

Ownership, Investment and Governance: The Costs and Benefits of Dual Class Shares

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Abstract

In this paper we show that dual-class shares can be an answer to agency conflicts rather than a result of agency conflicts. When a firm issues voting shares to raise funds, it increases the risk that manager-controlling shareholder could lose control of the firm and lose the associated private benefits. Thus, the incumbent may be willing to forgo positive NPV investments to maximize his overall welfare. Non-voting shares allow a firm to raise funds without diluting manager's control rights; hence, it can alleviate the underinvestment problem. But use of non-voting shares dilutes dividends and facilitates entrenchment, reducing value-enhancing takeover bids. We obtain conditions under which the benefit of using non-voting shares outweighs its costs. We integrate the dual-class decision into the rich body of research on capital structure and underinvestment, and show that dual-class structures can be a solution to an agency driven underinvestment problem.

Keywords: Blockholders, Controlling shareholders, Dual-class shares, Hostile takeovers, Ownership structure, Private benefits of control, Non-voting shares, Shareholder welfare, Takeover defenses, Underinvestment, Voting rights.

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1 Introduction

In recent years, equalizing the voting power and influence of common shares has become a touchstone of good governance and shareholder democracy in much of the world. Corporate charter provisions that explicitly limit the rights of minority shareholders are completely antithetical to this viewpoint, although they widely exist globally.¹ Thus, it is not surprising that dual-class provisions, which create a second class of common stock with reduced voting power, have come under fire. More generally, activist shareholders have voiced serious concern about the disenfranchisement of investors holding inferior voting classes of shares.² The corporate democracy movement has led policy makers, most vocal among them a high-level group of EU company law experts and Indian corporate activists, to warn of the threats posed by dual-class ownership structure.

Yet, the theoretical literature on dual-class shares is quite limited. Models by Grossman and Hart (1988), Harris and Raviv (1988) and Ruback (1988), have analyzed non-voting equity in the context of control contests and find that a dual-class share structure yields a negative shareholder wealth effect. These authors trace this negative wealth effect to the unbundling of voting rights and cash flow rights – arguing that the unbundled votes can act as an anti-takeover device. When the likelihood of a successful takeover is significantly diminished, it allows managers to deviate from actions that enhance shareholders’ wealth. For example, Shleifer and Wolfenzon (2002) show that firms with weaker shareholder protection have lower valuation, which is consistent with investors anticipating that some profits are likely to be diverted. As the market for corporate control weakens as a disciplining mechanism, investors recognize that managers have greater latitude to extract private benefits. Thus, according to the existing literature dual class share structures hurt firm value and shareholder wealth.

¹Companies like Berkshire Hathaway, Blackstone Group, Clearwire, Dolby, Echostar, Facebook, Ford, Fox News, Google, MasterCard, News Corp, Rosetta Stone, VISA, VMWare, and WebMD have multiple classes of shares. Dual-class structures are widely used in countries like Brazil, Canada, Denmark, Finland, Germany, Italy, Norway, Sweden, and Switzerland. In Canada 5 to 6% of listed companies, like Metro, Bombardier, Gluskin Sheff, Air Canada, Exfo, Cossette, and Celestica, have multiple classes of shares.

²Institutional Shareholder Services (ISS) recently recommended that dual-class structures be eliminated entirely for all newly listed companies. Also, ISS wants corporate laws to be changed to require sunset provisions for companies with dual class structure, such that all shares will revert to common shares after a pre-specified time, unless a majority of inferior-class shareholders vote to reaffirm the dual-class structure.

We argue for a more nuanced view of the role of dual-class shares: Although a dual-class structure weakens the incentives associated with the market for corporate control, at the same time it helps to mitigate the underinvestment problem resulting from the non-contractibility of a firm's investment policy. For example, a scale-expanding investment project generally requires a sizable issue of new shares as part of its overall financing of the project. If a manager is forced to issue voting shares, then these newly issued votes dilute a manager's proportional voting power and severely impede his ability to resist future takeover attempts. Thus, a rational manager owning voting shares and extracting private benefits of control has an incentive to reject many profitable scale-expanding investments. Wruck (1989) finds average management-controlled holdings fall by 1.5% around the time of a typical private equity sale, while public offers are likely to dilute manager voting rights much more, given SEOs typically larger size. On the other hand, newly issued non-voting shares do not affect the manager's control rights and thus, do not discourage value maximizing investments.³

We show that an incumbent manager, who has the option to use non-voting shares, will not automatically use non-voting shares to fund all new investments. This is because issuance of non-voting shares comes at a cost: Non-voting shares dilute the cash flow rights of all existing shareholders, including the incumbent manager, as more non-voting shares relative to voting shares must be issued to raise the same amount of project funding. For example, Faccio and Masulis (2005) find incentives to use non-voting shares to fund an acquisition, which tend to be large investments, are strongest when the target firm's ownership is concentrated and a bidder's controlling shareholder has an intermediate level of voting power – a range where he is most vulnerable to a loss of control under a stock-financed acquisition.

Similarly, an incumbent who has no option but to use voting shares to fund new investments will not necessarily underinvest. Because the manager is also a shareholder, he bears part of the cash flow loss from the firm forgoing positive NPV projects. Thus, it is possible for this cash flow loss

³According to the Library of Canadian Parliament, "Undeniably, some of the best-performing companies in Canada have multiple-voting shares. Thus, not all shareholders are concerned with the voting rights attached to a share. They may be more interested in the potential of sharing the company's wealth or trading on future prospects by buying cheaper, subordinated shares." Also, according to Barry Reiter of Bennett Jones LLP, "some dual-class firms are created to favor Canadian ownership in strategic or culturally sensitive fields. Many foreign investors have happily bought into structures of this sort."

to outweigh his expected control-related benefits from this underinvestment. Hence, an incumbent manager faces a clear trade-off when choosing the security type to use to fund the new investment projects. The manager faces the choice of dilution of his control rights from accepting the project and dilution of his cash flow rights from forgoing profitable investments.

To add yet another wrinkle to the decision process, we observe that outside shareholders, who are assumed to have approval rights, also face a trade-off: the expected costs of greater management entrenchment against a higher firm value from more investments. If the outside shareholders allow an incumbent manager to issue either voting and non-voting shares, then the incumbent invests in all available positive NPV projects; but outside shareholders also know that when this additional investment is funded using non-voting shares, the incumbent manager is relatively more entrenched because his private benefits play an enhanced role in potential takeover contests. Hence, future takeover attempts are made more costly, which results in lower expected takeover premiums and, consequently, lower firm values. However, if outside shareholders force the incumbent to use only voting shares to fund new investments, then he may forgo some positive NPV projects, which is also a cost for outside shareholders. For example, DeAngelo and DeAngelo (1985) indicate that an underinvestment problem can exist if the managers are faced with increasing risk of losing control when they fund new projects with voting equity.

Taking into consideration all of these possibilities, we propose that differences in investment opportunity sets may help to explain the considerable variation in the effects of dual-class share issues both within and among firms. Others have shown that deviations from “one share-one vote” can be optimal, but our model is the first to integrate the dual-class decision (heretofore viewed simply as problem for outside shareholders in the control literature) into the rich body of research on capital structure and underinvestment. We focus specifically on the firm’s decision to forgo positive NPV investment opportunities.⁴

⁴For example, Burkart et al. (1998) show that a dual-class share structure is optimal if the expected benefits of a higher likelihood of tender offers outweigh the expected costs of less efficient tender offers; Burkart and Raff (2012) show that even active boards destroy acquisition opportunities for rival managers, thus forcing all firms to pay higher incentives to incumbent managers; Blair et al. (1989) show that a market for votes can increase efficiency during control contests in the presence of taxes; Neeman and Orosel (2006) show that a voting contest for votes in addition to a contest for shares can have efficiency advantages; Sercu and Vinaimont (2008) demonstrate with

In a control context, the problem of underinvestment differs considerably from standard underinvestment scenarios: Here underinvestment results from a manager's fear of diluting his control rights – which reduces his likelihood of retaining control. For example, Masulis et al. (2009) find in quite a few firms with dual-class shares, that the largest shareholder does not own a majority of the voting rights. However, if the manager owns an insignificant block of voting shares or owns a majority of votes well in excess of 50%, it is easy to show that there is no scope for significant control dilution and hence, there is no underinvestment problem. Thus, only when manager voting power is in an intermediate range does an underinvestment problem exist.⁵

One possible solution to underinvestment problem is to issue debt rather than equity. However, debt does not solve the underinvestment problem, because it carries with it the risk of bankruptcy. Issuing more debt can require stricter covenants, which in turn risks a loss of control for the incumbent manager. Why? Because when a covenant is violated, the creditor often demands that the managers be replaced before agreeing to either a covenant revision or a voluntary debt restructuring. For example, DeAngelo and DeAngelo (1985) find evidence that dual-class firms infrequently resort to increasing leverage to retain control. Instead, dual-class firms seek to keep leverage low, consistent with their desire to minimize the risk of creditors taking control of the firm. The issuance of non-voting stock to fund new investment does not result in a dilution of a manager's control rights and at the same time it has none of the adverse effects of debt covenants.

A dual-class structure can be particularly helpful for a smaller firm facing large, profitable investment opportunities. It enables a firm to significantly increase shareholder wealth by not passing up profitable investments and thereby improves a firm's overall economic efficiency. However, as mentioned earlier the costs associated with issuing non-voting equity limit its effectiveness in

the help of simulations that one share-one vote is never optimal from an entrepreneur's point of view; and more recently, Edmans (2009) and Edmans and Manso (2011) show that shareholders who hold non-voting shares can exert influence through the threat of exit. Also, Brav and Mathews (2011) show that separate vote trading can improve overall efficiency. In contrast, Masulis et al. (2011) show that control can also be maintained by pyramid structured business groups, which may act as substitute for nonvoting shares. In a recent paper Laux (2012) shows that a suboptimal vesting period in CEO incentive compensation contracts can induce myopic investment behavior similar to ours.

⁵Underinvestment and its causes have been studied in a number of papers: Debt-induced underinvestment is considered by Galai and Masulis (1976), Henderson (1993), Myers (1977), and Berkovitch and Kim (1990). Myers and Majluf (1984) and Cooney and Kalay (1993) derive conditions under which an undervalued firm forgoes positive NPV investments.

solving the underinvestment problem.⁶

Our model predicts that high growth firms, rather than firms with lots of assets-in-place, are more likely to use dual-class shares and this prediction is consistent with existing empirical findings in Lehn et al. (1990) and Dimitrov and Jain (2006).⁷ Our analysis produces three new testable implications. First, restrictions on equity security design can reduce shareholder value. Such restrictions can lead to severe underinvestment and at times may outweigh the positive value effects that voting shares have on control contests. Next, we show that the likelihood of issuing non-voting shares is initially rising and then falling in a manager's shareholdings. If manager shareholdings are small, then there is no scope for control dilution since the manager fully invests even if only voting shares are allowed. If his holding size is large, then he bears a large part of the cost of underinvestment and dividend dilution. Hence, it is more likely that his expected cost of control dilution is outweighed by the benefits of full investment, even if only voting shares can be issued.

The remainder of this article is organized as follows: In Section 2 we develop a detailed numerical example capturing the essence of our insight. In Sections 3, 4 and 5 we show that the basic intuition of our numerical example can be formally modeled and analyzed. In these sections, we characterize the underinvestment problem and further analyze the effect of underinvestment on outside shareholders and the incumbent manager. Possible extensions are discussed in Section 6. Conclusions are presented in Section 7. Some of the more cumbersome results and an extensive numerical exercise are delegated to the appendix.

⁶If shareholders collectively make the firm's investment decisions, then underinvestment is no longer an issue. However, this solution would undercut efficient investment decision making because it makes it next to impossible to prevent competitors from gaining access to important proprietary information. If less stringent information requirements are imposed, then a manager can insure underinvestment by withholding crucial information from shareholders. Also, if it is possible to directly contract with a manager across all states of the world, again underinvestment can be avoided. However, such a contractual solution would require all a firm's investment opportunities to be known to its shareholders. Furthermore, these investment opportunities must be verifiable - imposing added verification costs on shareholders.

⁷There are a few more empirical implications that are partly or fully tested in prior papers. For example, our model corroborates that, conditional on the same level of assets and investment activities, a dual-class firm is less valuable than a comparable single-class firm – a prediction partly tested in Claessens et al. (2002), Boone and Mulherin (2007) and Gompers et al. (2010). Also, our model shows that a dual-class firm is less likely to become a takeover target, but conditional on a takeover bid, the premium offered for voting shares is likely to be higher. Papers like Seligman (1986), Jarrell and Poulsen (1988), Ambrose and Megginson (1992), Smart and Zutter (2003) and Krishnan and Masulis (2011) have empirically tested these implications.

2 Numerical Example

Consider a firm that has a public value of \$2.00 million, generates a private value for the incumbent of \$0.20 million and has 100 shares outstanding. The value of the existing firm, both public and private, is the same under both the incumbent and the rival manager. The incumbent owns 50 shares in the firm and the incumbent manager is wealth constrained. Given our assumption that the incumbent owns half the shares in the firm, there is a zero probability of a change in control of the firm, ($\phi = 1$), without the incumbent's consent.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns, plus the expected private benefits of control; that is, the value of the incumbent's stake in the firm is \$1.20 million ($= 1/2 \times \$2.00 + \phi \times 0.20 = \$1.00 + 1.0 \times 0.2$). The value of the shares owned by outside shareholders is the probability of the incumbent's retaining control times the public value of the firm under the incumbent, plus the probability of the rival's gaining control times the price paid by the rival. Thus, the value of the shares owned by the existing outside shareholders is \$1.00 million.

To keep the numerical example simple, we assume that the incumbent has to choose from three discrete investment levels: invest \$0, invest \$1.00 million, or invest \$2.00 million. If the incumbent invests nothing, there is no addition to the value of the firm and no new shares are issued. If the incumbent invest \$1.00 or \$2.00 million, the resulting value of the firm, and the additional public and private value generated under the incumbent and a rival manager are summarized in Table I below.

Investment in the projects adds to the public value of the firm and to the private benefits of the manager-in-control at the end of the investment horizon. We assume that the rival manager is strictly better than the incumbent: The rival manager can generate a public value that is higher than the sum of the public and private value that the incumbent can generate. For example, if the incumbent invests \$1.00 million and he is the manager-in-control at the end of the investment horizon, then the public value is \$1.10 million and his private benefit is \$0.06 million, giving an

Table I

Initial number of shares outstanding 100; incumbent manager owns 50							
Existing Firm Value		Investment Opportunity					Number of New Shares Issued
Public Value	Private Value	Investment	Incumbent Manager		Rival Manager		
			Addition to Public Value	Addition to Private Value	Addition to Public Value	Addition to Private Value	
2.00	0.20	0.00	0.00	0.00	0.00	0.00	0.00
2.00	0.20	1.00	1.10	0.06	1.18	0.00	50.00
2.00	0.20	2.00	2.12	0.07	2.20	0.00	100.00

Figure 1: This table summarizes the value of the existing firm and the additional public and private value created by the new investments under the incumbent and the rival manager. The first row depicts the no investment case; the second and the third rows depict the cases where the incumbent invests \$1.00 and \$2.00.

