Smart Buyers*

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Abstract

We study transactions in which sellers fears being underpaid because their outside option is better known to the buyer. We rationalize various observed contracts as solutions to such smart buyer problems. Key to these solutions is granting the seller upside participation. In contrast, the lemons problem calls for granting the buyer downside protection. But, in either case, the seller (buyer) receives a convex (concave) claim. Thus, contracts usually associated with the lemons problem, such as debt or cash-equity offers, can be equally well manifestations of the smart buyer problem, although the two information asymmetries have opposite cross-sectional implications. (JEL D82, D86)

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A commonly made assumption is that sellers are better than potential buyers at evaluating an asset. In this paper, we study bilateral transactions under the opposite assumption that the potential buyer of the asset is better informed about the seller’s outside option. We refer to these situations as “smart buyer problems.” As an example, imagine an inexperienced start-up founder who seeks capital. After scrutinizing the start-up’s business plan, an experienced venture capitalist is likely in a better position than the founder to assess the (latent) price the company’s shares might garner from future investors. Alternatively, a strategic acquirer may have a better idea of the start-up’s stand-alone potential in the product market.

The problem in these situations is that the “smart” buyer is tempted to understate the seller’s outside option, and this causes a rational seller to be suspicious of any proposed terms of trade. This contrasts with the lemons problem in which a better informed seller is tempted to overstate the buyer’s inside option. Formally, both are problems of one-sided asymmetric information about a common value, except that the identity of the informed party differs. Despite this apparent difference, the problems are intimately related: by switching the numeraire from cash to the asset (and hence the role of “buyer” and “seller”), the lemons problem can be cast as a smart buyer problem and vice versa. Indeed, mechanism design with common values studies equilibrium existence and efficiency without being explicit about the particular role (e.g., buyer or seller) that the informed agents play (Maskin and Tirole 1992; Jehiel and Moldovanu 2001); the general conditions for truthful mechanisms do not depend on the role of the informed party or whether the informed party wants to overstate or understate the common value.

Numerous papers in economics and finance examine the one-sided asymmetric information problem in specific transactions or (institutional) settings. This applied literature focuses on the “shape” of the optimal contract to explain observed real-world contracts. A prominent example in finance is Myers and Majluf (1984)’s explanation of debt as the optimal security for raising capital from less informed investors. Interestingly, almost all of the work in this literature – starting with the seminal papers by Akerlof (1970) and Spence (1973) – has focused on the lemons problem (see, e.g., the surveys by Riley 2001 and Hörner 2008).

We extend this applied perspective to the smart buyer problem for two reasons: First, in many transactions or settings it is plausible that the buyer, as opposed to
the seller, is better informed. Examples, apart from the aforementioned venture capitalist or strategic investor, include management buyout teams, commercial land developers, or publishers. It is of interest to learn which contractual arrangements help to resolve the smart buyer problem and to which extent they match real-world contracts. Second, it is further of interest to compare contractual solutions to the lemons and smart buyer problems given their close connection. This comparison is an indispensable step for correctly identifying or discriminating the two information frictions in empirical work.

Our analysis yields three main insights: First, purchase offers are persuasive once they include upside participation for the seller. This mutes the buyer’s incentives to underestimate the asset value, and most effectively so through a contract that gives the seller a convex claim. By means of illustration, consider a publisher who knows the market potential of a manuscript. The author suspects that the price is too low and is reluctant to accept a pure cash offer. To overcome this suspicion, the publisher has to combine the cash offer with royalties. Incentive compatibility (truth-telling) requires an inverse relationship between the cash price and royalties such that, for a valuable manuscript, the publisher is willing to pay a higher cash price to keep a larger share of the revenues. Conversely, for a less valuable manuscript, keeping the larger revenue share does not sufficiently compensate the publisher for the higher cash price. Furthermore, under the optimal contract, the author’s royalty share is adjusted upward when the sales volume is very high. Such convex royalties are used in practice and known as “escalation clauses.”

Second, contracts commonly associated with the lemons problem may well be manifestations of smart buyer problems. To overcome the lemons problem, a contract must provide downside protection to the buyer. To outside observers, such a contract is isomorphic to a contract that provides upside participation to the seller. One example of this observational equivalence is debt. As mentioned, Myers and Majluf (1984) show that issuing debt is optimal when investors are less informed. We show that an entrepreneur faced with better informed investors wants to retain the upside. That is, the start-up founder of our opening example would want to issue a more senior claim to the seasoned venture capitalist or even opt for a venture loan.¹ Thus, debt can be optimal, both when investors know less

¹Solomon, S. D. “The wisest entrepreneurs know how to preserve equity,” New York Times,
and when they know more than the issuer. Ownership retention, cash-equity bids, and contingent value rights are other examples of such observational equivalence.

Third, the smart buyer problem and the lemons problem can have opposite cross-sectional implications. An uninformed buyer is most reluctant to trade at high prices, whereas an uninformed seller is most reluctant to trade at low prices. Thus, the relationship between price and quantity is the opposite in the lemons and smart buyer problems. For example, in Leland and Pyle (1977), the informed entrepreneur signals a high firm value by selling a smaller equity stake to uninformed investors. In the smart buyer setting, the informed investor signals a low firm value by buying a smaller equity stake from the uninformed entrepreneur. While the ownership stake retained by the seller is a signal in either case, more retention signals a higher firm value in the lemons problem, but a lower firm value in the smart buyer problem.

Such conflicting predictions are not specific to this example and pose a challenge for empirically testing for the role of asymmetric information. Consider an empiricist who equates asymmetric information frictions (in a particular setting) with the lemons problem and does not find supporting cross-sectional evidence in the transactions data. The empiricist may falsely conclude that asymmetric information is not important, although, in fact, the lemons and smart buyer problems may be simultaneously present in the data such that the cross-sectional traces of asymmetric information wash out in the average effect. This has implications for empirical design: one should not associate a specific contract design with a particular asymmetric information structure (such as debt with the lemons problem). Further, instead of trying to infer the asymmetric information problem from the data, one should try to a priori classify individual transactions into different information structures and separately analyze the cross-sectional patterns for the resultant subsamples.

As mentioned earlier, the financial contracting literature typically assumes that the seller is better informed about the asset value. The main exception is a series of papers showing that in auctions the seller optimally issues debt to informed bid-

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ders (Hansen 1985; Rhodes-Kropf and Viswanathan 2000; DeMarzo, Kremer, and Skrzypacz 2005; Axelson 2007). Confining the competing bidders to debt claims in the underlying asset minimizes differences in their willingness to pay, thereby maximizing the seller’s expected payoff. If, instead, the bidders could choose how to bid, they would keep equity to maximize their own rents (DeMarzo, Kremer, and Skrzypacz 2005).3 By contrast, in the smart buyer problem, buyers may voluntarily confine themselves to debt because doing so most effectively reveals their private information and induces the seller to accept the offer.4

Theories of share repurchases are the other exception. In these theories a firm wants to reveal a high value of its own shares through repurchases, either (1) because it cares about its (interim) share price (e.g., Bhattacharya 1980; Vermaelen 1984; Miller and Rock 1985) or (2) because it plans to subsequently issue shares to fund investment (e.g., John and Williams 1985; Ambarish, John, and Williams 1987; Williams 1988). Clearly, these are not smart buyer problems in which buyers want to conceal high valuations. However, in a recent paper, Bond and Zhong (2016) construct partially separating equilibria in which some firm types pool with types that repurchase shares for the second reason above, with the intention to acquire shares at a bargain – like a smart buyer.

There are, to our knowledge, only two papers (both outside of finance) that explicitly deal with the smart buyer problem: Beggs (1992) studies licensing contracts between a better informed developer and an inventor and shows that the developer can reveal information through linear royalties. We show that the optimal royalty is nonlinear (escalation clauses) and consider additional applications. In independent work, Dari-Mattiacci, Onderstal, and Parisi (2010) compare pool-

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3It has also been shown that sellers may prefer not to issue debt when the bidders have private information about investment costs (Che and Kim 2010) or when there is competition among sellers (Gorbenko and Malenko 2011).

4Related to the above auction literature, DeMarzo (2005) analyzes whether multiple assets should be pooled or tranché prior to a sale. Considering both more and less informed sellers, he shows that less informed sellers pool assets to avoid “cherry-picking” by better informed buyers. A few other papers with privately informed investors, on closer inspection, are applications of the lemons problem. In Inderst and Mueller (2006), a lender privately learns the borrower’s inside option (payoff from borrowing). Their setting is thus a lemons problem: the lender “sells” a loan product that may or may not be good for the borrower. In the takeover models of Hansen (1987) and Eckbo, Giannoni, and Heinke (1990), acquirers have private information, but this information pertains to their own assets. They want to overstate their value, which creates a lemons problem for the target shareholders.
ing outcomes in the generic smart buyer problem to those in the lemons problem. We study pooling and separating equilibria in the smart buyer problem and show that equilibrium refinement selects the optimal separating contract.5

1 Framework

We formulate a generic bilateral trade model that accommodates a variety of transactions involving indivisible and divisible goods, as well as assets that generate verifiable or nonverifiable returns. For each type of transaction, we provide real-world examples and explore whether the smart buyer can signal the common value of the good.

1.1 Model

A buyer approaches a seller, who possesses one unit of a tradable asset. A transaction is characterized by a pair \((q, t)\) where \(q \in Q \subset [0, 1]\) is the traded quantity and \(t\) is the total (net) cash transfer from the buyer to the seller. The payoffs of the seller and buyer from a completed trade are, respectively,

\[
U(q; \cdot) = \theta_x (1 - q) + t, \\
V(q; \cdot) = \theta_x q + z(q; \cdot) - t.
\]

The parameter \(\theta_x\) captures the seller’s outside option, whereas \(z(q; \cdot)\) reflects the gains from trade. That is, \(\theta_x\) is a common value and \(z(q; \cdot)\) is the buyer’s private value component. For example, \(\theta_x\) could be the (expected) price a latent alternative buyer would pay, in which case \(z(q; \cdot)\) would be the value-added by the present buyer relative to the latent alternative buyer. The linearity of the seller’s valuation simplifies the analysis, but is not critical to the qualitative insights.

