Wolf Pack Activism *

Alon Brav[†] Amil Dasgupta[‡] Richmond Mathews[§]

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Abstract

Blockholder monitoring is key to corporate governance, but blockholders large enough to exercise significant unilateral influence are rare. Mechanisms that enable small blockholders to exert collective influence are therefore important. It is alleged that institutional blockholders sometimes implicitly coordinate their interventions, with one acting as "lead" activist and others as peripheral "wolf pack" members. We present a model of wolf pack activism. Our model formalizes a key source of complementarity across the engagement strategies of activists and highlights the lead activist's catalytic role. We also characterize share acquisition in pack formation, providing testable implications on ownership and price dynamics. **JEL code:** G34, G23

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[†]Duke and NBER

[‡]London School of Economics, CEPR, and ECGI

[§]University of Maryland, College Park

1 Introduction

Starting with Shleifer and Vishny (1986), economists have recognized the key role of blockholders in monitoring and engaging with management to ameliorate problems arising from the separation of ownership and control in public corporations. In particular, the concentration of ownership in the hands of a single large shareholder has been shown to enhance firm value, and more so the larger is the block. However, while blockholding is widely prevalent in the U.S., most blockholders are not large enough to exert significant unilateral influence in the face of recalcitrant management. Holderness (2009) documents that 96% of U.S. firms have at least one blockholder with 5% ownership. Yet, La Porta, Lopes de Silanes, and Shleifer (1999) document that 80% of the largest U.S. firms lack any single blockholder with a stake of at least 20%, a level that they argue generates effective control. Using data on a broader sample from Dlugosz et al (2006), we find that fewer than 15% of U.S. firms have a 20% outside blockholder.[1] As a result, mechanisms that enable small blockholders to gain collective influence are key to effective monitoring.

In this paper, we theoretically examine how small blockholders may gain collective influence. Our study is of particular applied relevance because market observers allege that institutional investors—who are the majority of blockholders in U.S. corporations today—*do* act in concert to magnify each other's influence over management. For example, lawyers allege that activist hedge funds often implicitly team up with other institutional investors to form so-called "wolf packs" (e.g., Briggs 2006, Nathan 2009, Coffee and Palia 2015). While activist funds are widely attributed with creating fundamental change (Brav et al 2008, Klein and Zur 2009), often in the face of hostile managers, they typically own only around 6% of the company's shares (Brav, Jiang,

[1]LaPorta et al (1999) also consider a smaller threshold of 10% for robustness and find no greater incidence of controlling blocks in the US. Using the 10% threshold in the Dlugosz et al (2006) data, we find that over half of US firms have no controlling outside blockholder.

and Kim 2010). The use of the wolf pack tactic to overcome management resistance has therefore attracted a great deal of attention. For example, legistlation recently proposed in the U.S. Senate in response to the rise of hedge fund activism (the Brokaw Act) cites protecting businesses from activist wolf packs as a central goal. In addition, U.S. courts have upheld the use of unconventional measures undertaken by corporations to defend against wolf packs.[2]

The wolf pack phenomenon is puzzling in light of the fact that share price appreciation—the key consequence of a successful activism campaign—is poorly suited to fostering collective action. Indeed, any given owner's incentive to expend costly effort to engage with management is *decreased* by the engagement of others if share price appreciation is the sole source of benefits to activists. This is because, if sufficiently many others engage, then activism succeeds and price appreciation accrues to each owner regardless of their individual actions.

We present a model of wolf pack activism that provides a foundation for coordinated engagement. In our model, wolf pack members are delegated portfolio managers that compete for the capital of investors who, in turn, choose among managers based on perceived skill. We show that such competition is sufficient to generate strategic complementarities that form a basis for group activism. Our model also provides a dynamic characterization of trading decisions that anticipate the emergence of group activism, and generates testable implications on ownership and price dynamics.

We model activism in a target firm by many investors: one large investor and many small ones. Our large investor is intended to represent, for example, an activist hedge fund (e.g., Pershing Square or TCI) which crosses the 5% ownership threshold and files SEC form 13-D. Our small investors may be other hedge funds—activist or otherwise—with smaller stakes or other types of institutional investors (e.g., event-driven funds) who may provide support to the lead activist via soft, "behind the scenes" engagement

[2]Third Point LLC vs Ruprecht, 2014.

(McCahery, Sautner, and Starks, 2016). These institutions play a smaller, supporting role in activism, and we refer to them throughout as small institutions for short.

Our modeling choice of asymmetry among activists—a single leader who is significantly larger than her many followers as opposed to a handful of large, similarly-sized, co-leaders—is motivated by applied relevance. First, joint interventions amongst multiple large players are rare. Between 1994 and 2011 there were over 2,500 activism events involving hedge funds in the US, but fewer than 10% involved two or more funds with stakes large enough to warrant a 13D filing. Even within this 10%, the median length of time across filings was over 150 days, which is far longer than the short-horizon pack formation described by Briggs (2006) and Nathan (2009). Second, we have direct evidence that activist hedge funds are supported by other, usually event-driven, institutional investors with much smaller stakes. For example, in conversation with us, Thomas Kirchner of Quaker Funds, an event-driven fund that buys small stakes in target firms in the immediate aftermath of 13-D filings by activist hedge funds, described the process by which lead and supporting activists interact as follows: "Lead activists are very well aware that there may be followers with smaller stakes like Quaker that will support them in a campaign, yet it's formally uncoordinated. Investors understand the activist's playbook and how their interests are aligned."

The lack of formal coordination across institutions arises from the legal constraints in the activism process: U.S. disclosure rules (Regulation 13D) require investors to file together as a group when their activities are formally coordinated. A group filing would potentially reduce trading profits and also risk triggering poison pills at an earlier stage, and therefore restrict the total holdings that can be achieved by the group. A distinct way to explicitly coodinate activities would be for the activist hedge fund to raise capital from other institutions ex ante and then unilaterally build up a stake that generates sufficient influence. However, in practice, this would be difficult, both because of poison pills and because crossing a 10% ownership threshold would render the activist fund an insider according to SEC regulation[3] and subject their trades to greater scrutiny. Accordingly, our model features no pooling of capital and no direct communication between players, but rather provides a positive analysis that formalises the origins of implicit, endogenous coordination across stakeholders of different sizes.

We start our analysis at the activism stage, taking ownership stakes as given. At this stage, each owner must decide whether to engage the target, which we interpret to be exerting influence through talking with management or other (less active) institutional owners, proposing new actions, etc., to try to improve the firm's decisions, and hence its value. McCahery, Sautner, and Starks (2016) document that formal and informal discussions with management and board members is the most commonly used engagement strategy amongst institutional investors. Activism is successful in raising firm value if the aggregate shareholdings of owners that choose to engage is sufficient to deliver value enhancement given the target managers' inclination and ability to resist. Engagement is costly because it requires time and effort. For group activism to be salient, there must be complementarity between different owners, that is, the potential engagement of others must encourage each owner to engage. As discussed above, any given owner's incentive to engage with management is *decreased* by the engagement of others if share price appreciation is the sole source of benefits to activists. Thus, coordiation requires the existence of some excludable, i.e., private, benefit from participation in activism.

While private benefits from successful activism (e.g., via board seats acquired during a proxy fight) are apparent for the lead activist, the source of excludable rents for small institutions is less obvious. Why would small institutions who have little unilateral influence ever expend effort in support of an activist campaign? Recognizing that wolf pack members are delegated portfolio managers offers a potential answer. The empirical literature documents that a wide spectrum of money managers are subject

[3]15 USC Chapter 2B § 78p.

to so-called "flow performance" relationships: their success relies on the approval of their investors.[4] Building on this, we model excludable rents as arising from a reputational mechanism in which small institutions can gain a reputation for being skilled via their selection of activist campaigns to support. Skilled institutions have better information-gathering abilities than unskilled institutions, and are therefore better able to predict the viability of an activist campaign. Investors observe institutions' engagement choices as well as the overall engagement outcome and make inferences about their ability. They believe that the information-gathering abilities of skilled institutions will result in higher future returns. As a result, a sufficient improvement in perceived ability leads to additional inflow of capital for the institution, which represents an excludable rent. Since reputation for skill is an equilibrium quantity, these rents are *endogenous*. We show that, in equilibrium, the reputational mechanism generates strategic complementarity: rents arise only from participating in a successful activism campaign where success, in turn, is generated by sufficient participation. The key reason is that institutions who discover themselves to be unskilled *choose* never to engage, and thus it is only possible to stand out from the crowd by engaging. Engagement, in turn, delivers reputational rewards only in the case in which activism succeeds.

Our model of activism also demonstrates that the presence of a lead activist can have a catalytic effect on engagement. We show that, holding the aggregate size of skilled institutional ownership constant, the presence of a large activist improves the level of coordination and leads to value-increasing engagement more often. An implication of this result is that, even when a significant number of shares are held by potential small activists, the arrival of a "lead" activist who holds a larger block may be a necessary catalyst for a successful campaign, which is consistent with the activist strategies that are well documented in the empirical literature. It is important to note that this positive effect of ownership concentration is very different from that in

[4]See, for example, Chevalier and Ellison (1997) for mutual funds or Lim, Sensoy, and Weisbach (2016) for hedge funds.

Shleifer and Vishny (1986): there, concentration encourages actions by the large blockholder, while in our setting it encourages engagement by other small shareholders. A related catalytic effect of a large player in a coordination game has been shown to arise in the context of speculative currency attacks by Corsetti, Dasgupta, Morris, and Shin (2004). In that paper, however, complementarity across strategies is exogenous, whereas in ours it arises endogenously.

Our model of engagement takes ownership stakes in the target firm as given. In the second component of our analysis, we develop a simple trading model that builds on our engagement model to characterize target share purchases by the lead activist and small institutions. Market observers highlight the dynamic nature of wolf pack formation, referring to a degree of unusual turnover around the declaration of a campaign by an activist hedge fund. For example, Nathan (2009) writes:

> The market's knowledge of the formation of a wolf pack (either through word of mouth or public announcement of a destabilization campaign by the lead wolf pack member) often leads to additional activist funds entering the fray against the target corporation, resulting in a rapid (and often outcome determinative) change in composition of the target's shareholder base seemingly overnight.

A recent study by Jetley and Ji (2015) shows that a substantial number of firms subject to 13D filings have more than 10% abnormal turnover between the day the filer crosses the 5% threshold and the day the 13D is filed, suggesting there could be some pre-filing information leakage that prompts wolf pack trading.[5] Using activism data from 1994 to 2011, focusing on the ten-day period following 13D filings, we find that for the

[5]An interesting related issue concerns whether and when a lead activist might want to notify potential wolf pack members of their intentions. In our model this is not a significant issue given that we assume transparent markets. See Kovbasyuk and Pagano (2014) for a theoretical analysis of the optimal strategy for publicizing arbitrage opportunities.

largest tercile of firms—where activists are most likely to require the support of wolf packs—there is an additional average abnormal turnover of over 30% of the activist's typical stake, suggesting that non-trivial wolf pack trading continues after the public declaration of activism.

Our model generates endogenous turnover in target firm shares because we show that the initial owners of a target firm—before the market becomes aware that the target is amenable to activism—must be institutions who know themselves to be unskilled. Since (as described above) unskilled institutions are never willing to engage management in equilibrium, these initial owners cannot earn reputational rewards. There are thus gains from trade (even in the absence of any market frictions) between these initial owners and potential entrants in the form of institutions who are unsure of whether they are skilled, because the latter assign positive probability to the prospect of earning reputational rewards.

The formation of a wolf pack is therefore synonymous in our model with turnover in the ownership of the target firm. There is evidence that target firms' managers themselves view abnormal turnover around 13D filings as evidence of wolf pack formation. In their study of how target managers defend against activist wolf packs, Boyson and Pichler (2016) find that a one standard deviation increase in abnormal turnover is associated with a 22% increase in the probability of the adoption of anti-coordination measures (e.g., strengthening of poison pills) by target firms. In addition to microfounding endogenous turnover in wolf pack formation, our model also provides a number of testable predictions about the timing of purchases and stock price movements surrounding coordinated activism events. These are discussed in Section 5.