Table II

Voting Rights Ownership and Probability of Retaining Control under Different Types of Equity Financing and Different Investment Levels				
Investments	Voting Shares Issued to Finance New Investment		Nonvoting Shares Issued to Finance New Investment	
	Managerial Ownership of Voting Rights	Probability of Retaining Control	Managerial Ownership of Voting Rights	Probability of Retaining Control
0.00	50.00%	1.00	50.00%	1.00
1.00	33.00%	0.95	50.00%	1.00
2.00	25.00%	0.79	50.00%	1.00

Figure 2: The first half of the table shows the likelihood of retaining control if voting shares are used to fund the new investment. The second half of the table shows the likelihood of retaining control if non-voting shares are used to fund the new investment.

aggregate value \$1.16. Whereas, if the incumbent invests \$1.00 million and the rival manager is the manager-in-control at the end of the investment horizon, then the public value is \$1.18 > \$1.10 + \$0.06 million.

If non-voting equity is used to finance the investment, the incumbent's proportional ownership of the control rights (votes) remains at 50% and the incumbent retains the ability to prevail in all control contests. If voting shares are issued to finance the investment, the incumbent's proportional ownership of the control rights drops to either 33% ($= 50/150$) or 25% ($= 50/200$) depending on the level of investment. We assign exogenously probabilities $\phi^1(x=1) = 0.95$ and $\phi^1(x=2) = 0.79$. These probabilities are essentially the ability of the incumbent to prevail in a control contest when he owns 33% and 25% of the voting shares. Basically, if the incumbent invests \$1 million in the new project and issues 33% new voting shares to finance the investment, then the likelihood he is able to buy these outside shares and retain control drop by 5%. Table II summarizes this information.

The expected value of the incumbent's stake in the firm is the sum of the expected public value of the shares that he owns plus the expected private benefits of control. The expected public value of a share in the firm is the probability that the incumbent retains control times the public value of the firm under the incumbent, plus the probability that the rival manager gains control times the public value of the firm under the rival manager. For investment level of \$1 million, the expected public value is equal to the expected NPV under the incumbent plus the expected NPV under the rival manager; that is, $2 + (0.95 \times (1.1 - 1) + 0.05 \times (1.18 - 1))$ or \$2.104 million.

The expected private benefit extracted by the incumbent is the private benefit of control times the probability of remaining in control. For investment level of \$1.00 million, the expected private benefit is 0.95×0.26 or \$0.247 million. Therefore, the expected value of the incumbent's stake if he invests \$1.00 million is $0.5 \times (2.104) + 0.247$ or \$1.299 million. For the investment level \$1.00, the shareholders' expected wealth is 0.5×2.104 or \$1.052 million. If the incumbent has a choice regarding the type of equity to issue to finance the project, the incumbent will issue non-voting equity to invest \$1.00 and the expected value of the shares owned by existing outside shareholders is \$1.05. For investment level of \$2.00, the expected public value of the firm is $2 + 0.79 \times (2.12 - 2) + 0.21 \times (2.20 - 2)$ or \$2.1368 million. The expected private benefit extracted by the incumbent is 0.79×0.27 or \$0.2133 million. Therefore, the expected value of the incumbent's stake given an investment level of \$2.00 is $0.5 \times 2.1368 + 0.2133$ or \$1.2817 million. The expected wealth of the outside shareholders is 0.5×2.1368 or \$1.0684 million. Table III summarizes this information.

The incumbent's expected wealth is maximized at an investment level of \$1.00 million when the investment is financed using voting shares and at an investment level of \$2.00 million when the investment is financed using non-voting shares. The decision made by the existing outside shareholders is related to the type of shares that the firm can issue to finance the new investment. If the incumbent is required to finance the investment by issuing voting shares, the incumbent will invest \$1.00 million and the expected value of the shares owned by existing outside shareholders is \$1.052 million. From the table we see that there are situations in which it is value increasing

Table III

Payoff of Incumbent Manager and Outside Shareholders' under Different Types of Equity Financing and Different Investment Levels				
Investments	Voting Shares Issued to Finance New Investment		Nonvoting Shares Issued to Finance New Investment	
	Manager's Payoff	Outside Shareholders' Payoff	Manager's Payoff	Outside Shareholders' Payoff
0.00	1.2000	1.0000	1.2000	1.0000
1.00	1.2990	1.0520	1.3100	1.0500
2.00	1.2817	1.0684	1.3300	1.0600

Figure 3: The first half of the table shows the expected payoff of the incumbent and outside shareholders when voting shares are issued to finance the investment, and the second half of the table shows payoff of the incumbent and outside shareholders when non-voting shares are issued to finance the investment. Outside shareholders always want the manager to invest in all positive NPV projects. But if forced to use voting shares, then the incumbent's optimal response is to invest \$1.00 rather than \$2.00 given his payoff from investing \$1.00 is \$1.2990 which is strictly greater than \$1.2817 – incumbent's payoff from investing \$2.00.

for outside shareholders to allow the incumbent to issue non-voting shares to finance investments. This increases the outside shareholders' wealth from \$1.052 million to \$1.06 million. This is true regardless of the fact that non-voting shares are likely to entrench the incumbent and prevent better rivals from taking over the firm. The difference in the value of the shares owned by the existing outside shareholders when voting and non-voting shares are used to finance the investment is a cost of entrenchment (for investment level \$1 million, the cost of entrenchment is $\$1.052 - \$1.05 = \$0.002$) per million dollar of investment.

Allowing the manager to issue non-voting shares will raise the value of the shares owned by existing outside shareholders when the loss in value from underinvestment is larger than the loss in value from entrenchment. Examples of this situation are firms that have many growth opportunities and firms in relatively new industries. For firms that have relatively few growth opportunities, the above result is unlikely to hold. In these firms underinvestment is less likely to be a problem and constraining these firms to issue voting shares have a smaller negative impact on the value of these firms.

Does a contractual solution to the underinvestment problem work? Often it may be possible to make a side payment to the manager to induce him to undertake the investment. This alternative requires the outside shareholders to compensate the manager for the decrease in expected wealth

associated with an investment of \$2 financed using voting shares. In the case presented above, contractual solution does not work. The increase in the outside shareholders expected wealth, $\$1.0684 - \$1.052 = \$0.0164$, if investment level goes up from \$1.00 to \$2.00, is smaller than the drop in the incumbent's expected wealth, $\$1.2817 - \$1.299 = -\$0.0173$. Hence, cross-subsidization is not feasible in this case.

3 Model Preliminaries

The model considers a firm that faces an investment opportunity. Our firm is a typical publicly traded firm with a sizable insider holding. Initially, our firm has only one class of share the commons. Each common share has equal percentage claim to a firm's total cash flows as well as to the total voting rights. We assume that the shareholders are able to make decisions under a simple majority vote rule about broad corporate objectives and policies such as changes in the board of directors, changes in control of the firm, and the menu of securities that the firm can issue to raise new capital. We highlight four players in our model – (i) the incumbent manager (I), (ii) outside shareholders, (iii) potential new investors, and (iv) the manager of a potential rival firm (R).

The incumbent is the one who searches for new investment opportunities, does the initial evaluation, and decides what investment projects to undertake. Like Jensen and Meckling (1976), Myers (1977), Cooney and Kalay (1993) and Zwiebel (1996), we assume that the incumbent maximizes the market value of the firm as well as the private benefits he derives from being in control. In addition, we assume that the incumbent is a “block” shareholder of the firm, such that the incumbent owns β fractions of the existing N commons (or voting shares).⁸

There can be two types of “closely held” ownership structures: The incumbent has a large minority block, which exceeds that of any other shareholders; or the incumbent owns an absolute

⁸Although only about 20% of the major exchange-listed public firms are closely held in the United States, a vast majority of U.S. corporations are closely held. Also, a study of top 27 stock markets finds that only 36% of the largest publicly traded firms are widely held – that is, there is no single shareholder controlling more than 20% of the total votes. Most large publicly traded firms (64%) have a controlling shareholder, which may be a family (30%), the state (19%), or another firm (15%). Among smaller companies the share of closely held firms is even higher. For detailed discussion see, for example, La Porta et al. (1999) and Claessens et al. (1999).

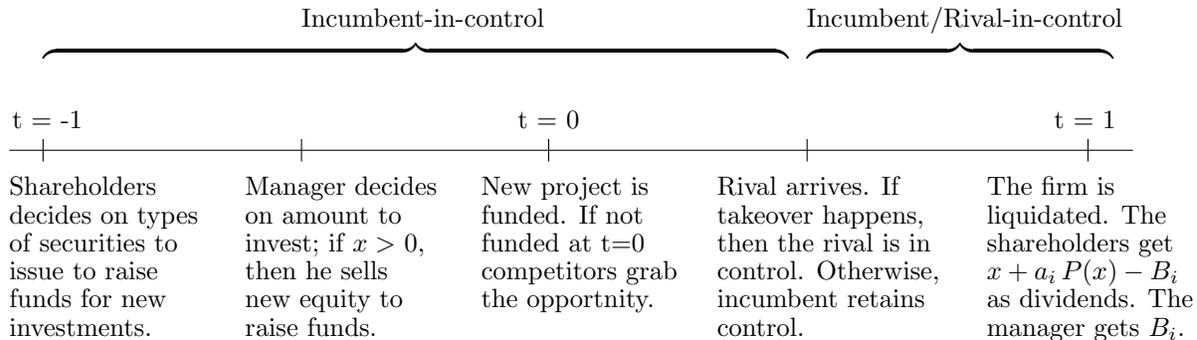
majority of the votes. Initially, we consider the case where the incumbent has a large minority block which exceeds that of any other shareholder; that is, $0 < \beta < 1/2$ implying that the incumbent has effective control rather than absolute control.⁹ The remaining $1 - \beta$ fraction of the common shares are diffusely held by outside shareholders. Each individual outside shareholder wants to maximize the value of his holdings.

The incumbent manager needs to issue equity to raise investment funds. Potential new investors are the ones who buy the securities that the firm issues, if any, to finance a new investment project. We do not restrict the existing outside shareholders from purchasing the newly issued securities, although we do assume that the incumbent manager is wealth constrained and cannot buy enough newly issued shares to keep his ownership fraction constant. Thus, if the firm invests by issuing common shares, the incumbent's ownership fraction declines. Also, we rule out any kind of preemptive rights offer.

The final player is the rival manager, who controls the rival firm. The rival manager, if he values our firm higher than the incumbent, offers to buy the firm. We rule out a "manager-rival negotiated" takeover: The only way to acquire the firm is through a market transaction, in an open market purchase of at least 50% plus of the voting shares. All participants are risk-neutral and the discount rate is zero; all securities have prices equal to their expected payoffs.

The temporal evolution of events is as follows: Shareholders decide on the types of securities that the firm can issue to finance the new investment opportunity. Next, the incumbent decides the level of investment, x , and if $x > 0$ the firm issues securities to finance the new investment. A rival arrives, and if he can take over the firm, he bids for the firm and gains control. The actual investment is undertaken and subsequently, the firm is liquidated in the final period and the public value is paid out to the investors as a dividend. The manager-in-control obtains the private benefits. The quality of the rival is uncertain at the beginning of the scenario, but is revealed at the time of his arrival. The figure below depicts the timeline described above:

⁹All of our results can be reproduced if we consider the case where the incumbent has absolute control; that is, he owns a simple majority of the votes ($\approx 50\%+$). We develop this case as a numerical example in the Appendix 8.



3.1 New Project

The project generates public value for the shareholders of the firm and a private benefit that accrues to the firm’s manager. The realized value of the project is $x + a_i P(x) + \varepsilon_x$. The random variable, ε_x , is uniformly distributed over the interval $(-\sigma_x, +\sigma_x)$, with a mean zero and variance $\sigma_x^2/3$. $P(x)$ is a concave function, differentiable everywhere with a unique maximum at \bar{x} . Thus, the maximized expected value of the new project is $\bar{x} + a_i P(\bar{x})$.

The parameter a_i is a measure of the manager-in-control’s ability to generate cash flows from the new project. Henceforth, we call the parameter a_i “public quality” of the manager-in-control at the end of the investment process, where the manager-in-control is either the incumbent (I) or the potential rival manager (R). We assume that the public quality of the incumbent is common knowledge and $a_I \in [0, 1]$. Initially, the potential rival manager’s public quality is unknown; thus, we assume that a_R is a random variable drawn from a *uniform distribution* with support 0 and 1. The lowest public quality manager is the one with $a_i = 0$, and the resulting NPV of the new project is 0. The highest public quality manager is the one with $a_i = 1$, and the resulting NPV of the new project is $P(x)$.

Also, we assume that the manager-in-control (whether incumbent or rival) can appropriate some benefits that are not shared by outside shareholders – a private benefit of control. This private benefit is not verifiable; otherwise, if it is verifiable, it will be relatively easy for outside shareholders to stop the manager from appropriating it. The realized value of the private benefit is $B_i = b_i a_i P(x)$, where $i = I, R$. The parameter b_i measures the manager-in-control’s ability to

convert one unit of NPV into his private benefit. Henceforth, we will call the parameter b_i “ability to extract private benefits” of the manager-in-control. We avoid the problem of over-investment by assuming that private benefits are also maximized at \bar{x} .¹⁰ Like his public quality, we assume that incumbent’s ability to extract private benefits, $b_I \in [0, 1]$, is common knowledge and the potential rival manager’s ability to extract private benefits b_R is a random variable drawn from a uniform distribution with support 0 and 1.

3.2 Firm Value

We normalize the initial value of the firm, $V_0 = 0$; hence, the present value of all the cash flows generated from the new investment is the only source of future dividends for the shareholders, adjusted for the extraction of private benefits of control. What are the costs of these private benefits? There is a direct loss to the shareholders a dollar worth of private benefit is a dollar less for the outside shareholders we call it the value effect of the private benefits. There is also an important indirect loss to the shareholders: Private benefit does not directly reduce value of the firm, but allows the manager to use the private benefits to stall potential value-enhancing takeovers activities we call this the entrenchment effect of private benefits. In both cases, these private benefits are direct gains for the manager. We focus on both these effects of private benefits.¹¹

If the firm invests in this new project and the manager-in-control at the liquidation date is of (a_i, b_i) type, then the expected firm value, denoted by FV_i , is

$$FV_i = \text{Investment} + \text{NPV} - \text{Private Benefits} = x + a_i P(x) - b_i a_i P(x) = x + a_i (1 - b_i) P(x), \quad (1)$$

where $i = I, R$. We assume \tilde{a}_R and \tilde{b}_R are independent random variables. Outside shareholders want to maximize firm value; hence, they want a manager who is best in terms of public quality

¹⁰Otherwise, the ability to issue nonvoting shares to fund new investments may encourage managers to invest in negative NPV projects that help to enhance their private benefits.

