protecting private benefits from control. We also consider the real-world relevance of debt-financed groups, but highlight the role of taxes instead. Taxes directly determine the shape of the group through their influence on the allocation of debt across units that are separately incorporated. Other ownership theories focus on the share of outside shareholders in complex organizations, rather than their structure. In Almeida and Wolfenzon (2006) and Almeida et al. (2011), separate incorporation allows the entrepreneur to use the cash flows from both the parent company and other shareholders in the funding of projects with a lower net
present value. In Zingales (1995), the controlling entity separates ownership from control through a pyramid, to strengthen its bargaining power in future negotiations. In our model, where units have equal expected cash flows and there are no private benefits from control, value is insensitive to ownership dispersion, as in Demsetz and Lehn (1985).

This analysis advances our understanding of capital structure in connected units. Previous studies explain that a merger allows for both a higher debt and higher tax shield, with respect to independent units, because of support between its diversified segments. However, the contagion costs may offset the gains stemming from the tax shield (Leland, 2007). This is why a parent that supports its fully owned subsidiary, after honoring its own obligations, obtains both a tax shield and diversification benefits while avoiding contagion. Such a hierarchical group displays a highly leveraged subsidiary and an unlevered parent when the subsidiary has zero dividend payout (Luciano and Nicodano, 2014). This paper illustrates that the parent company in a hierarchical group may display positive leverage, when the subsidiary has a positive dividend payout. Moreover, it points out that zero leverage firms that specialize in providing support to other affiliates may also belong to horizontal groups.

This paper also provides the first theoretical analysis of taxes targeted at complex organizations. Morck (2005) argues that the introduction of IDT, which is still present in the US tax code, improved on corporate governance during the New Deal by making pyramidal groups more expensive, thereby discouraging such complex ownership structures. Our model indicates that, when full intercorporate ownership is optimal, prior to the introduction of IDT, a sufficiently high IDT rate transforms a wholly owned subsidiary into a partially owned one. Thus, IDT may give rise to a pyramid, unless the statutory IDT tax rate decreases in the ownership share of the parent company. Our model confirms that IDT dismantles

\[\text{Eq. 1}\]

Several papers analyze the effect of personal dividend taxes on the dividend payout, investment and equity issues (Chetty and Saez, 2010), ignoring intercorporate links and leverage. We fix payout, investment and equity issues and analyze how IDT affects intercorporate links and leverage.

This observation provides a rationale for the statutory IDT tax rate in the US tax code, which is decreasing in the ownership share.
pyramidal groups when ownership irrelevance prevails, prior to the introduction of IDT. In this case, however, such an ownership transformation does not affect the corporate leverage and expected default costs.

In the light of our analysis, taxes may help explain some contrasting features of groups in the European Union (EU) and in the US. The EU tax authorities do not tax intercorporate dividends; instead, they cap interest deductions. Hence, EU parent units display a higher leverage than their subsidiaries and often own 100% of their affiliates (Bloch and Kremp, 1999; Bianco and Nicodano, 2006; Faccio and Lang, 2002). Moreover, the association between larger intercorporate dividend payments with parent debt financing is visible in France (De Jong et al., 2012). On the other hand, in the US, intercorporate dividends are taxed unless parent ownership exceeds a high threshold. Accordingly, evidence on family ownership (Amit and Villalonga, 2009; Masulis et al., 2011) shows that direct control via a horizontal structure is most common in the US, while hierarchical ownership is predominant in Europe.

Our model ignores control issues and real synergies across activities to highlight pecuniary gains stemming from the tax bankruptcy trade-off. Despite such a limited focus, some additional ownership patterns also appear to be broadly consistent with its implications. Like the parent company in our ownership irrelevance proposition, the private equity fund is unlevered and contributes to debt restructurings of its highly leveraged portfolio firms. Tax savings in the Leveraged Buyout (LBO) deals contribute to value creation (Acharya et al., 2009; Kaplan, 1989; Renneboog et al., 2007), as in our results. A financial conduit appears as another ownership mutation that optimally exploits the tax bankruptcy trade-off. Selling the cash flow rights of the supported subsidiary to outsiders avoids both IDT and TC rules applying to proper subsidiaries, while enjoying interest deductions.8

---

8In a financial conduit, the sponsoring unit and the investors agree upon the state contingent subsidization of the vehicle, beyond the formal obligations of the sponsor. Conduits, that can be incorporated either as proper subsidiaries or as orphan Special Purpose Vehicles (SPVs), are structured to be tax neutral, as they
The rest of the paper is organized as follows. Section 2 presents the model and characterizes the optimal intercorporate ownership, bailout probability and leverage choices, without additional frictions. Section 3 examines how ownership and capital structure adapt to additional taxes and rules associated with connected units. Section 4 concludes. All proofs are in the Appendix.

2. The model

This section describes our set-up, that follows Leland (2007) in modelling endogenous leverage and bankruptcy costs. The following section provides details on the intercorporate linkages.

At time 0, a controlling entity owns two units, \( i = P,S \). Each unit has a random operating cash flow \( X_i \) which is realized at time \( T \). We denote, with \( G(\cdot) \), the cumulative distribution function; \( f(\cdot) \) represents the density of \( X_i \), identical for the two units; \( g(\cdot,\cdot) \) is the joint distribution of \( X_P \) and \( X_S \). At time 0, the controlling entity selects the face value \( F_i \) of the zero-coupon risky debt to issue, so as to maximize the total arbitrage free value \( \nabla_{PS} \) of equity, \( E_i \), and debt, \( D_i \):

\[
\nabla_{PS} = \max_{F_P,F_S} \sum_{i=P,S} E_i + D_i. \tag{1}
\]

At time \( T \), realized cash flows are distributed to financiers. Equity is a residual claim: shareholders receive operational cash flow net of corporate income taxes and the face value of debt paid back to lenders. A unit is declared insolvent when it cannot meet its debt obligations. Its income, net of the deadweight loss due to default costs, is distributed first to the tax authority and then to the lenders.

The unit pays a flat proportional income tax at an effective rate \( 0 < \tau_i < 1 \) and suffers

---

9The subsidiary, \( S \), can be thought of as the consolidation of all other affiliates.

would otherwise be subject to double taxation (Gorton and Souleles, 2006). Han et al. (2015) illustrates that securitization increases with the corporate tax rate (i.e., with incentives to exploit the tax shield).
proportional dissipative costs \( 0 < \alpha_i < 1 \), in the case of default. Interest on debts are deductible from taxable income.\(^{10}\) The presence of a tax advantage for debt generates a trade-off for the unit: on the one hand, increased leverage results in tax benefits, while on the other, it leads to higher expected default costs since – everything else being equal – a highly levered unit is more likely to default. Maximizing the value of debt and equity is equivalent to minimizing the cash flows the controlling entity expects to lose in the form of taxes \((T_i)\) or of default costs \((C_i)\):

\[
\nu_{PS} = \min_{F_P, F_S} \sum_{i=s_F} T_i + C_i. \tag{2}
\]

The expected tax burden of each unit is proportional to the expected taxable income, that is to the operational cash flow \(X_i\), net of the tax shield \(X_i^Z\). In turn, the tax shield coincides with interest deductions, which are equal to the difference between the nominal value of debt \(F_i\), and its market value \(D_i\): \(X_i^Z = F_i - D_i\). The tax shield is a convex function of \(F_i\).

The expected tax burden in each unit separately – each taken as a stand-alone (SA) unit – is equal to (Leland, 2007):

\[
T_{SA}(F_i) = \tau_i \phi \mathbb{E}[(X_i - X_i^Z)^+] \tag{3},
\]

where the expectation is computed under the risk neutral probability\(^{11}\) and \(\phi\) is the discount factor. Increasing the nominal value of debt increases the tax shield, thereby reducing the tax burden because the market value of debt, \(D_i\), increases with \(F_i\) at a decreasing rate (reflecting a higher risk).

Similarly, expected default costs are proportional to cash flows when a default takes place

\(^{10}\)No tax credits or carry-forwards are permitted.

\(^{11}\)This allows for incorporating a risk premium in the pricing of assets without having to specify a utility function.
(i.e., when net cash flow is insufficient to reimburse lenders). A default occurs when the level of realized cash flow is lower than the default threshold, \( X_i^d = F_i + \frac{\tau_i}{1-\tau_D} D_i \):

\[
C_{SA}(F_i) = \alpha_i \phi \mathbb{E} \left[ X_i 1_{(0<X_i<X_i^d)} \right].
\]  

Default costs represent a deadweight loss to the economy. They increase in the default cost parameter, \( \alpha_i \), as well as in (positive) realized cash flows when the unit goes bankrupt. A rise in the nominal value of debt, \( F_i \), increases the default threshold, \( X_i^d \), thereby increasing the expected default costs.