Our analysis is related to the theoretical literature on the influence of blockholders in corporate governance. Papers in this literature tend to focus either on blockholders who, as in our model, exercise "voice" by directly intervening in the firm's activities (Shleifer and Vishny, 1986; Kyle and Vila, 1991; Burkart, Gromb, and Panunzi, 1997, Bolton and von Thadden, 1998; Maug, 1998; Kahn and Winton, 1998; Faure-Grimaud and Gromb, 2004), or those who use informed trading, also called "exit," to improve stock price efficiency and encourage correct actions by managers (Admati and Pfleiderer, 2009; Edmans, 2009). Dasgupta and Piacentino (2015) show that the ability to use exit as a governance mechanism is hindered when the blockholder is a flow-motivated fund manager. Flow motivations, modeled via reputational concerns, also play a key role in our paper. In contrast to Dasgupta and Piacentino (2015), in our paper reputational concerns play a positive role in creating a basis for group activism. Piacentino (2017) also demonstrates a positive role of reputational concerns in corporate finance in the context of feedback effects of prices on investment decisions. Some other papers suggest that blockholders improve decisions by directly providing information to decision makers (see Cohn and Rajan, 2012; Edmans, 2011). Our paper is distinct from all of these in its focus on implicit coordination between different block investors in their value creating activity.[6]

Several existing papers discuss the implications of having multiple blockholders, but from very different perspectives. Winton (1993) was the first to show that disaggregation of a block among multiple shareholders make it harder to overcome free rider problems in monitoring. Zwiebel (1995) models the sharing of private control benefits as part of a coalitional bargaining game, and derives the equilibrium number and size of blockholders who try to optimally capture these benefits. Edmans and Manso (2011) model a group of equal-size block holders and ask whether their impact on corporate governance through both exit and voice is larger or smaller than if the same block were held by a single entity. Their main result is that while having a disaggregated stake makes voice less productive due to free rider problems, it helps make the exit channel more effective since the blockholders trade more aggressively when competing

[6]Doidge, Dyck, Mahmudi, and Virani (2015) document explicit coordination among institutional investors in Canadian firms through an organization named the Canadian Coalition for Good Governance, and show that such coordinated action can have significant effects.

for trading profits. We take a very different perspective, asking how the activities of blockholders of different size affect their ability to implicitly coordinate around a target, and how it affects their initial decision to buy a block.

Noe (2002) studies a model in which strategic traders may choose to monitor management, which improves value. In his model, monitoring activities by different investors are perfect substitutes (i.e., if any one investor monitors, the full improvement in value is achieved), and the strategic investors play mixed strategies, where they generally mix between monitoring and buying vs not monitoring and selling. Instead of studying coordination among these monitors, therefore, Noe focuses on showing that there can be multiple monitors despite the substitutability because of trading opportunities in noisy financial markets. Cornelli and Li (2002) focus on how one or more traders (arbitrageurs) can accumulate enough shares to provide a solution to the free-rider problem in a takeover game by tendering their acquired shares. Like in Noe (2002), they focus on how noise in the market allows such traders to hide their trades in order to acquire the requisite number of shares, profiting from their privately held knowledge of entry. In contrast, we consider a noise-free market, in which all trades are transparent. In such markets, the phenomena modelled by Noe and Cornelli and Li cannot arise. Our focus, therefore, is not on how one or more shareholders are incentivized to become large by the possibility of trading profits, but rather on how a large number of infinitesimal traders can support a single large trader in her engagement efforts in the aftermath of trading, and how the endogenous private benefits generated by this engagement game can induce them to buy shares ex ante.

2 The Model

Consider a publicly traded firm that may become amenable to shareholder activism, in which case value can be created by inducing a change in management's policies. Such a change can be induced only if activist investors own or acquire shares and successfully engage with management. There are four dates, $t = 0, 1, 2, 3$, and all players are risk neutral.

The firm has a continuum of shares outstanding of measure 1, of which a measure $\bar{A} \in (0, 1)$ represents the "free float". The remaining shares can be thought to be owned by insiders, say management or founders, who are committed to the current operating strategy and never sell. Shares in the free float of the firm can be traded at any time at fair prices that reflect expected cash flows.

The firm enters the model in a state of "non-amenability" wherein it is commonly understood that no improvements can be made to its current operating strategy. Firm value at the end of period $t = 3$ is P_ℓ in that state and there is no scope for profitable activism. At the beginning of $t = 0$ there may be a shift in the firm's state: there is a probability p_A that a public signal will arrive indicating that the firm has become "amenable" to activism, in which case the current strategy is found to be suboptimal and there is scope for activism to improve value by changing strategy. If the firm shifts to the amenable state, then it is characterized by a stochastic fundamental η which measures the degree of difficulty in implementing changes in strategy. A natural source of such difficulty—which may vary across firms—is the willingness of the current management team to fight any proposed changes (e.g., by influencing the board, adopting poison pills, or piling pressure on institutional shareholders who have business ties with the firm). We therefore sometimes refer to the fundamental η as *entrenchment*.

Activism takes the form of engaging management behind the scenes to modify corporate strategy and succeeds if and only if a sufficient number of shareholders engage, given η. McCahery, Sautner, and Starks (2016) document that such behind the scenes dialogue with managers and directors is the preferred engagement strategy of a majority of institutional investors. We assume that engagement succeeds if the measure of shares that engage, e, is no smaller than η: if $e \geq \eta$, the firm's value at the end of $t = 3$ will rise to $P_h > P_\ell$. This "threshold" characterisation is meant to capture the idea that, for any given level of entrenchment, there is some level of pressure from shareholders that will induce target management to modify strategy, i.e., to "settle" with activists (instead of fighting them), perhaps because they become convinced that ultimate victory is unlikely enough should a formal proxy fight arise.[7] Bebchuk et al (2016) document that a large and increasing number of activist campaigns result in such settlements rather than in formal proxy fights. Since $e \in \left[0, \bar{A}\right]$, conditional on the amenable state, activism has some chance of being successful if and only if $\eta \leq \bar{A}$.

The firm fundamental η and our threshold characterisation can be more broadly interpreted. For example, η could measure the technological feasibility or complexity of making changes to the firm's strategy, while activist investors are those whose ideas are valuable in overcoming such technological challenges. For a given technological hurdle, change can be successfully implemented if sufficiently many valuable ideas are brought to the table.

To emphasize the difference between the ex ante certainty of the (stable) firm in the non-amenable state and the uncertainty introduced by the possibility of value enhancing activism, we model the public signal of amenability as being highly noisy (in terms of the conditional variance of firm value) by assuming that $\eta \sim N\left(\bar{A}, \frac{1}{\alpha_\eta}\right)$, where α_η is the precision of η, implying that conditional on the arrival of the amenability signal, there is a 50% chance that activism has some possibility of success.[8]

There are two types of investors in the model: a large pool of institutional investors who can each devote only relatively little capital to the firm, and a large activist institution, L, who is able to devote larger amounts of capital. Note that while we model the institutional investors as a continuum, we think of them empirically as small blockholders owning a non-trivial amount of stock in the target firm. The continuum assumption is justified by the idea that they are dispersed enough to make explicit

[7] Accordingly, η does not necessarily correspond to a particular voting threshold.

[8] Our qualitative results do not require that η has a mean of $\bar{A}$. This is further dicussed in section 6.3.

coordination difficult (or costly from a legal standpoint), and small enough not to have a unilaterally pivotal impact on the outcome of engagement. This is consistent with the characterizations of wolf packs discussed in the introduction.

The large activist, L, is available for activism with probability p_L, in which case she enters the model at $t = 1$ and considers whether to acquire a stake in the firm. L faces a capital constraint $A_L << \bar{A}$. Conditional on being available for activism, L has an opportunity cost of k_L (i.e., what she could earn by using her attention and capital elsewhere). If L is not available for activism, nothing happens at $t = 1$. If L is available for activism, risk neutrality together with fair prices implies that—if she decides to purchase shares—she will buy up to her capital constraint. Accordingly, we treat A_L as L's position size throughout. The events at $t = 1$ are publicly observed.

Institutional investors all have the potential to be activists, and exist ex ante in two pools: a large pool of unskilled institutions (who know they are unskilled), and a pool of measure 1 of *potentially* skilled institutions. More concretely, all institutional investors are one of two types: $\theta \in \{G, B\}$. Type G institutions who acquire a position in the firm before $t = 3$ observe η with small amounts of idiosyncratic noise at the beginning of $t = 3$. The noise in observing entrenchment can be thought to be the result of (potentially imperfect) due diligence (research) carried out by each institution into the target firm. Each such type G institution i receives a private signal $x_{s,i} = \eta + \frac{1}{\alpha_s}\epsilon_i$ where ϵ_i is standard normal, independent of η, and iid across institutions. The parameter α_s measures the precision of the signal. Type G institutions also have the potential to find profitable outside investment opportunities if they (instead) invest their capital and information gathering effort on something other than the target firm. The expected value of these outside opportunities is k_i, where $k_i \in \left[\underline{k}, \overline{k}\right]$ is uniformly distributed across the population of type G institutions (and each potentially skilled institution knows their k_i). The opportunity cost is sunk immediately upon the purchase of target shares by an institution. Type B (bad) institutions cannot find useful signals about fundamentals, and have no profitable outside opportunities. The large pool of unskilled institutions know that they are type B ex ante. The pool of potentially skilled institutions do not know their type, but are known to have probability γ of being type G.[9]

Small institutions are aware that there is a date $t = 1$ when L may enter and seek to establish a position in the firm. They may, in turn, buy shares in the firm, either before they know whether L will be available for activism but after observing that the firm is amenable to activism, i.e. at $t = 0$, or after they know whether L is available for activism and whether she has established a position in the firm, i.e., at $t = 2$. Each institution may only acquire shares once, but those institutions who do not acquire shares at $t = 0$ have the option of acquiring shares at $t = 2$.

At $t = 3$, each outside owner of shares, whether small or large, has the option of engaging or not engaging firm management in order to induce value enhancing changes in the firm. Not engaging is a costless action for both large and small owners.

Small institutions can potentially enjoy private benefits from acquiring a reputation for being type G. If they own a stake at time $t = 3$ then investors will update their beliefs about the institution's type to some posterior $\widehat{\gamma}$ after they observe the outcome of the activism game and the institution's individual action (engage or not). If the posterior is measurably higher than the prior, that is, $\hat{\gamma} \geq B$ for some $B \in (\gamma, 1)$, the institution gets reputational benefit R from participating in the game. Otherwise, the institution gets zero private benefits. The reputational benefit R could arise, for example, from fees on additional funds invested in the institution by investors who believe that skilled institutions' information gathering abilities will translate into higher returns in the future.[10] We discuss these reputational benefits further in Section 6.2. In addition, the institution receives a payoff of P_h if engagement is successful—capturing a

[9]For parsimony we do not consider investors who already know they are type G. Including a mass of such agents would not affect the model's qualitative results.

[10]Note that B can be arbitrarily close to γ.

free rider benefit in case they did not themselves engage—and a payoff of P_ℓ otherwise.

Choosing to engage the target costs a small institution c_s. There are at least two natural interpretations for this. The first—which we use in the baseline model—is that c_s represents the effort cost for formulating and articulating arguments for changes in target strategy, or—in the case of a campaign led by a large activist—the effort cost for conducting research to support the effort of the lead activist and of credibly communicating support for the campaign to target management. An alternative interpretation—which we discuss in Section 6.1—is that c_s captures portfolio diversification costs associated with holding a concentrated position in the target firm over the course of the activism stage, which in real life could take many months.

If the large activist does not engage she receives a payoff of $A_L P_h$ if any engagement by others is successful, and a payoff of $A_L P_\ell$ otherwise. Engagement entails a private effort cost of c_L. This may represent effort spent on pressuring management via discussion, visible publicity campaigns, and proxy proposal formulation and sponsorship. If the large activist engages she receives an additional payoff of β_L if the engagement is successful, where $\beta_L > c_L$ represents the excludable benefits earned from successful engagement. For example, if an activist campaign succeeds in appointing new board members, these board members are more likely to be friendly to the lead activist who installed them. In many cases, activist hedge funds managers appoint themselves to corporate boards as part of a successful campaign. This can then also endow them with soft information that leads to valuable trading strategies or other private benefits.[11]

[11] While β_L can also be interpreted, similar to the above, as reputational benefits that accrue to a large activist hedge fund manager from leading a successful activist campaign, we do not explicitly model a reputation mechanism for the large activist since there are likely many sources of private benefits for a successful lead activist.