¹¹In Section 8 of the paper, we develop an extensive numerical example that deals only the “entrenchment effect” of private benefits on investment decision. Also, in a different paper titled “Strategic Underinvestment and Ownership Structure of a Firm” we formally developed the case that deals with only the entrenchment effect of private benefits on new investments.

and has the least ability to convert shareholders value into private benefits; that is, a manager with $a_i = 1$ and $b_i = 0$. If $a_i = 1$ and $b_i = 0$, then the expected firm value, $x + P(x)$ for any level of investment x , is maximized. If $a_i = 1$ and $b_i = 1$, that is, the manager-in-control is the best manager in terms of public quality, but also extracts the most private benefits; hence, the resultant firm value is $x + (1 - b_i)P(x)$, which is strictly less than $x + P(x)$ unless $b_i = 0$ implying that other outside mechanisms to deter private benefit extraction are perfect. Thus, if $1 \gg 0$, then there must exist a rival manager with the same/lower public quality and lower ability to extract private benefits than the incumbent such that $a_R(1 - b_R) > (1 - b_I)$. This is important! This condition implies that if such a rival takes control of the firm, then he can generate a higher firm value than the incumbent can. For example, using an average estimate of 14% shareholder wealth appropriation by the manager reported by Dyck and Zingales (2004), we find that a rival with public quality, $a_R = 0.62$ and the ability to extract private benefits, $b_R = 0.5$, can generate higher cash flows than the incumbent with $a_I = 0.75$ and $b_I = 1$.

4 Potential Control Contest

The potential control contest is a critical element in the process. To gain control of the firm, the rival has to offer outside shareholders a higher price for their shares than the incumbent. If the rival cannot offer more, then he does not bid and the incumbent retains control. If the rival can offer a greater amount, then he pays shareholders an amount slightly higher than what the incumbent can offer and the rival takes control of the firm. Initially, we assume that the incumbent does not tender shares in the control contest.¹² In Subsection 6.1 we relax this assumption and allow the manager to tender if a rival makes an offer. We show that the results are qualitatively similar if the incumbent is allowed to tender in the control contest.

At this stage we introduce some additional notation: A superscript $j \in \{0, 1\}$ on variables indicates the value of the variable if the firm issues new shares with j votes per-share; e.g., $j = 0$

¹²This is justified because firm insiders' stock sales are subject to restrictions by securities regulatory authorities and these restrictions may severely affect the incumbent manager's ability to tender in a control contest. For example, the incumbent manager may hold "restricted voting" shares, which will create hurdles for tendering in a control contest.

corresponds to non-voting shares, while $j = 1$ corresponds to conventional voting shares. Let

- n^j = number of new shares issued if j -voting shares are issued to finance the investment;
- ϕ^j = probability of no takeover, if j -voting shares are issued to finance the investment;
- V_D^j = public value per-share if j -voting shares are issued to finance the investment; and
- V_{vote}^j = value of a pure vote claim if j -voting shares are issued to finance the investment.
- V_k^j = value of k -voting share when j -voting shares are issued to finance the new investment.

If the new investment is financed using voting shares, only one type of share is outstanding and its aggregate value is denoted by V_1^1 . If non-voting shares are issued to fund new investments, then two different types of shares are outstanding and their values are given by V_1^0 , for the old voting shares, and V_0^0 , for the newly issued non-voting shares. The value of the voting shares is equal to the value of the dividend received plus the value of the vote, while the value of the non-voting shares is equal to the value of the dividend received. Thus, the value of the voting shares if the new shares are also voting shares, the value of the voting shares if the new shares are non-voting shares and the value of the newly issued non-voting shares are respectively given by

$$V_1^1 = V_D^1 + V_{vote}^1, \quad V_1^0 = V_D^0 + V_{vote}^0, \quad \text{and} \quad V_0^0 = V_D^0 \quad (2)$$

We assume that the existing voting shares and non-voting shares are paid the same dividends; but the per-share dividend is different if newly issued shares have voting rights as opposed to being non-voting, because of the lower price of non-voting shares. Our analysis is restricted to two types of securities – non-voting shares and “commons” or voting shares. The effect of multiple classes of securities and the problem of optimal security design (the “best” combination of dividend and vote) are not formally addressed here. A discussion later in the Section 6 briefly addresses these issues. That is, $V_D^1 \neq V_D^0$. This is because the number of new shares that the firm needs to issue to finance x dollars of investment, $n^j = \frac{x}{V_i^j}$, depends on the type of security issued given their different prices.

Proposition 1. *The number of new non-voting shares, n^0 , needed to be issued to raise x dollars is always at least weakly greater than the number of voting shares, n^1 , needed to be issued to raise the same dollar amount.*

Proof. Follows directly from Equation (2). □

By design, voting shares and non-voting shares have equal dividends. Since the value of vote is nonnegative (vote premium is similar to option premium), the value of one voting share, which is equal to the expected value of dividend plus the value of vote, has to be at least weakly greater than value of one non-voting share, which is just the expected value of dividend. See, for example, Smith and Amoako-Adu (1995) for detailed discussion on relative prices of voting versus non-voting shares.

4.1 The Decision Problems

Both the incumbent manager and the outside shareholders are assumed to maximize their expected wealth. For the manager, the decision variable is the level of investment, x . Given that the manager does not tender his shares to the rival, this level is equivalent to

$$\max_x W_I(x) = \max_x \{\beta N V_D^j(x) + \phi^j b_I a_I P(x)\}. \quad (3)$$

The objective function above has two parts: The first part is related to the public value of the firm and reflects the fact that the manager is similar to any other shareholder. The second part is related to the incumbent manager's private benefit of control, and is realized only if the manager retains control of the firm. The solution to the manager's problem yields the manager's optimal response to restrictions on the type of security that the firm can issue.

Let \hat{x}^j be the solution to the manager's optimization problem given that he issues j -voting shares to finance the investment. Outside shareholders maximize the value of their shares, picking the type of security that the manager can issue while taking the manager's optimal response function

as given. Thus, the decision problem of the outside shareholders is

$$\max_{j=0,1} V_1^j(\hat{x}^j). \quad (4)$$

To solve the two optimization problems, given by Equations (3) and (4), we need (i) the probability that there is no takeover, (ii) the value of the dividend, and (iii) the value of a vote, for the case where the firm issues voting shares and for the case where the firm issues non-voting shares to fund new investments respectively.

4.2 Potential Control Payoffs

A change in control occurs when the rival can offer a higher per-share price for the outside voting shares than the incumbent. The incumbent retains control if he can offer higher price for the outside voting shares than the rival. We assume that the rival only bids if he is sure to win. We separately consider the cases of financing the new investment with voting shares and non-voting shares. The question that the rival asks: What is the maximum price the incumbent can pay to buy the outside voting shares? Obviously, the incumbent has to pay the public value per share to the outside voting shareholders. Thus, he can be forced to pay a significant part of the present value of all his private benefits to prevent the rival from gaining control of the firm. Thus, the maximum price the incumbent is willing to offer for the outside voting shares equals

$$\frac{[\text{Public value of the firm under the incumbent's control}]}{[\text{Number of outstanding cash flow claims}]} + \frac{[\text{Incumbent's total private benefits}]}{[\text{Number of outstanding outside votes}]} \quad (5)$$

The incumbent can retain control of the firm only if the value shown in Equation (5) is weakly greater than the potential rival's maximum offer. If voting shares are used to finance the investment,

this condition is equivalent to

$$\frac{FV_I}{N+n^1} + \frac{b_I a_I P(x)}{(1-\beta)N+n^1} \geq \frac{FV_R}{N+n^1} + \frac{b_R a_R P(x)}{(1-\beta)N+n^1}. \quad (6)$$

The first term on the LHS of Equation (6) describes the per-share public value that is generated with the incumbent in control. The second term on the LHS of Equation (6) describes the per-share public value that is generated with the incumbent in control. The second term on the LHS is related to the incumbent's private benefits: denominator of the second term is smaller than denominator of the first term because the incumbent's private benefits are distributed only to the outside voting shareholders given that the incumbent does not tender. The RHS terms are related to the public and private benefits per-share generated under the rival's control. For example, if we have a situation where $b_I = b_R$ and $a_I < a_R$, then the rival will win the control contest. Substituting Equation (1) into (6) and simplifying we obtain

$$a_I (1 + \kappa^1 b_I) \geq a_R (1 + \kappa^1 b_R), \quad (7)$$

where $\kappa^1 = \frac{N\beta}{(1-\beta)N+n^1}$. If non-voting shares are issued to finance new investments, then the incumbent retains control of the firm if

$$\frac{FV_I}{N+n^0} + \frac{b_I a_I P(x)}{(1-\beta)N} \geq \frac{FV_R}{N+n^0} + \frac{b_R a_R P(x)}{(1-\beta)N}. \quad (8)$$

The private benefits are captured only by those outside shareholders who own voting shares. Simplifying Equation (8) we obtain

$$a_I (1 + \kappa^0 b_I) \geq a_R (1 + \kappa^0 b_R), \quad (9)$$

where $\kappa^0 = \frac{N\beta+n^0}{(1-\beta)N}$.¹³ After simplifying, Equations (7) and/or (9) can be expressed as

$$b_R \leq \underline{b}_R^j = \frac{1}{\kappa^j} \left(\frac{a_I}{a_R} - 1 \right) + b_I \frac{a_I}{a_R}, \quad j = 0, 1, \quad (10)$$

where \underline{b}_R^j is the lowest value of b_R such that takeover is not possible. Given that $\underline{b}_R^j \in [0, 1]$, we simplify and rearrange Equation (10) to derive \underline{a}_R^j and \bar{a}_R^j :

$$\bar{a}_R^j = a_I(1 + \kappa^j b_I) \quad \text{and} \quad \underline{a}_R^j = \frac{a_I(1 + \kappa^j b_I)}{1 + \kappa^j} = \frac{\bar{a}_R^j}{1 + \kappa^j}. \quad (11)$$

The likelihood of takeover and the role of private benefits in a takeover contest depends on \bar{a}_R^j , \underline{a}_R^j and \underline{b}_R^j . Proposition below formally states these observations.

Proposition 2. (i) Rivals with public quality a_R higher than \bar{a}_R^j can gain control of the firm regardless of their ability to extract private benefits (i.e., even if $b_R = 0$); whereas (ii) rivals with public quality lower than \underline{a}_R^j cannot gain control of the firm, even if they have the highest possible ability to extract private benefits (i.e., even if $b_R = 1$).

Proof. Directly follows from Equations (10) and (11). □

If the rival's public quality is significantly higher (lower) than the public quality of the incumbent, then the control contest will be decided based only on the public quality of the contestants, and their private qualities will not play a part in the control contest. If the rival manager's public quality is high ($a_R > \bar{a}_R^j$), then the rival manager gains control; whereas, if the rival manager's public quality is low ($a_R < \underline{a}_R^j$), then the incumbent retains control.¹⁴ Ability to extract private benefits plays a role in the control contest only when the rival's public quality is in the intermediate range of values, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$. Rival managers with public quality, $a_R \in [\underline{a}_R^j, \bar{a}_R^j]$ and ability to

¹³From Equations [??] we see that the rival can take over the firm from an incumbent whose public quality, $a_I = 1$ and ability to extract private benefits, $b_I = 1$. This is because the incumbent can always offer *at least as much as any rival* and hence, keep control of the firm. The corporate control market fails to work – firm value is lower than what it could be under a host of rivals! Also, this result is independent of the type of security that the incumbent uses to fund the new investments.

¹⁴There can be many (a_I, b_I) combinations such that $\bar{a}_R^j > 1$. This simply implies that there exists no rival who can take over the firm based only on his public quality, a_R .

extract private benefits, $b_R \in [\underline{b}_R^j, 1]$ can rest control of the firm from the incumbent. These control regions, which are based on the potential rival's public quality and his ability to extract private benefits, are depicted in Figure 4.

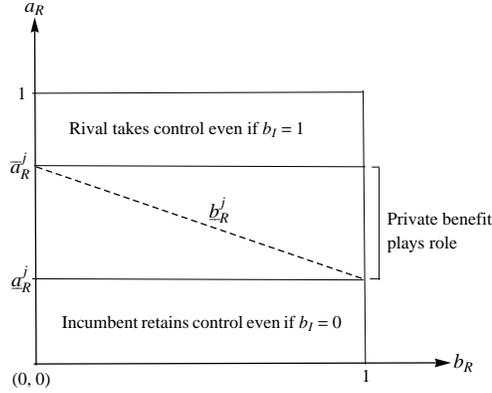


Figure 4: This figure depicts critical control regions as a function of potential rival's public quality, a_R and ability to extract private benefits, b_R . Private benefits do not play any role in the takeover contest if the potential rival's public quality is sufficiently high or sufficiently low so that either $[\bar{a}_R^j, 1]$ or $[0, \underline{a}_R^j]$. If the potential rival's public quality is drawn from the intermediate range $[\underline{a}_R^j, \bar{a}_R^j]$, then only the private benefits of the rival manager plays a role in the takeover contest. If the potential rival's ability to extract private benefits, $b_R > \underline{b}_R^j$ (dashed line), then the incumbent loses control of the firm; otherwise, incumbent retains control.

4.3 Effect of Investment on the Control Contest

Next, consider the effects of increasing investment, x , on \underline{a}_R^j , \bar{a}_R^j , and \underline{b}_R^j . These bounds determine the outcome of the control contest: The likelihood that the incumbent retains control of the firm after the investment depends on the level of investment. We show in Appendix B that $\partial n^j / \partial x > 0$, $\forall j$ implies that $\frac{\partial \kappa^0}{\partial x} > 0$ and $\frac{\partial \kappa^1}{\partial x} < 0$. Thus,

$$\frac{\partial}{\partial x} \bar{a}_R^j = a_I b_I \frac{\partial \kappa^j}{\partial x} \begin{cases} > 0 & \text{if } j = 0 \\ < 0 & \text{if } j = 1, \end{cases} \quad (12)$$

and

$$\frac{\partial}{\partial x} \underline{a}_R^j = \frac{\partial}{\partial \kappa^j} \underline{a}_R^j \times \frac{\partial \kappa^j}{\partial x} = -\frac{a_I(1-b_I)}{(1+\kappa^j)^2} \times \frac{\partial \kappa^j}{\partial x} \begin{cases} < 0 & \text{if } j = 0 \\ > 0 & \text{if } j = 1 \end{cases} \quad (13)$$

We formally state these results in the Propositions 3 and 4 below.