We assume that \(z = z(q; \theta_x, \theta_z)\) and is twice continuously differentiable. That

\footnote{There is also a large literature on bilateral trading with one-sided asymmetric information about private values (c.f. Bolton and Dewatripont 2004). In such settings, efficiency is restored by giving full bargaining power to the informed party, who then simply sets the price equal to the counterparty’s reservation value. By contrast, shifting bargaining power in our common value setting does not restore efficiency.}
is, the buyer’s private gains depend on the traded quantity \( q \), factors determining the common value (captured by \( \theta_x \)), and other factors (captured by \( \theta_z \)). For example, \( \theta_x \) is the objective quality of a car as the determinant of its market price, and \( \theta_z \) is the buyer’s idiosyncratic pleasure from driving.

**Assumption 1.** \( z \geq 0, z_q > 0, z_{\theta_x} > 0, z_{\theta_z} \geq 0, z_{\theta_x \theta_z} \geq 0, \) and \( z_{q \theta_z} \geq 0 \).

The buyer’s private gains are nonnegative and increase in \( q, \theta_x, \) and \( \theta_z \). Further, they increase marginally more in \( q \) when \( \theta_x \) or \( \theta_z \) are larger. Assumption 1 implies that the single-crossing condition holds and that the efficient outcome is full trade (\( q = 1 \)). For the analysis, \( u(q; \cdot) = \theta_x (1 - q) \) and \( v(q; \cdot) = \theta_x q + z(q; \cdot) \) denote valuations excluding transfers. Last, while not strictly necessary for our results, it makes sense to assume \( z(0; \cdot) = 0 \); if no trade occurs, there are no gains from trade.

The parameters \( \theta_x \) and \( \theta_z \) are continuously distributed on \( \Theta_x \times \Theta_z = [\theta_x, \overline{\theta}_x] \times [\theta_z, \overline{\theta}_z] \) according to a commonly known distribution. The true parameters \( (\theta_x, \theta_z) \) are realized prior to the offer. We now introduce our central assumption.

**Assumption 2.** Only the buyer observes \( \theta_x \).

Put differently, the seller knows less than the buyer about its own outside option. With respect to \( \theta_z \), our analysis presumes symmetric information, so that the buyer’s type (information advantage) is one-dimensional and given by \( \theta_x \in \Theta_x \). For simplicity, we assume that \( \theta_z \) is common knowledge. Nonetheless, the buyer has superior information both about the seller’s outside option \( \theta_x \) and about the gains from trade \( z \), though \( \theta_x \) is a sufficient statistic for the buyer’s superior information about both. (Assuming instead that neither party observes \( \theta_z \) does not change the qualitative results.)

Events unfold as in a standard signaling game: the buyer privately observes \( \theta_x \) and then makes a take-it-or-leave-it offer to the seller. The seller decides whether to accept the offer. If the offer is accepted, the transaction is consummated. Otherwise, the game ends without a transaction. We use perfect Bayesian equilibrium as the solution concept and later the intuitive criterion to refine the set of equilibria.

We now introduce two conditions that render our model applicable to a range of bilateral trade situations, as the examples in the next section illustrate.
**Condition** V: \( v(q; \cdot) \) is verifiable.

Condition V (for “verifiability”) is satisfied when a third party, such as a court, can verify the buyer’s valuation (only) after trade has taken place. If Condition V holds, cash transfers can be made contingent on the buyer’s valuation; otherwise, only fixed transfers (for a given quantity) are feasible. Note that, under Condition V, the buyer’s valuation is verifiable as a whole, but not as individual components.

We also consider two extreme cases of divisibility. We assume that the asset is either indivisible or satisfies Condition D.

**Condition** D: \( Q = [0, 1] \).

Condition D (for “divisibility”) states that the asset is perfectly divisible. When Condition D is not satisfied, the asset is traded in its entirety or not at all.

To see the difference between Conditions V and D, consider, for example, a firm as a collection of real assets. Condition D means that the firm’s real assets can be broken up among different parties (asset sales). Condition V means that the firm can be “shared” by issuing claims to the firm’s cash flow, while the management of the assets is undivided (security issuance). Once Condition V holds, Condition D loses relevance (unless there are restrictions on security design), as trading all of the asset(s) becomes optimal, as we will show in Section 2.3.

Given this characterization of the trade, a purchase contract takes the form

\[
C = (q, t_0, \tau(v))
\]

where \( q \) is the quantity traded, \( t_0 \) is a fixed cash transfer, and \( \tau(v) \) is a cash transfer contingent on the ex post realization of \( v \). The total cash transfer is thus \( t = t_0 + \tau(v) \). When Condition V is violated, \( \tau(v) = 0 \). When Condition D is violated, \( q = 1 \) for any nontrivial purchase offer. Last, let \( U(C; \theta_x) \) and \( V(C; \theta_x) \) denote, respectively, the buyer’s and seller’s expected payoffs for a given common value \( \theta_x \) under contract \( C \).

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*With restrictions on security design, such as linear sharing rules, the buyer may use quantity rationing or profit sharing (or both) as signals, depending on which signaling mechanism is cheaper. This, in turn, depends on the shape of \( z_q(q; \theta_x, \theta_z) \) relative to \( z(1; \theta_x, \theta_z) \). For instance, if the assets are highly complementary, splitting the assets is highly inefficient and therefore an inferior signaling device.*
1.2 Applications

To motivate our premise that a buyer may in certain situations have better knowledge of the seller’s outside option than the seller, consider the following examples:

**A1: Art collector/investor.** An experienced art collector wants to buy a work from a novice artist. The collector derives hedonic utility from complementing an existing collection with this work but also has more experience in assessing its latent market value. Here, \( \theta_z \) reflects idiosyncratic hedonic utility, while \( \theta_x \) reflects the market value. Neither Condition V nor Condition D is satisfied.

**A2: Securities trading.** A sophisticated investor wants to buy securities from a market maker. The investor gains from the trade partly because it hedges risk exposures specific to the current portfolio. These hedging gains are nonverifiable and cannot be shared with the market maker. The investor also has private information about the fundamental value of the securities. Here, \( \theta_z \) captures the idiosyncratic hedging demand, while \( \theta_x \) reflects the securities’ fundamentals. Condition D is satisfied.

**A3: Patent.** A company wants to buy a patent from a scientist to improve its products. The scientist knows less about how valuable the patent is for improving such products. Here, \( \theta_z \) may reflect the company’s product market share, while \( \theta_x \) reflects the patent’s latent market value. Condition V is satisfied, but Condition D is violated if the patent cannot be split up into several saleable parts.

**A4: Restructuring.** The controlling shareholder of a bankrupt firm offers to inject new capital in exchange for partial debt forgiveness. While there is consensus that restructuring is efficient, the controlling shareholder has superior information about the going concern value and the liquidation value. The creditors question the proposed terms. Here, \( \theta_z \) captures the shareholder’s managerial ability, while \( \theta_x \) reflects the liquidation value. Condition V is satisfied since the postrestructuring firm value is verifiable, but condition D is violated insofar as control is not divisible.

**A5: Takeover.** A firm has entered an industry with new promising ideas but little experience, and is now the takeover target of an established rival. As the rival submits its offer, the target is wary of the buy-side valuation of its stand-alone value. Here, \( \theta_z \) reflects acquirer characteristics, while \( \theta_x \) reflects the target’s stand-alone potential. Condition V is satisfied since the posttakeover value of the merged
firm is verifiable, but condition $D$ is violated insofar as control is not divisible.

$A6$: Venture capital. A seasoned venture capitalist wants to invest in a start-up firm and help develop its business and take it public. The venture capitalist contributes useful experience but also knows more about the start-up’s potential market value. The firm founders fear conceding too large a stake. Here, $\theta_z$ captures the venture capitalist’s value-added, while $\theta_x$ reflects the start-up’s intrinsic potential. While Condition V is satisfied, Condition D is violated if the operations of the start-up cannot be split up.

$A7$: Movie rights. A film studio wants to buy the movie rights to a novel. Compared to the seller (writer and/or publishing company), the studio can better assess the box office potential of the novel. The seller is concerned about giving up a “hidden gem.” Here, $\theta_z$ captures the studio’s movie-making capacities, while $\theta_x$ reflects the novel’s box office potential. Condition D is violated if rights to different parts of the novel cannot be sold separately, and Condition V is satisfied since the revenues from the rights to the novel can be shared.

$A8$: Hiring talent. A music producer wants exclusive rights to produce the records of a new musician. While the musician is inexperienced, the producer has a track record of developing new talent. The musician has reservations about some of the contract terms and wonders whether a better deal could be obtained elsewhere or later. Here, $\theta_z$ captures the producer’s capability, and $\theta_x$ reflects the musician’s talent. Condition V is satisfied, while Condition D would be satisfied only if the musician could commit part of the creative output to this producer through a nonexclusive contract.\footnote{In Chari (1982), firms have private information about their own productivity, while workers’ outside options (reservation wages and unit labor costs) are commonly known. Since productivity shocks are observed after labor contracts are written, tension arises between (ex ante) insuring risk-averse workers against shocks and (ex post) truthfully revealing realized shocks. By contrast, no such tension would arise if firms knew their productivity before contracting or workers were risk neutral (as in our model), in which case workers would be paid their outside option. Indeed, the outcome would be efficient as the information advantage concerns a private value. The friction in our “smart employer” setting is different: the firm has private information (not about (its own but) the worker’s productivity. In Spence (1973), of course, workers know their own productivity, and this constitutes a lemons problem.}
2 Persuasive Purchase Offers

In this section, we focus on fully revealing equilibria.\(^8\) By the revelation principle, we can restrict attention to direct mechanisms. A direct mechanism, denoted \(C(\hat{\theta}_x)\), maps a buyer’s self-reported type \(\hat{\theta}_x \in \Theta_x\) into a contract \(C\). In a fully revealing mechanism, the buyer’s optimal report, or contract proposal, satisfies

\[
\arg \max_{\hat{\theta}_x \in \Theta_x} V(C(\hat{\theta}_x), \theta_x) = \theta_x
\]

and also

\[
U(C(\theta_x), \theta_x) \geq \theta_x
\]

for all \(\theta_x \in \Theta_x\). The incentive compatibility constraint (1) states that the buyer’s type is truthfully revealed in the offer, that is, in equilibrium \(\hat{\theta}_x = \theta_x\), while the participation constraint (2) requires that the seller is willing to accept the offer.