2.1. Internal Bailouts and Ownership

This section provides details on intercorporate linkages. We first model intercorporate ownership and bailout transfers that characterize complex organizations. Next, we assess how the two links impact on both the tax burden and the default costs of the group, given exogenous debt levels.

The parent owns a fraction, \( \omega \), of its subsidiary’s equity. The subsidiary distributes its profit after paying the tax authority and lenders, \((X^n_S - F_S^*)^+\), where \( X^n_S \) are its cash flows, net of corporate income taxes. Assuming a unit payout ratio, the parent receives a share \( \omega \) of the subsidiary profits at time \( T \). Let the effective (i.e., gross of any tax credit) tax rate on the intercorporate dividend be equal to \( 0 \leq \tau_D < 1 \). Thus, the IDT is equal to a fraction \( \omega \tau_D \) of the subsidiary cash flows. Consequently, the expected present value of the intercorporate dividend net of taxes is equal to:

\[
ID = \phi \omega \mathbb{E} \left[ (1 - \tau_D)(X^n_S - F_S^*)^+ \right].
\]  

The cash flow available to the parent, after receiving the intercorporate dividend, increases to:

\[
X^n_P^{\omega\tau_D} = X^n_P + (1 - \tau_D)\omega(X^n_S - F_S^*)^+.
\]
Equation (6) indicates that dividends provide the parent with an extra buffer of cash that can help it remain solvent in adverse contingencies in which it would default as a stand alone company. It follows that the dividend transfer generates an internal rescue mechanism within the unit combination, whose size increases in the parent ownership, \( \omega \), and falls in the dividend tax rate, \( \tau_D \), given the capital structure.

We do not analyze personal dividend and capital gains taxation levied on shareholders (other than the parent). Therefore, we assume that the positive personal dividend (and capital gains) tax rate is already included in \( \tau_i \), which is indeed an effective tax rate. We also assume that the personal tax rate on distributions is equal across the parent and the subsidiary, so as to rule out straightforward tax arbitrage between the two. Similarly, we focus on the controlling entity’s choice of direct versus indirect ownership, without explicitly involving minority shareholders.

As for the internal bailout probability, we model it following Luciano and Nicodano (2014). The parent chooses the probability of the ex-post cash transfer to the other unit. This promise implies a transfer equal to \( F_S - X^n_S \) from the parent to its subsidiary, if the subsidiary is insolvent but profitable \( (0 < X^n_S < F_S) \) and if the parent stays solvent after the transfer \( (X^n_S - F_P \geq F_S - X^n_S) \). Lenders perceive the promise as being honored with probability \( \pi \). This conditional bailout differs from both internal loans and unconditional guarantees, even when it occurs with certainty. Both alternatives help the subsidiary service its debt, but impair the service of the parent debt. Moreover, both internal loans and contractual guarantees are typically not contingent on positive subsidiary cash flows.\(^{12}\)

We can now show how dividends and the bailout promise affect default costs and the tax burden of the group.

\(^{12}\)In a static model, the ex-post enforcement of bailouts must rely on courts. In practice, enforcement mechanisms vary from reputation (Boot et al., 1993) to the purchase of the junior tranche by the sponsoring parent (De Marzo and Duffie, 1999).
2.2. The Tax Bankruptcy Trade-Off in Complex Organizations

This section adds to the set up intercorporate links, i.e., the presence of a bailout in favor of the subsidiary and intercorporate ownership. In particular, we now analyze how they affect the tax bankruptcy trade-off, given the debt levels, \( F_P \) and \( F_S \). Equations (3) and (4) define the expected tax burden, \( T_{SA}(F_i) \), and default costs \( C_{SA}(F_i) \) for each unit as a stand alone unit. These coincide with group values when there is zero intercorporate ownership (\( \omega = 0 \)) and no bailout promise (\( \pi = 0 \)). Default costs in the subsidiary, \( C_S \), are lower, due to the bailout transfer from the parent. Such a reduction in expected default costs (\( \Gamma \)) is equal to:

\[
\Gamma = C_{SA}(F_S) - C_S = \pi \alpha_S \phi \mathbb{E} \left[ X_S 1_{\{0 < X_S < X_P^d, X_P \geq h(X_P)\}} \right] \geq 0. \tag{7}
\]

Subsidiary expected default costs are lower the higher the probability of the bailout promise and the greater the ability of the parent to rescue its subsidiary. The indicator function \( 1_{\{} \) defines the set of states of the world in which the rescue occurs (i.e., when both the subsidiary defaults without transfers (first term) and the parent has sufficient funds for rescue (second term)). The function \( h \), which is defined in the Appendix, implies that the rescue by the parent is likelier the smaller the parent debt, \( F_P \).

Subsidiary dividends impact on the default costs of the parent, as follows. The cum-dividend cash flow in the parent – defined in equation (6) – is larger when intercorporate ownership, \( \omega \), is larger. Such additional cash flow raises both the chances that the parent is solvent and lenders’ recovery rate in insolvency. Expected default costs saved by the parent, \( \Delta C \), are equal to:

\[
\Delta C = C_{SA}(F_P) - C_P = \alpha_P \phi \mathbb{E} \left[ X_P \left( 1_{\{0 < X_P < F_P\}} - 1_{\{0 < X_P^{\omega} < F_P\}} \right) \right]^+ \geq 0. \tag{8}
\]

The first (second) term in square brackets measures the amount of parent cash flows that
is lost in the default, without (with) the dividend transfer. It is easy to prove that the parent default costs fall in the dividend receipts, net of taxes. These, in turn, increase in $\omega(1 - \tau_D)$ and decrease in subsidiary debt.

Finally, when intercorporate dividends are taxed, the group tax burden increases, relative to the case of two stand alone units. We denote this change as $\Delta T$, defined as:

$$\Delta T = T_S + T_P - T_{SA}(F_P) - T_{SA}(F_S) = \phi \omega \tau_D \mathbb{E}[(X_S^n - F_S)^+] \geq 0. \quad (9)$$

This is positive and increasing in the subsidiary dividend. In turn, the dividend increases in the profits after the service of debt, $(X_S^n - F_S)^+$, and in the intercorporate ownership $\omega$.

### 2.3. Optimal Intercorporate Links and Leverage

This section determines the capital structure ($F_P$ and $F_S$) and intercorporate links ($\pi, \omega$) that maximize the value (i.e., that minimize total default costs and tax burdens of the two units) defined as in Equation (2), solving:

$$\min_{F_S, F_P, \omega, \pi} T_S + T_P + C_S + C_P. \quad (10)$$

Throughout the paper, we maintain the standard technical assumption of the convexity of the objective function, with respect to the face values of debt.\textsuperscript{13} We report the Kuhn-Tucker conditions associated with the minimum program at the beginning of Appendix B. The value-maximizing organization may result in two stand alone units, with no links ($\pi^* = 0, \omega^* = 0$). It may also be a complex hierarchical group, with both intercorporate ownership and a bailout mechanism ($\pi^* > 0, \omega^* > 0$) or an organization with internal bailouts, but no intercorporate ownership ($\pi^* > 0, \omega^* = 0$), as in horizontal groups or in subsidiaries fully

\textsuperscript{13}Extensive numerical analysis shows that convexity in the relevant solution range holds true, unless the effective tax rate is much higher than the proportional default cost rate. In our analysis, fixing the tax rate to 20\%, following Leland’s (2007) calibration, this occurs if the default cost rate is lower than 15\%. 

13
financed by outsiders. Finally, it can be a structure with partially owned subsidiaries, but no bailout promises ($\pi^* = 0, \omega^* > 0$)\textsuperscript{14}. Before proceeding, we introduce the following lemma that summarizes the properties of $\Delta C$ and $\Delta T$, with respect to the debt levels.

**Lemma 1.** Default costs of the parent are non-increasing in intercorporate ownership, $\omega$. The additional tax burden, due to intercorporate dividend taxation (when $\tau_D > 0$), $\Delta T$, is non-increasing in the subsidiary debt, insensitive to parent debt, $F_P$, and non-decreasing in intercorporate ownership $\omega$.