Our model requires no restriction on the relative values of β_L and R and of c_L and c_s. However, we believe that a natural interpretation is that β_L and c_L are larger than R and c_s respectively. This is because leading an activist campaign is likely to be both more costly and more rewarding than simply participating in one.

The large activist observes η perfectly at the beginning of $t = 3$, reflecting the fact that she specializes in activist strategies and enjoys an information advantage relative to smaller institutions.[12]

We now solve the game by backward induction. We first take the ownership structure of the firm as given, and solve for the activism game at $t = 3$. Subsequently, we solve for the endogenous stake purchase and sale decisions of each type of owner. Before doing so, we outline two natural parameter restrictions:

Assumption 1.

a. $R \in (c_s, 2c_s)$

b. $\underline{k} < \frac{R-c_s}{2} < \overline{k}$.

Assumption 1(a) ensures that the potential reputational rents are commensurate to the effort required for activism. Assumption 1(b) ensures that small institutions with the lowest opportunity costs prefer to buy into the target firm, while those with the highest opportunity costs prefer not to do so. These parameter restrictions are further discussed in Section 6.3.

3 Activism

In this section we analyze the engagement game. When analyzing engagement, we take the ownership structure of the firm as given. However, we begin with a simple preliminary characterisation of initial ownership of the firm. Given the set of agents in our model, the ownership of the free float of the target firm when it enters the model can, in principle, be made out of potentially skilled institutions or institutions that know themselves to be unskilled. We first show:

[12] It would be conceptually straightforward, though algebraically tedious, to generalise the information structure to cases where L's information was imperfect but superior to that of the small institutions.

Lemma 1. *As long as* $p_A < \frac{2k}{R-c_s}$*, the initial owners of the free float* $\bar{A}$ *are institutions that know themselves to be unskilled.*

All proofs are in the appendix. Intuitively, since potentially skilled institutions have expected opportunity costs, they will buy into the target firm if and only if their potential payoffs from doing so exceed these opportunity costs. If amenability is rare (p_A is small), it will not be in the interest of potentially skilled institutions to buy before amenability is known. As a result, the initial free-float of the target must be held by institutions that know themselves to be unskilled. The condition $p_A < \frac{2k}{R-c_s}$ is sufficient to guarantee that the upper bound on payoffs from ownership prior to learning amenability for potentially skilled institutions is smaller than the lower bound on their opportunity costs. This assumption is not essential for our qualitative analysis, but for ease of exposition we maintain this upper bound on p_A throughout the paper.[13]

From here forward we focus on the interesting case in which the target firm is in the amenable state (i.e., a public signal of amenability arrived at $t = 0$). In our model, a proportion $1 - \gamma$ of potentially skilled institutions who buy into the firm will discover at $t = 3$ that they are, in fact, unskilled. For technical reasons we assume that a small measure λ of these institutions randomize non-strategically in the coordination game, engaging with probability 1/2. In the sequel to Proposition 1 we let $\lambda \to 0$. The introduction of these randomising types ensures that an unskilled type can never gain reputation by taking the *wrong* action (i.e., engaging when engagement fails).[14]

[13] If p_A is larger than this bound, it is feasible for some potentially skilled institutions to be initial owners of the target free float. This would simply reduce the quantitative size of the turnover characterised in Section 4 without changing the qualitative properties of our analysis.

[14] When skilled players have noise in their signals of η, with some probability they will make a mistake and engage when engagement fails. If in a proposed equilibrium all unskilled types are supposed to *not* engage, then choosing to engage can result in the inference that you are a good type even when you took the wrong action, i.e., that you are a skilled type who happened to get an incorrect signal. Adding a small amount of randomization that is commensurate with the amount of noise in the signals eliminates this unrealistic possibility.

Let A_s denote the measure of potentially skilled institutions who purchased shares at $t = 0$ or $t = 2$. Apart from the large activist, if present, there are then four groups of owners of the firm at $t = 3$: (i) Skilled ($\theta = G$) institutions in measure $A_s\gamma$, (ii) unskilled ($\theta = B$) strategic institutions in measure $A_s (1-\gamma)(1-\lambda)$, (iii) unskilled randomizing institutions in measure $A_s (1-\gamma)\lambda$, and (iv) initial owners that have not yet had an opportunity to sell, in measure $\bar{A} - A_s$. By Lemma 1, these initial owners are institutions who know themselves to be unskilled. Since agents in groups (ii) and (iv) are therefore identical (none of them receive signals), we refer to them jointly as "unskilled institutions".

We look for equilibria in monotone strategies—each skilled institution i engages if and only if his private signal $x_{s,i}$ is weakly below some threshold—and allow for arbitrary symmetric strategies for unskilled institutions.

Proposition 1. *For* $\lambda < \min\left[\frac{2\gamma(1-B)}{(1-\gamma)B}, \frac{2(B-\gamma)}{(1-\gamma)B}\right]$, *there exists* $\underline{\alpha}(\lambda) \in \mathbb{R}_+$ *such that for all* $\alpha_s \geq \underline{\alpha}(\lambda)$ *in equilibrium:*

(i) unskilled small institutions never engage

(ii) skilled small institutions engage iff their signal is below a unique threshold x_s^*,

(iii) engagement succeeds iff the target entrenchment is below a unique threshold η_s^* *and*

(iv) the large activist, if present, engages if and only if $\eta \leq \eta_s^*$.

In the limit as $\alpha_s \to \infty$, *the thresholds are given by:*

$$x_s^* = \eta_s^* = \mathbf{1}_L A_L + \gamma A_s \left(1 - \frac{c_s}{R}\right) + \frac{1}{2} A_s (1-\gamma)\lambda.$$

We provide intuition for the result in the limiting case in which $\alpha_s \to \infty$. We first note that whenever skilled institutions employ monotone strategies with threshold x_s^*, there exists a critical threshold level of η, which we label η_s^*, such that engagement succeeds if and only if $\eta \leq \eta_s^*$. Further, it is easy to check that as $\alpha_s \to \infty$, $x_s \to \eta$ and $x_s^* \to \eta_s^*$. In other words, in threshold equilibria, skilled institutions always make correct choices in the limit as noise vanishes. This means that unskilled institutions can never earn reputational rents by engaging when engagement fails or not engaging when it succeeds.

Now consider the possibility that unskilled institutions always engage in equilibrium. Then, when engagement succeeds, the only non-engaging owners are randomising unskilled institutions. When λ is small enough, almost all institutions, whether skilled or unskilled, choose to engage. Thus, the posterior update to reputation from engaging in the case engagement succeeds is very small, and not enough to generate reputational rents R. Yet, since skilled institutions never engage when engagement fails as $\alpha_s \to \infty$, there are also no reputational rents arising from engagement in case of failure. In effect, there are no reputational rents to be earned from engaging. Given that security benefits are non-exclusive, and do not require engagement, this implies that no unskilled institution would wish to pay the positive cost of engaging. Thus, it cannot be an equilibrium for unskilled institutions to always engage in equilibrium.

Next, consider the possibility that unskilled institutions never engage in equilibrium. Then, by a similar argument to the previous case, there are no reputational rents to non-engagement as $\alpha_s \to \infty$ and for small enough λ. Engaging however, does deliver reputational rewards in case of success, because all skilled institutions engage in this case if $\alpha_s \to \infty$, whereas, for small λ, essentially no unskilled institution does. Thus, unskilled institutions would wish to deviate and engage if the expected reputational benefit from engagement exceeds its cost. Viewed from the perspective of uninformed unskilled institutions, the expected benefit is never larger than $\Pr\left(\eta \leq \bar{A}\right) R = R/2$, however, whereas the cost is c_s. Thus, since $R < 2c_s$, the deviation is unattractive, and it is indeed an equilibrium for unskilled instutions to never engage. The key intuition is that for those institutions who decided to gamble on establishing a reputation for being skilled (i.e., those whose expected opportunity costs were not too high), but subsequently discovered themselves to be unskilled, the best bet is to sit tight and not expend any resources on trying to "pretend" to be skilled. An important economic

Excludable payoffs	Engagement succeeds	Engagement fails
Engage	$R - c_s$	$-c_s$
Not Engage	0	0

Table 1: Excludable payoffs for skilled institutions

implication of this is that reputational rents can be achieved only by participating in a successful activism campaign. There are never rents for remaining inactive, even when activism fails. The proof in the appendix also shows that no mixed equilibria can arise.

We now turn to the skilled institutions. As a first step, we consider the case where the large activist is absent, or—equivalently—where $A_L = 0$. The choice whether to engage or not is only affected by the reputational payoffs associated with these choices. As explained above, since unskilled institutions never engage in equilibrium, skilled institutions can only earn reputational rewards by engaging when engagement succeeds. As a result, their repuational rents can be summarised as in Table 1.

The payoffs in Table 1 take the form of a standard binary action coordination game. If it were common knowledge that $\eta \in (0, \gamma A_s)$, so that the engagement of the available skilled institutions could overcome entrenchment, then—given these payoffs—there would be multiple equilibria, one in which *all* skilled institutions engage, and one in which *none* do. However, with incomplete information about η as in our game, the equilibrium behavior of skilled institutions is uniquely pinned down. To understand why this is the case, it is important to recognize that the payoffs of any given skilled institution are determined jointly by the exogenous fundamental, η, and the endogenous measure of other skilled institutions who engage, which we label e_s. In other words, both uncertainty about firm fundamentals and uncertainty about the actions of other skilled institutions, i.e., *strategic uncertainty*, is relevant to each institution. When η is common knowledge, there is neither uncertainty about firm fundamentals nor strategic uncertainty. In the $\alpha_s \to \infty$ limit, uncertainty about firm fundamentals vanishes. However, strategic uncertainty does *not* vanish. As $\alpha_s \to \infty$, each skilled institution remains highly uncertain about his *relative* ranking in the population of skilled institutions. In particular, each skilled institution has *uniform* beliefs over the *proportion* of skilled institutions who have received signals about η which are lower than his own. The presence of such strategic uncertainty limits the precision with which skilled agents can coordinate with each other and eliminates multiplicity. This insight derives from the literature on global games (Carlsson and van Damme 1993, Morris and Shin 1998). In the global games literature, however, complementarities across players' strategies is taken as given. In our model, complementarities arise *endogenously* via the reputational concerns of small institutions: the payoffs for skilled institutions in Table 1 arise as a result of the equilibrium behavior of unskilled institutions.

Using the characterization of strategic uncertainty described above in the $\alpha_s \to \infty$ limit delivers a heuristic method for computing the threshold η_s^*, as follows. The skilled institution with signal x_s^* must be indifferent between engaging and not engaging. Further, all skilled institutions with signals lower than his will wish to engage. Thus, the proportion of skilled institutions with signals lower than his is simply e_s. In the limit as $\alpha_s \to \infty$, the skilled institution with signal x_s^* believes that $e_s \sim U(0,1)$. Consider the case where the large activist is absent and $\lambda \to 0$, so that there are now no randomising unskilled institutions. Then, since unskilled institutions do not engage, this skilled institution's evaluation of the probability of successful engagement is simply $\Pr(\gamma A_s e_s \geq \eta_s^*)$. Since $e_s \sim U(0,1)$ this can be rewritten as $1 - \frac{\eta_s^*}{\gamma A_s}$, giving rise to the indifference condition:

$$R\left(1 - \frac{\eta_s^*}{\gamma A_s}\right) = c_s,$$

which immediately implies that $\eta_s^* = \gamma A_s \left(1 - \frac{c_s}{R}\right)$, which is exactly the value of η_s^* in Proposition 1 for $\mathbf{1}_L = \lambda = 0$.

Finally, we turn to the large activist. While the strategy of the large activist is straightforward, since she knows η, the effect of her presence on smaller skilled institutions is not. Does the presence of a large activist have a tangible effect on the probability of successful engagement over and above the impact arising from the presence of dispersed skilled institutions? In order to isolate the potential effect cleanly we must control for total holdings by those owners who may engage—the large activist and the potentially skilled institutions—which we refer to as the "activist base". In other words, we must consider the change in the efficacy of activism when, for a given activist base, we replace the large activist by an equal measure of dispersed potentially skilled institutions.