Proposition 3. (i) *If the firm issues voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality, $[\bar{a}_R^1, 1]$ increases; whereas (ii) if the firm issues non-voting shares to fund new investments, then the set of rivals who can take over the firm using only their public quality, $[\bar{a}_R^0, 1]$, decreases.*

Proof. Follows directly from Equation (12). □

If the firm under the incumbent uses voting shares to fund the new investments, then the larger the investment, x , the larger is the likelihood that a rival can gain control regardless of his ability to extract private benefits (i.e., even if $b_R = 0$). An incumbent with relatively high ability to extract private benefits, b_I , will be especially concerned! As he issues more and more voting shares, his private benefits become less and less useful in the control contest! This is because issuing new voting shares shift the relative weights from the vote premium to dividends in any control contest. When voting shares are issued, the vote premium is divided among a larger number of outside vote holders, $(1 - \beta)N + n^1$, and as a result the per-share vote premium falls. In contrast, if the firm issues non-voting shares to raise funds for investments, the role of private benefits remains unchanged; the incumbent can use his private benefits to buy just the existing outside votes, $(1 - \beta)N$.

Proposition 4. (i) *If the firm under the incumbent issues voting shares to fund the new investment, then the region over which the incumbent retains control regardless of a rivals ability to extract private benefits, $[0, \underline{a}_R^1]$, increases; whereas (ii) if the firm issues non-voting shares, then the region over which the incumbent retains control of the firm regardless of a rivals ability to extract private benefits, $[0, \underline{a}_R^0]$, decreases.*

Proof. : Follows directly from Equation (13). □

The intuition is similar; if the firm uses voting shares, then the role of the incumbents private benefits in the control contest gets reduced (region II gets smaller). On the other hand, if the firm uses non-voting shares, then the role of the incumbents private benefits in the control contest increases (region II expands). Panel A and B of Figure 5 depicts the results stated in Propositions 3 and 4.

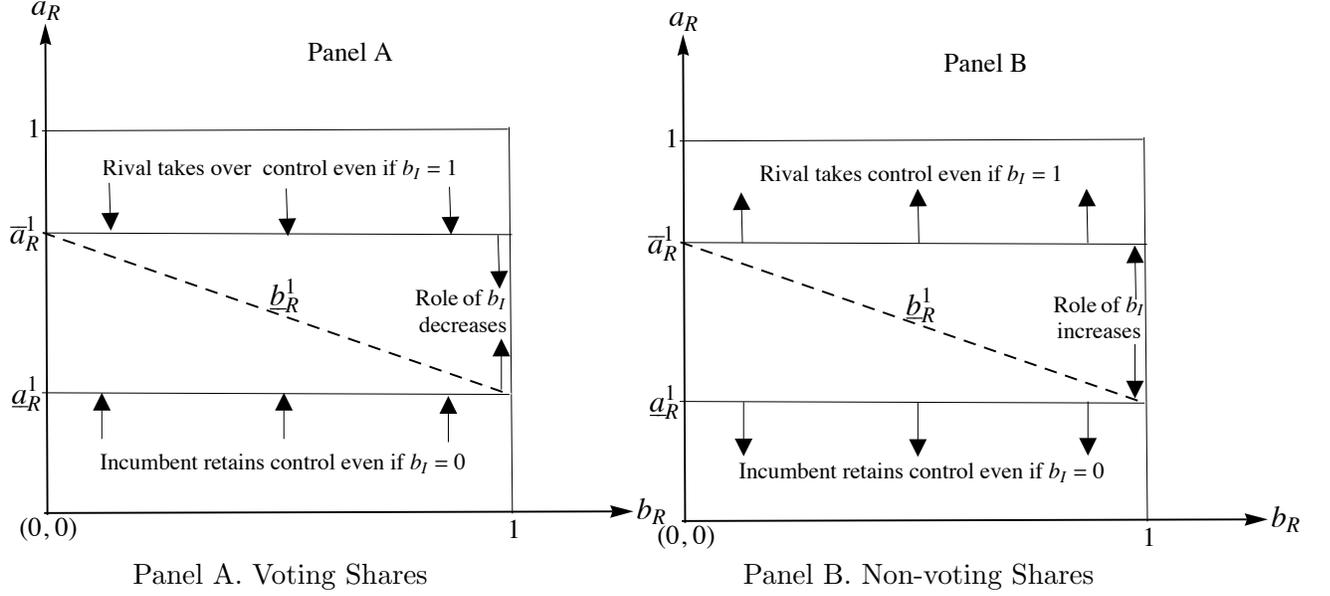


Figure 5: The effect of increasing x on \bar{a}_R^j , \underline{a}_R^j and \underline{b}_R^j . Panel A depicts the case when the incumbent issues voting shares to fund the firm's new investment. Panel B depicts the case when the incumbent issues non-voting shares to fund the firm's new investment. In both cases, arrows show the direction of movement of the upper bound, \bar{a}_R^j , and lower bound \underline{a}_R^j , as x , increases. In panel A, as x increases region II shrinks, implying that the private benefit plays a relatively lesser role in the control contest when the firm's investment is finance using voting shares. In panel B, as x increases region II expands, implying that private benefit plays a more important role in the control contest when the firm's investment is finance using non-voting shares.

The effect of increasing investment, x , on the likelihood of incumbent's retaining control is not unambiguous: *Region I*, where the incumbent loses control for certain, expands. But *Region III*, where the incumbent retains control for sure, expands too. Thus, the net effect of increasing x on the probability of the incumbent retaining control, ϕ^j depends on the incumbent's public and private qualities relative to the average public and private qualities of a potential rival.

4.4 Probability of Control

The probability of the incumbent's retaining control of the firm after issuing j -th type shares is

$$\phi^j = \int_0^{\underline{a}_R^j} da_R + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_0^{\underline{b}_R^j} db_R da_R \quad \text{where } j = 0, 1. \quad (14)$$

The first term in Equation (14) is the region where the potential rival's public quality is very low. In this region the rival has no hope of gaining control regardless of his ability to extract private

benefits. The second term is the region where the rival's public quality is such that the incumbent retains control if the rival's ability to extract private benefits is lower than \underline{b}_R^j ; otherwise, the rival gains control. There is one more range $[\bar{a}_R^j, 1]$, but the incumbent has no hope of retaining control over this range, regardless of the rival's ability of the rival to extract private benefits. Integrating the expression in Equation (14) and further simplifying, we get the probability of the incumbent's retaining control,

$$\phi^j(a_I, b_I, \beta, N, , x) = a_I(1 + b_I k^j) \frac{\log(1 + k^j)}{k^j} = a_I \times \phi^{j'}(b_I, \beta, N, , x). \quad (15)$$

We use the chain rule to differentiate Equation (15) with respect to x and obtain

$$\frac{\partial \phi^j(x)}{\partial x} = a_I \times \frac{\partial \phi^{j'}(x)}{\partial k^j} \times \frac{\partial k^j}{\partial x} = a_I \times \frac{\frac{k^j(1+b_I k^j)}{1+k^j} - \frac{\log(1+k^j)}{(k^j)^2}}{(k^j)^2} \times \frac{\partial k^j}{\partial x}. \quad (16)$$

Thus, the public quality of the manager only has a "level effect" on the incumbent manager's likelihood of retaining control, given that the firm invests x . For any given value of a_I the change in the incumbent's likelihood of retaining control after this investment is function of the incumbent's ability to extract private benefits, b_I . For $b_I \geq 1/2$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is nonnegative. For all $b_I < 1/2$, $\frac{\partial \phi^j(x)}{\partial k^j}$ is strictly negative. Hence, the sign of $\frac{\partial \phi^j(x)}{\partial x}$ depends on the sign of $\frac{\partial k^j}{\partial x}$ and the value of b_I . Hence,

$$\frac{\partial \phi^1(x)}{\partial x} = \frac{\partial \phi^1(x)}{\partial k^1} \times \frac{\partial k^1}{\partial x} \begin{cases} < 0 & \text{if } b_I \geq 1/2 \\ > 0 & \text{if } b_I < 1/2. \end{cases} \quad (17)$$

Similarly,

$$\frac{\partial \phi^0(x)}{\partial x} = \frac{\partial \phi^0(x)}{\partial k^0} \times \frac{\partial k^0}{\partial x} \begin{cases} > 0 & \text{if } b_I \geq 1/2 \\ < 0 & \text{if } b_I < 1/2 \end{cases} \quad (18)$$

Propositions 5 and 6 describe the relationships among the incumbent's likelihood of retaining control of the firm, his ability to the extract private benefits, and the firm's level of investment.

Proposition 5. *Suppose that the incumbents ability to extract private benefits is greater than an*

average rivals ability to extract private benefits; that is, $b_I \geq 1/2$. (i) If the incumbent issues voting shares to fund the new investment, then the likelihood of the incumbent retaining control, ϕ^1 , decreases as the firms investment level, x , increases; whereas, if the incumbent issues non-voting shares to fund the new investment, then the likelihood of incumbent retaining control, ϕ^0 , increases as the firms investment level, x , increases.

Proof. Follows directly from Equation (17). □

Given the public quality of the incumbent, a_I , if his ability to extract private benefits, b_I , is greater than an average rivals ability to extract private benefits, $E(b_R) = 1/2$, then using voting shares to fund the new investment reduces the incumbents ability to retain control post-investment. This is because voting shares make *dividends* as opposed to the *vote premium* relatively more important in a control contest. On the other hand, non-voting shares make the *vote premium* relatively more important than the *dividends* in a control contest. Hence, an incumbent who has relatively weak ability to extract private benefits, $b_I < 1/2$ prefers to issue voting shares to fund the new investment so as to shift the weight away from the *vote premium* towards *dividends*.

Proposition 6. *Suppose that the incumbents ability to extract private benefits is lower than an average rivals ability to extract private benefits; that is, $b_I < 1/2$. (i) If the incumbent issues voting shares to fund the new investment, then the likelihood of the incumbent retaining control, ϕ^1 , increases as the firms level of investment, x , increases; whereas, if the incumbent issues non-voting shares to fund the new investment, then the likelihood of incumbent retaining control, ϕ^0 , decreases as the firms level of investment, x , increases.*

Proof. Follows directly from Equation (18). □

The main intuition from Proposition 5 and 6 are depicted in the Figure 6 below.¹⁵

¹⁵As x increases so does n^j – not linearly, but strictly monotonically; hence, we use n^j instead of x .

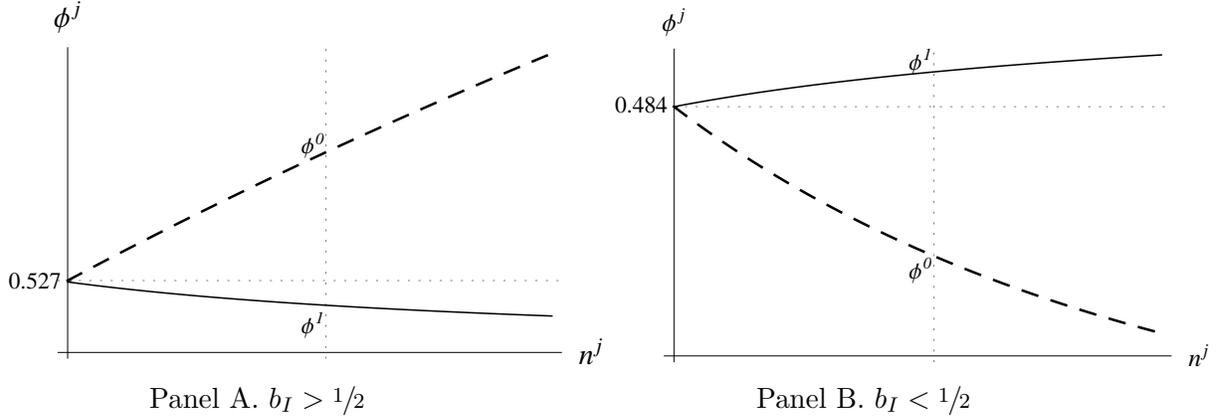


Figure 6: This figure depicts the likelihood that the incumbent retains control of the firm, ϕ^j , as a function of the size of the new equity issue, n^j . The solid black lines in both panels A and B correspond to the case where the new investment is financed using voting shares. The dashed lines and dotted lines, in both panels A and B, correspond to the case where the new investment is financed using non-voting shares.

4.5 Value of One Dividend Claim

The value of a pure dividend claim is equal to the expected dividend that the holder of the claim gets. This value depends on the manager-in-control's public quality and the type of security issued to finance the new investments. Thus, the value of the per-share pure dividend claim is

$$V_D^j = \phi^j \frac{FV_I}{N + n^j} + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^1 \frac{FV_R}{N + n^j} db_R da_R + \int_{\bar{a}_R^j}^1 \int_0^1 \frac{FV_R}{N + n^j} db_R da_R, \quad (19)$$

where $j = 0, 1$. The first term is the probability that the incumbent retains control times the per-share public value of the firm under the incumbent. The second and third terms give the expected dividend under the rival. Rivals of intermediate public quality and relatively high ($> \underline{b}_R^j$) ability to extract private benefits populate the second region. Rivals of very high public quality, who can take over the firm regardless of their ability to extract private benefits, populate the third region. If ϕ^j decreases, then $1 - \phi^j$ increases and vice versa.

Given our assumption that all available projects are positive NPV projects, the level of dividend increases with the level of investment, x . If x increases, either it is more likely for the incumbent to retain control or it is more likely for the rival to wrest control from the incumbent. If non-voting shares are used to fund the new investment, then the average dividend per-share increases less than

if voting shares are used. This is because the number of new non-voting shares required to raise x is greater than the number of voting shares; hence, each dividend claim gets diluted more.