The higher is the subsidiary debt, the lower are the subsidiary dividends, given its exogenous cash flow. This implies lower taxes on incorporate dividends. As for ownership, the higher the share $\omega$, the lower the default costs in the parent, thanks to the dividend payment from its subsidiary. However, the tax burden associated with the intercorporate dividend increases, for a positive IDT tax rate.

The proposition in Lemma 2 deals with the joint determination of leverage and ownership structure, given the bailout promise.

**Lemma 2.** Let $\tau_D = 0$. Then

(i) there exists a $\bar{\omega} > 0$ such that, if $\frac{\tau_D}{\omega_S} < z(\bar{\omega})$, for every $\pi > \bar{\omega}$ the parent is unlevered ($F_P^* = 0$), the subsidiary is levered and the optimal intercorporate ownership share is indefinite;

(ii) there exists a $\bar{\pi} > 0$ such that, if $\frac{\tau_D}{\omega_S} \geq z'(\bar{\pi})$, for every $\pi < \bar{\pi}$ the parent is levered and it fully owns its subsidiary.

The bailout guarantee encourages debt shifting towards the subsidiary. Lemma 2 states that a high probability of a bailout is more likely to free the parent from debt and its associated default costs. This happens when the guarantee is effective enough, and when

\textsuperscript{14}For simplicity, we assume that there is no “piercing of the corporate veil” when intercorporate ownership reaches 100% (i.e., the parent enjoys limited liability vis-à-vis the lenders of its subsidiary also when it is the sole owner of its subsidiary).
the ratio between the parent effective tax rate and the subsidiary default cost is not too high. Indeed, both a low tax rate in the parent and high subsidiary default costs reduce the opportunity costs of a zero leverage parent that specializes in bailing out its subsidiary. If the conditions stated in part (i) hold, the value of the units is therefore insensitive to the intercorporate ownership and dividend receipts, as they do not affect the tax bankruptcy trade-off. If the bailout is unlikely, and/or if the ratio between the tax rate of the parent and the default cost rate in the subsidiary is high enough, part (ii) of Lemma 2 indicates that the value maximizing intercorporate ownership is 100%. This happens because the parent is no longer unlevered and subsidiary dividends help service its debt, thereby allowing the parent to increase its own tax shield. In such a case, setting up two stand alone units \((\omega = 0; \pi = 0)\) is suboptimal for the controlling entity. It is also suboptimal for the controlling entity to directly own shares in the subsidiary, and/or to allow outside shareholders to buy subsidiary shares \((\omega < 1)\).

It is now possible to characterize the optimal intercorporate ownership, the likelihood of the bailout and the associated capital structure.

**Theorem 1.** Assume \(\tau_D = 0\). Then the bailout occurs with certainty \((\pi^* = 1)\). (i) If \(\frac{\tau_S}{\alpha_S} < z(\pi^*)\), intercorporate ownership \((\omega^*)\) is indefinite. Moreover, optimal debt in the complex organization exceeds the debt of two stand-alone companies if and only if the ratio of the percentage default costs to the tax rate \(\frac{\alpha_S}{\tau_S}\) is lower than a constant \(Q\); (ii) if \(\frac{\tau_S}{\alpha_S} \geq z'(\pi^*)\) then the parent is levered and it fully owns its subsidiary.

Theorem 1 characterizes ownership and bailout links together with capital structure. It illustrates that a unit bailout probability is value maximizing. On the one hand, the bailout transfer reduces the subsidiary default likelihood, at any given debt level. Being conditional on positive subsidiary cash flows, the bailout reduces lenders’ recovery upon the default. This increases the spread, thereby reducing the subsidiary tax burden. These effects are
stronger, when the bailout probability is higher. As a consequence, a perfectly credible bailout commitment ($\pi = 1$) is always optimal.

Part (i) shows that the parent is unlevered for a sufficiently low ratio between the parent tax rate, $\tau_P$, and the subsidiary default cost rate, $\alpha_S$. As parent debt falls, the set of states in which the parent may provide a bailout transfer grows. This generates a higher value when the subsidiary default cost rate is higher. It also increases the tax burden in the parent company. That being said, this cost is limited for a sufficiently low tax rate.

The combination of an unlevered parent and a highly levered subsidiary reminds us of the private equity fund with its LBO firms. The private equity fund participates in the restructurings of the firms when they are insolvent, but profitable (a state contingent support mechanism). Leverage contributes to tax savings and value creation (Acharya et al., 2009; Kaplan, 1989; Renneboog et al., 2007) along with efficiency gains (Axelson et al., 2009), that are absent in our set up.

More importantly, part (i) also contains an ownership-irrelevance proposition. As long as the parent company is unlevered and there is no IDT, the controlling entity is indifferent between direct and indirect ownership - as well as to the sharing of subsidiary ownership with outsiders. This irrelevance result indicates that the extreme capital structure in Luciano and Nicodano (2014), obtained under a 100% subsidiary ownership, zero subsidiary dividend and a unit bailout probability, carries over to any intercorporate ownership with a positive payout. This result provides a rationale for zero leverage companies (Strebulaev and Yang, 2013) that do not always belong to hierarchical groups.

Part (ii) provides an explanation for the existence of hierarchical groups with wholly owned subsidiaries (Faccio and Lang, 2002), that are quite pervasive organizations. The parent company becomes levered and fully owns its subsidiary, thereby receiving all its dividends. These groups do appear to have both positive parent leverage and positive subsidiary dividend payouts (Bianco and Nicodano, 2006; De Jong et al., 2012). Part (ii) suggests that indirect ownership optimizes the tax-bankruptcy trade-off allowing for support to all levered
group affiliates, through both dividends and state-contingent bailouts.

Part (i) of the theorem highlights instances when the parent has zero leverage. In some other cases, the subsidiary may display zero optimal leverage, so as to transfer all of its operating income to its levered parent company. However, unreported extensive numerical analysis highlights that the bailout guarantee and dividends have asymmetric effects on the capital structure when the units display the same tax rates and default costs. Indeed, zero parent debt is optimal for a larger set of parametric combinations, implying that the bailout guarantee is more valuable than the dividend transfer. We trace this behavior back to the fact that the dividend transfer occurs even when the parent defaults, thereby increasing bankruptcy losses, while the bailout is conditional on the survival of the subsidiary.

In the next section, we will see how ownership irrelevance breaks down due to the presence of taxes or regulatory measures associated with the group structure.

3. Mutant Ownership

This section analyzes the effects of additional tax provisions on the ownership and capital structure of connected units. The analysis starts from IDT, as they may be able to dismantle the pyramidal groups through the double taxation of dividends (Morck, 2005). It then studies the effects of TC rules that directly cap interest deductions in guaranteed companies, thereby putting an upper bound on the incentive to lever up. We also discuss the effects of group synergies deriving from tax consolidation. Last, but not least, we explore the consequences of a ban on internal bailouts when combined with dividend taxes.

3.1. Neutrality of IDT

So far, we assumed no other tax provision, beside corporate income taxes and interest deductions. The following theorem characterizes optimal intercorporate links and capital structure in the presence of IDT.
**Theorem 2.** Let the tax rate on the corporate dividend be positive \((0 < \tau_D < 1)\). Then:

a) if \(\frac{\tau_P}{\alpha_S} < z(\pi^*)\), the optimal intercorporate ownership is zero \((\omega^* = 0)\), while the capital structure and the probability of bailouts are unchanged.

b) if \(\frac{\tau_P}{\alpha_S} \geq z'(\pi^*)\) then: i) if \(\tau_D > \tau_D\), optimal intercorporate ownership is less than full \((\omega^* < 1)\); ii) if \(\tau_D > \bar{\tau}_D\), then optimal intercorporate ownership is zero \((\omega^* = 0)\) with \(0 < \tau_D \leq \bar{\tau}_D < 1\).

Theorem 2 proves that IDT discourages full intercorporate ownership. Under the conditions of Theorem 2a), as soon as the tax rate \(\tau_D\) is non-null, the optimal intercorporate ownership drops to zero, so as to avoid the double taxation of dividends. Both the certain state contingent bailout and the associated capital structure remain optimal. Indeed, the bailout still ensures the optimal exploitation of the tax bankruptcy trade-off. Theorem 2b) states that zero intercorporate ownership may be optimal even when the parent is levered, provided that the dividend tax rate is high enough.

A real world counterpart of the organization envisaged by Theorem 2 is a horizontal group. The controlling entity and, possibly, outside shareholders, directly buy shares in the “subsidiary”. The latter exploits the interest deductions, thanks to a bailout guarantee from the other unit.