In our dynamic model, the share acquisition decisions of small institutions at $t = 0$ anticipate the potential arrival of the large activist which—if it occurs—may potentially spur further share acquisitions by other dispersed institutions. Thus, fixing an initial set of parameters, it is never the case in equilibrium that the total size of the activist base is identical with and without the presence of the large activist. Nevertheless, our model provides the basis for carrying out a comparative statics exercise which pinpoints the impact of the large activist. We compare the efficacy of activism under two potential ownership structures. Under the first ownership structure there are only small institutions in a total measure A^T (i.e., $A_s = A^T$). Under the second ownership structure a measure A_L of the small institutions are replaced by the single large activist L, so that $A_s + A_L = A^T$. For simplicity, let $\lambda \to 0$. By using Proposition 1, we can compare the fundamental levels below which activism succeeds under the two ownership structures:

Corollary 1. *There exists a range of entrenchment levels of measure* $A_L \left[1 - \gamma \left(1 - \frac{c_s}{R}\right)\right]$ *for which engagement is successful in a target firm if and only if a large activist is present.*

The result follows from comparing η_s^* (for $A_s = A^T$) and η_L^* (for $A_s = A^T - A_L$):

$$\eta_L^* - \eta_s^* = A_L + \gamma \left(A^T - A_L\right) \left(1 - \frac{c_s}{R}\right) - \gamma A^T \left(1 - \frac{c_s}{R}\right) = A_L \left[1 - \gamma \left(1 - \frac{c_s}{R}\right)\right] > 0.$$

In words, fixing the size of the activist base, if a measure of dispersed potentially skilled institutions is replaced by a single large activist, activism becomes more effective. To appreciate the forces behind this result, let us compare the engagement threshold of the skilled institutions. Under the ownership structure with only small institutions, this engagement threshold is $\gamma A^T\left(1-\frac{c_s}{R}\right)$, i.e., skilled institutions engage only when they (correctly) believe $\eta<\gamma A^T\left(1-\frac{c_s}{R}\right)$. Under the alternative ownership structure where a measure A_L of potentially skilled institutions are replaced by a single large activist, the engagement threshold *rises* to $A_L+\gamma\left(A^T-A_L\right)\left(1-\frac{c_s}{R}\right)$. In other words, the presence of a large activist in their midst makes skilled institutions more aggressive in their engagement strategy, and more so the larger is the lead activist's stake, A_L. That is, the presence of a large activist has a catalytic effect on smaller skilled institutions.[15] It may seem as though this result is a simple by-product of the fact that the lead activist has an informational advantage. However, it is important to note that the presence of the lead activist causes a discrete jump in the probability of successful engagement even in the limit where all skilled institutions are perfectly informed. Thus, it is not information alone that leads to the catalytic effect. The effect instead arises as a result of the subtle interaction of the informational advantage of the lead activist with the fact that her presence eases the coordination problem: since the lead activist unilaterally controls the engagement strategy of A_L shares, her presence eliminates any coordination issues within that block.

4 Trading Dynamics

We now turn to trading dynamics prior to the activism game. Throughout we focus on the limiting equilibrium from above where $\alpha_s\to\infty$ and $\lambda\to 0$. We model trading at

[15]Here we have assumed that part of the pool of *potentially* skilled institutions is replaced by the large activist. Qualitatively similar results are obtained if we assume part of the pool of *ex post* skilled institutions is replaced by the large activist.

all dates as a reduced form transparent market, where all participants share common information about the game and identity of all traders, and shares change hands at their expected non-excludable value. For example, this could be modeled as a Kyle (1985) type market with a risk neutral market maker and no noise trade.

Endogenous Valuation Differences: Despite the transparency of our market and the absence of noise, trade arises endogenously, as a consequence of two results proved above. First, as we show in Lemma 1, the free float of $\bar{A}$ is initially owned by institutions who know themselves to be unskilled. Second, given the results in Proposition 1, these unskilled institutions know that they will never choose to engage the target, and can thus realize only the non-excludable value of the shares. In contrast, potentially skilled institutions, upon learning that the target firm is amenable to activism, will attach positive probability to earning repuational rents if they own shares in the firm. As a result, the valuation of such potential buyers for target shares will be strictly higher than those of the initial owners, giving rise to trade.

4.1 Following the Lead Activist

We proceed backward, beginning with potential trading among institutional investors at $t = 2$, after it is known whether the large activist has entered or not. In particular, potentially skilled institutions who did not acquire a position in the firm at $t = 0$ have the option of doing so at $t = 2$. The strategy of these institutions at $t = 2$ is conditioned on the actions of the large activist, who chooses at $t = 1$. Institutions do not yet know their type and, as a result, their realized opportunity cost. Since the incentive to acquire is decreasing in the $t = 2$ expectation of such opportunity costs, γk_i, we focus on strategies in which small institutions acquire if and only if γk_i is below some threshold value, i.e., monotone strategies (as in the activism game). Accordingly, we characterize two thresholds: $K_2^*(A_L)$ and $K_2^*(0)$, representing the cases where the large activist holds a position in the firm and where she does not, respectively.

What about the potentially skilled institutions who acquired shares at $t = 0$, before knowing whether L would enter or not? As will become clear later, an institution will buy shares at $t = 0$ only if his expected opportunity cost, γk_i, is below the minimum $t = 2$ purchase threshold. Thus, denoting the threshold for purchase at $t = 0$ by K_0^*, we guess (and later verify) that $K_0^* \leq \min \{K_2^* (A_L), K_2^* (0)\}$. Further, we assume that if any institution is indifferent between entry at $t = 0$ and $t = 2$, they enter at $t = 0$. For example, this could be because there are small trading profits available if these institutions trade prior to the 13D announcement because they are better able than unskilled institutions to predict the availability of the lead activist. For parsimony, we do not model this asymmetric information trading game, but we believe it would not significantly alter the model's qualitative results.

From Lemma 1, we know that institutions who acquire a position in the firm at any date t purchase their shares from unskilled institutions. Since the unskilled institutions are rational, share the same information at the point of acquisition (recall that the skilled institutions' private signals are only received at the beginning of $t = 3$), and are only willing to trade at fair value, the sole source of gains for potentially skilled institutions arises from their net private reputational rents $(R - c_s)$ from successful activism. In other words, any potentially skilled institutions who choose to purchase shares and participate in the activism game do so solely to determine and advertise their type in an attempt to gain reputation. In turn, since the activism game at $t = 3$ is played with vanishing noise, institutions who turn out to be skilled engage only when engagement is successful. Thus, apart from any non-excludable rents, a potentially skilled institution can expect to receive $R - c_s$ in the event that they turn out to be skilled and engagement is successful, and nothing otherwise. Engagement succeeds whenever the level of entrenchment is below the relevant threshold, which in turn depends on the size of the activist base.

In case L is present, under our maintained hypothesis that $K_0^* \leq \min \{K_2^* (A_L), K_2^* (0)\}$, the mass of potentially skilled small institutions is given by

$$A_s = \Pr\left(\gamma k_i \le K_2^*\left(A_L\right)\right) = \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\overline{k} - \underline{k}}.$$

Proposition 1 implies that the entrenchment threshold in the activism game is then $A_L + \gamma \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)$, so that the expected payoff from share acquisition for any given potentially skilled institution is:

$$\gamma \Pr\left(\eta \le A_L + \gamma \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)\left(R - c_s\right),$$

while his expected opportunity cost is γk_s^i. The potentially skilled institution with opportunity cost $K_2^*\left(A_L\right)$ must be exactly indifferent, i.e., $K_2^*\left(A_L\right)$ is implicitly determined by

$$\gamma \Pr\left(\eta \le A_L + \gamma \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)\left(R - c_s\right) = K_2^*\left(A_L\right). \tag{1}$$

It is easy to see that as long as there is sufficient volatility in entrenchment levels, there exists a unique such threshold $K_2^*\left(A_L\right)$:

Lemma 2. *There exists $\overline{\alpha}_\eta \in \mathbb{R}_+$ such that if $\alpha_\eta \le \overline{\alpha}_\eta$ there is a unique solution to (1).*

The intuition for uniqueness is as follows: Both sides of the equation implicitly defining $K_2^*\left(A_L\right)$ are increasing in $K_2^*\left(A_L\right)$. Under these circumstances, a sufficient condition for uniqueness is that rates of change with respect to $K_2^*\left(A_L\right)$ are strictly ranked. The left hand side is a scaled probability in η. As long as the density function of η is sufficiently spread out, the left hand side will always increase slower than the right hand side (the 45 degree line), giving rise to uniqueness. The economic interpretation of this condition is that sufficient variation in potential entrenchment levels prevents small changes in the mass of activists from having too much influence on success probabilities. Sufficient variation in entrenchment levels is sufficient but not necessary for our qualitative results. We believe this is an economically reasonable assumption, as it implies that there is sufficient uncertainty over how large a wolf pack is needed.

In case L is absent, as long as $K_0^* \leq \min\left\{K_2^*\left(A_L\right), K_2^*\left(0\right)\right\}$, the mass of activists is given by $\frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}$. Given this mass of activists, Proposition 1 implies that the entrenchment threshold in the activism game is $\gamma\frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)$, so that $K_2^*\left(0\right)$ is implicitly defined by:

$$\gamma \Pr\left(\eta \leq \gamma\frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)\left(R-c_s\right) = K_2^*\left(0\right). \tag{2}$$

The sufficient condition for the uniqueness of $K_2^*\left(0\right)$ is similar to that for $K_2^*\left(A_L\right)$. Thus, we state without proof:

Lemma 3. *There exists $\widehat{\alpha}_\eta \in \mathbb{R}_+$ such that if $\alpha_\eta \leq \widehat{\alpha}_\eta$ there is a unique solution to (2).*

From this point onward, we assume that $\alpha_\eta \leq min\left(\overline{\alpha}_\eta, \widehat{\alpha}_\eta\right)$. Given Lemmas 2 and 3, we can now compare the thresholds $K_2^*\left(A_L\right)$ and $K_2^*\left(0\right)$ to determine the effect of the entry of the large activist on subsequent entry by potentially skilled institutions. We show:

Proposition 2. *$K_2^*\left(A_L\right) > K_2^*\left(0\right)$.*

The intuition for this result can be understood as follows. The reason potentially skilled institutions may acquire shares in the firm even though they trade with rational traders who charge the full expected continuation value is due to their expected future net reputational benefits from successful coordinated engagement. Such benefits must be offset against their expected opportunity costs, γk_s^i, giving rise to a threshold level of expected opportunity costs below which share acquisition occurs and above which it does not. Anything that increases expected reputational benefits, increases incentives to acquire blocks and moves the opportunity cost threshold upwards.

Consider the potentially skilled institution with opportunity cost $K_2^*\left(0\right)$. This institution is exactly indifferent between acquiring a share and not acquiring a share if the large activist does *not* participate, in which case—by monotonicity—exactly $\frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}$ institutions will participate, giving rise to a expected net benefit from share acquisition of

$$\gamma \Pr\left(\eta \leq \gamma \frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)(R-c_s).$$

However, imagine now that the large activist *does* participate. *Even if skilled institutions did not change their behavior*, the probability of successful engagement would rise to $\Pr\left(\eta \leq A_L + \gamma \frac{\frac{K_2^*(0)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)$, and thus the institution with opportunity cost $K_2^*(0)$ would no longer be exactly indifferent between acquiring a share or not: he would strictly prefer to acquire shares. By continuity, this means that some institutions with strictly higher expected opportunity costs would strictly prefer to participate. In other words, the threshold level of opportunity cost would increase since the value of gathering information about the target has increased.

The implication of this result is that the entry of a large activist spurs additional entry by potentially skilled institutions, that is, a wolf pack expands given the presence of a leader.

4.2 The Lead Activist

Given our earlier analysis, we know that if L enters, the size of the activist base will be $A_L + \frac{\frac{K_2^*(A_L)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}$, giving rise to an expected payoff for entry of:

$$A_L \Pr\left(\eta \leq A_L + \gamma \frac{\frac{K_2^*(A_L)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)(\beta_L - c_L) \tag{3}$$

which will be compared to L's opportunity cost k_L. We show

Proposition 3. *For a given* $(A_L, k_L, \beta_L, c_L, R, c_s, \gamma)$ *the large activist enters only if* $\overline{k}$ *and* $\underline{k}$ *are small enough.*

The smaller are $\overline{k}$ and $\underline{k}$, the larger is the expected size of the wolf pack of skilled institutions, because (i) fixing $\underline{k}$, reducing $\overline{k}$ reduces the mass of potentially skilled institutions with high opportunity costs who would not wish to buy in, while (ii) fixing $\overline{k}$, reducing $\underline{k}$ increases the mass of potentially skilled institutions with low expected opportunity costs who are mostly likely to buy in. Accordingly, the result shows that the large activist will enter only if the anticipated skilled institutional ownership is large enough.