We begin our analysis with the case in which the buyer’s payoff is nonverifiable and the asset may or may not be divisible (Section 2.1). Thereafter, we consider the case in which the buyer’s payoff is (ex post) verifiable (Sections 2.2 and 2.3). In either case, we characterize the key properties common to all fully revealing equilibria, and give special attention to the least cost separating (LCS) equilibrium, in which the seller’s participation constraint (2) binds for all \(\theta_x \in \Theta_x\). All mathematical proofs are relegated to the Appendix.

2.1 Upward-sloping supply

Suppose Condition V is violated, but Condition D is satisfied; transfers cannot be contingent, but the asset is divisible. So contracts are of the form \(C = (q, t_0, 0)\).

For a buyer of type \(\theta_x\), the payoff under a given contract is therefore

\[
V(C, \theta_x) = q\theta_x + z(q; \theta_x, \theta_z) - t_0
\]

\(^8\)Despite a slight abuse of terminology, we use the term fully revealing equilibria also for semi-separating equilibria in which not all types trade, but those types that do trade are fully revealed.
and the seller’s participation constraint is
\[ t_0 \geq q \theta_x. \] (4)

**Proposition 1.** *(Trade rationing)* Suppose only Condition D is satisfied. Deterministic fully revealing equilibria exist, and in all of them transfer \( t_0 \) and trade quantity \( q \) increase with buyer type \( \theta_x \). The LCS equilibrium is given by the differential equation
\[ \frac{\partial q}{\partial \theta_x} = \frac{q}{z_q(q; \theta_x, \theta_z)} \] (5)
and \( t_0 = q \theta_x \) under the boundary condition \( q = 1 \) for \( \theta_x = \overline{\theta}_x \).

Lower-valued buyer types purchase smaller quantities at lower prices. Quantity rationing is a means of relinquishing gains from trade to credibly signal a lower valuation. The single-crossing condition implied by Assumption 1 ensures that high-valued types have more (gains from trade) to lose by trading a smaller quantity and hence refrain from mimicking low-valued types. Conversely, low-valued types do not mimic high-valued types because the price increase outweighs their gains from trading more.

There exist multiple separating schedules, all of which satisfy the property that the trade quantity is increasing in the common value, that can be supported as perfect Bayesian equilibria. In any of these equilibria, the buyer appropriates no part of the common value, in turn, implying that the buyer’s type is not signaled by forgoing common value, but rather by relinquished private gains \( z \) when purchasing less than all the asset. The equilibria differ in the magnitude of the rents earned by the seller across buyer types. In the LCS equilibrium, characterized by (5), the seller never earns a rent. That is, the participation constraint binds for every buyer type and the unit price \( t_0/q \) equals the common value \( \theta_x \).

Since quantities and the unit price are increasing in \( \theta_x \), the “supply” curve is upward-sloping, or put differently, the seller demands a quantity premium rather

\[ ^9 \text{The boundary condition follows from the fact that the seller is always willing to sell the entire asset for } t_0 = \overline{\theta}_x \text{ because the true common value cannot be higher. Consequently, the highest buyer type cannot be held to a payoff less than the profit under this offer, which must therefore be the offer she makes in the LCS equilibrium.} \]
than a discount.\footnote{Maskin and Riley (1994) derive conditions under which a monopolistic seller offers quantity discounts to buyers who are better informed about their private valuation of the good. In their setting, however, the seller’s participation constraint above is absent: since there is no common value component, there is no smart buyer problem, and the seller’s reservation price does not increase with the buyer type. Furthermore, as mentioned earlier, giving buyers all the bargaining power in their setting restores efficiency.} By contrast, the lemons problem generates a downward-sloping “demand” curve, under which larger quantities are traded at lower unit prices. For example, capital market illiquidity has been modeled as downward-sloping demand (DeMarzo and Duffie 1999).

Example 1 (Illiquidity). Consider a simple two-period model of financial trade (A2). A buyer and a seller are each endowed with (zero-interest) cash to support trade. The seller is further endowed with one unit of a security that yields an uncertain payoff \( \theta_x \in \Theta_x \) later at date 1, where \( \Theta_x = (1, \theta_x) \).

The seller’s and the buyer’s consumption utilities are, respectively,

\[
    u(c) = c_0 + c_1 \quad \text{and} \quad v(c) = c_0 + (1 + \hat{\theta}_z)c_1,
\]

where \( c_t \) denotes date-\( t \) consumption, and \( \hat{\theta}_z \in \{-\theta_z, \theta_z\} \) is a consumption preference shock. If \( \hat{\theta}_z = -\theta_z \), the buyer is impatient and prefers consumption at date 0. If \( \hat{\theta}_z = \theta_z \), the buyer is patient and prefers consumption at date 1. By contrast, the seller is indifferent with respect to the timing of consumption.

When \( \hat{\theta}_z = -\theta_z \), the buyer uses wealth to consume at date 0, and there is no demand for trading the security. However, when \( \hat{\theta}_z = \theta_z \), the buyer would like to invest some wealth in the security to increase date 1 consumption. If both knew the realization of \( \theta_x \) at date 0, the buyer would simply offer \( t_0 = \theta_x \) and would enjoy additional benefits of \( z(1; \theta_x, \theta_z) = \theta_z \theta_x \). When only the buyer learns the true return \( \theta_x \), fully revealing equilibria are characterized by 1. In particular, condition (5) becomes

\[
\frac{\partial q}{\partial \theta_x} = \frac{q}{\theta_z \theta_x},
\]

since \( z(q; \theta_x, \theta_z) = \theta_z \theta_x q \) in this example. Integrating on both sides and using the
boundary condition $q = 1$ for $\theta_x = \bar{\theta}_x$, yields the LCS equilibrium schedule

$$q = \left( \frac{\theta_x}{\bar{\theta}_x} \right)^{1/\theta_z}. \quad (6)$$

Since the equilibrium per-unit price is $\theta_x$, (6) describes an upward-sloping supply curve. One can invert (6) to derive an equilibrium price function

$$P = \bar{\theta}_x q^{\theta_z}.$$  

The slope of this function $\partial P/\partial q = \theta_x \bar{\theta}_x q^{\theta_z-1}$ reflects the price impact of a given quantity order, akin to Kyle (1985)'s $\lambda$, though not a constant. Supply is upward-sloping: for $q > 0$ (buy orders), the price impact is positive (quantity premium).

Both trade quantity $q$ and price $P$ are strictly increasing in $\theta_z$ for all $\theta_x < \bar{\theta}_x$. (Recall that $q < 1$ for all $\theta_x < \bar{\theta}_x$ and $q = 1$ for $\theta_x = \bar{\theta}_x$.) When noninformational trade motives are less important (smaller $\theta_z$), the seller is more suspicious of buy offers, which translates into less trade and higher price impact (less liquidity).

The equilibrium when both Conditions D and V are violated constitutes a special case of Proposition 1. Under these conditions, a trade is consummated under a contract of the form $C = (1, t_0, 0)$. Given the trade quantity is fixed, there cannot be any deterministic separating equilibrium by the law of one price: two different prices for a given quantity cannot be sustained in equilibrium, since all bidder types would select the lower $t_0$.

To construct separating equilibria, one has to allow for stochastic contracts. Under a stochastic contract $\tilde{C}$, a deterministic contract $C$ is randomly implemented according to a probability distribution, $g(C)$. For a buyer of type $\theta_x$, the expected payoff under a stochastic contract $\tilde{C}$ is

$$\Pi(\tilde{C}; \theta_x) = p[\theta_x + z(1; \theta_x, \theta_z) - t_1^0] - (1-p)t_0^0$$

$$= p[\theta_x + z(1; \theta_x, \theta_z)] - \bar{t}_0.$$

where $p \equiv \Pr(q = 1)$, $t_1^0 \equiv E(t_0 | q = 1)$, $t_0^0 \equiv E(t_0 | q = 0)$, and $\bar{t}_0 \equiv E(t_0)$ under
the probability distribution $g$. The seller’s participation constraint becomes
\[ \bar{t}_0 \geq p\theta_x. \]

The payoff-relevant properties of $\tilde{C}$ are thus summarized by $p$ and $\bar{t}_0$.

Note that (7) and (8) are isomorphic to (3) and (4). This becomes apparent once one relabels $q$ as $p$ and chooses the private value function $z(q; \theta_x, \theta_z) = q\bar{z}$ with $\bar{z} \equiv z(1; \theta_x, \theta_z)$. Thus, Proposition 1 applies. The differential equation (5) characterizing the LCS schedule becomes $\frac{\partial p}{\partial \theta_x} = \frac{p}{\bar{z}}$, which has the explicit solution
\[ p = \exp \left[ -\int_{\theta_x}^{\bar{\theta}_x} [z(1; s, \theta_z)]^{-1} ds \right] \]
under the boundary condition $p = 1$ for $\theta_x = \bar{\theta}_x$. Furthermore, the expected transfer is $\bar{t}_0 = p\theta_x$.

Both trade probability $p$ and expected transfer $\bar{t}_0$ are increasing in buyer type $\theta_x$. A lower-valued buyer credibly reveals her type by accepting a higher risk of trade failure. With less to gain, lower-valued types are less keen on trading and so bid less aggressively. This generates a stochastic upward-sloping supply curve; a higher price makes it more likely that the seller supplies the asset. By contrast, in a lemons problem, higher prices go together with lower trade probabilities.

### 2.2 Equity retention

In this section, we assume that only Condition V holds; while the good cannot be divided, transfers can be made contingent on $v(\cdot)$. A classic application of the lemons problem in which Condition V holds is an entrepreneur who wants to issue equity, but faces less informed investors who are reluctant to buy (Leland and Pyle 1977). By contrast, we conceive of an inexperienced entrepreneur reluctant to sell equity to a sophisticated investor who is better at valuing the firm.

Feasible deterministic contracts now take the form $C = (1, t_0, \tau(v))$. We allow for more general contingent contracts below, but begin with the simple category

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11In the context of tender offers, Hirshleifer and Titman (1990) construct an analogous separating equilibrium in which higher prices go together with higher takeover probabilities.
of (unlevered) equity: linear sharing rules for which \( \tau(v) = (1 - \alpha)v \).