4.6 Value of One Vote Claim

Voting rights matter because they allow stockholders to have a say in *who runs the company* and *how it is run*.¹⁶ Voting power arises, especially at badly managed companies, when a challenge is mounted against the incumbent either from within (activist stockholders) or from outside (hostile acquisitions).¹⁷

The value of a pure vote claim is related to the extraction of private benefits from the rival in the form of a vote premium. To obtain an expression for the value of the vote, we classify rival managers into one of three types: The first type represents rivals who cannot gain control of the firm because they have very low public quality ($a_R < \underline{a}_R^j$). If this type of rival is drawn, no private benefit is extracted and the value of the vote is zero. Next consider the rivals who can gain control of the firm without having to pay out any of their private benefit – those with very high public quality ($a_R > \bar{a}_R^j$). Again, it is not necessary for the rival manager to give up any of his private benefits. Hence, the private benefit is extracted only when a rival manager is of an intermediate type ($\underline{a}_R^j < a_R^j < \bar{a}_R^j$). The rival needs to pay “takeover” premium that is greater than the dividend it will produce; hence, the payoff to the vote claim when the firm issues voting shares can be written as

$$\begin{cases} \frac{FV_I}{N+n^1} + \frac{B_I}{N(1-\beta)+n^1} - \frac{FV_R}{N+n^1} & \text{if } \underline{a}_R^1 \leq a_R \leq \bar{a}_R^1 \\ 0 & \text{otherwise.} \end{cases} \quad (20)$$

¹⁶Zingales (1995b) and Nenova (2003) estimate the value of a vote based on the price difference of shares in firms with unequal voting rights that have both classes publicly traded. They find that the value of the vote is positive and varies across countries. See also Smart et al. (2008) for implications of vote on IPO valuation.

¹⁷Institutional investors’ benign neglect of different voting share classes at Google is rationalized by the fact that they think the company is well managed and that control is therefore worth little or nothing. There is a kernel of truth to this statement: The expected value of control (and voting rights) is greater in badly managed companies than in well managed ones. However, if you are an investor for the long term, you have to worry about whether managers who are perceived as good managers today could be perceived otherwise in a few years.

Similarly, the payoff on the vote claim when the firm issues non-voting shares is

$$\begin{cases} \frac{FV_I}{N+n^0} + \frac{B_I}{N(1-\beta)} - \frac{FV_R}{N+n^0} & \text{if } \underline{a}_R^0 \leq a_R \leq \bar{a}_R^0 \\ 0 & \text{otherwise,} \end{cases} \quad (21)$$

The value of the vote is simply the expectation of these values,

$$V_{vote}^1 = \left(\frac{FV_I}{N+n^1} + \frac{B_I}{N(1-\beta)+n^1} \right) \int_{\underline{a}_R^1}^{\bar{a}_R^1} \int_{\underline{b}_R^1}^1 db_R da_R - \int_{\underline{a}_R^1}^{\bar{a}_R^1} \int_{\underline{b}_R^1}^1 \frac{FV_R}{N+n^1} db_R da_R \quad (22)$$

and

$$V_{vote}^0 = \left(\frac{FV_I}{N+n^0} + \frac{B_I}{N(1-\beta)} \right) \int_{\underline{a}_R^0}^{\bar{a}_R^0} \int_{\underline{b}_R^0}^1 db_R da_R - \int_{\underline{a}_R^0}^{\bar{a}_R^0} \int_{\underline{b}_R^0}^1 \frac{FV_R}{N+n^0} db_R da_R. \quad (23)$$

Using expressions (2), we can derive the value of voting shares when the new shares issued are voting, as well as when the new shares issued are non-voting shares. Next, we depict the type of manager who will underinvest if he is forced to finance the firm's investment using voting shares.

5 Entrenchment and Investment

The manager chooses the investment level to maximize his expected wealth. There are three terms in the manager's objective function that depend on the level of investment chosen: The value of the dividend, the probability of retaining control, and the private benefits of control. The value of the dividend increases with investment as we consider only *positive NPV* opportunities. By design, the private benefits of control also increase with investment. We also see, from Equations (17) and (18), that the likelihood that the incumbent retains control of the firm depends primarily on two things: Level of investment (x) and ability of the incumbent to extract private benefits (b_I).

5.1 Low Quality Managers and the Control Contest

First, we turn to the issue of economic efficiency already established in the existing literature. Grossman and Hart (1988) and other studies have found that non-voting shares allow control of the firm to remain in or pass to the hands of inferior managers, lowering economic efficiency. We show that it is true that non-voting shares allow inferior managers to win control contests.¹⁸

Our result is similar to their result. The principal difference between voting and non-voting shares is that non-voting shares cause the private benefits of managers to have a larger impact on the control contests outcome. Consider a rival with ability to extract greater private benefits higher than the incumbent, that is, $b_R > b_I$. Nonvoting shares favor the rival in a control contest, making it easier for him to gain control of the firm; that is, he can gain control for lower values of a_R , values in which he would lose the control contest if instead the investment was financed with voting shares. Similarly, if $b_R < b_I$; that is, if the incumbent has greater ability to extract private benefits than the rival, then non-voting shares would favor the incumbent in a control contest, making it easier for him to retain control of the firm. That is, an incumbent can keep control of the firm with lower levels of a_I , values in which he would lose control if the investment was financed with voting shares. The proposition below formalizes this result.

Proposition 7. *The minimum public quality required for an incumbent manager to retain control of the firm is lower in firms financed with dual-class shares.*

Proof. See the proof stated in Section 8.2. □

The fact that a manager of lower public quality could gain control of firms should be a serious concern for market regulators. If other mechanisms can be used to discipline managers, then the cost of this problem will be small. For example, Moyer et al. (1992) find that alternative monitoring mechanisms emerge in firms after they issue dual-class shares.¹⁹

¹⁸A statement on economic efficiency, though, requires analysis of a trade-off between the costs of underinvestment and the cost of inefficiently managed firms. This process requires assumptions regarding the ability of other firms to undertake projects that the firm under consideration has forgone. We leave this aspect of the problem to future research.

¹⁹Hollinger International presents a good example of the negative effects of dual-class shares. Former CEO Conrad

5.2 Incumbent's Ownership and Underinvestment

Next, consider the case in which the manager does not own any existing shares in the firm; that is, $\beta = 0$. Suppose investments are financed by issuing voting shares; because the incumbent has no shareholding in the firm, there is no possibility of dilution in his control rights. Thus, the likelihood of the incumbent retaining control of firm is unaffected by the firm's investment level. In fact, the likelihood that the incumbent retains control depends only on his public quality, a_I . Thus, the incumbent's objective function is strictly increasing in x . Since underinvestment is not an issue, there is no need to allow the incumbent to issue non-voting shares. The proposition below formalizes this result.

Proposition 8. *When investments are financed by issuing voting shares and the incumbent does not own any equity in the firm (i.e., $\beta = 0$), then the incumbent invests in all available positive NPV projects.*

Proof. See the proof stated in Section 8.3. □

This is a counter-intuitive result: When the incumbent owns part of the firm's equity, he bears part of the cost of underinvestment. The larger the incumbent's ownership, β , the larger his share of the cost of underinvestment. Thus, it seems if $\beta = 0$, then the incumbent cares only about the probability of retaining control and his private benefits! But Proposition 8 shows that if $\beta = 0$, then the incumbent always invests in all available positive NPV projects. From Equation (6), we see that if $\beta = 0$, then it does not matter whether the incumbent pays more dividends or pays some as vote premium, because the number of outside votes, $N + n^1$, is equal to number of dividend claims, $N + n^1$. Hence, the control contest depends only on how much cash flow the incumbent generates vis-à-vis the rival. Thus, he invests \bar{x} to maximize the cash flows and consequently, maximize his chance of retaining control.

Black controlled all of the company's class-B shares, which gave him 30% of the firm's equity and 73% of its voting power. He ran the company as if he were the sole owner, exacting huge management fees, consulting payments, and personal dividends.

5.3 Investment using Voting and Non-voting Shares

Next, we consider a firm's financing decision and its effect on the incumbent's likelihood of retaining control of the firm after the new investments are funded. From Equations (17) and (18) we know that the incumbent's ability to extract private benefits, b_I , determines whether the probability of retaining control, ϕ^j , increases or decreases with the investment level. Given any level of public quality of the incumbent, a_I , we can isolate two types of incumbent: An incumbent with relatively high ability to extract private benefits, $1/2 \leq b_I \leq 1$; and an incumbent with relatively low ability to extract private benefits, $0 \leq b_I < 1/2$.

If the incumbent has relatively high ability to extract private benefits and uses voting shares to fund the new investments, then two implications arise for the incumbent's objective function: First, his expected private benefit can decrease because if the incumbent has a relatively high ability to extract private benefits and uses voting shares to fund the new investments, then two implications arise for the incumbent's objective function: First, his expected private benefit can decrease because the likelihood that he retains control decreases as x increases. But new investment increases his dividend, $N\beta V_D^1$, as well as his private benefits, $b_I a_I P(x)$. Hence, the firm's investment level depends on the net effect of increasing x on his expected wealth. This result is formalized in the following proposition.

Proposition 9. *If voting shares are issued to fund new investments, then the necessary conditions for the incumbent manager to forgo some positive NPV projects are (i) $\beta > 0$ and (ii) $b_I \geq 1/2$.*

Proof. See the proof stated in Section 8.4. □

Even if the incumbent's expected private benefit falls as he invests more financed with voting shares, the higher value of the dividend claim resulting from the increased investment can increase the incumbent's objective function. In addition, by design the level of private benefit increases with the investment level. Thus, when the negative impact of investment on the likelihood of retaining control outweighs the positive effects on dividends and private benefits, the incumbent underinvests. Next, we derive the sufficiency condition for underinvestment.

Proposition 10. *When voting shares are issued to fund new investment and the incumbent manager owns some equity in the firm ($\beta > 0$), then the incumbent manager forgoes some positive NPV projects if his ability to extract private benefits, b_I , is weakly greater than \hat{b}_I , where*

$$\hat{b}_I = \min \left[\frac{\beta + \frac{2(1-\beta)^2 \log\left(1 + \frac{\beta}{1-\beta}\right) - \frac{2(1-\beta)^2}{\beta}}{\beta^2}}{2(1-\beta) - \frac{\beta^2}{1-\beta}}, 1 \right].$$

Proof. See the proof stated in Section 8.5. □

The condition provided in the Proposition 10 is a sufficient condition for underinvestment: The incumbent manager always forgoes some positive NPV investments if he is forced to fund the new investments with voting shares and his ability to extract private benefits is greater than \hat{b}_I . The incumbent manager may forgo some positive NPV projects even when he has weaker ability to extract private benefits, $\frac{1}{2} \leq b_I \leq \hat{b}_I$. One can interpret \hat{b}_I as a proxy for the likelihood of underinvestment: Incumbent managers with private qualities greater than \hat{b}_I are sure to forgo some positive NPV investments. As \hat{b}_I gets larger and approaches one, it becomes less likely that we will find an incumbent with the ability to extract private benefits greater than \hat{b}_I . Hence, it also becomes less likely that the incumbent will underinvest. As a shareholder the incumbent bears part of any underinvestment opportunity cost. The larger an incumbents ownership fraction, the larger is his share of the opportunity costs. Thus, the condition given in the Proposition 10 depends on β . If the manager owns 8% of the firm's equity, then the incumbent forgoes some positive NPV projects only if $b_I > 0.8689$. This implies that if we collect a sample of firms with an average 8% managerial ownership level, then we should expect to find underinvestment in 13.11% of the sample. Figure 7 depicts the likelihood of underinvestment, $1 - \hat{b}_I$, as a function of an incumbents ownership fraction, β , and the ineffectiveness of private benefit deterrent mechanisms, .

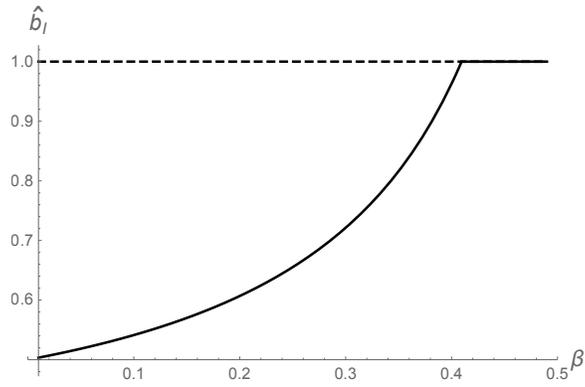


Figure 7: This figure depicts the likelihood of an incumbent manager underinvesting, $1 - \hat{b}_I$ as function of his shareholdings, β . In this figure we plot the likelihood of incumbent manager underinvestment as function of manager's initial ownership, β .

5.4 Welfare of Existing Shareholders and the Incumbent Manager

So far we obtain conditions under which investment increases or decreases if voting shares or non-voting shares are issued to raise funds. Increased investment financed with non-voting shares is not always in the best interests of either outside shareholders or the incumbent. There are costs to issuing non-voting equity. These costs are detailed below.

Since investors who buy non-voting shares are not entitled to any private benefits paid in a control contest as *vote premium*, they demand a lower per-share price for non-voting shares than for otherwise comparable voting shares. This means that a larger number of non-voting shares, n^0 , relative to voting shares, n^1 , must be issued to finance a given level of investment, x , reducing the per-share dividend that is available to existing shareholders. This is called the “dividend dilution” effect. Also, issuance of non-voting shares decreases the likelihood of a successful takeover bid which we call the entrenchment effect. Thus, holding constant the incumbents private benefits, a drop in the likelihood of a bid reduces the value of the voting rights. Although there is an offsetting gain namely a share of the extracted private benefits paid out as *vote premium* among a smaller number shareholders. To summarize, issuance of non-voting shares affects existing shareholders and the manager in three ways: (i) a lower per-share dividend; (ii) a lower probability of a change in control; and, (iii) a higher per-share takeover premium conditional on a successful takeover.

Recall that the firms value for any incumbent type (a_I, b_I) given an investment level, $x \in (0, \bar{x}]$,

financed with voting shares, $FV(x^1 = x) \geq FV(x^0 = x)$, the value of the same firm for the same level of investment financed with non-voting shares. Thus, existing outside shareholders will voluntarily allow an incumbent to issue non-voting shares only if the investment level, $\bar{x} - \Delta\bar{x}$, financed with voting shares leads to a firm value, $FV(x^1 = \bar{x} - \Delta\bar{x}) < FV(x^0 = \bar{x})$, which is the firm values given the same investment level financed with non-voting shares. The next proposition presents conditions under which the existing voting shares value is higher if new non-voting shares are used to fund new investments.

Proposition 11. *For all $b_I \geq \hat{b}_I$ and \bar{x} such that $n^0(\bar{x}) \leq N$, existing outside shareholders prefer the firm to finance its investment financed using non-voting shares if*

$$1 - \frac{P(x)}{P(\bar{x})} \geq \frac{a_I^2 b_I (2 + b_I (2\beta + 1) - 2\beta)}{2 a_I (1 - \beta)^2 (1 - b_I) - (a_I - a_I \beta (1 - b_I))^2 + (1 - \beta)^2}.$$

Proof. See the proof stated in Section 8.6. □

The result above gives the underinvestment level needed before existing shareholders voluntarily allow the incumbent manager to raise funds by issuing non-voting shares. The LHS of the above inequality is a measure of outside shareholders loses due to underinvestment, while the RHS is a measure of the costs of issuing non-voting shares. It is optimal for outside shareholders to allow the manager to issue non-voting shares only when the gains realized by reducing underinvestment outweigh the costs of issuing non-voting shares. If $a_I = 0.5$, $b_I = 0.75$, $\beta = 0.2$, and $\beta = 0.05$, outside shareholders will find the issuance of non-voting shares to finance an investment optimal, even if underinvestment is roughly 6.8% and the likelihood of underinvestment, $1 - \hat{b}_I$ is 27%.