Another organization implied by this theorem is a sponsor with its orphan SPV. In such an organization, the sponsoring parent and investors agree to the state contingent subsidization of the SPV, beyond the formal obligations of the sponsor (Gorton and Souleles, 2006).\(^{15}\) This ensures that the SPV exploits the tax-bankruptcy trade-off effectively, saving on IDT.

Corollary 1 summarizes the effects of IDT.

**Corollary 1.** The introduction of a tax on intercorporate dividend transforms a hierarchical group into either a pyramid, or a horizontal group depending on the dividend tax rate.

\(^{15}\)While guarantees may take several forms, sponsoring banks typically choose indirect credit enhancement methods that minimize the capital requirements (Jones, 2000).
However, such transformations affect neither the value nor the leverage if $\frac{\lambda_C}{\lambda_S} < z(\pi^*)$.

In line with Morck (2005), Corollary 1 highlights the ability of IDT to dismantle the hierarchical groups. In our setting, Corollary 1 points out that dismantling the hierarchical structure may affect neither tax revenues nor dissipative default costs when only the guaranteed unit levered up.

A few remarks are useful. First, the dismantling result holds, as long as the payout ratio is positive and inflexible. If the subsidiary payout ratio is set to zero, the hierarchical group survives following the introduction of IDT when the parent lenders can claim their share of profits in the subsidiary. Second, recall that we collapsed the personal dividend tax into the effective corporate income tax to avoid cumbersome notation. We also set tax rates equal for parent and subsidiary. Theorem 1, and thus, the previous corollary, hold as long as the personal tax rate on dividends from the parent is the same as the one on dividends from its subsidiary. Otherwise, the shift from intercorporate ownership to direct ownership may no longer be neutral, also with an unlevered supporting unit. Third, so far, there are no costs associated with ownership transformations. These can be sizable when real synergies explain the group structure. We discuss this case after considering the TC rules.

3.2. TC rules

Tax authorities know that guaranteed subsidiaries may have too little equity capital (that is, too high leverage), due to the exploitation of the tax shield. This is why they limit the interest deductions in guaranteed units through the so-called TC rules. These measures directly cap interest deductions in subsidiaries or indirectly restrict them by constraining debt/equity ratios below a certain level. Either way, they cause a departure from the optimal

\footnote{Dividend payouts for corporate shareholders appear not to adjust to corporate tax clienteles (Barclay et al., 2009; Dahlquist et al., 2014).}

\footnote{The benefits of ownership mutations may be large, as well. Cooper et al. (2015) illustrates that US business income migrates away from traditional corporations, with a 31.6% average tax rate, into pass-through partnerships, with a 15.9% rate.}
capital structure we described in the previous theorems. We now characterize the optimal capital structure and ownership following the introduction of TC rules.

**Theorem 3.** Let the leverage constraint in the guaranteed unit be binding \((F^*_S = K < F^*_S)\) and let \(0 < \tau_D \leq \bar{\tau}_D < 1\). If \(K \leq \tilde{K}(\alpha_S)\), then:

a) the parent is optimally levered;

b) intercorporate ownership is (a) full \((\omega^* = 1)\) if \(\tau_D = 0\); (b) less than full \((\omega^* < 1)\) if \(\tau_D > \tau_D\), (c) zero for \(\tau_D > \bar{\tau}_D\).

The first part of the theorem shows that debt shifts to the parent, if debt in the subsidiary is constrained to be lower than a level, \(\tilde{K}\). The forced reduction in subsidiary debt makes an unlevered parent suboptimal, even for a low ratio \(\frac{\tau_D}{\alpha_S}\). Forgone gains from using the tax shield are no longer offset by tax shield gains accruing to the subsidiary.

The second part of the theorem states that, for a sufficiently high dividend tax rate, the parent will no longer own all the shares in its subsidiary. This, in turn, may generate a pyramid if shares are sold to outsiders instead of being bought directly by the controlling entity.

Note that Theorem 3 holds true only if the tax authority enforces TC rules in every formally or informally supported unit. If it limits enforcement to proper subsidiaries in hierarchical groups, the neutrality Theorem 2a) characterizing IDT carries over to TC rules. Table 1 exemplifies the results in Theorem 3.

It reports the optimal ownership and capital structure for different values of the Inter-corporate Dividend Tax Rate when subsidiary debt is capped at the stand alone level. The case without TC is reported in the first column, following the Leland (2007) calibration, with \(\tau = 20\%\) and \(\alpha = 23\%\). In such a case, ownership is irrelevant because the parent is unlevered. Moreover, the subsidiary displays high debt and default costs along with a low tax burden. The table illustrates the levels of the dividend tax rate that, together with the TC cap on subsidiary debt, trigger the ownership changes described in Theorem 3. The parent
Table 1: This table reports the optimal ownership, leverage, value, default costs, tax burden and dividend tax levied with different levels of dividend taxation, $\tau_D$. Parameters used in the analysis follow Leland (2007): $\alpha = 23\%$, $\tau = 20\%$, $\phi = 0.78$, $r = 5\%$, $\rho = 0.2$, $\sigma = 0.22$, $X_0 = 100$.

owns 100% of the equity in the supported unit when $\tau_D = 0$; 87% when $\tau_D = 1\%$; 0% when $\tau_D = 7\%$. Hence, the group may be organized as a fully integrated hierarchical group, as a pyramid (with the presence of outside ownership), or as a horizontal group. Both parent leverage and value decrease with the dividend tax rate, because dividend support, which generates additional tax savings, is impaired by IDT. Default costs are dramatically lower than in the case without TC ($8.13$ vs. values ranging from $1.02$ to $1.56$, depending on the IDT rate). They are also decreasing in the dividend tax rate, highlighting the effectiveness of the combination of the two tax policies in containing them.

In summary, both the leverage and value respond to the ownership change induced by IDT. Parent leverage falls, because it receives lower after tax dividends. Value may fall as a consequence, if the reduction in default costs due to the lower leverage offsets the reduction in tax savings. Both Theorem 4 and Figure 1 compare groups to stand alone organizations.

**Theorem 4.** Let the leverage constraint in the subsidiary be binding to the optimal stand alone unit level, $F_{S^*} = F_{S_A}^*$, and $\tau_D > \bar{\tau}_D$. Then, the group shows both higher value and lower leverage than the stand alone organization.

The theorem states that a proper combination of TC rules and IDT may generate groups that are at the same time more valuable and less levered than stand alone organizations. Indeed, TC rules contain leverage in the subsidiary, while a sufficiently high IDT rate prevents
debt shifting onto the subsidiary. The bailout guarantee and dividend payouts allow the
group to obtain a higher value than the stand alone organization, thanks to a lower level of
default costs.

Figure 1 completes the comparison between the connected and stand alone units. In this
figure, the former display both higher leverage and value than the connected units. This
provides additional evidence that connected units are able to optimize the tax-bankruptcy
trade-off, even when subject to tax-related constraints. For instance, the combination of IDT
and TC rules that generates the horizontal group leads to the highest tax burden, and yet,
connecting the units via the bailout link creates value, relative to the case of unconnected
units.

The previous analysis implies that connected companies display extremely different cap-
ital structures, despite the similar tax rates, bankruptcy costs and cash flows. Such compa-
nies may display disparate ownership connection, but also usually have a lower tax burden
than unconnected ones. Group affiliates with a level of debt closer to that of stand alone
firms should systematically display larger intercorporate ownership. These predictions may
contribute to explain the heterogeneity in corporate leverage, first observed in the US by
Bernanke et al. (1990).

3.3. Hierarchical Group Synergies: Tax Consolidation

In the previous sections, group affiliates only exploit financial synergies. They enjoy in-
ternal support transfers and coordinate their capital structure choices to optimize the tax
shield. Intercorporate ownership can generate other synergies, relating for instance to in-
vestment choices (Stein, 1997 and Matvos and Seru, 2014) or to product market competition
and workers’ incentives (Fulghieri and Sevilir, 2011). Real synergies can make both the
ownership and leverage decisions of the firm less responsive to changes in tax rates.