For a buyer of type \( \theta_x \), the payoff from a profit sharing contract is

\[
\mathcal{V}(C, \theta_x) = \alpha[\theta_x + z(1; \theta_x, \theta_z)] - t_0.
\]

(9)

The seller’s participation constraint is \( t_0 + (1 - \alpha)[\theta_x + z(1; \theta_x, \theta_z)] \geq \theta_x \) and can be rearranged to

\[
t_0 \geq \alpha \theta_x - (1 - \alpha)z(1; \theta_x, \theta_z).
\]

(10)

The buyer’s objective function is essentially the same as under (stochastic) trade rationing; relabeling \( \alpha \) as \( p \) and \( t_0 \) as \( \bar{t}_0 \) shows that (9) is isomorphic to (7). The difference between profit sharing and trade rationing is the seller’s participation constraints (10) and (8). While \( \alpha \theta_x \) is the seller’s reservation price, both in (10) and (8), the upfront transfer \( t_0 \) can be smaller than \( \alpha \theta_x \) only in (10). This is because, under profit sharing, the seller also receives \( (1 - \alpha)z(1; \theta_x, \theta_z) \) of the gains from trade through the contingent payment in addition to the upfront transfer.

Emphasizing this point, when the seller’s share of revenue is large, the optimal cash payment may even be negative, in which case the transaction actually switches from a smart buyer problem into a lemons problem. To stay within the confines of the smart buyer problem, that is, to ensure a positive cash transfer \( t_0 \) in equilibrium, we impose the additional assumption that the lowest buyer type \( \theta_x \) generates no trade surplus:

Assumption 3. \( z(1; \theta_x, \theta_z) = 0 \).

This guarantees that our setting remains, in equilibrium, a “means-of-payment” problem in which the buyer pays the seller with an optimally chosen combination of cash and equity.

Proposition 2. (Linear sharing) Suppose only Condition V is satisfied and contingent transfers are restricted to linear sharing rules. Deterministic fully revealing equilibria exist, and in all of them upfront transfer \( t_0 \) and the buyer’s revenue share \( \alpha \) increase with her type \( \theta_x \). If

\[
\alpha = \frac{z_{\theta_x}(1; \theta_x, \theta_z)}{1 + z_{\theta_x}(1; \theta_x, \theta_z)}
\]

(11)
is strictly increasing in $\theta_x \in \Theta_x$, the LCS equilibrium exists and is given by (11) and the schedule $C(\theta_x) = (1, \alpha \theta_x - (1 - \alpha)z(1; \theta_x, \theta_z), (1 - \alpha)v(1; \theta_x, \theta_z))$.

In any equilibrium, there is a negative relationship between the common value $\theta_x$ and the fraction $1 - \alpha$ of equity retained by the seller. The buyer’s willingness to leave more equity with the seller credibly signals a low valuation. Under the equilibrium contract schedule, understating the common value is not profitable because the gains from paying a lower cash price are outweighed by the cost of conceding more equity. Conversely, overstating the common value is not profitable since the gains from a larger share of equity do not compensate for the higher cash price.

An illustration of Proposition 2 is an inexperienced entrepreneur who wants to issue equity. Confronted with smart investors, the entrepreneur is reluctant to give up (equity) ownership to the extent of being wary of being fooled into a bargain — and the suspicion is greater when the offered price is lower.\textsuperscript{12} By contrast, in the lemons problem studied by Leland and Pyle (1977), the relationship between common value and equity retention by the seller is the opposite. A better-informed entrepreneur retains equity because selling (more of) it would be a poor signal, and this concern is more relevant when the common value is higher.

If (11) is strictly increasing for all $\theta_x \in \Theta_x$, a LCS equilibrium exists in which (PC) binds for all types. This is the case when $z_{\theta_x, \theta_z} > 0$ for all $\theta_x \in \Theta_x$. Such convexity of $z$ in $\theta_x$ makes it increasingly unattractive for higher types to relinquish equity to the extent that lower-valued types need not relinquish more equity than required to satisfy the seller’s participation constraint without being mimicked. Otherwise, some buyer types must concede positive rents to the seller. Even so, the buyer can be made weakly better off by conceding rents than by not trading. Thus, an efficient equilibrium always exists in which the entire asset is traded (unlike under deterministic trade rationing).

**Example 2 (Means of payment in takeovers).** Consider a young firm facing a takeover proposal by an established rival (A5). The rival has conducted thorough due diligence, and knows enough to gauge its reservation value $v(1; \theta_x, \theta_z)$.

as well as the target’s stand-alone going-concern value \( \theta_x \). By contrast, being less experienced, the target is unsure about \( \theta_x \).

Suppose

\[
z(1; \theta_x, \theta_z) = (k\theta_x)^{\theta_z}
\]

where \( k > 0 \) and \( \theta_z > 1 \) are the acquirer’s productivity parameters and commonly known. Since

\[
z_{\theta_x\theta_z} = k^{\theta_z} \theta_z (\theta_z - 1) \theta_x^{\theta_z - 2} > 0,
\]

the LCS equilibrium exists and (11) becomes

\[
\alpha = \frac{k^{\theta_z} \theta_z \theta_x^{\theta_z - 1}}{1 + k^{\theta_z} \theta_z \theta_x^{\theta_z - 1}}
\]  

(12)

with

\[
\frac{\partial \alpha}{\partial \theta_x} = \frac{z_{\theta_x\theta_z}}{(1 + k^{\theta_z} \theta_z \theta_x^{\theta_z - 1})^2} > 0.
\]

Given the seller’s binding participation constraint (10), the upfront transfer is

\[
t_0 = \alpha \theta_x - (1 - \alpha) z(1; \theta_x, \theta_z).
\]

Using (12), the per-share cash price \( P \equiv t_0 / \alpha \) is

\[
P = \left(1 - \frac{1}{\theta_z}\right) \theta_x
\]

which is strictly positive and strictly increasing in \( \theta_x \). From \( \frac{\partial \alpha}{\partial \theta_x} > 0 \) and \( \frac{\partial P}{\partial \theta_x} > 0 \), it follows that \( \frac{\partial P}{\partial \alpha} > 0 \). That is, the per-share cash consideration is increasing in the fraction of equity acquired by the buyer (or decreasing in the fraction of equity “paid” to the seller).

The total value of the LCS cash-equity offer equals the seller’s reservation value \( \theta_x \). The fraction of the takeover consideration paid in cash can be expressed as

\[
\frac{t_0}{\theta_x} = \frac{\alpha P}{\theta_x} = \alpha \left(1 - \frac{1}{\theta_z}\right).
\]

Since \( \frac{\partial \alpha}{\partial \theta_x} > 0 \), the “cash ratio” is positively related to the total takeover valuation \( \theta_x \). This is consistent with the empirical evidence that takeover announcement returns are negatively correlated with the proportion of the takeover consideration paid in equity. By contrast, a lemons problem with a better-informed acquirer will
generate the opposite equilibrium relationship. Indeed, to match the empirical pattern, the means-of-payment literature has resorted to models with two-sided asymmetric information (Hansen 1987; Eckbo, Giammarino, and Heinkel 1990).\textsuperscript{13} Our analysis shows that one-sided private information on the part of the acquirer can also explain the empirical pattern.

2.3 Security design

We now relax the restriction to linear sharing rules and allow for general contingent transfers $\tau(\cdot)$. The buyer chooses $C = (1, t_0, \tau(v))$ to maximize

$$V(C; \theta_x) = \theta_x + z(x; \theta_x, \theta_z) - t_0 - E[\tau(v) | \theta_x].$$

(13)

Given that $\theta_z$ is commonly known, the buyer knows exactly how large the contingent payment will be. In fact, $E[\tau(v) | \theta_x] = \tau(v(1; \theta_x, \theta_z))$, which greatly simplifies the contracting problem.

**Proposition 3.** *(Security design)* Suppose Condition V is satisfied. A fully revealing equilibrium exists in which buyer type $\theta_x$ acquires the good in exchange for a fixed transfer $t_0 = \theta_x$ and a contingent transfer

$$\tau(v) = \begin{cases} 
0 & \text{if } v \leq v(1; \theta_x, \theta_z) \\
\ v - v(1; \theta_x, \theta_z) & \text{if } v > v(1; \theta_x, \theta_z)
\end{cases},$$

(14)

thereby retaining the entire trade surplus, as under symmetric information.

Under the contract characterized by (14), the seller is awarded additional payments if the payoff exceeds a certain threshold. If the threshold is set optimally, granting the seller such upside participation prevents the buyer from understating the value of the asset. As a result, all buyer types acquire the entire asset and extract the full surplus.\textsuperscript{14}

\textsuperscript{13}Models in which the means of payment serve to deter potential rival bidders are a notable exception (e.g., Fishman 1988, 1989).

\textsuperscript{14}From a mechanism design perspective, it is not surprising that the buyer can extract the entire gains from trade through the use of contingent payments. In a setting with interdependent values, Mezzetti (2007) shows that mechanisms in which transfers can be conditioned on (reports of) realized payoffs generally allow the principal to extract the entire surplus.
Formally, for \( q = 1 \) and a given \( \theta_z \), the buyer's valuation \( v \) is a one-to-one mapping from \( \Theta_x \) to \([v(1; \theta_x, \theta_z), v(1; \hat{\theta}_x, \theta_z)]\). Moreover, \( v \) is strictly increasing in \( \theta_x \): the buyer incurs a penalty if and only if the realized \( v \) is larger than \( v(1; \hat{\theta}_x, \theta_z) \), the valuation implied by the reported type \( \hat{\theta}_x \). The penalty specified in Proposition 3 requires the buyer to pay the difference \( v - v(1; \hat{\theta}_x, \theta_z) \), and this amounts to writing a call option with strike price \( v(1; \hat{\theta}_x, \theta_z) \). In sum, under this contract, the seller’s overall claim \( t = \tau + t_0 \) is monotone \((\partial t/\partial v \geq 0)\), convex \((\partial^2 t/\partial v^2 \geq 0)\), and respects the buyer’s limited liability \((t \leq v)\).