4.3 Anticipating the Lead Activist

At $t = 0$ institutions have the option of buying into the firm before they know whether L will enter, or to wait until uncertainty over L's presence is resolved. Note that since there is a $1 - p_L$ probability that L is unavailable for activism, there is always ex ante uncertainty with regard to L's presence. The behavior of potentially skilled institutions is characterized by a threshold: institutions with expected opportunity costs, γk_i, below K_0^* will enter early (by our tie-breaking assumption) and those with higher opportunity costs will wait until $t = 2$. Note that, since it is costless to wait and verify whether L is present (because the transaction price for share acquisition is always fair and the reputational benefits are received after $t = 3$), a potentially skilled institution can only wish to buy a share at $t = 0$ if his k_i is low enough that he would prefer to own regardless of whether L enters or not. In other words, K_0^* is defined by:

$$\gamma \Pr\left(\eta \leq \gamma \frac{\frac{K_2^*(0)}{\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)(R - c_s) = K_0^*,$$

which has a unique solution if $\alpha_\eta \leq \widehat{\alpha}_\eta$. But notice that this condition is identical to (2), and thus $K_0^* = K_2^*(0)$. Thus, we have $K_0^* \leq \min\left\{K_2^*(A_L), K_2^*(0)\right\}$ as conjectured above.

5 Wolf Pack Formation

In this section, we summarize the empirical implications of our model for the dynamics of wolf pack formation. Our predictions can be classified into implications for ownership dynamics and price dynamics.

5.1 Ownership dynamics

In the unique equilibrium of our model:

- Some small institutions (those with low expected opportunity costs) acquire positions in the target firm at $t = 0$ in potential anticipation of the large activist's arrival.
- If the large activist is available for activism at $t = 1$, she acquires a stake in the firm if and only if she expects that there will be a sufficiently large activist base given her opportunity cost (i.e., if she believes that the total expected mass of small institutions at $t = 3$ is large enough).
- Conditional on the large activist's entry at $t = 1$ there will be additional entry by small institutions with higher opportunity costs.

Imagine that the entry of the large activist is synonymous with the filing of a 13D. Then, combining these dynamic implications delivers several empirical implications:

Remark 1. Firms in which 13Ds are filed will have substantially higher activist presence (measure $A_L + \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\bar{k} - \underline{k}}$) than firms in which they are not (measure $\frac{\frac{K_2^*(0)}{\gamma} - \underline{k}}{\bar{k} - \underline{k}}$).

The empirical content of this depends on our definition of an activist. If we define an activist to be an institutional investor, as in the model, then this result captures the Brav et al (2008) finding that firms in which activist hedge funds file 13Ds have high institutional ownership.

Remark 2. There will be significant additional accumulation of activist shares following a 13D filing (a measure $\frac{\frac{K_2^*(A_L)}{\gamma}-\underline{k}}{\overline{k}-\underline{k}} - \frac{\frac{K_0^*}{\gamma}-\underline{k}}{\overline{k}-\underline{k}}$ of additional small institutions will enter conditional on the large activist's entry).

Thus, one should expect abnormal turnover in target shares following a 13D filing. This is consistent with the evidence of abnormal turnover around 13D filings discussed in the introduction. Further, there may be differences in institutions who buy into a target firm's shares before and after a 13D filing:

Remark 3. Late entrants to wolf packs have higher opportunity costs than early entrants.

One potential way to interpret this is that more concentrated, smaller, and more "specialized" vehicles (such as other activist or event-driven funds) may be more inclined to acquire a stake only after the filing of a 13D by a lead activist. This is in keeping with Nathan's (2009) description in the introduction.

5.2 Price dynamics

To examine the dynamics of prices in our model we first set up some additional notation. Let P_t denote the equilibrium price of the firm at t. Then, P_t is the date-t expected terminal ($t = 3$) value of the firm, taking into account the expected probability of successful engagement given the information available at t, and therefore $P_t \in [P_\ell, P_h]$. It is straightforward to show that the price reacts to information in the model as follows:

- At $t = 0$, uncertainty over whether the firm will be amenable to activism is resolved. The price is $P_0 = P_\ell$ if the firm is not in the amenable state (in which case $P_t = P_\ell$ for all t), and is strictly greater than P_ℓ if the firm is in the amenable state.

Conditional on the firm being amenable:

- At $t = 1$, uncertainty on whether a large activist will be available is resolved. There are two cases:

  - Case (A) The large activist will not acquire a stake even if she arrives (e.g., because the expected size of the wolf pack is too small). In this case the arrival of the large activist is immaterial and $P_1 = P_0$.
  - Case (B) The large activist will acquire a stake if she arrives. In this case, if the large activist arrives and acquires a stake the price rises (i.e., $P_1 > P_0$). If the large activist is not available, the price falls (i.e., $P_1 < P_0$).

- At $t = 2$, if the large activist has acquired a stake then additional small institutions enter. If the large activist has not acquired a stake, there is no additional entry of small institutions. Since the number of entrants is perfectly predictable based on the large activist's actions at $t = 1$, the price does not change (i.e., $P_2 = P_1$).

- At $t = 3$, uncertainty on the level of entrenchment, and therefore the outcome of engagement, resolves. The price rises if engagement succeeds (i.e., $P_3 = P_h > P_2$) and falls otherwise (i.e., $P_3 = P_l < P_2$).

As above, if the entry and acquisition of a large activist is synonymous with the filing of a 13D, then we have the following empirical implication.

Remark 4. Targets experience positive returns upon the filing of a 13D (i.e., conditional on the entry of a large activist, $P_1 > P_0$).

This implication has wide support in the empirical literature on hedge fund activism. A significant number of papers find that targets experience positive abnormal short-term returns conditional on the filing of a 13D (see Brav et al 2010 for a survey of this literature).

The lack of price dynamics at $t = 2$ derives from the fact that there is no uncertainty with regard to how small institutions will respond to the lead activist's entry decision. This is a consequence of our modelling choice to have a continuum of small institutions. In reality there is likely to be some unpredictability with respect to the number of small institutions with low enough opportunity costs to find entry attractive following the large activist's stake purchase. One way to incorporate this into our framework is to introduce some aggregate uncertainty with respect to small institutions' opportunity costs. For example, assume the value of outside opportunities is Δk_i, where k_i is as before and $\Delta \in \{1 - \delta, 1 + \delta\}$ represents an aggregate shock to outside opportunities, where $\delta \in (0, 1)$. Further assume that the realization of Δ, which equals $(1 + \delta)$ with probability p_Δ, is publicly revealed at $t = 2$. Finally, modify Assumption 1(b) as follows: $(1 + \delta)\,\underline{k} < \frac{R - c_s}{2} < (1 - \delta)\,\overline{k}$. Now, it is straightforward to show that all elements of our analysis go through, and in addition we have the following result.

Proposition 4. *Assume there is aggregate uncertainty on small institutions' opportunity costs. Then $P_2 - P_1$ is decreasing in Δ, i.e., target returns following the filing of a 13D are increasing in the size of the wolf pack.*

Intuitively, this result arises from the following modified price dynamics at $t = 2$:

- At $t = 2$, uncertainty on the aggregate shock to the opportunity costs for small institutions resolves. The price rises if opportunity costs fall and many small institutions enter (i.e., $P_2 > P_1$). The price falls if opportunity costs rise and few small institutions enter (i.e., $P_2 < P_1$).

We are aware of no systematic evidence for this implication, which therefore represents a testable prediction of our model. Further, this implication separates our story from pure herding or other information-based stories of institutional share acquisition following a 13D filing. In such stories, the post-13D entrants add no value to the target and should have no price impact. In our model, the post-13D entrants are key participants in the value enhancement process and thus the price reacts positively to higher levels of entry.

6 Discussion

6.1 Alternative Interpretations of Engagement

Throughout the paper we have maintained the interpretation that engagement on the part of both large and small activists entails “behind the scenes” discussions with management or other shareholders, or other influence activities designed to increase pressure on management. An alternative interpretation is that wolf pack members “engage” by maintaining their presence as continued small blockholders of the firm’s shares who support the lead activist and will vote with her if the engagement process leads to a proxy contest. Since activist campaigns can last many months, this is a costly action to small institutions that may have an outsized portion of their capital committed to the target and thus suffer from underdiversificaiton or further opportunity costs (in addition to those paid to initially buy shares and investigate the firm) to remaining invested throughout the campaign.

The only change required in our model to accommodate such an interpretation would be the addition of a final trading round after institutions receive their signals, at which point they must simultaneously choose whether to exit (not engage) or maintain their investment in the target’s shares (engage). Formally, since there is a continuum of institutions, a threshold equilibrium in which those with signals below the threshold sell and those with signals above the threshold hold will fully reveal the firm’s fundamental and the number of engaging institutions. Thus, the price at this trading round will be exactly equal to the post-engagement firm value. Each institution at the margin has no influence on the price or success probability, so will, as in the current model, choose its engagement strategy without taking into account any potential trading profits or any marginal effect on the probability of success, and will thus trade off its underdiversification cost against the potential reputational benefit of engaging when engagement is successful. Semantically, one could argue that it is unrealistic to assume that institutions have no chance to change their minds once the fundamental has been revealed through the price, but this trading round is meant to represent private trading decisions that take place over a significant length of time in a noisy market. Thus, we think this alternative specification is reasonable. Also note that engagement in this model is visible as long as potential investors can see changes in institutions' holdings over some reasonable time window.

6.2 Microfounding reputational rents

In our model, reputational concerns play a key role in generating excludable rents from engagement, thus making wolf packs feasible. Reputation is generated endogenously in our model: the value of posteriors is determined by the equilibrium behavior of institutions who, in turn, correctly anticipate these posteriors. Rewards from reputation are obtained as follows: if the posterior on an institution's type is measurably higher than the prior, that is, $\hat{\gamma} \geq B$ for some $B \in (\gamma, 1)$, the institution gets private benefit R from participating in the game. Otherwise, the institution gets no private benefits. In this section we discuss how the parameters R and B could be endogenized.

Imagine that there is a class of investors who are of high intrinsic ability, i.e., "smart money". The smart money is able to invest directly without paying fees, and thus—given its high intrinsic skills—finds it optimal to delegate only to funds whose reputation for being informed is sufficiently high to indicate a sufficient level of outperformance in the future. The remaining investors represent "dumb money" which has no direct investment skills and does not monitor fund reputation. The dumb money is therefore happy to invest in any fund.

Now, if we normalize the returns to funds from attracting only dumb money in the future to be zero, define B to be the ability threshold required to attract smart money, and set R to be the additional payoff (from fees etc.) to funds from being able to attract smart money, we obtain exactly the structure described above.

Needless to say, a full determination of equilibrium values for R and B would require a fully dynamic model, which is beyond the scope of this paper. In such a model R could represent, for example, the benefit obtainable by newly reputable institutions from attracting part of the steady state reallocation of smart money due to the deaths of existing reputable institutions.

6.3 Parameter Constraints

A key parameter constraint we have imposed throughout is that the reward to reputation not be too large relative to the cost of engagement, i.e. Assumption 1(a), $R \in (c_s, 2c_s)$. The purpose of this constraint is to ensure that unskilled institutions will never find it optimal to deviate from the pure strategy equilibrium in which they never engage. If $R > 2c_s$ then for some parameter constellations there would be no pure strategy equilibrium for the unskilled types, which would significantly complicate the analysis. As a result we use the sufficient (but not necessary) condition $R \in (c_s, 2c_s)$ to maintain tractability. In addition, we believe that this constraint is economically reasonable as it simply states that the rewards to gaining reputation are not excessive compared to the costs. This should generally be true in settings where reputation is a useful mechanism in equilibrium, as otherwise it would be extremely difficult to prevent unskilled types from trying to mimic skilled types, i.e, the reputation mechanism would break down. We should also note that this restriction is tied to the prior distribution of our entrenchment variable $\eta \sim N\left(\bar{A}, \frac{1}{\alpha_\eta}\right)$. If we allow for $R > 2c_s$ but raise the prior mean of η commensurately, our analysis will be unchanged: even with the prospect of higher reputational gain will be offset by the lower probability of successful engagement (from the perspective of uninformed institutions) and thus leave their behavior unchanged.