Another important and widespread group-related synergy is tax consolidation, by which
a profitable parent can use subsidiary losses to reduce its taxable income, and viceversa. We
Taxes and optimal connected units

Figure 1: This figure shows value, tax burden, debt and dividend in connected units with: a) no tax provisions (ownership irrelevance); b) Thin Capitalization (TC) rules (TC, full ownership); c) TC rules and 1% Intercorporate Dividend Tax (IDT) rate (TC+IDT, pyramid); d) TC rules and 7% IDT rate (TC+IDT, horizontal group) and e) unconnected units. Parameters used in the analysis follow Leland (2007): $\alpha = 23\%$, $\tau = 20\%$, $\phi = 0.78$, $r = 5\%$, $\rho = 0.2$, $\sigma = 0.22$, $X_0 = 100$. Unconnected units have $\omega = 0$, $\pi = 0$.

discuss the consequences of consolidation in this section. Suppose that the group can exploit the consolidation option whenever its intercorporate ownership exceeds a certain threshold, $\bar{\omega} > 0$.\(^{18}\) Such option is valuable, because it implies that the tax burden of the group never exceeds the one of stand alone units, and is typically smaller. However, a trade-off, involving the choice of leverage, may emerge between the optimization of consolidation gains on the one hand and that of tax shield gains on the other. The controlling entity can avoid such trade-off by setting up separate vehicles, characterized by a low $\frac{\tau}{\alphaS}$ ratio, that sell cash

---

\(^{18}\)Tax consolidation is an option at the Federal level in the US and in other EU jurisdictions, such as France, Italy and Spain, provided intercorporate ownership exceeds some predetermined thresholds. It is forbidden in certain jurisdictions, such as the UK and some US states.
flow rights to outsiders and optimize the tax shield, while the rest of the group exploits consolidation. In this case, the analysis in the previous sections holds for these separate “tax arbitrage vehicles”.

Without ad hoc vehicles, the ownership irrelevance result of Theorem 1i) holds for a sufficiently high cash flow correlation between the units, because the tax shield option is more valuable, relative to the consolidation option, the higher is correlation. For a lower correlation, we conjecture that the case of ownership irrelevance disappears: the minimum optimal intercorporate ownership is at least equal to the prescribed ownership threshold for consolidation, $\bar{\omega} > 0$, in order to trigger consolidation gains on top of tax shield optimization.

The introduction of IDT generates the above-mentioned trade-off. Increasing ownership up to the prescribed threshold, $\bar{\omega}$, lowers the tax burden through consolidation on the one hand, but increases taxes paid on intercorporate dividends on the other. Given a certain $\frac{z_F}{\alpha_3}$ ratio, the presence of consolidation synergies implies that intercorporate ownership drops below 100% for a higher cutoff level of the IDT tax rate in Theorem 2b). At the consolidation threshold, $\bar{\omega}$, a discontinuous increase in the IDT tax rate in order to dismantle the hierarchical group.

Tax rules may help rationalize cross-country ownership patterns, according to the previous analysis. Consolidation benefits, along with the absence of IDT and the presence of TC rules, provide an additional reason for the existence of wholly owned subsidiaries in EU non-financial groups, as well as larger debt raised by parent companies. In the US, the threshold for consolidation ($\bar{\omega} = 80\%$) triggers a zero tax rate on intercorporate dividends. This tax design eliminates the above mentioned trade-off associated with intercorporate ownership. It implies a discontinuity in the presence of hierarchical groups above this consolidation threshold, with higher debt raised by the parent companies. Below this threshold, horizontal groups should be more common.

---

19 A minority interest may however be sufficient for financial conduits.
3.4. No Bailouts

This section analyzes optimal ownership and capital structure when there is no bailout mechanism between the parent and its affiliate. This may represent the outcome of recent prudential rules, because both the Volcker Rule and the Vickers Committee limit the possibility for banking units to bail out their affiliates, incorporated as SPVs.\textsuperscript{20} Such rules set the probability of subsidiary bailout to zero. In our model, this implies that the parent company is levered, because the following lemma, that extends the result in Lemma 2ii), holds:

**Lemma 3.** Let $\tau_D = 0$. Then the parent company is levered if the bailout probability, $\pi$, is lower than a certain level, $0 \leq \pi < \hat{\pi}$.

With enforcement of the Volcker Rule ($\pi = 0$), it never pays to concentrate leverage in the subsidiary. As in Lemma 2ii), the levered parent has an incentive to fully own its subsidiary, because subsidiary dividends reduce the likelihood of its default, without affecting the set of states in which subsidiary defaults. However, the introduction of IDT changes such optimal ownership structure.

<table>
<thead>
<tr>
<th>Table 2: Effects of IDT on ownership and leverage, $\pi = 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ownership ($\omega$)</strong></td>
</tr>
<tr>
<td>$\tau_D = 0%$</td>
</tr>
<tr>
<td>100%</td>
</tr>
<tr>
<td><strong>Value ($\nu$)</strong></td>
</tr>
<tr>
<td>163.67</td>
</tr>
<tr>
<td><strong>Parent Debt ($F_P$)</strong></td>
</tr>
<tr>
<td>94</td>
</tr>
<tr>
<td><strong>Subsidiary Debt ($F_S$)</strong></td>
</tr>
<tr>
<td>42</td>
</tr>
<tr>
<td><strong>Default Costs ($C$)</strong></td>
</tr>
<tr>
<td>1.67 ($1.36;0.31$)</td>
</tr>
<tr>
<td><strong>Tax Burden ($T$)</strong></td>
</tr>
<tr>
<td>34.77 ($16.33;18.44$)</td>
</tr>
<tr>
<td><strong>Dividend Tax (IDT)</strong></td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

Table 2: This table reports the optimal ownership, leverage, value, default costs, tax burden and dividend tax levied with different levels of dividend taxation, $\tau_D$ when bailouts are not allowed, $\pi = 0$. Parameters used in the analysis follow Leland (2007): $\alpha = 23\%$, $\tau = 20\%$, $\phi = 0.78$, $r = 5\%$, $\rho = 0.2$, $\sigma = 22\%$, $X_0 = 100$.

Table 2 numerically illustrates the effects of a ban on bailouts, with different levels of the IDT rate. We return to the parametrization of Leland (2007) that, as discussed in the

---

\textsuperscript{20}\textsuperscript{20}See the discussion in Segura (2014).
previous sections, delivers high leverage in the subsidiary and an unlevered parent when the bailout probability is at its optimal level, \( \pi^* = 1 \). With a zero IDT rate, a ban on bailouts causes a reduction in the subsidiary debt, from 220 in the unrestricted case (see the first column of Table 1) to 42. At the same time, there is some debt shifting towards the parent bank, from zero up to 94. Optimal ownership of the subsidiary is no longer indefinite, and becomes full. The parent company is able to generate additional tax savings by increasing its leverage, while dividends from the subsidiary help reduce the default costs. Thus, the ban on bailouts does reduce default costs from 8.13% to 1.67% of cash flows.

As the IDT tax rate increases, the optimal ownership structure changes. A higher dividend tax \( (\tau_D = 1\%) \) generates a pyramid, in which parent ownership of the subsidiary is 90%. In this case, total debt issued by the organization is slightly higher than in the absence of dividend taxes (137 vs. 136), but more balanced between the two units (86 in the parent, 51 in the subsidiary). However, value is lower (163.20 vs. 163.67) as both the tax burden (34.89 vs. 34.77) and default costs (1.89 vs. 1.67) are higher. If \( \tau_D \) is high (7%), as in the third column, optimal ownership of the subsidiary by the parent falls to zero. In this case, the two units never transfer funds internally and their optimal capital structure decisions coincide with those of stand alone firms. Surprisingly, the outcome of this last situation is an increase in the the default costs with respect to the zero IDT tax rate (1.78 vs. 1.67), even though the overall leverage decrease (114 vs. 136). This exercise suggests that IDT can increase financial instability when a ban on internal bailouts is at work.

4. Summary and Concluding Comments

This paper investigates the link between the ownership and the capital structure of complex organizations, abandoning the textbook fiction of the stand alone firm. It indicates how tax provisions generate the ownership structures of connected units that we commonly observe. The results reveal that fully integrated hierarchical groups emerge when the parent tax rate is sufficiently high, relative to its subsidiary default cost rate. Such groups exploit
all internal supporting mechanisms, including state contingent subsidiary bailouts and subsidiary dividends. However, the introduction of IDT will discourage indirect ownership, thus generating pyramids or horizontal groups.

Separation between leverage and ownership structure decisions holds with unconstrained internal bailouts and interest deductions, when the tax rate of the supporting unit is small, relative to the bankruptcy costs of the supported affiliate. These conditions are likely to characterize the tax arbitrage vehicles, that may reduce the effective tax rate of the supporting unit through either non-debt tax shields or incorporation in low tax jurisdictions. Their unbalanced capital structure, with debt concentrated in one unit that receives support from another unit (firm, or fund or sponsor), allows for high interest deductions from the corporate income tax of the guaranteed unit. These vehicles will display mutant ownership.