Such call options resolve the smart buyer problem even when \( \theta_z \) is unobserved by either party. Under Assumption 1, the support of \( v \) also differs across buyer types for unobservable \( \theta_z \). The maximum of the support \( \tau(\theta_x, \theta_z) \) would be strictly increasing in \( \theta_x \), and a penalty scheme almost identical to the one in Proposition 3 would implement the efficient outcome. But, even if the support of \( v \) were identical for all types, an incentive-compatible penalty scheme would exist under additional restrictions on the set of distributions and could be implemented with debt and levered equity (Burkart and Lee 2015).

Example 3 (“20-against-20”). Consider an example of “hiring talent” (A8). A film studio wants an actor for a lead role in a new movie. The studio is better informed about industry factors that determine the actor’s latent outside options \( (\theta_x) \), and it can better estimate the movie’s box office potential \((v(1; \theta_x, \theta_z))\). A producer and a director are already contracted, both well-known and experienced (high \( \theta_z \)).

Otherwise reluctant to commit to the project in hopes of better options, the actor asks for a high salary \((t_0^0)\). Finding the actor’s demand too high \((t_0^0 > \theta_x)\), the studio instead offers to pay the larger of a cash salary \( t_0 \) and a share \( \alpha \) of the revenues \( v \): \( \max\{t_0, \alpha v\} \). In essence, this compensation package amounts to a fixed salary supplemented by a fraction of revenues, provided that the revenues exceed a certain threshold.

Such convex compensation exists in the film industry. One better-known exam-

\(^{15}\)While other contingent payment schemes exist that support an efficient outcome, the proposed scheme is the monotonic claim that minimizes ex post payments.
ple is the so-called “20-against-20” contract, whereby an actor effectively receives the larger of $20 million and 20% of the movie’s gross revenues.\footnote{For example, Tom Cruise signed a 20-against-20 contract for \textit{Valkyrie}.} This creates upside participation; the payoff is flat until the revenues reach $100 million but thereafter increases linearly with further revenues. Weinstein (1998) indeed argues that one explanation for these contracts is that studios are better informed than are actors. Thus, the smart buyer problem offers an alternative explanation for convex compensation contracts, which are usually attributed to moral hazard problems.

The contingent payments in a “20-against-20” contract are, in a way, nonlinear royalties. Such royalties also exist in other intellectual property transactions. Publishing contracts often contain so-called escalation clauses, whereby the publisher pays the author a royalty rate that is increasing in sales. For example, a higher royalty rate may kick in once sales surpass a pre-specified target.\footnote{Goetzmann et al. (2007) find that the (ex post) most successful screenplays traded at higher and less contingent prices, experienced screenwriters are most likely to receive fixed payments, and film studios forecast the box office success of scripts well. These empirical patterns are consistent with our results, suggesting smart buyer problems in the market for screenplays.}

In mergers and acquisitions, nonlinear contingent payments are referred to as contingent value rights (CVRs). In the words of practitioners, when an “acquirer’s offer is spurned as too low by a target corporation, the deal can be \textit{sweetened} [emphasis added] by using CVRs to promise future rewards.”\footnote{Stocker, M. “Shadowy shares: The dark side of contingent value rights,” \textit{Forbes}, May 9, 2011.} For example, earnout clauses specify supplementary payments when the target’s operational or financial performance exceeds predetermined threshold levels within a given time period after the acquisition. Similarly, “antiembarrassment” clauses specify supplementary payments when the buyer resells the asset at a higher price within a prespecified period. Such payments amount to an adjustment of the original price, protecting the original seller from the possible “embarrassment” of having sold the asset for too low a price.\footnote{In the past land purchase programs in India have led to violent protests because the sellers—mainly rural farmers—have felt cheated out of their land. The government acted as a straw buyer to buy the land cheaply at the behest of private developers. In hindsight, the indignant farmers wanted damages in recompense. As for new deals, they have demanded price appreciation rights, that is, a convex claim on the land value. See, for example, Barman, A. “Get the government out of land deals,” \textit{The Economic Times}, September 21, 2010. Venkatesan, J. “Return Noida land
As already mentioned in the previous section, a salient concern of entrepreneurs is that they let investors buy-in at bargain prices. This concern can be alleviated by retaining the (most) convex claim. Entrepreneurs of venture capital financed firms usually obtain more cash flow rights when company performance improves, while the venture capitalists hold cash flow rights senior to those held by the entrepreneur when the firm performs poorly (Kaplan and Stromberg 2003).

Propositions 2 and 3 illustrate two effects that verifiability of $v$ (Condition V) and the use of contingent transfers have on the solution to the smart buyer problem. First, trade becomes efficient, since revenue sharing replaces more wasteful means of relinquishing gains from trade, such as rationing. Second, the buyer appropriates more of the trade surplus in the absence of restrictions on the contract form, since security design enhances the buyer’s ability to commit to truthful behavior.

A noteworthy proviso is that both results rely on the implicit assumption that contingent transfers do not affect the surplus. This is debatable in some applications. For example, CVRs can reduce an acquirer’s incentives to improve a target’s posttakeover value, creating a tension between signaling and incentive provision.

3 Lemons or Smart Buyers?

3.1 Observationally equivalent contracts

The difference between the smart buyer and the lemons problem is reflected in opposite signaling incentives and the respective solutions: an informed seller wants to convey a high value. As a result, downside protection, which recompenses the uninformed buyer if expectations are not met, must be provided. In contrast, an informed buyer wants to convey a low value, which calls for upside participation, whereby the uninformed seller is recompensed if expectations are surpassed. Accordingly, the security design solution of Proposition 3 provides the uninformed seller with a convex claim, $\tau(v) = \max\{0, v - v(1; \theta_x, \theta_z)\}$. As $v$ increases from zero, the payoff from this claim is constant until $v = v(1; \theta_x, \theta_z)$ and thereafter...
Figure 1: Optimal securities

The graph illustrates the security design solution. It plots the value of the contingent claims allocated to the seller and the buyer as functions of total realized value \(v\). The smart buyer optimally reveals her information by offering levered equity to the seller (solid), and consequently retains a debt claim (dashed).

Increases linearly. The informed buyer’s payoff is thus concave, increasing linearly until \(v = v(1; \theta_x, \theta_z)\) and constant thereafter. As Figure 1 illustrates, these claims represent standard securities, debt for the buyer, and levered equity for the seller.

Interestingly, this is also the optimal claim structure when the seller is better informed. For example, according to the pecking order theory, debt best protects less informed investors from buying overvalued securities (Myers and Majluf 1984; DeMarzo and Duffie 1999). In our setting, debt best protects the issuer from selling undervalued securities to better informed investors. Thus, the lemons problem and the smart buyer problem can lead to observationally equivalent contracts.

There are other examples of this equivalence. Consider cash-equity payments in mergers and acquisitions. The standard explanation for the use of equity as a means of payment is based on the assumption that target shareholders have private information about the target (e.g., Hansen 1987; Eckbo, Giammarino, and Heinkel 1990).\(^{21}\) However, the use of equity is, as we have shown, just as rational when the acquirer has private information about the target. Similarly, it has been shown that royalties can convey not only information from licensors to licensees (Gallini and Wright 1990), but also vice versa (Beggs 1992). Analogous arguments apply to other contractual provisions, such as earnouts and “20-against-20.”\(^{22}\) It is therefore fallacious to build empirical tests on the presumption that any of the above contracts – say, debt – are (only) a sign of better-informed sellers.

\(^{21}\)In both models, while both the target and the acquirer are privately informed, the target’s private information gives equity a role as a means of payment. If only the acquirer were privately informed, cash would be the only means of payment in equilibrium.

\(^{22}\)For example, Datar, Frankel, and Wolfson (2001) argue that earnouts resolve the lemons problem, and Weinstein (1998) acknowledges that nonlinear actor salaries could, in principle, also reflect a lemons problem.

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Figure 2: Opposite cross-sectional predictions
The two graphs illustrate that the smart buyer problem and the lemons problem can have opposite cross-sectional implications. In the case of trade rationing, a larger trade quantity signals a higher value in the smart buyer problem, but a lower value in the lemons problem (2A). In the case of ownership structure, a larger stake retained by the issuer signals a lower value in the smart buyer problem, but a higher value in the lemons problem (2B).

### 3.2 Opposite cross-sectional implications

In view of the equivalence discussed above, real-world contracts may be insufficient to empirically identify the underlying information problem. Hence, in addition to the contract shape, one could try to take into account the division of surplus. Signaling costs are borne by the seller in the lemons problem, and by the buyer in the smart buyer problem. For instance, identifying the party willing to pay for third-party verification, such as due diligence or fairness opinions, could help to discriminate between the two information problems. However, in practice, it is difficult to attribute such expenses to one or the other party, because they may be laid out by one party, but accounted for in the transaction price.

Alternatively, one can study how contracts relate to (revealed) common value, which reflects the distribution of rents across common value types. This relation changes with the identity of the informed party. Let us extend the financial trade application (Example 1) by giving the informed party an endowment that it wants to sell in case of impatience, thereby introducing a lemons problem. In equilibrium, the relationship between trade quantity $q$ and common value $\theta_x$ is positive when the informed party wants to buy, but negative when it wants to sell. Identifying trade “direction” is therefore important.\(^{23}\)

Accounting for trade direction also matters in other settings, such as those discussed in Section 2.2. In equity issues, the price is positively related to the

\(^{23}\)The slope of the transaction curves in Figure 2A is similar to the “price impact” in market microstructure models such as Kyle (1985) or Glosten and Milgrom (1985). This is not surprising since market makers in those models deal with better informed buyers and sellers. Unlike empirical contract theory or corporate finance, empirical market microstructure research has therefore long tried to identify trade direction, that is, whether a trade is an informed buy or sell (e.g., Lee and Ready 1991).
stake retained by a better informed issuer, as shown by Leland and Pyle (1977). The opposite relationship obtains in our framework with a less informed issuer. Similarly, in mergers and acquisitions (Example 3), the relationship between target value and the cash-equity ratio reverses when the identity of the informed party switches from buyer to seller.