We have also assumed $\underline{k} < \frac{R-c_s}{2} < \overline{k}$ (Assumption 1(b)). This is to avoid corner solutions whereby there are either no institutions who want to buy into the target firm, or all institutions do. This does not affect our qualitative results. Similarly, our assumption that the probability with which targets become amenable to activism is not very high, i.e., $p_A < \frac{2\underline{k}}{R-c_s}$, is used to highlight the endogenous trading motive, wherein institutions who want to demonstrate their skill will displace existing holders with no such motive. Changing this assumption would simply make that dynamic less stark; there could be some initial owners who are potentially skilled, but a mass of unskilled initial owners would still be displaced in equilibrium.

7 Conclusion

The possibility of coordinated engagement by shareholders has important implications for corporate governance. In this paper we show that implicit coordination among institutional shareholders can play a powerful role in activist campaigns. One of the key characteristics of institutional investors, who now own a majority of corporate equity, is that they are delegated portfolio managers who rely on the continued approval of their investor base to be successful. As Franklin Allen emphasized in his AFA Presidential Address (Allen 2001), the incentives faced by money managers can have a significant impact on financial markets. Our study indicates that these incentives can have even wider-ranging implications, for example by affecting the nature of shareholder activism. In particular, we show that money managers' competition for investor capital can give rise to strong strategic complementarity in their engagement strategies, providing a basis for coordinated shareholder activism.

Our analysis provides a lens through which to view activist wolf packs, a controversial tactic which hedge funds are alleged to use to accomplish coordinated activism. In addition to formalizing a theoretical basis for implicit coordination among wolf pack members, we also demonstrate that the emergence of a lead activist has an important catalytic effect on the aggressiveness of other institutional shareholders. Finally, we show that empirically demonstrated trading dynamics are consistent with our model of implicit coordination, and provide further testable hypotheses.

Our results should enable empirical researchers to better study the mechanics and implications of wolf pack tactics. Future work could also examine the role that explicit collusion or intentional information leakage might play in either substituting for or complementing the implicit coordination mechanism we model.

Appendix

Proof of Lemma 1: Since all trades occur at fair prices, before amenability is known, an upper bound on the returns from buying a share in the firm is given by the product of the probability of amenability (p_A), the maximum probability that engagement is successful ($Pr\left(\eta \leq \overline{A}\right) = 1/2$ since activist ownership is bounded above by $\overline{A}$), and the net reputational payoff from successful engagement conditional on engaging only when engagement succeeds ($R - c_s$), i.e., $p_A \frac{1}{2}\left(R - c_s\right)$. A lower bound on the expected opportunity cost for buying a share is $\left(1-\delta\right)\underline{k}$. Since $p_A < \frac{2(1-\delta)\underline{k}}{R-c_s}$, the lower bound is always higher than the upper bound, and hence no potentially skilled institution would own the firm.

Proof of Proposition 1: Denote by $\mathbf{1}_L$ the indicator function that is equal to 1 if the large activist is present. Denote the probability with which each unskilled institution engages by $p_e \in [0,1]$. p_e is formally a function of $\mathbf{1}_L$, but we suppress this dependence here for notational brevity as we shall show below that the strategies of the small unskilled institutions are independent of the presence of the large activist in equilibrium. The strategies of the skilled small institutions will depend on $\mathbf{1}_L$, p_e and λ. Denote the threshold by $x_s^*\left(\mathbf{1}_L, p_e, \lambda\right)$. Finally, define $\hat{A} = \bar{A} - \mathbf{1}_L A_L$, the measure of shares that is jointly owned by small institutions, skilled or unskilled. Since $x_{s,j}|\eta \sim N\left(\eta, \frac{1}{\alpha_s}\right)$, for each η, the measure of engagement by small institutions is given by

$$A_s\gamma \Pr\left(x_{s,j} \leq x_s^*\left(\mathbf{1}_L, p_e, \lambda\right)|\eta\right) + \left(A_s\left(1-\gamma\right)\left(1-\lambda\right) + \left(\hat{A} - A_s\right)\right)p_e + A_s\left(1-\gamma\right)\frac{\lambda}{2}.$$

The large activist will engage if present if and only if

$$A_L + A_s\gamma \Pr\left(x_{s,j} \leq x_s^*\left(\mathbf{1}_L, p_e, \lambda\right)|\eta\right) + \left(A_s\left(1-\gamma\right)\left(1-\lambda\right) + \left(\hat{A} - A_s\right)\right)p_e + A_s\left(1-\gamma\right)\frac{\lambda}{2} \geq \eta.$$

Thus, engagement is successful if and only if

$$\mathbf{1}_L A_L + A_s\gamma\Phi\left(\sqrt{\alpha_s}\left(x_s^*\left(\mathbf{1}_L, p_e, \lambda\right) - \eta\right)\right) + \left(A_s\left(1-\gamma\right)\left(1-\lambda\right) + \left(\hat{A} - A_s\right)\right)p_e + A_s\left(1-\gamma\right)\frac{\lambda}{2} \geq \eta.$$

The LHS is decreasing in η, the RHS is increasing in η, so there exists $\eta_s^*(p_e, \lambda)$ such that engagement is successful if and only if $\eta \leq \eta_s^*(p_e, \lambda)$, where $\eta_s^*(p_e, \lambda)$ is defined by

$$\begin{array}{c} \mathbf{1}_L A_L + A_s \gamma \Phi\left(\sqrt{\alpha_s}\left(x_s^*(\mathbf{1}_L, p_e, \lambda) - \eta_s^*(\mathbf{1}_L, p_e, \lambda)\right)\right) + \\ \left(A_s(1-\gamma)(1-\lambda) + \left(\hat{A} - A_s\right)\right) p_e + A_s(1-\gamma)\frac{\lambda}{2} \end{array} = \eta_s^*(\mathbf{1}_L, p_e, \lambda). \tag{4}$$

Which implies that

$$x_s^*(\mathbf{1}_L, p_e, \lambda) = \begin{array}{c} \eta_s^*(\mathbf{1}_L, 0, \lambda) + \\ \frac{1}{\sqrt{\alpha_s}} \Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L, 0, \lambda) - \mathbf{1}_L A_L - \left(A_s(1-\gamma)(1-\lambda) + \left(\hat{A} - A_s\right)\right) p_e - A_s(1-\gamma)\frac{\lambda}{2}}{A_s \gamma}\right) \end{array}.$$

Note that this implies that as $\alpha_s \to \infty$, $x_s^*(\mathbf{1}_L, p_e, \lambda) \to \eta_s^*(\mathbf{1}_L, 0, \lambda)$.

We now compute the posterior reputation of each small institution in equilibrium. Since individual small institutions may engage (E) or not (N), and engagement may succeed ($S := \{\eta \leq \eta_s^*(p_e, \lambda)\}$) or fail ($F := \{\eta > \eta_s^*(p_e, \lambda)\}$), there are four possible posterior reputations: $\hat{\gamma}(S, E), \hat{\gamma}(F, E), \hat{\gamma}(S, N)$, and $\hat{\gamma}(F, N)$.

$$\begin{aligned}
\hat{\gamma}(S, E) &= \Pr(\theta = G | S, E) \\
&= \frac{\frac{A_s \gamma}{\hat{A}} \Pr(S, E | \theta = G)}{\frac{A_s \gamma}{\hat{A}} \Pr(S, E | \theta = G) + \frac{A_s(1-\gamma)(1-\lambda)}{\hat{A}} \Pr(S) p_e + \frac{A_s(1-\gamma)\lambda}{\hat{A}} \frac{1}{2} + \frac{\hat{A} - A_s}{\hat{A}} \Pr(S) p_e} \\
&= \frac{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda), S)}{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda), S) + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right) \Pr(S) p_e + A_s(1-\gamma) \Pr(S) \frac{\lambda}{2}} \\
&= \frac{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | S)}{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | S) + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right) p_e + A_s(1-\gamma) \frac{\lambda}{2}}.
\end{aligned}$$

By analogy

$$\begin{aligned}
\hat{\gamma}(F, E) &= \frac{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | F)}{A_s \gamma \Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | F) + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right) p_e + A_s(1-\gamma) \frac{\lambda}{2}}, \\
\hat{\gamma}(S, N) &= \frac{A_s \gamma \Pr(x_s > x_s^*(\mathbf{1}_L, p_e, \lambda) | S)}{A_s \gamma \Pr(x_s > x_s^*(\mathbf{1}_L, p_e, \lambda) | S) + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right)(1 - p_e) + A_s(1-\gamma) \frac{\lambda}{2}}, \\
\hat{\gamma}(F, N) &= \frac{A_s \gamma \Pr(x_s > x_s^*(\mathbf{1}_L, p_e, \lambda) | F)}{A_s \gamma \Pr(x_s > x_s^*(\mathbf{1}_L, p_e, \lambda) | F) + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right)(1 - p_e) + A_s(1-\gamma) \frac{\lambda}{2}}.
\end{aligned}$$

Denoting by I the information set of a given player and by $\mathbf{1}$ the indicator function which is equal to one if its argument is true, the payoffs from engagement are given by:

$$\Pr(S|I)\left[\mathbf{1}\left(\hat{\gamma}(S,E)\geq B\right)R+P_h\right]+\left(1-\Pr(S|I)\right)\left[\mathbf{1}\left(\hat{\gamma}(F,E)\geq B\right)R+P_l\right]-c_s,$$

whereas the payoffs from non-engagement are given by:

$$\Pr(S|I)\left[\mathbf{1}\left(\hat{\gamma}(S,N)\geq B\right)R+P_h\right]+\left(1-\Pr(S|I)\right)\left[\mathbf{1}\left(\hat{\gamma}(F,N)\geq B\right)R+P_l\right].$$

First consider the unskilled small institutions, so that $I=\varnothing$. We first show that:

Lemma 4. *For* $\lambda<\min\left[\frac{2\gamma(1-B)}{(1-\gamma)B},\frac{2(B-\gamma)}{(1-\gamma)B}\right]$ *there exists* $\underline{\alpha}_{III}(\lambda)\in\mathbb{R}_+$ *such for all* $\alpha_s\geq\underline{\alpha}_{III}(\lambda)$*, unskilled small institutions must choose* $p_e=0$ *in equilibrium.*

Proof of Lemma: First we show that for sufficiently precise signals, $p_e=0$ is a best response by unskilled institutions to a monotone strategy with threshold $x_s^*(0,\lambda)$ used by skilled institutions. For $p_e=0$ the posteriors are as follows:

$$\hat{\gamma}(S,E)=\frac{\gamma\Pr\left(x_s\leq x_s^*(\mathbf{1}_L,0,\lambda)|S\right)}{\gamma\Pr\left(x_s\leq x_s^*(\mathbf{1}_L,0,\lambda)|S\right)+(1-\gamma)\frac{\lambda}{2}}\underset{\alpha_s\to\infty}{\to}\frac{\gamma}{\gamma+(1-\gamma)\frac{\lambda}{2}}.$$

For $\lambda<\frac{2\gamma(1-B)}{(1-\gamma)B}$, $\frac{\gamma}{\gamma+(1-\gamma)\frac{\lambda}{2}}>B$, and thus there exists $\underline{\alpha}_1(\lambda)\in\mathbb{R}_+$ such that for $\alpha_s\geq\underline{\alpha}_1(\lambda)$, $\hat{\gamma}(S,E)\geq B$.

$$\hat{\gamma}(F,E)=\frac{\gamma\Pr\left(x_s\leq x_s^*(\mathbf{1}_L,0,\lambda)|F\right)}{\gamma\Pr\left(x_s\leq x_s^*(\mathbf{1}_L,0,\lambda)|F\right)+(1-\gamma)\frac{\lambda}{2}}\underset{\alpha_s\to\infty}{\to}0.$$