Their ownership adaptations are able to neutralize new taxes targeted to specific ownership forms, allowing them to keep their leverage and their tax burden unchanged. IDT, for instance, may transform their ownership shape, if the dividend payout is not flexible, without affecting their leverage. TC rules, if limited to proper subsidiaries, are also unable to contain leverage and tax savings. On the contrary, TC rules targeted to every supported unit may give rise to hierarchical ownership structures.

Our model sidesteps control and agency issues that determine the fraction of equity sold to outside shareholders, so as to highlight the tax motives of complex ownership. Both agency and control measures appear in past empirical studies of complex ownership. Hopefully, future investigations will include tax and bankruptcy provisions as well.
Appendix A. Definition of the $h(\cdot)$ function

The function $h(X_S)$ defines the set of states of the world in which the parent company has enough funds to intervene in saving its affiliate from default while at the same time remaining solvent. The rescue happens if the cash flows of the parent $X_P$ are enough to cover both the obligations of the parent and the remaining part of those of the subsidiary. The function $h(X_S)$, which defines the level of parent cash flows above which the rescue occurs, is defined in terms of the cash flows of the subsidiary as:

$$h(X_S) = \begin{cases} X_P^d + \frac{F_S}{1-\tau} - \frac{X_S}{1-\tau} & X_S < X_S^Z, \\ X_P^d + X_S^d - X_S & X_S \geq X_S^Z. \end{cases}$$

When $X_S < X_S^Z$ the cash flow $X_S$ of the subsidiary does not give rise to any tax payment, as it is below the tax shield generated in that unit.

Appendix B. Proofs

*Kuhn-Tucker conditions of the minimum program*

Before proving the results presented in the paper, let us provide the set of Kuhn-Tucker conditions of the minimization program (10). To keep the notation simple, here and in the following proofs, we only report the dependence of the functions on the parent and subsidiary
debt, specifying the computations at $\omega^*$ and $\pi^*$, when necessary.

\[
\begin{align*}
\frac{dT_{SA}(F_P)}{dF_P} + \frac{dC_{SA}(F_P)}{dF_P} - \frac{\partial \Gamma(F_P,F_S^*)}{\partial F_P} - \frac{\partial C(F_P,F_S^*)}{\partial F_P} + \frac{\partial \Delta T(F_P,F_S^*)}{\partial F_P} &= \mu_1, \\
F_P^* &\geq 0, \\
\mu_1 F_P^* &= 0, \\
\frac{dT_{SA}(F_S^*)}{dF_S} + \frac{dC_{SA}(F_S^*)}{dF_S} - \frac{\partial \Gamma(F_P,F_S^*)}{\partial F_S} - \frac{\partial C(F_P,F_S^*)}{\partial F_S} + \frac{\partial \Delta T(F_P,F_S^*)}{\partial F_S} &= \mu_2, \\
F_S^* &\geq 0, \\
\mu_2 F_S^* &= 0, \\
\mu_1 \geq 0, \mu_2 &\geq 0 \\
- \frac{\partial \Delta C(F_P,F_S^*)}{\partial \omega} + \frac{\partial \Delta T(F_P,F_S^*)}{\partial \omega} &= \mu_3 + \mu_4 \\
\omega^* - 1 &\leq 0 \\
\omega^* &\geq 0 \\
\mu_3 (\omega^* - 1) &= 0 \\
\mu_4 (\omega^*) &= 0 \\
\mu_3 \leq 0, \mu_4 &\geq 0 \\
- \frac{\partial \Gamma(F_P,F_S^*)}{\partial \pi} &= \mu_5 + \mu_6 \\
\pi^* - 1 &\leq 0 \\
\pi^* &\geq 0 \\
\mu_5 (\pi^* - 1) &= 0 \\
\mu_6 (\pi^*) &= 0 \\
\mu_5 \leq 0, \mu_6 &\geq 0 \\
\end{align*}
\]

(B.1)
Proof of Lemma 1

The integral expressions of $\Delta C$ and $\Delta T$ are:

$$
\Delta C = \alpha P \phi \int_{X_S^d}^{+\infty} \int_{X_S^d}^{X_P^d} \frac{xg(x,y)dx}{(X_P^d - \omega(1-\tau_D)[(1-\tau_S)y + \tau S X_S^Z - F_S])} + \frac{xg(x,y)dx}{(X_P^d - \omega(1-\tau_D)[(1-\tau_S)y + \tau S X_S^Z - F_S])} \times
$$

$$
\Delta T = \phi \omega \tau_D \int_{X_S^d}^{+\infty} \left[(1-\tau_S)x + \tau S X_S^Z - F_S\right]f(x)dx.
$$

We now compute the first derivatives of $\Delta C$ and $\Delta T$ with respect to $F_S$ and $F_P$, and we prove our statement:

$$
\frac{\partial \Delta C}{\partial F_P} = \alpha P \phi \frac{\partial X_P^d}{\partial F_P} \int_{X_S^d}^{+\infty} X_P^d g(X_P^d, y)dy + \frac{\partial X_P^d}{\partial F_P} - \omega(1-\tau_D) \int_{X_S^d}^{+\infty} X_P^d \frac{\partial g(X_P^d, y)dy}{X_P^d} \left[(1-\tau_S)y + \tau S X_S^Z - F_S\right] \times
$$

$$
\frac{\partial \Delta T}{\partial F_P} = \phi \omega \tau_D \frac{\partial X_S^Z}{\partial F_P} \int_{X_S^d}^{+\infty} \tau S f(x)dx \geq 0,
$$

$$
\frac{\partial \Delta T}{\partial F_S} = \phi \omega \tau_D \left[\tau S \frac{d X_S^Z}{d F_S} - 1\right] (1 - G(X_S^d)) \leq 0.
$$
The previous set of expressions result from the fact that $\frac{\partial X^d_P}{\partial F_P} \leq 0$, $\frac{\partial X^Z_S}{\partial F_S} \geq 0$.

$$\frac{\partial \Delta C}{\partial \omega} = \alpha P \phi \int_{X^d_S}^{X^d_P} \left[ (1 - \tau_D) \left( (1 - \tau_S)y + \tau_S X^Z_S - F_S \right) \right] \times$$
$$\times \left( X^d_P - \omega (1 - \tau_D) \left( (1 - \tau_S)y + \tau_S X^Z_S - F_S \right) \right) \times$$
$$\times g \left( X^d_P - \omega (1 - \tau_D) \left( (1 - \tau_S)y + \tau_S X^Z_S - F_S \right), y \right) dy \geq 0. \quad (B.3)$$

$\Delta C$ is non-decreasing in $\omega$, as default costs saved in the parent through dividends are higher the higher the dividend transfer from the subsidiary. The change in the tax burden due to IDT is always non-decreasing in $\omega$ as well, as – ceteris paribus – higher dividend taxes are paid when the ownership share is higher:

$$\frac{\partial \Delta T}{\partial \omega} = \phi \tau_D \int_{X^d_S}^{+\infty} \left( x(1 - \tau_S) + \tau_S X^Z_S - F_S \right) f(x) dx \geq 0. \quad (B.4)$$

This derivative has zero value when $\tau_D = 0$.

**Proof of Lemma 2**

Consider the Kuhn-Tucker conditions (i) to (xiii) in (B.1). Under our convexity assumption, these conditions are necessary and sufficient. We investigate the existence of a solution in which $F^*_P = 0$ and $F^*_S > 0$. This implies $\mu_1 \geq 0$ and $\mu_2 = 0$. We focus on condition (iv) first. We have to prove that the term $-\frac{\partial \Delta C(F^*_P = 0, F^*_S)}{\partial F_S} + \frac{\partial \Delta T(F^*_P = 0, F^*_S)}{\partial F_S}$ has a negative limit as subsidiary debt, $F_S$ tends to zero, and a positive limit when $F_S$ goes to infinity, since the rest of the l.h.s. does, under the technical assumptions that $xf(x)$ converges as $x \rightarrow +\infty$ (see Luciano and Nicodano, 2014).