Being aware of both “contractual equivalence” and “opposite cross-sectional implications” is important when testing for the relevance of asymmetric information frictions. Suppose a takeover study searches for evidence of the lemons problem, but finds that takeover premiums (or posttakeover performance) are not decreasing in the cash-equity ratio. This does not warrant the conclusion that asymmetric information is negligible for the choice of means of payment. In fact, the evidence could be consistent with the smart buyer problem, or the average effect could be weak because both information asymmetries with their countervailing effects are present in the data.\footnote{Interestingly, Chang (1998) documents that stock offerings, on average, have a positive announcement effect on the bidder when the target company is privately held, but a negative one when the target is publicly held. One possible explanation, suggested also by Chang (1998), is that the two settings have opposite information asymmetries.}

3.3 Robustness to competition

One may argue that the smart buyer problem is resolved through competition and therefore lacks relevance in practice. In the model, the problem would indeed disappear in the presence of a second (equally well) informed buyer; the seller could guarantee herself the common value by committing to sell the good in an auction.\footnote{It is worthwhile noting that homogeneous competition with commitment also eliminates the lemons problem: buyer’s informational disadvantage becomes immaterial once they face two equally informed sellers offering identical goods.} However, this case may not arise in practice for several reasons.

First, competition may not emerge, especially in a common value environment, when it is costly to become informed. Once one bidder is already informed, any potential rival does not incur the cost of acquiring information unless it expects to have substantially larger gains from trade. In fact, the potential buyer with the largest expected gains from trade has the strongest incentive to become informed first so as to preempt competition.

\footnote{Interestingly, Chang (1998) documents that stock offerings, on average, have a positive announcement effect on the bidder when the target company is privately held, but a negative one when the target is publicly held. One possible explanation, suggested also by Chang (1998), is that the two settings have opposite information asymmetries.}

It is worthwhile noting that homogeneous competition with commitment also eliminates the lemons problem: buyer’s informational disadvantage becomes immaterial once they face two equally informed sellers offering identical goods.
Second, competition may only be latent. A seller may have to wait or search for a second buyer to receive a competing offer. This is costly and has unknown benefits to the seller. The expected (net) return from turning latent into actual competition is one interpretation of the seller’s outside option in our model.

Third, the smart buyer problem persists even with actual competition if the seller is unable to commit to sell (to the winning bidder). Without commitment, the seller’s beliefs conditional on bid increases matter for the seller’s willingness to trade. This may allow for perfect Bayesian equilibria with suitably chosen off-equilibrium beliefs that deter all bidders from raising their bid above a “pooling” price equal to the (conditional) expected common value. In such an equilibrium, certain bidder types overpay (or underpay), and some may be shut out. In other words, a variant of the smart buyer problem persists.

This is a fortiori the case when bidders have heterogeneous information. In such a setting, the winning bid may be below the winner’s best estimate of the common value. Since the seller cannot fully infer the winner’s best estimate from the bidding process, the two parties may find themselves in a smart buyer problem once the bidding contest is over.

4 Pooling Outcomes

In related work, Dari-Mattiacci, Onderstal, and Parisi (2010) show that in pooling equilibria, a market with informed buyers collapses from the bottom, unlike a market with informed sellers, which collapses from the top. That is, informed buyers find it impossible to buy low-quality assets at low prices, while informed sellers find it impossible to sell high-quality goods at high prices. In our framework, pooling equilibria – like fully revealing equilibria – always exist. However, only fully revealing equilibria survive the intuitive criterion.

Since the security design solution in Proposition 3 achieves the first-best outcome, we must exclude it to create a role for pooling outcomes. We thus abstract from verifiability and focus on trade rationing contracts; only Condition D is satisfied, and contracts take the form $C = (q, t_0, 0)$.

Our analysis focuses on pooling equilibria in which passive buyer types offer the no-trade contract $C_0 \equiv (0, 0, 0)$ and all active buyer types offer a uniform
contract $C_P = (q_P, t_P, 0) \neq C_\emptyset$. Either contract offer must meet the participation constraints of the buyer and the seller. Clearly, $C_\emptyset$ satisfies this condition. Contract $C_P$ meets the buyer’s participation constraint if and only if the buyer’s type $\theta \in \Theta$ satisfies

$$q_P \theta_x + z(q_P; \theta_x, \theta_z) \geq t_P. \quad (15)$$

For a given $C_P$, let $\Theta^P$ denote the subset of buyer types for which (15) holds.

Similarly, $C_P$ satisfies the seller’s participation constraint if and only if

$$t_P \geq q_P E[\theta_x | C_P]. \quad (16)$$

In words, the seller must believe that the offered transfer at least matches the forgone common value, given beliefs that are conditional on the observed offer. To determine the conditional expectation in (16), the seller must conjecture which subset of $\Theta$ prefers $C_P$ over $C_\emptyset$. Let $\hat{\Theta}^P$ denote this conjecture.

In a perfect Bayesian equilibrium, $\{C_P, C_\emptyset\}$ must satisfy (15) and (16), subject to the rational expectations condition $\hat{\Theta}^P = \Theta^P$ and out-of-equilibrium beliefs that prevent deviations to contracts other than $C_\emptyset$ and $C_P$.

**Proposition 4.** Suppose only Condition D is satisfied. Pooling equilibria exist.

Depending on the model specifications, there can be an equilibrium in which all buyer types are active and make the same offer ($\Theta^P = \Theta$). Such an uninformative equilibrium can only exist when the average common value, $E(\theta_x)$, is so low as to permit a sufficiently low pooling price. Otherwise, low-valued buyer types prefer not to trade ($\Theta^P \subset \Theta$), so that the buyer’s decision (not) to trade partially reveals the type. In general, there also can be partially revealing equilibria in which active buyer types are partitioned into several subsets each offering a different pooling contract.

As is common in signaling games, fully revealing and pooling equilibria coexist, though pooling outcomes can be eliminated by applying the intuitive criterion.

**Proposition 5.** Only fully revealing equilibria survive the intuitive criterion.

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26 Despite a slight abuse of terminology, we use the term pooling equilibria also for equilibria in which some types actually choose to be passive and do not trade in equilibrium, while the other types make a uniform offer.
In any equilibrium in which some buyer types pool on the same offer, some types pay more, and others less, than their respective common values. As it turns out, a deviating offer with a lower price and smaller quantity that is attractive only to the overpaying types always exists. Under the intuitive criterion, such a deviation can be attributed only to these types. Among such off-equilibrium beliefs, none can sustain pooling outcomes.

Thus, pooling equilibria are not robust to the intuitive criterion unless overpaying types cannot deviate to a lower quantity due to an exogenous constraint. Yet, imposing a minimum quantity is generally not sufficient. Similar restrictions also would have to be applied to other signaling devices, such as trading probabilities or, if feasible, revenue sharing.

5 Concluding Remarks

Our analysis of bilateral trade frictions and their contractual resolution premises that the buyer is better informed about the seller’s outside option. This outside option, which we posit in reduced form, could be the seller’s (counterfactual) payoff either when retaining the good indefinitely or when seeking out alternative buyers to eventually sell the good. In the latter case, our implicit assumption is that searching for alternative buyers is costly and that the initial buyer has private information about the costs and benefits of doing so. A natural microfoundation for this assumption would be to embed the current model into a search market, in which participants on one side of the market are informed about each other’s valuations, whereas participants on the other side of the market know only their individual valuations. In such a setting, every meeting between potential trading partners results in a smart buyer problem – since one has private information about the other’s outside option – the severity of which would depend on the severity of the search frictions.

It also may be interesting to explore the role of intermediaries when smart buyer and lemons problems coexist. In practice, laypeople frequently employ experts as agents to negotiate trades with the other (better informed) side of the market, often motivated by the fear of otherwise being short-changed. Conversely, better-informed parties sometimes use “front men” to trade on their behalf to avoid
suspicion. This use of third parties by both buyers and sellers has possibly interesting implications for market structure, intermediary contracts, and firm boundaries. These issues, as well as more specific applications of the smart buyer framework, are left for future research.
Appendix

Proof of Proposition 1

A fully revealing equilibrium is characterized by the following optimization problem:

$$\max_{\hat{\theta} \in \Theta} q(\hat{\theta}_x)\theta_x + z(q(\hat{\theta}_x); \theta_x) - t_0(\hat{\theta}_x),$$

subject to the incentive compatibility constraint (IC)

$$q(\theta)x + z(q(\theta); \theta) - t_0(\theta) \geq q(\hat{\theta}_x)\theta_x + z(q(\hat{\theta}_x); \theta_x) - t_0(\hat{\theta}_x) \quad \forall \hat{\theta}_x \in \Theta_x$$

and the participation constraint (PC)

$$[1 - q(\theta_x)]\theta_x + t_0(\theta_x) \geq \theta_x.$$ 

Since the parameter \(\theta_z\) is common knowledge, we subsequently omit it.

In our setting, since the conditions of Theorem 3 in Mailath and von Thadden (2013) are met, incentive-compatibility implies differentiability of the optimal contract schedule. Hence, we can adopt the first-order approach without loss of generality. In doing so, we follow the steps outlined in Baron and Myerson (1982).

Claim. A fully revealing equilibrium schedule \(\{q(\cdot), t_0(\cdot)\}\) is characterized by

$$\max_{\hat{\theta} \in \Theta} q(\hat{\theta}_x)\theta_x + z(q(\hat{\theta}_x); \theta_x) - t_0(\hat{\theta}_x)$$

subject to

- (FOC) \[ q'(\theta_x)\theta_x + zq'(\theta_x) - t'_0(\theta_x) = 0 \]
- (M) \[ q'(\theta_x) > 0 \]
- (PC) \[ [1 - q(\theta_x)]\theta_x + t_0(\theta_x) \geq \theta_x \]

for all \(\theta_x \in \Theta_x\).

Proof. We show that (FOC) and (M) are jointly necessary and sufficient conditions for (IC).