Next, we consider the incumbent's expected wealth. Use of non-voting shares to fund the new investments lowers the per-share dividend and thus, affects the incumbent's wealth negatively. If $b_I \geq 1/2$, then non-voting shares lower the probability of a successful takeover and thus, non-voting shares help to increase expected wealth of the incumbent.²⁰ Given the incumbent's type (a_I, b_I)

²⁰The increase in the takeover premium does not affect the manager because we have assumed that he does not tender in a takeover.

and his objective function, underinvestment will occur if the incumbent is forced to issue only voting shares to fund new investment such that $W_I(x^1 = \bar{x}) < W_I(x^0 = \bar{x})$ and there exists a $\Delta \bar{x}$ such that $W_I(x^1 = \bar{x} - \Delta \bar{x}) = W_I(x^0 = \bar{x})$. The propositions below provide results on the types of managers who are better off if the firm issues non-voting stock.

Proposition 12. *For all $1/2 \leq b_I \leq 1$ and \bar{x} such that $n^0(\bar{x}) \leq N$, the incumbent prefers investment financed by non-voting equity if $b_I \geq \hat{b}_I$, where*

$$\hat{b}_I = \frac{(1 - \beta) \left((2 - \beta)^2 \beta^2 (1 + \beta) + 2(1 - \beta)^2 \left((1 + \beta) \log \left(1 + \frac{\beta}{1 - \beta} \right) - \beta \log \left(1 + \frac{1 + \beta}{1 - \beta} \right) \right) \right)}{\beta(1 + \beta) \left(4(1 - \beta)^2 \log \left(1 + \frac{1 + \beta}{1 - \beta} \right) - \beta(1 + \beta) - 4(1 - \beta)^2 \log \left(1 + \frac{\beta}{1 - \beta} \right) \right)}.$$

Proof. See the proof stated in Section 8.7. □

From Equation (18) we see that investment using non-voting shares increases the likelihood that the incumbent retains control of the firm if $b_I \geq \frac{1}{2}$. From Proposition 12 we know that the incumbent is better off if non-voting shares can be issued and $b_I \geq \hat{b}_I$. The divergence exists because of the dividend dilution due to the issuance of lower priced non-voting shares. If non-voting shares are issued, then the aggregate dividend is divided among $N + n^0$ shares, which is strictly greater than $N + n^1$ shares if voting shares are issued. If $\beta = 0.2$ and $\beta = 0.4$, the manager prefers non-voting shares if b_I is greater than 0.59, even though the incumbents likelihood of retaining control increases with investment for all $b_I \geq 0.5$. For all $\beta > 0.22$ and for all $\beta \leq 0.2$ the value of $\hat{b}_I \geq 1$, implies that the incumbent never prefers non-voting shares over this range of parameter values.

From Propositions 10, 11, and 12 we can derive some interesting observations. Consider the case when $b_I \in [1/2, 1]$. It is easy to show for all β and \bar{x} such that \hat{b}_I and \hat{b}_I are within the range $1/2$ and 1, that $\hat{b}_I > \hat{b}_I$. We depict the comparable values of \hat{b}_I and \hat{b}_I in Figure 8 below. Thus, we can divide the range of the incumbents private qualities into four regions: $[0, 1/2)$, $[1/2, \hat{b}_I)$, $[\hat{b}_I, \hat{b}_I)$, and $[\hat{b}_I, 1]$. If the incumbent is constrained to use voting shares, then the incumbent with $b_I \in [\hat{b}_I, 1]$ underinvests.²¹ Otherwise, he invests in all available positive NPV projects. If the incumbent is

²¹If the incumbents $b_I \in [\hat{b}_I, \hat{b}_I]$, he may or may not underinvest. This range is indeterminate as we are only able to solve the sufficient condition for underinvestment along with the incumbents choice of voting vs. non-voting shares.

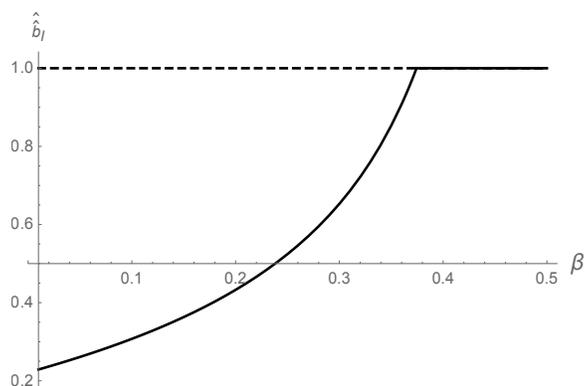
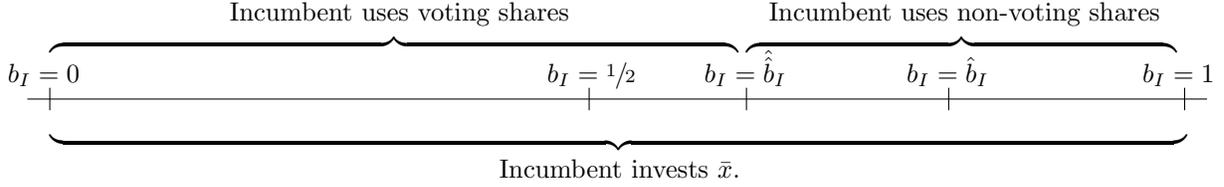


Figure 8: This figure depicts parameter values such that the incumbent prefers non-voting shares. In panel A we find that for parameter values such that the likelihood of underinvestment is positive, $1 - \hat{b}_I > 0$, we find $\hat{b}_I > \hat{b}_I$. This implies that for all parameter values such that the manager underinvests, the manager also prefers financing with non-voting shares. In panel B we depict the minimum investment level b_I such that the incumbent prefers non-voting shares as a function of his ownership fraction, β . As incumbent shareholding rise, the incumbent is less likely to prefer non-voting shares.

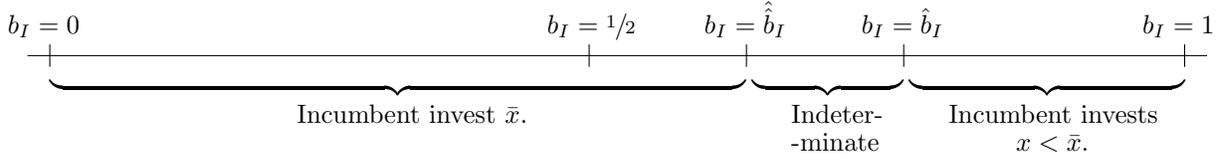
given a choice of using either voting shares or non-voting shares, then the incumbent always invests in all available positive NPV projects. The manager uses voting shares to fund the investment, if his ability to extract private benefits $b_I \in [0, \hat{b}_I)$. Otherwise, the manager uses non-voting shares to fund the investment, if his ability to extract private benefits $b_I \in [\hat{b}_I, 1]$. Figure 9 below depicts these b_I ranges.

The final question that we answer in this subsection is in what instances do firms issue non-voting shares. The answer depends on the balance of power between the manager and the shareholders. If shareholders have the upper hand and can force the manager to issue a particular type of security, the condition given in Proposition 11 determine when the firm issues non-voting shares. In contrast, if shareholders can only specify a menu of securities, then the conditions in Proposition 10 and Proposition 12 must both be satisfied before the firm issues non-voting shares. The next section discusses extensions to our model. It also looks at the effect of relaxing some of our initial assumptions.

Allowed to use either voting shares or non-voting shares



Forced to use only voting shares



6 Extensions

In this section we consider three related issues. First, we allow the incumbent to tender his holdings in a control contest. This is important because it helps to make the incumbent more entrenched. Second, we address the issuance of shares with fewer than one vote per-share. The reason that these questions may be useful to explore is that they could have lower costs related to dividend dilution than zero-vote shares. Firms in many countries allow firms to issue multiple classes of shares with different voting rights just like dual class shares. Third, we discuss the costs and benefits of multiple classes of shares.

6.1 Entrenchment and Investment when the Incumbent Tenders

A change in control occurs when the rival manager offers a higher per-share value to outside shareholders than the incumbent offers. Earlier, we assumed that the incumbent does not tender his shares in the control contest. We now relax this assumption. If the manager tenders his shares in a control contest, then the rival's private benefit is divided over a larger number of shares, $N + n$ as opposed to $(1 - \beta)N + n$ outside voting shares when the manager is not able to tender. This puts

the rival at a disadvantage relative to the incumbent.²² The probability of a takeover is obtained by separately considering the cases of non-voting shares and voting shares. The incumbent retains control if he offers a higher price per share than the rival. If voting shares are used to finance the investment, this contest is equivalent to

$$\frac{x + a_I P(x)}{N + n^1} - \frac{b_I a_I P(x)}{N + n^1} + \frac{b_I a_I P(x)}{(1 - \beta) N + n^1} \geq \frac{x + a_R P(x)}{N + n^1} - \frac{b_R a_R P(x)}{N + n^1} + \frac{b_R a_R P(x)}{N + n^1}. \quad (24)$$

The first two terms on the LHS of Equation (6) are the per-share public value that is generated with the incumbent in control. The third term on the LHS is related to the incumbent's private value. The denominator is smaller here than in the first two terms because the private benefit is distributed only to the outside shareholders. The RHS terms are related to the public and private benefit per-share generated under the rival. The private benefits of the potential rival are divided among all the firm's shareholders. Simplifying Equation (24) we obtain

$$a_I (1 + \kappa^1 b_I) \geq a_R, \quad (25)$$

where $\kappa^1 = \frac{N\beta}{(1-\beta)N+n^1}$. From Equation (11) we know that $\bar{a}_R^1 = a_I (1 + \kappa^1 b_I)$. Thus, the incumbent retains control of the firm if $a_R \in [0, \bar{a}_R^1]$. The ability to extract the rivals private benefits plays no role in the control contest, whereas the ability to extract the incumbents private benefits plays a significant role in the control contest. When the incumbent can tender, the range of the rivals public quality over which the incumbent retains certain control, $[0, \bar{a}_R^1]$, is much higher than $[0, \underline{a}_R^1]$, the range when the incumbent does not tender. Since the range over which the incumbent certainly loses control, $[\bar{a}_R^1, 1]$, remains the same, the likelihood that the incumbent retains control after investing x , say $\hat{\phi}^1(x)$, when the incumbent can tender is greater than $\phi^1(x)$, the likelihood that the incumbent retains control after investing x , given the incumbent is not able to tender.

What happens to the incumbents likelihood of retaining control if x increases? Because $\frac{\partial \kappa^1}{\partial x}$ is

²²See, for example, Burkart et al. (1998) for more discussions.

negative, $\frac{\partial \hat{\phi}^1}{\partial x}$ is also negative. Hence, the incumbents likelihood of retaining control decreases as the investment level rises.

If non-voting shares are issued to finance the investment and if the manager can tender his shares in a control contest, the rivals private benefit is divided over a larger number of shares, N rather than $(1 - \beta)N$. Then the incumbent retains control if

$$\frac{x + a_I P(x)}{N + n^0} - \frac{b_I a_I P(x)}{N + n^0} + \frac{b_I a_I P(x)}{(1 - \beta)N} \geq \frac{x + a_R P(x)}{N + n^0} - \frac{b_R a_R P(x)}{N + n^0} + \frac{b_R a_R P(x)}{N}. \quad (26)$$

The private value is distributed to all shareholders who own voting shares. The holders of the non-voting shares do not get a share of the private value because they cannot affect the outcome of the control contest. Simplifying Equation (26) we get

$$a_I (1 + \kappa^0 b_I) \geq a_R (1 + \hat{\kappa}^0 b_R), \quad (27)$$

where $\kappa^0 = \frac{N\beta + n^0}{(1-\beta)N}$ and $\hat{\kappa}^0 = \frac{n^0}{N}$. Also note that $\kappa^0 - \hat{\kappa}^0 = \frac{\beta(N+n^0)}{(1-\beta)N} > 0$. This gives

$$\hat{a}_R^0 = a_I(1 + \kappa^0 b_I) = \bar{a}_R^0 \quad \text{and} \quad \underline{a}_R^0 = \frac{a_I(1 + \kappa^0 b_I)}{1 + \hat{\kappa}^0} = \frac{\bar{a}_R^0}{1 + \hat{\kappa}^0} > \underline{a}_R^0. \quad (28)$$

From Equation (28), we see that the range of rival's qualities over which the incumbent retains certain control, $[0, \hat{a}_R^0]$, is larger than $[0, \underline{a}_R^0]$, rival's public quality over which the incumbent retains control when the incumbent is not able to tender. Since the range over which the incumbent loses control, $[\bar{a}_R^0, 1]$, remains the same in this case, the incumbent's likelihood of retaining control for a given investment level, $\hat{\phi}^0(x)$, is at least weakly greater when the manager can tender, relative to when the manager cannot tender. This is formally stated in the proposition below.

Proposition 13. *Regardless of the type of investment financing choice, namely voting or non-voting shares, the ability of the incumbent to extract private benefits, b_I , plays a relatively more decisive role in the control contest and makes the incumbent more entrenched, when he is able to tender his shares in the control contest.*

Proof. Follows directly from Equations (25) or (28). □

Like our initial setup, these bounds determine the outcome of the control contest: The likelihood that the incumbent retains control of the firm after investment depends on the level of investment, and the incumbent can choose the size of x . Because the number of new shares needed to finance the investment rises with the investments size, $\frac{\partial n^0}{\partial x} > 0$, then $\frac{\partial \hat{\kappa}^0}{\partial x} > 0$, it follows that $\frac{\partial n^0}{\partial x} > 0$, then $\frac{\partial \hat{\kappa}^0}{\partial x} > 0$. Thus,

$$\frac{\partial}{\partial x} \hat{a}_R^0 = \frac{\partial}{\partial n^0} \hat{a}_R^j \times \frac{\partial n^0}{\partial x} = -\frac{a_I N(1 - \beta - \frac{b_I}{2}(1 - \beta))}{(N + n^0)^2(1 - \beta)} \times \frac{\partial n^0}{\partial x} < 0. \quad (29)$$

Thus, the implications of increasing investment on the incumbent manager's likelihood of retaining control are qualitatively similar to the case when the manager is unable to tender.

6.2 Optimal vote-dividend combination

The optimal vote-dividend combination can be viewed in two different ways. The first is to consider shares that have one unit of dividend and θ votes, and to find the optimal value of θ . The second is to allow the firm to simultaneously issue both voting and non-voting shares. We first consider θ -vote shares.