The derivative $\frac{\partial \Delta C(0,F^*_S)}{\partial F_S} = 0$ for every $F_S$. Moreover, $\frac{\partial \Delta T}{\partial F_S}$ is always lower than, or equal to, zero, and has a negative limit as $F_S$ goes to zero, since $\lim_{F_S \rightarrow 0} \frac{\partial X^Z_S}{\partial F_S} = 1 - \phi(1 - G(0)) > 0$. When $F_S$ goes to infinity, $\frac{\partial \Delta T}{\partial F_S}$ goes to zero, as $G(X^d_S) \rightarrow 1$. Hence, we proved that, when $F^*_P = 0$ there exists an $F^*_S > 0$, which solves the equation that equates the l.h.s. of
condition (iv) to zero.

As for condition (i), notice that the derivative \( \frac{\partial \Delta C}{\partial F} \) vanishes at \( F^*_P = 0 \). Hence, we look for conditions for the l.h.s. to be positive and set it equal to \( \mu_1 \) to fulfill the condition. A sufficient condition for the l.h.s. of (i) to be positive is

\[
\frac{\tau_P(1 - \tau_P)G(0)(1 - G(0))}{1 - \tau_P G(0)} \frac{1}{\alpha_S} \leq \pi \left[ \int_{0}^{X^{z_0}_S} xg(x, \frac{F^*_S}{1 - \tau_S} - \frac{x}{1 - \tau_S}) dx \right] + \int_{X^{z_0}_S}^{X^{z_0}_S} xg(x, X^{z_0}_S^d - x) dx . \tag{B.5}
\]

Both sides of this inequality are non-negative. The l.h.s. is increasing in \( \tau_P \) and decreasing in \( \alpha_S \). The r.h.s. is increasing in \( \pi \). When \( \pi = 0 \), it can be satisfied only by \( \tau_P = 0 \). When \( \pi > 0 \), there exists a certain combination of \( \tau_P \) low enough and \( \alpha_S \) high enough such that this condition is satisfied. Thus, we define \( z(\pi) \) as the level of \( \frac{\tau}{\alpha_S} \) below which this condition is satisfied and focus on this situation. As such, for all \( \pi \geq \tilde{\pi} \) the condition is satisfied.

\[\pi \geq \tilde{\pi} \text{ is then a necessary – and sufficient, given our convexity assumption – condition, given } F^*_S, \text{ for the existence of a solution in which } F^*_P = 0.\]

When \( \pi \) is above \( \tilde{\pi} \) and \( \tau_D = 0 \), the dividend from the subsidiary to the parent does not affect the value of the parent, as it does not affect its default costs (\( \Delta C = 0 \) because \( X^{d}_P = 0 \)). Also, \( \Delta T = 0 \) when \( \tau_D = 0 \). Intercorporate ownership \( \omega \) has no effect on the default costs: notice that when \( F^*_P = 0 \), condition (viii) is always satisfied, for any \( \omega \). The tax burden of the subsidiary and its value are independent of \( \omega \): \( \omega^* \) is indefinite and part (i) of our proposition is proven.

Let us now prove part (ii). Let us consider again the Kuhn-Tucker condition (i). We define the level of \( \frac{\tau}{\alpha_S} \) above which such condition is not satisfied at \( (F_P = 0, F_S = F^*_S) \) as \( z'(\pi) \), where \( z'(\pi) \), like \( z(\pi) \), is an increasing function of \( \pi \). Then, when \( \pi > \tilde{\pi} \), leverage is optimally raised by the parent, as there exists no solution in which \( F^*_P = 0 \). We consider now \( \omega^* \) when \( F^*_P > 0 \). When \( \omega^* = 0 \), \( \mu_4 \geq 0 \) and \( \mu_3 = 0 \). Condition (viii) is violated, since the l.h.s. is negative at \( \omega = 0 \) from (B.4). The existence of an interior solution, \( 0 < \omega^* < 1 \), requires both \( \mu_3 = 0 \) and \( \mu_4 = 0 \). Condition (viii) is satisfied only for \( \omega^* \to \infty \), which violates condition (ix). Hence, no interior solution satisfies the Kuhn-Tucker conditions.
Finally, let us analyze the corner solution $\omega^* = 1$, which requires $\mu_3 \leq 0, \mu_4 = 0$. Condition (viii) is satisfied for an appropriate $\mu_3$: all other conditions can be satisfied at $F_S^*, F_P^*, \omega^* = 1$. It follows that $\omega^* = 1$ when $\tau_D = 0$; as such, part (ii) is proven.

Proof of Theorem 1

We first illustrate that the probability of bailouts is equal to 1. First of all, we remark that $-\frac{\partial \pi}{\partial \tau}$ is always negative, as one can easily check from equation (7). It follows that the only value of $\pi^*$ that satisfies the Kuhn-Tucker conditions is $\pi^* = 1$. If $\pi^* \neq 1$, indeed, the right hand side of condition (xvi) is either zero or positive, leading to a violation of the conditions.

It follows immediately from Lemma 2i) that, if $\frac{\tau}{\alpha_S} \leq z(\pi = 1)$, $F_P^* = 0$ and that $\omega^*$ is indefinite. We define $z(\pi^*) = z(\pi = 1)$. As for $F_S^* + F_P^* > 2F_{SA}^*$ if $\alpha/\tau > Q$, we know that $F_S^* > 2F_{SA}^*$ if $\pi = 1, \omega = 1$ and $\alpha/\tau > Q$ (Luciano and Nicodano, 2014). Here, we have $\pi^* = 1$, $F_P^* = 0$ and $F_S$ depends on $\omega$, only through the parent debt. Analogously, part ii) of the Theorem follows directly Lemma 2ii). Consequently, the statement is true.

Proof of Theorem 2

Theorem 1 proves that optimal organizations, absent IDT, are characterized by $\pi^* = 1$, and that, in that case, $F_P^* = 0$ if $\frac{\tau}{\alpha_S} \leq z(\pi^*)$. Let us now introduce IDT under these conditions. When $\tau_D > 0$, $\omega^* = 0$ is the only value of $\omega$ which does not lead to a contradiction of condition (viii). In fact, $\frac{\partial C(0,F_S)}{\partial \omega} = 0$ for every $F_S$, while $\frac{\partial T}{\partial \omega}$ is strictly positive as soon as $\tau_D > 0$, leading to contradiction unless $\omega^* = 0$, and hence, $\mu_3 = 0$. The controlling entity who can freely select ownership optimally sets $\omega^* = 0$ as soon as $\tau_D > 0$, with no influence on the value in the optimal arrangement. Indeed, when $\omega = 0$ both $\Delta C$ and $\Delta T$ are 0 for every $(F_P, F_S)$ couple. An analogous discussion of the Kuhn-Tucker conditions w.r.t. Lemma 2 part (i) allows us to state that, as soon as $\pi > \bar{\pi}$, there exists a solution in which $F_P^* = 0, F_S^* > 0$, even when $\tau_D > 0$, because $\omega^* = 0$. Moreover, we know from Theorem 1
that \( \pi^* = 1 \), the result being independent of \( \tau_D \). As a consequence, the presence or absence of IDT is irrelevant at the optimum for value, capital structure choices, default costs and welfare and part a) is proven.

To prove part b), recall first that, according to Lemma 2, \( \frac{\tau_P}{\alpha_S} > z^* = 1 \) implies \( F_P^* > 0 \). We consider this case and we look for a condition on \( \tau_D \) such that \( \omega^* = 0 \). This implies \( \mu_4 \geq 0, \mu_3 = 0 \) in (B.1). Condition (viii) in (B.1) when \( \omega^* \to 0 \) reads

\[
\begin{align*}
&- \alpha_P \phi (1 - \tau_D) \int_{X^d_S}^{+\infty} \left[ (1 - \tau_S) y + \tau_S X^Z_S - F^*_S \right] X^d_P g(X^d_P, y) dy + \\
&+ \phi \tau_D \int_{X^d_S}^{+\infty} (x(1 - \tau_S) + \tau_S X^Z_S - F^*_S) f(x) dx = \mu_4,
\end{align*}
\]

where we considered that the upper limit of integration, \( \frac{X^d_P}{\omega(1 - \tau_D)(1 - \tau_S)} + X^d_S \), tends to \( +\infty \) when \( \omega \) goes to 0 and we denoted with \( X^Z_i \) and \( X^d_i \) for \( i = P, S \) the thresholds evaluated at the optimum. The l.h.s. of the above equation is non-positive for \( \tau_D = 0 \) and it is increasing in \( \tau_D \), since its first derivative with respect to \( \tau_D \) is strictly positive. It follows that a necessary condition for the existence of a solution where \( \omega^* = 0 \), for given \( F^*_S \) and \( F^*_P \), is that \( \tau_D \) is higher than a certain level \( \bar{\tau}_D \). This quantity depends on \( \alpha_P, \sigma, \rho, \tau_S, \tau_H, \phi, \mu \). If \( \tau_D < \bar{\tau}_D \), then \( \omega^* > 0 \). This proves part i).