Necessity. Consider two arbitrary types, \(\theta^h_x\) and \(\theta^l_x < \theta^h_x\). Using (IC), the downstream incentive compatibility constraint, which ensures that type \(\theta^h_x\) does
This implies
\[ q(\hat{\theta}_x^h)\hat{\theta}_x^h + z(q(\hat{\theta}_x^l); \hat{\theta}_x^h) - t_0(\hat{\theta}_x^h) \geq q(\hat{\theta}_x^l)\hat{\theta}_x^l + z(q(\hat{\theta}_x^l); \hat{\theta}_x^h) - t_0(\hat{\theta}_x^l). \]

Similarly, the upstream incentive compatibility constraint, which ensures that type \( \hat{\theta}_x \) does not report \( \hat{\theta}_x = \theta_x^h \), is
\[ q(\hat{\theta}_x^l)\hat{\theta}_x^l + z(q(\hat{\theta}_x^l); \hat{\theta}_x^l) - t_0(\hat{\theta}_x^l) \geq q(\hat{\theta}_x^h)\hat{\theta}_x^h + z(q(\hat{\theta}_x^h); \hat{\theta}_x^l) - t_0(\hat{\theta}_x^h). \]

Isolating \( t_0(\hat{\theta}_x^h) - t_0(\hat{\theta}_x^l) \) on one side in each of the two constraints, we can combine them into
\[
[q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^h + z(q(\hat{\theta}_x^h); \hat{\theta}_x^h) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^h) \geq t_0(\hat{\theta}_x^h) - t_0(\hat{\theta}_x^l) \geq [q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^l + z(q(\hat{\theta}_x^h); \hat{\theta}_x^l) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^l). \quad (17)
\]

This implies
\[
[q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^h + z(q(\hat{\theta}_x^h); \hat{\theta}_x^h) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^h) \geq [q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^l + z(q(\hat{\theta}_x^h); \hat{\theta}_x^l) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^l),
\]

which can only hold if \( q(\hat{\theta}_x^h) > q(\hat{\theta}_x^l) \). For \( q(\hat{\theta}_x^h) < q(\hat{\theta}_x^l) \), \([q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^h < [q(\hat{\theta}_x^h) - q(\hat{\theta}_x^l)]\hat{\theta}_x^l\) and \( z(q(\hat{\theta}_x^h); \hat{\theta}_x^h) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^h) \leq z(q(\hat{\theta}_x^h); \hat{\theta}_x^l) - z(q(\hat{\theta}_x^l); \hat{\theta}_x^l) \) since \( z_{x\hat{\theta}_x} \geq 0 \) by Assumption 1, so (17) would be violated. For \( q(\hat{\theta}_x^h) = q(\hat{\theta}_x^l) \), both types would prefer the offer with the smaller transfer. Dividing (17) and \( q(\hat{\theta}_x^h) > q(\hat{\theta}_x^l) \) each by \( (\hat{\theta}_x^h - \hat{\theta}_x^l) \) and taking the limit \( (\hat{\theta}_x^h - \hat{\theta}_x^l) \to 0 \) implies, respectively, (FOC) and (M).

**Sufficiency.** Differentiating the objective function with respect to the choice variable \( \hat{\theta}_x \) yields
\[ q'(\hat{\theta}_x)\hat{\theta}_x + z_q(q'(\hat{\theta}_x), \theta_x)q'(\hat{\theta}_x) - t'_0(\hat{\theta}_x). \]

Using \( t'_0(\hat{\theta}_x) = q'(\hat{\theta}_x)\hat{\theta}_x + z_q q'(\hat{\theta}_x) \) from (FOC), the derivative becomes
\[ q'(\hat{\theta}_x)[\theta_x - \hat{\theta}_x + z(q(\hat{\theta}_x); \theta_x) - z(q(\hat{\theta}_x); \hat{\theta}_x)]. \]
By (M), \( q' (\hat{\theta}_x) > 0 \). Then, since \( z_x \theta_x \geq 0 \) by Assumption 1, the derivative is positive for all \( \hat{\theta}_x \leq \theta_x \) and negative for all \( \hat{\theta}_x \geq \theta_x \). In other words, given (FOC) and (M), the objective function is quasiconcave in \( \hat{\theta}_x \). This, in turn, implies that the maximum is fully identified by the local optimality conditions, that is, (FOC) and (M) are sufficient for (IC).

\[ \square \]

Claim. A schedule \( \{q(\cdot), t_0(\cdot)\} \) exists that satisfies (FOC), (M), and (PC) for all \( \theta_x \in \Theta_x \).

Proof. Rewrite (PC) as

\[ q(\theta_x) \theta_x = t_0(\theta_x) + r_S(\theta_x), \]

where \( r_S(\cdot) \geq 0 \) is a “seller’s rent” function. Restricting attention to differentiable contract schedules, we differentiate both sides with respect to \( \theta_x \), which yields

\[ q'(\theta_x) \theta_x + q(\theta_x) = t'_0(\theta_x) + r'_S(\theta_x). \]

Using \( t'_0(\theta_x) = q'(\theta_x) \theta_x + z_x q'(\theta_x) \) from the (FOC) and rearranging yields

\[ q'(\theta_x) = \frac{q(\theta_x) - r'_S(\theta_x)}{z_q(q(\theta_x); \theta_x)}. \] (18)

To construct a fully revealing equilibrium schedule, the seller’s marginal rent function \( r'_S(\cdot) \) has to be chosen such that the right-hand side of (18) is strictly positive for all \( \theta_x \) so as to also satisfy (M). Across (buyer types with) strictly positive trades, this can be made to hold for \( r(\cdot) = 0 \), which leads to the LCS equilibrium defined by (5). Rent functions exist with \( r(\theta_x) > 0 \) for some \( \theta_x \) that also satisfy (M), in which case the seller earns a strictly positive rent from some buyer types in equilibrium.

In regard to existence, (18) is a differential equation of the form

\[ \frac{\partial q}{\partial \theta_x} = F(q, \theta_x). \]

The boundary condition that the highest type trades all the asset defines an initial value \( (q^0, \theta^0_x) = (1, \bar{\theta}_x) \). Since \( z \) is twice continuously differentiable, \( F(q, \theta_x) \) is
continuously differentiable in \((q, \theta_x) \in [0, 1] \times [\theta_x, \bar{\theta}_x]\). So a unique solution exists to the initial value problem defined by (18) and \((q^0, \theta_x^0) = (1, \bar{\theta}_x)\) (cf. Sydsæter et al. 2008, 217).

We conclude the proof by describing how the buyer’s payoff varies with her type: by the envelope theorem, the equilibrium payoff \(V^*\) satisfies

\[
\frac{\partial V^*}{\partial \theta_x} = q(\theta_x) + z_{\theta_x}(q(\theta_x); \theta).
\]

Since the highest type, \(\bar{\theta}_x\), earns its full information payoff, \(z(1; \bar{\theta}_x)\), we have

\[
V^*(\theta_x) = z(1; \bar{\theta}_x) - \int_{\theta_x}^{\bar{\theta}_x} q(u) + z_{\theta_x}(q(u); u)\,du.
\]

Since \(q(\theta_x) + z_{\theta_x}(q(\theta_x), \theta_x) > 0\), the buyer’s equilibrium payoff is strictly increasing in \(\theta_x\). Furthermore, there may be model specifications in which buyer types below a unique threshold do not trade, in which case the equilibrium is semiseparating: not all types trade, but all types with strictly positive trades are fully revealed.

**Proof of Proposition 2**

A fully revealing equilibrium schedule is characterized by the following optimization problem:

\[
\max_{\hat{\theta}_x \in \Theta_x} \alpha(\hat{\theta}_x)[\theta_x + z(1; \theta_x)] - t_0(\hat{\theta}_x),
\]

subject to the incentive compatibility constraint (IC)

\[
\alpha(\theta_x)[\theta_x + z(1; \theta_x)] - t_0(\theta_x) \geq \alpha(\hat{\theta}_x)[\theta_x + z(1; \theta_x)] - t_0(\hat{\theta}_x) \quad \forall \hat{\theta}_x \in \Theta_x
\]

and the participation constraint (PC)

\[
t_0(\theta_x) + [1 - \alpha(\theta_x)] [\theta_x + z(1; \theta_x)] \geq \theta_x.
\]

As in the proof of Proposition 1, we omit \(\theta_z\) and use the first-order approach.
Claim. A fully revealing equilibrium schedule \( \{\alpha(.), t_0(.)\} \) is characterized by

\[
\max_{\theta_x \in \Theta_x} \alpha(\hat{\theta}_x)[\theta_x + z(1; \theta_x)] - t_0(\hat{\theta}_x)
\]

s.t. \( \alpha'(\theta_x)[\theta_x + z(1; \theta_x)] - t'_0(\theta_x) = 0 \) \hspace{1cm} (FOC)

\( \alpha'(\theta_x) > 0 \) \hspace{1cm} (M)

\( t_0(\theta_x) + [1 - \alpha(\theta_x)] [\theta_x + z(1; \theta_x)] \geq \theta_x \) \hspace{1cm} (PC)

for all \( \theta_x \in \Theta_x \).

Proof. We must show that (FOC) and (M) are a necessary and sufficient condition for (IC).

Necessity. Consider two arbitrary buyer types, \( \theta^h_x \) and \( \theta^l_x < \theta^h_x \). By (IC), the downstream incentive compatibility constraint is

\[
\alpha(\theta^h_x)[\theta^h_x + z(1; \theta^h_x)] - t_0(\theta^h_x) \geq \alpha(\theta^l_x)[\theta^h_x + z(1; \theta^h_x)] - t_0(\theta^l_x).
\]

Similarly, the upstream incentive compatibility constraint is

\[
\alpha(\theta^l_x)[\theta^l_x + z(1; \theta^l_x)] - t_0(\theta^l_x) \geq \alpha(\theta^h_x)[\theta^l_x + z(1; \theta^l_x)] - t_0(\theta^h_x).
\]