The optimality of θ -vote shares, with $0 < \theta < 1$, will depend on the size of the firm's investment opportunity. For a class of shares, the vote has value only if a sufficient mass of votes in that class exists, so that these shares can be used by the manager to block a takeover. This means that managers issue θ -vote shares only if the investment opportunity is large enough that $\beta N + n\theta \geq \frac{1}{2}(N + n\theta)$.

The reasoning captured by the above inequality is as follows: Consider a firm with two share classes, one-vote and θ -vote shares outstanding. Suppose $n\theta$ is small so that the above inequality is not fulfilled. In this case, the manager has no incentive to bid for the θ -vote class of shares; blocking the rival only requires the manager to bid for the full-vote shares. The rival has no incentive to bid for the θ -vote shares either. The outcome of the control contest is determined solely by the

owners of the voting shares. This causes the vote to have zero value in the θ -vote shares, giving the manager no incentive to issue θ -vote shares.

If \bar{x} is small, $\theta = 0$ is likely to be optimal. This is because the number of shares that are issued is going to be small for small \bar{x} , and the total number of votes held by shareholders in that class will be insufficient to meet the above condition. Our model has assumed that shareholders are homogeneous. Heterogeneity among shareholders may result in cases where θ -vote shares may become optimal even when \bar{x} is small.

Allowing firms to simultaneously issue both non-voting and voting shares will increase the set of firms that find it optimal to issue dual-class shares. This assertion is based on the following line of reasoning: Existing one-vote shareholders prefer non-voting shares when the level of underinvestment is high; that is, $1 - \frac{P(x)}{P(\bar{x})} \geq g(\beta, \kappa^0, b_I, a_I)$. From Equation (P11-5) we know that the RHS of the above inequality is an increasing function of κ^0 which itself is an increasing function of $n^0(\bar{x})$. Hence, as $n^0(\bar{x})$ decreases, the outside shareholders find it optimal to allow the manager to finance investments using non-voting shares, even for low levels of underinvestment. If the investments are partly financed using voting shares and this analysis is carried out over the remaining projects, the relevant $n^0(\bar{x})$ will have a smaller value, implying that the existing shareholders, the owners of the voting shares, would be more willing to allow managers to issue non-voting shares. In this case, the existing shareholders could allow the manager the choice of issuing voting shares or a mix of θ -voting shares per non-voting share issued.

6.3 Multiple classes of shares

We considered a firm that issues only two classes of shares: Voting and non-voting shares. One logical extension to this model is to consider multiple classes of shares. Is it optimal either for the manager or for existing shareholders to issue multiple classes of shares? Consider shares that give their owners fractional voting rights can be considered. Now, the firm can simultaneously issue shares with $\theta_0, \theta_1, \theta_2$, and θ_3 votes (an example can be $\theta_0 = 0, \theta_1 = 0.33, \theta_2 = 0.5$, and $\theta_3 = 1$). In the above framework, the shares with fractional votes are issued if the fractional votes have value.

The fractional votes have value if there is a sufficient mass of each of these share classes outstanding so that the rival is forced to buy them to take control of the firm.

The manager can raise the cost of a takeover for the rival by issuing multiple classes of shares. However, this does not mean that it is optimal for the manager to issue multiple share classes. The manager bears a cost when he issues multiple share classes, which is in the form of lower dividends. Thus, the existing shareholders are likely to find multiple classes of shares detrimental to their interest. As the number of share classes increases the probability of a change in control is likely to decrease very quickly. The compensating factor, investment, is unlikely to go up fast enough to increase the value of the shares held by outside shareholders. Thus, multiple share classes are unlikely to be optimal for old shareholders.

7 Conclusions

This study provides a theoretical justification for easing the prohibitions on the issuance of dual-class shares, which has recently been proposed or enacted in a number of developed and developing countries. We analyze a firm's decision problem when a set of positive NPV projects are available. We show that if a firm requires outside equity financing to undertake profitable investment projects, then in some cases managers find separation of voting and dividend claims optimal. Raising equity capital has two effects: (i) The value of the firm increases as positive NPV projects are undertaken and (ii) the proportion of the firm's shares owned by the manager decreases, raising the likelihood that the manager loses control of the firm. Thus, a manager, who values control because of the private benefits it offers, may find it optimal to forgo some positive NPV projects. Non-voting shares enables the manager to finance the investment without diluting his control, and thus increasing his chances of retaining control of the firm, which in turn increases the manager's willingness to undertake all positive NPV projects.

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8 Appendix A

8.1 Basic Results

Before we present the proofs of the propositions, we first show some basic results that we will use repeatedly. By definition we have

$$\kappa^0(x) = \frac{\beta N + n^0(x)}{(1 - \beta)N} \text{ and } \kappa^1(x) = \frac{\beta N}{(1 - \beta)N + n^1(x)}, \quad (\text{BR-1})$$

where $n^j(x) = 0$ at $x = 0$, and $n^j(x) > 0, \forall x \in (0, \bar{x}]$, and $j = 0, 1$. Also, $\frac{\partial n^j(x)}{\partial x} > 0$. Thus,

$$\kappa^0(0) = \kappa^1(0) = \frac{\beta}{1 - \beta}. \quad (\text{BR-2})$$

For $x > 0$ we get

$$\kappa^0(x) = \frac{\beta}{1 - \beta} + \frac{n^0(x)}{(1 - \beta)N} > \frac{\beta}{1 - \beta} = \kappa^0(0),$$

If $n^0(\bar{x}) \leq N$, then

$$\kappa^0(x) \leq \frac{\beta}{1 - \beta} + \frac{1}{(1 - \beta)}. \quad (\text{BR-3})$$

$$\kappa^1(x) = \frac{\beta}{1 - \beta} - \frac{\beta n^1(x)}{(1 - \beta)N [(1 - \beta)N + n^1]} < \frac{\beta}{1 - \beta} = \kappa^1(0). \quad (\text{BR-4})$$

Thus $\kappa^0(x) \geq \kappa^1(x) \forall x \in [0, \bar{x}]$. Differentiating $\kappa^0(x)$ and $\kappa^1(x)$ with respect to x we get

$$\frac{\partial \kappa^0}{\partial x} = \frac{\partial}{\partial x} \left(\frac{N\beta + n^0}{(1 - \beta)N} \right) = \frac{1}{(1 - \beta)N} \frac{\partial n^0}{\partial x} > 0. \quad (\text{BR-5})$$

and

$$\frac{\partial \kappa^1}{\partial x} = \frac{\partial}{\partial x} \left(\frac{N\beta}{(1-\beta)N+n^1} \right) = \frac{-1}{((1-\beta)N+n^1)^2} \frac{\partial n^1}{\partial x} < 0. \quad (\text{BR-6})$$

8.1.1 Dividend claims

We can simplify Equation (19) and rewrite as

$$\begin{aligned} (N+n^j)V_D^j &= x + \phi^j \times a_I(1-b_I)P(x) + \int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^1 (a_R(1-b_R)P(x)) db_R da_R \\ &+ \int_{\bar{a}_R^j}^1 \int_0^1 (a_R(1-b_R)P(x)) db_R da_R. \end{aligned} \quad (\text{BR-7})$$

Integrating the third term of the above expression we get

$$\int_{\bar{a}_R^j}^1 \int_0^1 (a_R(1-b_R)P(x)) db_R da_R = \frac{2-}{4} \left(1 - (\bar{a}_R^j)^2\right) P(x). \quad (\text{BR-8})$$

Integrating the second term of the above expression we get

$$\int_{\underline{a}_R^j}^{\bar{a}_R^j} \int_{\underline{b}_R^j}^1 (a_R(1-b_R)P(x)) db_R da_R = A(x, \kappa^j) + B(x, \kappa^j), \quad (\text{BR-9})$$

where $A(x, \kappa^j) = \frac{(\bar{a}_R^j)^2(2+(4-5)\kappa^j+(6-)(\kappa^j)^2)P(x)}{4\kappa^j(1+\kappa^j)}$ and $B(x, \kappa^j) = \frac{\bar{a}_R^j P(x)}{2\kappa^j} \ln(1+\kappa^j)$.

8.1.2 Value of existing voting shares when voting shares are issued

Using Equations (19) and (22) we can express total value of all voting shares when voting shares are issued to finance the new projects:

$$(N+n^1)V_1^1 = (N+n^1)V_D^1 + (N+n^1)V_{\text{vote}}^1 \quad (\text{BR-10})$$

$$\begin{aligned} (N+n^1)V_1^1 &= \int_0^{\bar{a}_R^1} \int_0^1 FV_I db_R da_R + \int_{\bar{a}_R^1}^1 \int_0^1 FV_R db_R da_R + \int_{\underline{a}_R^1}^{\bar{a}_R^1} \int_{\underline{b}_R^1}^1 \frac{(N+n^1)B_I}{(1-\beta)N+n^1} db_R da_R \\ &= x + \frac{1}{2}P(x) \left((a_I(1-b_I) + (1+\kappa^1) \left(1 - \frac{\ln(1+\kappa^1)}{\kappa^1}\right) \bar{a}_R^1) \right) \\ &+ \frac{1}{2}P(x) (1-1/2) (1 - (\bar{a}_R^1)^2). \end{aligned} \quad (\text{BR-11})$$

If the firm issues voting stock to finance the investment, and because all new securities are issued at zero expected profit, $n^1 V_1^1 = x$, we get

$$N V_1^1 = \frac{1}{2} P(x) \left((a_I(1 - b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1} \right) \bar{a}_R^1 + (1 - \beta) (1 - (\bar{a}_R^1)^2) \right). \quad (\text{BR-12})$$

8.1.3 Value of existing voting shares when non-voting shares are issued

If non-voting shares are used to finance the investment, then using Equations (19) and (22), the value of an existing voting share is

$$\begin{aligned} (N + n^0) V_1^0 &= (N + n^0) V_D^0 + (N + n^0) V_{\text{vote}}^0 \\ &= \frac{P(x)}{2} \left((a_I(1 - b_I) + \left((1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left(1 - \frac{\ln(1 + \kappa^0)}{\kappa^0} \right) \bar{a}_R^0 \right) \\ &\quad + \frac{P(x)}{2} (1 - \beta) (1 - (\bar{a}_R^0)^2). \end{aligned} \quad (\text{BR-13})$$

8.2 Proof of Proposition 7

Using Equations (7) and (9) we get the incumbent's minimum public quality required to win the control contest: If financed using voting shares, it is

$$\underline{a}_I^1 \geq \frac{a_R (1 + \kappa^1 b_R)}{(1 + \kappa^1 b_I)}, \quad (\text{P7-1})$$

and if financed using non-voting shares, it is

$$\underline{a}_I^0 \geq \frac{a_R (1 + \kappa^0 b_R)}{(1 + \kappa^0 b_I)}. \quad (\text{P7-2})$$

We need to show that $\underline{a}_I^0 \leq \underline{a}_I^1$. Thus, we obtain

$$\underline{a}_I^0 - \underline{a}_I^1 = - \frac{a_R (b_I - b_R) (\kappa^0 - \kappa^1)}{(1 + \kappa^1 b_I) (1 + \kappa^0 b_I)} \leq 0. \quad (\text{P7-3})$$

From Equation (BR-1) we know that $\kappa^0 - \kappa^1 > 0$. Thus, if $b_I \geq b_R$ then $\underline{a}_I^0 < \underline{a}_I^1$. Hence, the proof.

8.3 Proof of Proposition 8

The manager chooses the investment level to maximize his objective function. We show that the first derivative of the manager's objective function evaluated at \bar{x} is nonnegative, which implies that the incumbent invests \bar{x} .

When investments financed using voting shares and $\beta = 0$, then from Equation (11) we get $\underline{a}_R^1 =$

$\bar{a}_R^1 = a_I$. This is because $\kappa^1 = \frac{\beta N}{(1-\beta)N+nI} = 0$. Thus,

$$\phi^1(x) = \int_0^{\bar{a}_R^1} da_R + \int_{\bar{a}_R^1}^{\bar{a}_R^1} \int_0^{b_R^1} db_R da_R = \int_0^{a_I} da_R = a_I. \quad (\text{P8-1})$$

Incumbent manager's objective function if he finances the new investment issuing j-type shares, say MO^j , is

$$MO^j(x) = \beta N V_D^j(x) + \phi^j(x) b_I a_I P(x). \quad (\text{P8-2})$$

Differentiating the incumbent's objective function with respect to x gives

$$\frac{\partial MO^j(x)}{\partial x} = \beta N \frac{\partial V_D^j(x)}{\partial x} + b_I a_I \left(\frac{\partial \phi^j(x)}{\partial x} P(x) + \phi^j(x) \frac{\partial P(x)}{\partial x} \right). \quad (\text{P8-3})$$

Substituting $j = 1$ and $\beta = 0$ we get

$$\frac{\partial MO^1(x)}{\partial x} = b_I a_I \left(\frac{\partial \phi^1(x)}{\partial x} P(x) + \phi^1(x) \frac{\partial P(x)}{\partial x} \right). \quad (\text{P8-4})$$

But $\frac{\partial \phi^1(x)}{\partial x} \Big|_{\beta=0} = \frac{\partial a_I}{\partial x} = 0$. Hence,

$$\frac{\partial MO^1(x)}{\partial x} \Big|_{\beta=0} = b_I a_I \phi^1(x) \frac{\partial P(x)}{\partial x}. \quad (\text{P8-5})$$

Since by definition $\frac{\partial P(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial P(x)}{\partial x} = 0$ for $x = \bar{x}$, it must be the case that $\frac{\partial MO^1(x)}{\partial x} > 0$ for all $x \in [0, \bar{x})$ and $\frac{\partial MO^1(x)}{\partial x} = 0$ for $x = \bar{x}$. Thus, the incumbent manager invests in all available positive NPV projects. Hence, the proof.

From Equation (17) we know that if voting shares are issued and $b_I < 1/2$, then $\frac{\partial \phi^1(x)}{\partial x} > 0$ for all x . Hence, the sufficient condition for the incumbent to invest \bar{x} using voting shares is $b_I < 1/2$. Similarly, from Equation (18) we know that if non-voting shares are issued and $b_I \geq 1/2$, then $\frac{\partial \phi^0(x)}{\partial x} > 0$ for all x . Hence, the sufficient condition for the incumbent to invest \bar{x} using non-voting shares is $b_I \geq 1/2$. Hence, the proof.