Opposite considerations apply when checking solutions where \( \omega^* = 1 \). Condition (viii), evaluated at \( \omega^* = 1 \) is

\[
\begin{align*}
&- \alpha_P \phi \int_{X^d_S}^{x^d_P} \left[ (1 - \tau_D) (1 - \tau_S) y + \tau_S X^Z_S - F^*_S \right] X^d_P g(X^d_P, y) dy \\
&\times \left[ (X^*_P - (1 - \tau_D) (1 - \tau_S) y + \tau_S X^Z_S - F^*_S) \right] \\
&\times g \left( X^d_P - (1 - \tau_D) [(1 - \tau_S) y + \tau_S X^Z_S - F^*_S], y \right) dy + \\
&+ \phi \tau_D \int_{X^d_S}^{x^d_P} (x(1 - \tau_S) + \tau_S X^Z_S - F^*_S) f(x) dx = \mu_3,
\end{align*}
\]
and $\mu_3 \leq 0$. When $\tau_D = 0$ the first term of the sum on the l.h.s. of the equation is negative and the second term disappears, whereas when $\tau_D = 1$ the first term disappears, while the second term is positive. Hence, by continuity, there exists a level of $\tau_D$ that we denote as $\tau_D^*$, above which no solution at $\omega^* = 1$ is present. Notice that under the additional assumption that $g(\cdot, \cdot)$ is non-decreasing in the first argument below $X_D^*$, then $\tau_D^* \leq \tau_D$. This concludes our proof of part b) of the theorem.

Proof of Theorem 3

We prove part (i) of the theorem first. The presence of a cap on subsidiary debt introduces a further constraint in the optimization program: $F_T^{**} \leq K$, where $K$ is the imposed cap and $(F_T^{**}, F_S^{**}, \omega^{**}, \pi^{**})$ denotes the solution to such a constrained program. We thus consider the set of Kuhn-Tucker conditions in (B.1) and modify them appropriately:

$$(iv)' : \frac{\partial T_{SA}(F_S^*)}{\partial F_S} + \frac{\partial C_{SA}(F_S^*)}{\partial F_S} - \frac{\partial \Gamma(F_P^*, F_S^*)}{\partial F_S} - \frac{\partial \Delta C(F_P^*, F_S^*)}{\partial F_S} + \frac{\partial \Delta T(F_P^*, F_S^*)}{\partial F_S} = \mu_2 - \mu_3,$$

$$(vii)' : \mu_1 \geq 0, \mu_2 \geq 0, \mu_3 \geq 0$$

$$(xx)' : \mu_3 (F_S^* - K) = 0.$$

Let us consider the case in which the newly introduced constraint (xx)' is binding, so that $F_S^{**} = K$. We look for the conditions under which the parent can be unlevered. Hence, $\mu_1 \geq 0, \mu_2 = 0, \mu_3 \geq 0$. We focus on condition (i), and we refer the reader to the proof of Lemma 2 for the discussion of other conditions, which is immediate. Condition (i), when
The first term is negative, the second is negative and increasing in \( K \) (as \( X^d_S \) is increasing and convex in \( F_P \)), while the third one is null when \( K = 0 \) and is increasing in \( K \), since its derivative with respect to \( K \) is:

\[
\frac{\partial Y_d^F(0, K)}{\partial F_P} + \int_{X^d_S(0, K)}^{+\infty} \tau_S f(x) dx = \mu_1
\]

The first term is negative, the second is negative and increasing in \( K \) (as \( X^d_S \) is increasing and convex in \( F_P \)), while the third one is null when \( K = 0 \) and is increasing in \( K \), since its derivative with respect to \( K \) is:

\[
\alpha_S \phi \frac{\partial X^d_P(0, F_S)}{\partial F_P} \left( \frac{\partial X^d_F(0, F_S)}{\partial F_S} X^d_S(0, F_S) f(X^d_S, 0) \right) > 0.
\]

It follows that condition (i) can be satisfied only for a \( K \) high enough. We define \( \bar{K}(\alpha_S) \) as the cap above which the parent is optimally unlevered. It solves the following equation:

\[
\alpha_S \phi \frac{\partial X^d_P(0, \bar{K})}{\partial F_P} \left[ \int_0^{X^d_S(0, \bar{K})} xg(x, 1) - \int_{X^d_S(0, \bar{K})}^{+\infty} \tau_S f(x) dx \right] = \mu_1 + \tau_P(1 - G(0)) \frac{\partial X^d_P(0, \bar{K})}{\partial F_P}
\]

Considerations similar to the unconstrained case apply to condition (iv)', which is met at \( F^*_S = K \) by an appropriate choice of \( \mu_3 \). Notice that the higher \( \alpha_S \), the lower the required
cap level K that allows for the presence of an optimally unlevered parent company. This concludes our proof of part (i).

Part (ii) descends directly from Theorem 2b), because it can be easily noticed that its proof relies on the parent being levered.

Proof of Theorem 4

We know from Luciano and Nicodano (2014) that conditional guarantees are value increasing. As a consequence, as soon as \( \pi > 0 \), the value of the parent/subsidiary structure is \( \nu_{PS}(F_{P}^{*}, F_{SA}) \geq 2\nu_{SA}(F_{SA}) \), where \( \nu_{SA}(F_{SA}) = \nu_{PS}(F_{SA}, F_{SA}, \pi = 0, \omega = 0) \). A fortiori, such value is greater than that of two stand alone units when \( \pi^{*} = 1 \) is optimally chosen.

We know from the previous considerations that the f.o.c. for a solution to the PS problem when \( F_{P}^{**} > 0 \) and \( \pi = \pi^{*} = 1 \) include:

\[
\frac{\partial T_{SA}(F_{PS}^{**})}{\partial F_{P}} + \frac{\partial C_{SA}(F_{PS}^{**})}{\partial F_{P}} - \frac{\partial T(F_{P}^{**}, F_{SA}^{*})}{\partial F_{P}} - \frac{\partial \Delta C(F_{P}^{**}, F_{SA}^{*})}{\partial F_{P}} - \frac{\partial \Delta T(F_{P}^{**}, F_{SA}^{*})}{\partial F_{P}} = 0. \tag{B.7}
\]

The equivalent equation in the stand-alone case is simply

\[
\frac{\partial T_{SA}(F_{SA}^{*})}{\partial F_{SA}} + \frac{\partial C_{SA}(F_{SA}^{*})}{\partial F_{SA}} = 0.
\]

We also know that \( \frac{\partial T(F_{P}^{**}, F_{SA}^{*})}{\partial F_{P}} \leq 0 \), since the guarantee is more valuable the lower \( F_{P} \) is, and non-zero as soon as \( \pi > 0 \). Also, when \( \tau_{D} > \tau_{D}, \Delta C = 0 \) and \( \Delta T = 0 \) for all \( F_{P} \) and \( F_{S} \) since \( \omega^{*} = 0 \). Since by our assumption \( T_{SA} + C_{SA} \) is convex in the face value of debt, it follows that \( F_{P}^{**} < F_{SA}^{*} \).

Proof of Lemma 3

Let us analyze the Kuhn-Tucker condition (i) when \( \pi = 0 \) and \( F_{P} = 0 \). The condition is violated for every \( F_{S} \), because \( \frac{\partial T_{SA}(0)}{\partial F_{P}} + \frac{\partial C_{SA}(0)}{\partial F_{P}} < 0 \) because of convexity and the other three terms on the l.h.s. are zero when \( F_{P} = 0 \). As \( \pi \) increases, the l.h.s. increases, since \( \frac{\partial T(0, F_{S})}{\partial F_{P}} \).
is increasing in $\pi$. By continuity, there exists a $\hat{\pi}$ such that condition (i) is satisfied. Then, the other conditions follow as in the proof of Theorem 1. When $\pi < \hat{\pi}$, $F^*_p > 0$, i.e., the parent optimally raises debt.

**References**


Her Majesty’s Revenue and Customs, INTM541010 - Introduction to thin capitalisation (legislation and principles), http://www.hmrc.gov.uk.