Rearranging these constraints to

\[
[\alpha(\theta^h_x) - \alpha(\theta^l_x)][\theta^h_x + z(1; \theta^h_x)] \geq t_0(\theta^h_x) - t_0(\theta^l_x)
\]

\[
t_0(\theta^h_x) - t_0(\theta^l_x) \geq [\alpha(\theta^h_x) - \alpha(\theta^l_x)][\theta^l_x + z(1; \theta^l_x)]
\]

implies

\[
[\alpha(\theta^h_x) - \alpha(\theta^l_x)][\theta^h_x + z(1; \theta^h_x)] \geq [\alpha(\theta^h_x) - \alpha(\theta^l_x)][\theta^l_x + z(1; \theta^l_x)], \tag{19}
\]

which can only hold if \( \alpha(\theta^h_x) > \alpha(\theta^l_x) \). For \( \alpha(\theta^h_x) < \alpha(\theta^l_x) \), (19) would be violated because \( \theta^h_x + z(1; \theta^h_x) > \theta^l_x + z(1; \theta^l_x) \), which follows from \( z_{\theta_x} > 0 \) (Assumption 1). For \( \alpha(\theta^h_x) = \alpha(\theta^l_x) \), both types would prefer the offer with the smaller fixed transfer. Dividing (19) and \( \alpha(\theta^h_x) > \alpha(\theta^l_x) \) each by \( (\theta^h_x - \theta^l_x) \) and taking the limit \( (\theta^h_x - \theta^l_x) \to 0 \) implies, respectively, (FOC) and (M).
**Sufficiency.** Consider the first derivative of the objective function with respect to the choice variable $\hat{\theta}_x$:

$$
\alpha'(\hat{\theta}_x)[\theta_x + z(1; \theta_x)] - t'_0(\hat{\theta}_x).
$$

Using $t'_0(\hat{\theta}_x) = \alpha'(\hat{\theta}_x)[\hat{\theta}_x + z(1; \hat{\theta}_x)]$ from (FOC), the derivative becomes

$$
\alpha'(\hat{\theta}_x)[\theta_x - \hat{\theta}_x + z(1; \theta_x) - z(1; \hat{\theta}_x)].
$$

By (M), $\alpha'(\hat{\theta}_x) > 0$, and since $z_{\theta_x} > 0$ (Assumption 1), the derivative is positive for all $\hat{\theta}_x \leq \theta_x$ and negative for all $\hat{\theta}_x \geq \theta_x$. In other words, given (FOC) and (M), the objective function is quasi concave in $\hat{\theta}_x$. This, in turn, implies that the maximum is fully identified by the local optimality conditions.

**Claim.** A schedule $\{\alpha(.), t_0(.)\}$ exists that satisfies (FOC), (M), and (PC) for all $\theta_x \in \Theta_x$.

**Proof.** Rewrite (PC) as

$$
t_0(\theta_x) + [1 - \alpha(\theta_x)] [\theta_x + z(1; \theta_x)] = \theta_x + r_S(\theta_x),
$$

where $r_S(.) \geq 0$. Rearranging and differentiating on both sides with respect to $\theta_x$ yields

$$
t'_0(\theta_x) = \alpha'(\theta_x)\theta_x + \alpha(\theta_x) + \alpha'(\theta_x)z(1; \theta_x) - [1 - \alpha(\theta_x)] z_{\theta_x}(1; \theta_x) + r'_S(\theta_x).
$$

Using $t'_0(\theta_x) = \alpha'(\theta_x)[\theta_x + z(1; \theta_x)]$ from the (FOC) and rearranging yields

$$
\alpha(\theta_x) = \frac{z_{\theta_x}(1; \theta_x) - r'_S(\theta_x)}{1 + z_{\theta_x}(1; \theta_x)}.
$$

(20)

To construct a fully revealing equilibrium schedule, the seller’s marginal rent function $r'_S(.)$ has to be chosen such that the right-hand side of (20) is strictly increasing in $\theta_x$ so as to also satisfy (M). Differentiating (20) with respect to $\theta_x$ yields

$$
\alpha'(\theta_x) = \frac{z_{\theta_x,\theta_x}(1; \theta_x) - r''_S(\theta_x)[1 + z_{\theta_x}(1; \theta_x)] + r'_S(\theta_x)z_{\theta_x,\theta_x}(1; \theta_x)}{[1 + z_{\theta_x}(1; \theta_x)]^2}.
$$

(21)
Two cases exist: First, if $z_{\theta,x}(1; \theta_x) > 0$ for all $\theta_x \in \Theta_x$, we can set $r_{S}(.) = 0$ such that (PC) is *binding* for every type. This yields the LCS equilibrium defined by (11) in the proposition. Second, if $z_{\theta,x}(1; \theta_x) \leq 0$ for some $\theta_x \in \Theta_x$, there exists no incentive-compatible schedule under which (PC) binds for every type. Instead, the seller’s rent function $r_{S}(.)$ must be strictly positive for some $\theta_x \in \Theta_x$. The latter requirement implies that $r_{S}(.)$ is strictly decreasing in $\theta_x$ over some interval in $\Theta_x$. ■

By the envelope theorem, the buyer’s equilibrium payoff $\mathcal{V}^*$ satisfies

$$\frac{\partial \mathcal{V}^*}{\partial \theta_x} = \alpha(\theta_x)[1 + z_{\theta_x}(1; \theta_x)].$$

Since the highest type, $\bar{\theta}_x$, earns its full information payoff, $z(1; \bar{\theta}_x)$, we have

$$\mathcal{V}^*(\theta_x) = z(1; \bar{\theta}_x) - \int_{\theta_x}^{\bar{\theta}_x} \alpha(u)[1 + z_{\theta_x}(1; u)]du.$$ Since $\alpha(\theta_x)[1 + z_{\theta_x}(1; \theta_x)] > 0$, the buyer’s equilibrium payoff is strictly increasing in $\theta_x$. Again, there may be model specifications in which buyer types below some unique threshold do not trade. ■

**Proof of Proposition 3**

A buyer of type $\theta_x$ receives the payoff $z(1; \theta_x, \theta_z)$ when making a truthful offer with the fixed transfer $\theta_x$. Now consider the buyer’s payoff when mimicking a lower-valued type $\theta'_x < \theta_x$. By Assumption 1, $v(1; \theta'_x, \theta_z) < v(1; \theta_x, \theta_z)$. Hence, when mimicking type $\theta'_x$, type $\theta_x$ would incur a penalty $\tau > \theta_x - \theta'_x$ and its payoff would be less than $z(1; \theta_x, \theta_z)$. Now consider the payoff from mimicking any type $\theta''_x > \theta_x$. By Assumption 1, $v(1; \theta''_x, \theta_z) > v(1; \theta_x, \theta_z)$. Hence, mimicking would not trigger a penalty, but type $\theta_x$ would pay a fixed transfer of $\theta''_x$, which is higher than the fixed transfer $\theta_x$ under its truthful offer. ■
Proof of Proposition 4

We establish the existence of a pooling equilibrium in which all active buyer types offer the same contract \((q_P, t_P)\). We first consider, in turn, the buyer’s and the seller’s participation constraints, then use a fixed point argument, and conclude with specifying the necessary off-equilibrium beliefs.

**Buyer’s participation constraint.** For a given transfer \(t_P\), the buyer types that prefer to be active are defined by the buyer’s participation constraint

\[
q_P \theta_x + z(q_P; \theta_x) - t_P \geq 0.
\]

Since \(z_{\theta_x} > 0\), the left-hand side is strictly increasing in \(\theta_x\). Thus, unique cutoff type \(\theta^c_x = f(t_P)\) exists, defined by

\[
q_P \theta^c_x + z(q_P; \theta^c_x) - t_P = 0
\]

such that all and only types \(\theta_x \geq \theta^c_x\) make the offer. Note that \(f(.)\) is continuous and increasing and satisfies \(f(q_P \theta_x^c) = \theta_x\), and \(f(q_P \bar{\theta}_x) < \bar{\theta}_x\), where \(q_P \bar{\theta}_x\) is the (smallest) price the seller would accept regardless of her beliefs about the common value.

**Seller’s participation constraint.** For a given cutoff type, the seller accepts a given offer \((q_P, t_P)\) if her participation constraint holds:

\[
t_P \geq \int_{\theta^c_x}^{\bar{\theta}_x} \frac{h(u)}{1 - H(\theta^c_x)} q_P u du \equiv g(\theta^c_x).
\]

This inequality defines a set of acceptable prices. We focus on the lowest acceptable price \(g(\theta^c_x)\). Note that \(g(.)\) is continuous and increasing and satisfies \(g(\bar{\theta}_x) = q_P E(\theta_x) > q_P \theta_x\), and \(g(\theta_x^c) = q_P \bar{\theta}_x\).

**Fixed point.** It follows directly from the properties of \(f(.)\) and \(g(.)\) that they intersect at least once in \(\Theta_x\). That is, there exists a fixed point at which \(f(g(\theta^c_x)) = \theta^c_x\), which defines an equilibrium offer: \(q_P\) and \(t_P = g(\theta^c_x) \leq q_P \bar{\theta}_x\).

**Off-equilibrium beliefs.** To support \((q_P, g(\theta^c_x))\) as a perfect Bayesian equilibrium, assume that the seller assigns any offer other than \((q_P, g(\theta^c_x))\) to the highest
type. Given these beliefs, even the highest type does not want to deviate if

\[ z(1; \theta_x) \leq q_P \theta_x + z(q_P; \bar{\theta}_x) - t_P. \]

This imposes a restriction on \( q_P \) but always holds, for example, for \( q_P = 1 \).

**Proof of Proposition 5**

In any equilibrium other than the fully revealing one, there is a subset \( \Theta^P \) with at least two types that choose the same contract \( C_P = (q_P, q_P P_P, 0) \), where \( P_P \) is the per-unit price. The seller’s participation constraint requires \( P_P \geq E(\theta_x | \theta_x \in \Theta^P) \). Denote the lowest type in this subset by \( \bar{\theta}^P \equiv \min \Theta^P \). Clearly, \( P_P > \bar{\theta}^P \).

Consider the contract \( C^d = (q^d, q^d P^d, 0) \), with \( q^d = q_P - \delta \). A given type \( \theta_x \) prefers \( C^d \) over \( C_P \) if and only if

\[ q_P P_P - q^d P^d > v(q_P; \theta_x, \theta_z) - v(q^d; \theta_x, \theta_z). \]

Since the right-hand side of the inequality increases in \( \theta_x \) (Assumption 1), if the inequality holds for some type \( \theta_x \), then it also holds for all lower types. Hence, we can adjust \( P^d \) such that the inequality holds only for \( \theta_x \leq \bar{\theta}^P \). For very small \( \delta \), this requires a small change in the price such that \( P^d \) remains above \( \bar{\theta}^P \). Under the intuitive criterion, the seller assigns the deviation \( C^d \) to types \( \theta_x \leq \bar{\theta}^P \). Hence, given \( P^d \geq \bar{\theta}^P \), the seller never rejects the contract. Thus, only fully revealing equilibria survive the intuitive criterion.
References