Thus, for any λ, there exists $\underline{\alpha}_2(\lambda)\in\mathbb{R}_+$ such that for $\alpha_s>\underline{\alpha}_2(\lambda)$, $\hat{\gamma}(F,E)<B$.

$$\hat{\gamma}(S,N)=\frac{A_s\gamma\Pr\left(x_s>x_s^*(\mathbf{1}_L,0,\lambda)|S\right)}{A_s\gamma\Pr\left(x_s>x_s^*(\mathbf{1}_L,0,\lambda)|S\right)+\left(A_s(1-\gamma)(1-\lambda)+\hat{A}-A_s\right)+A_s(1-\gamma)\frac{\lambda}{2}}\underset{\alpha_s\to\infty}{\to}0.$$

Thus, for any λ, there exists $\underline{\alpha}_3(\lambda)\in\mathbb{R}_+$ such that for $\alpha_s\geq\underline{\alpha}_3(\lambda)$, $\hat{\gamma}(S,N)<B$.

$$
\begin{aligned}
\hat{\gamma}(F,N) &= \frac{A_s\gamma \Pr\left(x_s > x_s^*\left(\mathbf{1}_L,0,\lambda\right)|F\right)}{A_s\gamma \Pr\left(x_s > x_s^*\left(\mathbf{1}_L,0,\lambda\right)|F\right) + \left(A_s\left(1-\gamma\right)\left(1-\lambda\right)+\hat{A}-A_s\right) + A_s\left(1-\gamma\right)\frac{\lambda}{2}} \\
&\underset{\alpha_s\to\infty}{\to} \frac{A_s\gamma}{A_s\gamma + \left(A_s\left(1-\gamma\right)\left(1-\lambda\right)+\hat{A}-A_s\right) + A_s\left(1-\gamma\right)\frac{\lambda}{2}} \\
&= \frac{\gamma}{\gamma+\left(1-\gamma\right)\left(1-\frac{\lambda}{2}\right)+\frac{\hat{A}}{A_s}-1} \leq \frac{\gamma}{\gamma+\left(1-\gamma\right)\left(1-\frac{\lambda}{2}\right)}, \text{ since } \hat{A}\geq A_s.
\end{aligned}
$$

For $\lambda < \frac{2(B-\gamma)}{(1-\gamma)B}$, $\frac{\gamma}{\gamma+(1-\gamma)\left(1-\frac{\lambda}{2}\right)} < B$, and thus there exists $\underline{\alpha}_4(\lambda) \in \mathbb{R}_+$ such that for $\alpha_s > \underline{\alpha}_4(\lambda)$, $\hat{\gamma}(F,N) < B$. Now, setting

$$
\underline{\alpha}_I(\lambda) := \max\left[\underline{\alpha}_1(\lambda), \underline{\alpha}_2(\lambda), \underline{\alpha}_3(\lambda), \underline{\alpha}_4(\lambda)\right],
$$

for $\alpha_s \geq \underline{\alpha}_I(\lambda)$, we can write the payoffs for unskilled small institutions from engaging as follows:

$$
\Pr(S)\left(R+P_h\right)+(1-\Pr(S))P_l - c_s,
$$

whereas payoffs from not engaging are

$$
\Pr(S)\,P_h + (1-\Pr(S))P_l.
$$

Thus, $p_e = 0$ is optimal whenever

$$
\Pr(S) \leq \frac{c_s}{R},
$$

which is always satisfied because $\Pr(S) = \Pr(\eta \leq \eta_s^*(0,\lambda)) < \Pr(\eta \leq 1) = \frac{1}{2}$ since $\eta_s^*(0,\lambda) < 1$, whereas $\frac{c_s}{R} \geq \frac{1}{2}$ since $R \leq 2c_s$.

Next we show that $p_e = 1$ cannot arise in equilibrium. For $p_e = 1$ the posteriors are as follows:

$$
\begin{aligned}
\hat{\gamma}(S,E) &= \frac{A_s\gamma \Pr\left(x_s \leq x_s^*\left(\mathbf{1}_L,1,\lambda\right)|S\right)}{A_s\gamma \Pr\left(x_s \leq x_s^*\left(\mathbf{1}_L,p_e,\lambda\right)|S\right) + A_s\left(1-\gamma\right)\left(1-\lambda\right)+\hat{A}-A_s + A_s\left(1-\gamma\right)\frac{\lambda}{2}} \\
&\underset{\alpha_s\to\infty}{\to} \frac{A_s\gamma}{A_s\gamma + A_s\left(1-\gamma\right)\left(1-\lambda\right)+\hat{A}-A_s + A_s\left(1-\gamma\right)\frac{\lambda}{2}} \\
&= \frac{\gamma}{\gamma+\left(1-\gamma\right)\left(1-\frac{\lambda}{2}\right)+\frac{\hat{A}}{A_s}-1} \leq \frac{\gamma}{\gamma+\left(1-\gamma\right)\left(1-\frac{\lambda}{2}\right)}.
\end{aligned}
$$

This is identical to the case for $p_e = 0$ and $\hat{\gamma}(F, N)$. Thus, for $\alpha_s > \underline{\alpha}_4(\lambda)$, $\hat{\gamma}(S, E) < B$. Similarly it is easy to see that for $\alpha_s > \underline{\alpha}_3(\lambda)$, $\hat{\gamma}(F, E) < B$ while for $\alpha_s > \underline{\alpha}_2(\lambda)$, $\hat{\gamma}(S, N) < B$. Finally,

$$\hat{\gamma}(F, N) = \frac{\gamma \Pr(x_s > x_s^*(\mathbf{1}_L, 1, \lambda) | F)}{\gamma \Pr(x_s > x_s^*(\mathbf{1}_L, 1, \lambda) | F) + (1-\gamma)\frac{\lambda}{2}} \underset{\alpha_s \to \infty}{\rightarrow} \frac{\gamma}{\gamma + (1-\gamma)\frac{\lambda}{2}},$$

which is again identical to the case for $p_e = 0$ and $\hat{\gamma}(S, N)$. Thus, for $\alpha_s \geq \underline{\alpha}_1(\lambda)$, $\hat{\gamma}(F, N) \geq B$. Now, for $\alpha_s \geq \underline{\alpha}_I(\lambda)$, we can write the payoffs for unskilled institutions from engaging as follows:

$$\Pr(S) P_h + (1 - \Pr(S))P_l - c_s,$$

whereas payoffs from not engaging are

$$\Pr(S) P_h + (1 - \Pr(S))(P_l + R).$$

Since $\Pr(S) P_h + (1 - \Pr(S))(P_l + R) > \Pr(S) P_h + (1 - \Pr(S))P_l$, $p_e = 1$ can never be a best response to $x_s^*(1, \lambda)$.

Finally, we show that $p_e \in (0, 1)$ also cannot arise in equilibrium. For $p_e \in (0, 1)$ the posteriors are given by the general expressions above. Note that since $\hat{\gamma}(F, E)$ and $\hat{\gamma}(S, N)$ are bounded in p_e, there exist $\underline{\alpha}_5(\lambda) \in \mathbb{R}_+$ and $\underline{\alpha}_6(\lambda) \in \mathbb{R}_+$ such that, for any p_e, for $\alpha_s \geq \underline{\alpha}_5(\lambda)$, $\hat{\gamma}(F, E) < B$ and for $\alpha_s \geq \underline{\alpha}_6(\lambda)$, $\hat{\gamma}(S, N) < B$. Now consider $\alpha_s \geq \underline{\alpha}_{II}(\lambda) := \max[\underline{\alpha}_5(\lambda), \underline{\alpha}_6(\lambda)]$. For any $p_e \in (0, 1)$, λ:

$$\lim_{\alpha_s \to \infty} \hat{\gamma}(S, E) = \frac{A_s \gamma}{A_s \gamma + \left(A_s(1-\gamma)(1-\lambda) + \hat{A} - A_s\right) p_e + A_s(1-\gamma)\frac{\lambda}{2}}.$$

Either:

Case A: there exists a $\overline{p}_e > 0$ such that $\lim_{\alpha_s \to \infty} \hat{\gamma}(S, E) > B$ for $p_e \leq \overline{p}_e$ or

Case B: There exists no such $\overline{p}_e > 0$.

First, consider Case B. Note first that since $\hat{\gamma}(S, E)$ is increasing in $\Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | S)$ and $\Pr(x_s \leq x_s^*(\mathbf{1}_L, p_e, \lambda) | S)$ is increasing in α_s, $\hat{\gamma}(S, E) < B$ for all α_s.Thus, for any

$\alpha_s > \underline{\alpha}_{II}(\lambda)$, the payoff to engaging is $\Pr(S) P_h + (1 - \Pr(S))P_l - c_s$. But the payoff to not engageing is never less than $\Pr(S) P_h + (1 - \Pr(S))P_l$. Thus, $p_e \in (0,1)$ cannot arise in equilibrium.

Now consider Case A. Given the argument for Case B, $p_e > \overline{p}_e$ cannot arise in equilibrium either. The only possibility is that $p_e \in (0, \overline{p}_e]$. Fix such a p_e, and suppose there exists some $\alpha_s \geq \underline{\alpha}_{II}(\lambda)$ such that for such a pair (p_e, α_s) we have $\hat{\gamma}(S, E) > B$. There are two possibilities:

Either for that (p_e, α_s), $\hat{\gamma}(F, N) \leq B$, in which case the payoffs to engaging are:

$$\Pr(S)(R + P_h) + (1 - \Pr(S))P_l - c_s,$$

whereas payoffs from not engaging are

$$\Pr(S) P_h + (1 - \Pr(S))P_l.$$

Having, $p_e \in (0,1)$ requires that

$$\Pr(S) = \frac{c_s}{R},$$

which is impossible because $\Pr(S) < \frac{1}{2}$ and $\frac{c_s}{R} \geq \frac{1}{2}$.

The other possibility is that for that (p_e, α_s), $\hat{\gamma}(F, N) > B$ in which case the payoffs to engaging are

$$\Pr(S)(R + P_h) + (1 - \Pr(S))P_l - c_s,$$

whereas payoffs from not engaging are

$$\Pr(S) P_h + (1 - \Pr(S))(P_l + R).$$

Having, $p_e \in (0,1)$ requires that

$$\begin{aligned} \Pr(S) R - c_s &= (1 - \Pr(S))R \\ \text{i.e., } \Pr(S) &= \frac{1}{2} + \frac{c_s}{2R}, \end{aligned}$$

which is again impossible because $\Pr(S) < \frac{1}{2}$. Thus, for any λ and $\alpha_s \geq \underline{\alpha}_{II}(\lambda)$, $p_e \in (0,1)$ cannot arise in equilibrium.