8.4 Proof of Proposition 9

Differentiating Equation P8-2, substituting $j = 1$ and further simplifying we obtain

$$\frac{\partial MO^1(x)}{\partial x} = \beta N \frac{\partial V_D^1(x)}{\partial x} + b_I a_I \left(\frac{\partial \phi^1(x)}{\partial x} P(x) + \phi^1(x) \frac{\partial P(x)}{\partial x} \right). \quad (\text{P9-1})$$

Let us consider following situations:

1. $\beta = 0$ and $b_I \in [0, 1]$;
2. $\beta > 0$ and $b_I \in [0, 1/2]$; and,
3. $\beta > 0$ and $b_I \in [1/2, 1]$.

We have shown in Proposition 9 that regardless of the incumbent manager's ability to extract private benefits, the incumbent will fund all available positive NPV projects if $\beta = 0$. Hence, $\beta > 0$ is a necessary condition for underinvestment. If $\beta > 0$, then $\frac{\partial \phi^1(x)}{\partial x} < 0$ if $b_I \geq 1/2$ and $\frac{\partial \phi^1(x)}{\partial x} > 0$ if $b_I < 1/2$.

Thus, if $b_I < 1/2$, then all three, $\frac{\partial V_D^1(x)}{\partial x}$, $\frac{\partial P(x)}{\partial x}$, and $\frac{\partial \phi^1(x)}{\partial x}$ in Equation P9-1, are increasing in all $x \in [0, \bar{x}]$; hence, $\frac{\partial MO^1(x)}{\partial x}$ is increasing in x . If $b_I \geq 1/2$, then $\frac{\partial V_D^1(x)}{\partial x}$ and $\frac{\partial P(x)}{\partial x}$ are still increasing in x , but $\frac{\partial \phi^1(x)}{\partial x}$ is decreasing in x . Thus, only way $\frac{\partial MO^1(x)}{\partial x}$ can be negative is when $\frac{\partial \phi^1(x)}{\partial x} < 0$. But the only way $\frac{\partial \phi^1(x)}{\partial x} < 0$ is if $\beta > 0$ and $b_I \geq 1/2$. Hence, the proof.

8.5 Proof of Proposition 10

Using Equation (??) we find that the total value of voting shares,

$$N V_1^1 = \frac{1}{2} P(x) \left((a_I(1 - b_I) + (1 + \kappa^1) \left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1} \right) \bar{a}_R^1 + (1 - 1/2) (1 - (\bar{a}_R^1)^2) \right), \quad (\text{P10-1})$$

and the total value of vote is

$$N V_{\text{vote}}^1 = \frac{P(x)}{2} (1 + \kappa^1) \left(\frac{1 - \ln(1 + \kappa^1)}{\kappa^1} \right) \bar{a}_R^1. \quad (\text{P10-2})$$

Hence, the total value of the dividend claim is

$$N V_D^1 = N V_1^1 - N V_{\text{vote}}^1 = \frac{1}{2} P(x) \left((a_I(1 - b_I) + (1 - 1/2) (1 - (\bar{a}_R^1)^2)) \right). \quad (\text{P10-3})$$

Thus, MO^1 can be expressed as

$$MO^1 = \beta N V_D^1 + \phi^1 b_I a_I P(x) = P(x) \left(\frac{\beta}{2} \left((a_I(1 - b_I) + (1 - 1/2) (1 - (\bar{a}_R^1)^2)) \right) + \phi^1 b_I a_I \right). \quad (\text{P10-4})$$

Let $A_{p11} = \frac{\beta}{2} \left((a_I(1 - b_I) + (1 - 1/2) (1 - (\bar{a}_R^1)^2)) \right) + \phi^1 b_I a_I$. Differentiating Equation (P10-4) with respect to x and rearranging terms, we have

$$\frac{\partial MO^1}{\partial x} = A_{p11}(\kappa^1) \frac{\partial P(x)}{\partial x} + P(x) \frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} \frac{\partial \kappa^1}{\partial x}. \quad (\text{P10-5})$$

From Equation (P10-5) we know that $\frac{\partial MO^1}{\partial x} < 0$ at $x = \bar{x}$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} \frac{\partial \kappa^1}{\partial x} < 0$ because $\frac{\partial P(x)}{\partial x} = 0$ at $x = \bar{x}$. Also, $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} \frac{\partial \kappa^1}{\partial x} < 0$ when $\frac{\partial \kappa^1}{\partial x} < 0$ implies that $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} > 0$. Differentiating A_{p11} with respect to κ^1 and rearranging terms, we have

$$\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} = \frac{(1 + b_I \kappa^1)}{\kappa^1(1 + \kappa^1)} + \frac{b_I \log(1 + \kappa^1)}{\kappa^1} - \frac{(1 + b_I \kappa^1) \log(1 + \kappa^1)}{(\kappa^1)^2} - \left(1 - \frac{1}{2} \right) \beta (1 + b_I \kappa^1). \quad (\text{P10-6})$$

For $\frac{\partial A_{p11}(\kappa^1)}{\partial \kappa^1} > 0$, we need

$$\frac{(1 + b_I \kappa^1)}{\kappa^1(1 + \kappa^1)} + \frac{b_I \log(1 + \kappa^1)}{\kappa^1} > \frac{(1 + b_I \kappa^1) \log(1 + \kappa^1)}{(\kappa^1)^2} + \left(1 - \frac{\beta}{2}\right) \beta(1 + b_I \kappa^1). \quad (\text{P10-7})$$

Solving for b_I such that the condition in Equation (P10-7) is satisfied,

$$b_I > \frac{\frac{2 \log(1 + \kappa^1)}{(\kappa^1)^2} + (2 - \beta) - \frac{2}{\kappa^1(1 + \kappa^1)}}{2 \left(\frac{2}{1 + \kappa^1} - (2 - \beta) \beta \kappa^1 \right)}. \quad (\text{P10-8})$$

We do not know the exact value of κ^1 , but we do know the RHS of expression (P10-8) is strictly increasing in κ^1 . Hence, we replace κ^1 by its maximum value $\frac{\beta}{1 - \beta}$, and get the sufficient condition for underinvestment as follows:

$$b_I > \frac{(1 - \beta) \left(\frac{2(1 - \beta)^2(1 - (1 - \beta)) \log\left(1 + \frac{\beta}{1 - \beta}\right)}{2 - \beta(4 - \beta((1 - \beta)(2 - \beta) +))} - \beta \right)}{2 \beta^2}. \quad (\text{P10-9})$$

8.6 Proof of Proposition 11

Consider the values of b_I for which the manager invests in all available positive NPV projects if non-voting equity is used to finance the investment. Assume that the manager invests some x if voting equity is used to finance the investment. We have to obtain conditions such that $V_1^0(\bar{x}) \geq V_1^1(x)$. Substituting for $V_1^0(\bar{x})$ and $V_1^1(x)$ from Equations (??) and (BR-12), we get

$$\begin{aligned} V_1^0(\bar{x}) - V_1^1(x) &= \frac{a_I(1 - b_I)}{2} (P(\bar{x}) - P(x)) + \frac{2 - \beta}{4} (1 - a_I^2 - a_I^2 b_I \kappa^0(2 + b_I \kappa^0)) P(\bar{x}) \\ &\quad - \frac{2 - \beta}{4} (1 - a_I^2 - a_I^2 b_I \kappa^1(2 + b_I \kappa^1)) P(x) - \frac{P(x)}{2} (1 + \kappa^1) \left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1}\right) \bar{a}_R^1 \\ &\quad + \frac{P(\bar{x})}{2} \left((1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left(1 - \frac{\ln(1 + \kappa^0)}{\kappa^0}\right) \bar{a}_R^0 \end{aligned} \quad (\text{P11-1})$$

But $\bar{a}_R^0 - \bar{a}_R^1 = b_I a_I (\kappa^0 - \kappa^1) > 0$ and $P(\bar{x}) - P(x) > 0 \forall x \neq \bar{x}$. Thus, ignoring these terms we substitute maximum value of κ^1 and minimum value of κ^0 , as both are strictly increasing in κ , and simplifying we get

$$\begin{aligned} &\left((1 + \kappa^0) - \frac{1}{1 - \beta} \right) \left(1 - \frac{\ln(1 + \kappa^0)}{\kappa^0}\right) - (1 + \kappa^1) \left(1 - \frac{\ln(1 + \kappa^1)}{\kappa^1}\right) \\ &= \frac{(1 - \beta) \log\left(\frac{\beta}{1 - \beta} + 1\right) - \beta^2 - \beta(1 - \beta)^2 \log\left(\frac{\beta}{1 - \beta} + 1\right)}{(1 - \beta)\beta} > 0, \end{aligned} \quad (\text{P11-2})$$

for all $\in [0, 1]$ and for all $\beta \in (0, 1/2)$. Using this result we can ignore these terms and rewrite Equation P11-1 as

$$\begin{aligned} V_1^0(\bar{x}) - V_1^1(x) &= \frac{a_I(1-b_I)}{2} (P(\bar{x}) - P(x)) + \frac{2-}{4} (1 - a_I^2 - a_I^2 b_I \kappa^0(2 + b_I \kappa^0)) P(\bar{x}) \\ &\quad - \frac{2-}{4} (1 - a_I^2 - a_I^2 b_I \kappa^1(2 + b_I \kappa^1)) P(x). \end{aligned} \quad (\text{P11-3})$$

After simplifying and rearranging we get

$$1 - \frac{P(x)}{P(\bar{x})} \geq \frac{\frac{2-}{4} ((1 - a_I^2 - a_I^2 b_I \kappa^1(2 + b_I \kappa^1)) - (1 - a_I^2 - a_I^2 b_I \kappa^0(2 + b_I \kappa^0)))}{\frac{2-}{4} (1 - a_I^2 - a_I^2 b_I \kappa^1(2 + b_I \kappa^1)) + \frac{a_I(1-b_I)}{2}}. \quad (\text{P11-4})$$

The expression, say $J_{p11}^1 = \frac{2-}{4} (1 - a_I^2 - a_I^2 b_I \kappa^1(2 + b_I \kappa^1))$, is a decreasing function of κ^1 and the entire expression is an increasing function of J_{p11}^1 . Thus, we substitute minimum value of κ^1 into the above expression. Similarly, the expression, say $J_{p11}^0 = \frac{2-}{4} (1 - a_I^2 - a_I^2 b_I \kappa^0(2 + b_I \kappa^0))$, is a decreasing function of κ^0 , but the entire expression is a decreasing function of J_{p11}^0 . Thus, substituting maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ and simplifying we get

$$1 - \frac{P(x)}{P(\bar{x})} \geq \frac{a_I^2 b_I (2-)(2 + b_I(2\beta+) - 2\beta)}{2 a_I (1-\beta)^2 (1 - b_I) - (2-)(a_I - a_I \beta(1 - b_I))^2 + (2-)(1-\beta)^2}. \quad (\text{P11-5})$$

8.7 Proof of Proposition 12

We prove this proposition in two parts: First we hold the investment level fixed and obtain conditions under which the manager is better off if the investment is financed using non-voting shares; then, we show that the manager remains better off if the investment level is increased. From Proposition 11 we know that $MO^j = A_{p1j} P(x)$, where j stands for the type of shares issued. To show that $MO^0 \geq MO^1$, we need to obtain conditions under which $A(k^0) \geq A_{p11}$. Substituting for A_{p10} and A_{p11} from Equations and simplifying we get

$$\frac{\beta}{2} (a_I(1 - b_I) + (1 - /2) (1 - (\bar{a}_R^0)^2)) + \phi^0 b_I a_I > \frac{\beta}{2} ((1 - /2) (1 - (\bar{a}_R^1)^2)) + \phi^1 b_I a_I. \quad (\text{P12-1})$$

Rearranging the terms we get

$$\phi^0 b_I a_I - \phi^1 b_I a_I > \frac{\beta}{2} ((1 - /2) (1 - (\bar{a}_R^1)^2)) - \frac{\beta}{2} ((1 - /2) (1 - (\bar{a}_R^0)^2)). \quad (\text{P12-2})$$

After substituting for \bar{a}_R^j from Equation (11) and ϕ^j from Equation (15) and on further simplification,

$$4 \kappa^1 (1 + b_I \kappa^0) \log(1 + \kappa^0) - 4 \kappa^0 (1 + b_I \kappa^1) \log(1 + \kappa^1) - (2-)\beta \kappa^0 \kappa^1 (\kappa^0 - \kappa^1) (2 + b_I(\kappa^0 + \kappa^1)) > 0. \quad (\text{P12-3})$$

Next, we solve for b_I such that the above inequality holds:

$$b_I \geq \hat{b}_I = \frac{(2-) \beta(\kappa^0 - \kappa^1) + 4 \frac{1}{\kappa^0} \log(1 + \kappa^0) - 4 \frac{1}{\kappa^1} \log(1 + \kappa^1)}{(4 \log \frac{1+\kappa^1}{1+\kappa^0} + (2-) \beta(\kappa^0 - \kappa^1)(\kappa^1 + \kappa^0))}. \quad (\text{P12-4})$$

As κ^1 increases for \hat{b}_I , so we substitute the maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ into the expression. But \hat{b}_I is also an increasing function of κ^0 and unfortunately κ^0 is unbounded:

$$\kappa^0 = \frac{\beta N + n^0}{(1-\beta)N} = \frac{\beta}{1-\beta} + \frac{n^0}{(1-\beta)N}. \quad (\text{P12-5})$$

If we assume that \bar{x} is such that $n^0(\bar{x}) \leq N$, (i.e., if new shares issued are no greater than the existing number of shares), then the maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$. Substituting maximum value of $\kappa^1 = \frac{\beta}{1-\beta}$ and assumed maximum value of $\kappa^0 = \frac{\beta}{1-\beta} + \frac{1}{1-\beta}$ we obtain

$$\hat{b}_I = \frac{(1-\beta) \left((2-) \beta^2(1+\beta) + 2(1-\beta)^2 \left((1+\beta) \log \left(1 + \frac{\beta}{1-\beta} \right) - \beta \log \left(1 + \frac{(1+\beta)}{1-\beta} \right) \right) \right)}{\beta(1+\beta) \left(4(1-\beta)^2 \log \left(1 + \frac{(1+\beta)}{1-\beta} \right) - (2-) \beta(1+\beta) - 4(1-\beta)^2 \log \left(1 + \frac{\beta}{1-\beta} \right) \right)}. \quad (\text{P12-6})$$

Equation P12-6 ensures that the manager is better off if the level of investment remains the same. The second part of the proof requires us to show that the manager is better off if the level of investment increases. This requires that $MO^0(x_2) > MO^0(x_1)$, where $x_2 > x_1$. But we know that if $b_I \geq \frac{1}{2}$, then $\frac{\partial MO^0(x)}{\partial x} > 0$. Hence, $\hat{b}_I \geq \frac{1}{2}$ is sufficient condition for full investment. If $b_I \geq \hat{b}_I$, then the incumbent is better off if he is allowed to use non-voting shares to fund new projects.