Defining $\underline{\alpha}_{III}(\lambda) := \max\left[\underline{\alpha}_I(\lambda), \underline{\alpha}_{II}(\lambda)\right]$ completes the proof of the Lemma.□

For the remainder of the proof, consider $\lambda < \min\left[\frac{2\gamma(1-B)}{(1-\gamma)B}, \frac{2(B-\gamma)}{(1-\gamma)B}\right]$ and $\alpha \geq \underline{\alpha}_{III}(\lambda)$, so that we can use the above characterisation of the strategies of unskilled institutions. Consider the putative equilibrium thresholds for the skilled institutions which are given by $x_s^*(0,\lambda)$. The payoffs from engagement are given by:

$$\Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_{s,j}\right)(R + P_h) + \left(1 - \Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_{s,j}\right)\right)P_l - c_s,$$

whereas the payoffs from non-engagement are given by:

$$\Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_{s,j}\right)P_h + \left(1 - \Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_{s,j}\right)\right)P_l.$$

Thus, the net expected payoff from engagement is given by

$$\Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_{s,j}\right)R - c_s$$

which is clearly decreasing in $x_{s,j}$. The existence of the dominance regions and continuity jointly imply that there exists $x_s^*(0,\lambda) \in \mathbb{R}$ such that

$$\Pr\left(\eta \leq \eta_s^*(\mathbf{1}_L, 0, \lambda)\,|x_s^*(\mathbf{1}_L, 0, \lambda)\right)R - c_s = 0.$$

Further, since $\eta|x_{s,j} \sim N\left(\frac{\alpha_\eta\mu_\eta + \alpha_s x_{s,j}}{\alpha_\eta + \alpha_s}, \frac{1}{\alpha_\eta + \alpha_s}\right)$, we have the following condition:

$$\Phi\left(\sqrt{\alpha_\eta + \alpha_s}\left(\eta_s^*(\mathbf{1}_L, 0, \lambda) - \frac{\alpha_\eta\mu_\eta + \alpha_s x_s^*(\mathbf{1}_L, 0, \lambda)}{\alpha_\eta + \alpha_s}\right)\right) = \frac{c_s}{R}. \tag{5}$$

Solving (4) for $x_s^*(\mathbf{1}_L, 0, \lambda)$ at $p_e = 0$ gives

$$x_s^*(\mathbf{1}_L, 0, \lambda) = \eta_s^*(\mathbf{1}_L, 0, \lambda) + \frac{1}{\sqrt{\alpha_s}}\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L, 0, \lambda) - \mathbf{1}_L A_L - A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right).$$

Substituting into (5) gives:

$$\Phi\left(\sqrt{\alpha_\eta + \alpha_s}\left(\eta_s^*(\mathbf{1}_L, 0, \lambda) - \frac{\alpha_\eta\mu_\eta + \alpha_s\left(\eta_s^*(\mathbf{1}_L, 0, \lambda) + \frac{1}{\sqrt{\alpha_s}}\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda) - \mathbf{1}_L A_L - A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right)}{\alpha_\eta + \alpha_s}\right)\right) = \frac{c_s}{R},$$

$$\text{i.e., } \Phi\left(\eta_s^*(\mathbf{1}_L, 0, \lambda)\frac{\alpha_\eta}{\sqrt{\alpha_\eta + \alpha_s}} - \frac{\alpha_\eta\mu_\eta}{\sqrt{\alpha_\eta + \alpha_s}} - \frac{\sqrt{\alpha_s}}{\sqrt{\alpha_\eta + \alpha_s}}\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L, 0, \lambda) - \mathbf{1}_L A_L - A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right) = \frac{c_s}{R}. \tag{6}$$

Taking the derivative of this relative to $\eta_s^*(\mathbf{1}_L, 0, \lambda)$ we obtain:

$$\begin{array}{c} \phi\left(\eta_s^*(\mathbf{1}_L,0,\lambda)\frac{\alpha_\eta}{\sqrt{\alpha_\eta+\alpha_s}} - \frac{\alpha_\eta\mu_\eta}{\sqrt{\alpha_\eta+\alpha_s}} - \frac{\sqrt{\alpha_s}}{\sqrt{\alpha_\eta+\alpha_s}}\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda)-\mathbf{1}_LA_L-A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right)\times \\ \left(\frac{\alpha_\eta}{\sqrt{\alpha_\eta+\alpha_s}} - \frac{\sqrt{\alpha_s}}{\sqrt{\alpha_\eta+\alpha_s}}\frac{1/A_s\gamma}{\phi\left(\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda)-\mathbf{1}_LA_L-A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right)}\right) \end{array}.$$

As $\alpha_s \to \infty$ the above expression reduces to

$$\phi\left(-\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda)-\mathbf{1}_LA_L-A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right)\left(-\frac{1/A_s\gamma}{\phi\left(\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda)-\mathbf{1}_LA_L-A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right)}\right) < 0.$$

Continuity in α_s implies that there exists an $\underline{\alpha}_{IV}(\lambda) \in \mathbb{R}_+$ such that for $\alpha \geq \underline{\alpha}_{IV}(\lambda)$, the left hand side of (6) is monotone in $\eta_s^*(\mathbf{1}_L, 0, \lambda)$. Thus there can be only one solution $\eta_s^*(\mathbf{1}_L, 0, \lambda)$. Existence of a solution can be verified by taking the limit of (6) as $\alpha_s \to \infty$:

$$\Phi\left(-\Phi^{-1}\left(\frac{\eta_s^*(\mathbf{1}_L,0,\lambda)-\mathbf{1}_LA_L-A_s(1-\gamma)\frac{\lambda}{2}}{A_s\gamma}\right)\right) = \frac{c_s}{R},$$

so that

$$\eta_s^*(\mathbf{1}_L,0,\lambda) = \mathbf{1}_LA_L + A_s\gamma\left(1-\frac{c_s}{R}\right) + A_s(1-\gamma)\frac{\lambda}{2}.$$

The proof is completed by setting $\underline{\alpha}(\lambda) := \max\left[\underline{\alpha}_{III}(\lambda), \underline{\alpha}_{IV}(\lambda)\right]$.■

Proof of Lemma 2: Define $\sigma_\eta^2 = \frac{1}{\alpha_\eta}$. The proof of existence is as follows. For $K_2^*(A_L) = \gamma\underline{k}$ the left hand side is given by

$$\gamma Pr(\eta \leq A_L)(R-c_s) = \gamma\Phi\left(\frac{A_L-\overline{A}}{\sigma_\eta}\right)(R-c_s) \underset{\sigma_\eta\to\infty}{\longrightarrow} \frac{1}{2}\gamma(R-c_s).$$

Since $\underline{k} < \frac{R-c_s}{2}$, this is bigger than the right hand side. For $K_2^*(A_L) = \gamma\overline{k}$ the left hand side is given by

$$\gamma Pr\left(\eta \leq A_L + \gamma\left(1-\frac{c_s}{R}\right)\right)(R-c_s) < \frac{1}{2}\gamma(R-c_s).$$

Since $\overline{k} > \frac{R-c_s}{2}$, the left hand side is smaller than the right hand side. Continuity then implies that there exists at least one crossing point.

The proof of uniqueness is as follows. Since $\eta \sim N\left(\overline{A}, \sigma_\eta^2\right)$, Taking the derivative with respect to $K_2^*(A_L)$ of the left hand side gives:

$$\frac{\gamma}{\overline{k}-\underline{k}} \frac{(R-c_s)^2}{R} \phi_{\overline{A},\sigma_\eta^2}\left(A_L + \gamma \frac{\frac{K_2^*(A_L)}{\gamma} - \underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right) > 0.$$

Since $\phi_{\overline{A},\sigma_\eta^2}(\cdot) < \frac{1}{\sqrt{2\pi}\sigma_\eta}$, for any given $\overline{k}$, $\underline{k}$, R, γ, and c_s, there exists a $\underline{\sigma}_\eta \in \mathbb{R}_+$ (and correspondingly, an $\overline{\alpha}_\eta \in \mathbb{R}_+$) such that if $\sigma_\eta \geq \underline{\sigma}_\eta$ (i.e., if $\alpha_\eta \leq \overline{\alpha}_\eta$) the rate of increase of the left hand side is strictly smaller than 1, the rate of increase of the right hand side. Then, the intersection point is unique.$\blacksquare$

Proof of Proposition 2: When $\alpha_\eta \leq min\left(\overline{\alpha}_\eta, \widehat{\alpha}_\eta\right)$, $K_2^*(0)$ is uniquely defined by (2) while $K_2^*(A_L)$ is uniquely defined by (1). Note first that for $A_L = 0$, (2) coincides with (1), so that

$$K_2^*(A_L)\,|_{A_L=0} = K_2^*(0).$$

Further note that the left hand side and right hand side of (1) are both increasing in $K_2^*(A_L)$ but only the left hand side is increasing in A_L. This implies that $\frac{dK_2^*(A_L)}{dA_L} > 0$, so that $K_2^*(A_L) > K_2^*(0)$.$\blacksquare$

Proof of Proposition 3: We first show that the threshold $K_2^*(A_L)$ is decreasing in $\overline{k}$ and $\underline{k}$. Consider (1) which implicitly defines $K_2^*(A_L)$. The result follows since the left hand side is decreasing in $\overline{k}$ and $\underline{k}$, and increasing in $K_2^*(A_L)$, while the right hand side is unaffected by $\overline{k}$ and $\underline{k}$.

Now note that each term in (3) is decreasing in $\overline{k}$ and $\underline{k}$, and increasing in $K_2^*(A_L)$, which in turn is decreasing in $\overline{k}$ and $\underline{k}$.$\blacksquare$

Proof of Proposition 4: We first solve for equilibrium behavior in this modified model by stating results parallel to the baseline case. It is straightforward to see that the following modification of Lemma 1 holds:

Lemma 1′. *As long as* $p_A < \frac{2(1-\delta)\underline{k}}{R-c_s}$, *the initial owners of the free float* $\bar{A}$ *are institutions that know themselves to be unskilled.*

Further, it is clear that Proposition 1 remains unmodified, since it relates solely to post-acquisition behavior.

Turning to the analysis at $t = 2$ we now let the two thresholds, $K_2^*(A_L, \Delta)$ and $K_2^*(0, \Delta)$, depend on the realization of Δ. Now, an institution will buy shares at $t = 0$ only if his *worst case* expected opportunity cost $(1+\delta)\gamma k_i$ is below the minimum $t = 2$ purchase threshold. Accordingly, we guess (and later verify) that $K_0^* \leq \min\{K_2^*(A_L, \Delta), K_2^*(0, \Delta)\}$. Following the same logic as in the main model, $K_2^*(A_L, \Delta)$ is implicitly determined by

$$\gamma \Pr\left(\eta \leq A_L + \gamma \frac{\frac{K_2^*(A_L,\Delta)}{\Delta\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)(R - c_s) = K_2^*(A_L, \Delta). \tag{7}$$

By a minor variant of the previous argument, it is easy to see that:

Lemma 2′. There exists a $\overline{\alpha}'_\eta \in \mathbb{R}_+$ such that if $\alpha_\eta \leq \overline{\alpha}'_\eta$ there is a unique solution to (7).

Similarly, $K_2^*(0, \Delta)$ is implicitly defined by:

$$\gamma \Pr\left(\eta \leq \gamma \frac{\frac{K_2^*(0,\Delta)}{\Delta\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)(R - c_s) = K_2^*(0, \Delta), \tag{8}$$

and

Lemma 3′. There exists a $\widehat{\alpha}'_\eta \in \mathbb{R}_+$ such that if $\alpha_\eta \leq \widehat{\alpha}'_\eta$ there is a unique solution to (8).

It further follows that:

Proposition 2′. $K_2^*(A_L, \Delta) > K_2^*(0, \Delta)$ for all Δ.

If L enters, the size of the activist base increases to $A_L + \frac{\frac{K_2^*(A_L,\Delta)}{\Delta\gamma} - \underline{k}}{\overline{k} - \underline{k}}$, and therefore she enters iff:

$$(1 - p_\Delta)\left[A_L \Pr\left(\eta \leq A_L + \gamma \frac{\frac{K_2^*(A_L,1-\delta)}{(1-\delta)\gamma} - \underline{k}}{\overline{k} - \underline{k}} \left(1 - \frac{c_s}{R}\right)\right)(\beta_L - c_L)\right]$$

$$+p_\Delta\left[A_L\Pr\left(\eta\le A_L+\gamma\frac{\frac{K_2^*(A_L,1+\delta)}{(1+\delta)\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)(\beta_L-c_L)\right]\ge k_L. \tag{9}$$

Finally, turning to $t=0$, K_0^* is defined by:

$$\gamma\Pr\left(\eta\le\gamma\frac{\frac{K_2^*(0,1+\delta)}{(1+\delta)\gamma}-\underline{k}}{\overline{k}-\underline{k}}\left(1-\frac{c_s}{R}\right)\right)(R-c_s)=K_0^*,$$

which has a unique solution if $\alpha_\eta\le\widehat{\alpha}'_\eta$. But notice that this condition is identical to (7) when we set $\Delta=(1+\delta)$, and thus $K_0^*=K_2^*(0,(1+\delta))$. Now note that $K_2^*(0,(1+\delta))<K_2^*(0,(1-\delta))$ is immediate, and from Proposition $2'$ we know that $K_2^*(0,\Delta)<K_2^*(A_L,\Delta)$. Thus, we have $K_0^*\le\min\{K_2^*(A_L,\Delta),K_2^*(0,\Delta)\}$ as conjectured above, completing the model solution.

Inspection of (7) and (8) reveals that $K_2^*(\cdot,\cdot)$ is decreasing in its second argument. Accordingly, regardless of whether the large activist enters, if $\Delta=1+\delta$ less small activists enter at $t=2$ than if $\Delta=1-\delta$. The resulting reduction in the mass of small activists decreases the probability of successful engagement and thus reduces P_2. In contrast, the realized value of Δ does not affect P_1. As a result, P_2-P_1 is decreasing in Δ.■

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