The Economics of Super Managers *

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Abstract

We study an agency model with a novel combination of features – agents (CEOs) differ in their ability, firms choose both the scope of the CEO's activities and their incentives, and there is free entry by firms. The outcome is an industry equilibrium in which firms are heterogenous in scope and output. That is, firms hiring more able CEOs complement higher ability with greater scope and stronger incentives, resulting in greater output. Pay has a strong "superstars" element in the sense that motivating higher ability CEOs to accept a job involving more effort and greater risk of managing greater scope, requires much greater rewards.

The model is a simple one that makes strong assumptions; this allows us to analyze it very completely and arrive at sharp conclusions. For example, we find that an increase in demand for the industry's product, e.g., a booming economy or opening of foreign economies, increases both the overall level and skewness of the cross section distribution of CEO compensation. The model suggests a variety of other empirical predictions.

Some preliminary empirical work suggests the model may prove quite useful for understanding some interesting trends in compensation. For example, our model provides an explanation for the recent increased level and dispersion in CEO compensation that is rooted in product market competition and rational board reaction to changes in the firm's environment.

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1 Introduction

The recent growth in CEO compensation, especially the dramatic increases for the top paying CEOs, have lead many to question whether CEOs have too much control over their own compensation. The academic literature on this issue is exploding, but far from reaching a consensus: while many view the large increases as a sign of CEO's abuse of power,[1] others argue that the large compensation packages can simply reflect market equilibrium where shareholders (boards) set CEO pay optimally[2]. The existing theoretical literature on CEO compensation in a market economy, however, either provides only qualitative descriptions (for example, Murphy and Zabojnik (2004a, b)) or relies on exogenous drivers such as increases in firm size to explain the observed increases in CEO compensation (as in Gabaix and Landier (2006)). Most papers in this literature ignore incentive pay, i.e., the most important component of CEO compensation.[3]

In this paper, we explore a model in which both firm size and CEO compensation are equilibrium phenomena, and the distribution of CEO incentive pay is the consequence of firms choosing incentives that are tailored to CEO skills and activities. The endogeneity of firm size and CEO compensation allows us to investigate how changes in the economic environment, e.g., an increase in product demand or an inflow of new labor, affect the distribution of firm size and CEO compensation. We are able to derive many testable implications, as well as to comment on a number of phenomena of interest, e.g., the so-called pay-for-luck phenomenon. The model also offers a straightforward comparison of the degree to which our simple model generates compensation patterns similar to those in the data.

The ingredients of the model are easy. Indeed, the originality in the paper stems from the novel combination of features, all of which have been individually explored. Firms operate in a competitive market with free entry and a limited supply of managerial talent. The output of each firm is subject to an idiosyncratic random shock to a technology having with managerial talent and effort as inputs. The impact of managerial ability on the firm's productivity is assumed to increase

[1]See, for example, Bebchuk and Fried (2003, 2004), Bertrand and Mullanaithan (2001).

[2]See, for example, Murphy and Zabojnik (2004a, b), Gabaix and Landier (2006), Kaplan and Rauh (2006).

[3]A number of papers that do examine CEO incentive pay in a market equilibrium have little implications on the distribution of incentive pay; see for example, Himmelberg and Hubbard (2000).

with firm size. In equilibrium, more talented managers work for larger (more "projects") firms and exert more effort. Higher ability makes each project more productive, but also causes the manager optimally to supervise more projects. In return, the higher ability manager receives a much larger salary and incentive pay (in dollar value), and enjoys a much higher total pay to compensate for the much greater risk. Like the familiar superstars effect, we find that firm sizes and CEO total compensation are both convex in managerial ability.

The observed increases in the level and the dispersion of CEO compensation over the last decade are very consistent with our analysis of a booming economy. Consider, for example, an increase in the demand for the industry's product. If labor supply is elastic, so that the workers' wages are not much affected by the increase in product demand (as may happen, for example, during the recovery stage of the economy), then in response to increasing demand, firms expand, with large firms growing more, and new firms enter the industry. With increased firm sizes, managers receive larger compensation packages. Increased skewness of firm sizes leads to increased skewness in managerial compensation. If, on the other hand, the supply of workers is quite limited (as may happen, for example, in the expansion stage of the economy), a positive demand shock causes wages to increase, which can offset the increase in product price. It may transpire that no new firms enter, and firm sizes remain unchanged, as does CEO incentive pay (although salary, total compensation and CEO utility grow). We conduct a similar analysis on changes in labor supply, uncertainty of future output, and the cost of managerial effort.

Our model also suggests a rational explanation for the documented pay-for-luck phenomenon (Bertrand and Mullanaithan (2001)), i.e., rewarding or penalizing managers for observable external shocks. According to the model, a positive product market demand shock increases the managers' compensation for two reasons. First, the shock increases the product price, increasing profits, which firms compete away to retain managerial talent. Second, the increased price induces managers to exert more effort, with more talented managers increasing effort even more. This makes their firms substantially more profitable. In equilibrium, managers are rewarded not only for the increased product price but also for their increased effort. Similar arguments apply to other positive shocks, e.g. an increase in labor supply or a reduction in cost of effort.

The model's simplicity allows us to calculate closed form solutions for a number of variables of interest, and so permits two types of empirical tests. The first explores to what extent our model is able to replicate the highly skewed distributions of firm size and CEO pay observed in the data. Keeping in mind that our model assumes homogeneous firms and a competitive product market, we perform our tests on a sample of *Execucomp* firms in one representative (retail) industry. We find that, even assuming a uniform (zero skewness) distribution of managerial ability, the model is able to generate cross-sectional distributions of salary and incentive pay that closely resemble those observed in the sample. For a wide range of parameters, however, the model with uniform ability produces total CEO pay and firm sizes that are substantially less skewed than the data, suggesting a compensation for the top earning CEOs that is substantially below the observed levels. Interestingly, we show that replicating the observed incentive pay requires a highly skewed ability distribution. This ability distribution generates firm sizes that are similar to those in the data, but the implied total CEO compensation is substantially more skewed than the data, suggesting compensation for the top earning CEOs substantially above the observed levels.

The second set of tests explores whether the closed form expressions that link CEO incentive pay (pay-for-performance sensitivity) to firm sizes, sales, CEO salary, and total CEO compensation fit the data. We conduct our tests using both the total sample of *Execucomp* firms (controlling for industry) and samples restricted to individual industry groups. Consistent with our predictions, we find that firm sizes, sales, CEO salary, and total CEO compensation are decreasing and extremely convex in CEO incentive pay. Additionally, we find that the functional forms suggested by the model largely outperform models that are flexible polynomials in CEO incentive pay. Restricting the sample to a specific industry group generally improves the model's performance; as would be expected from the model, it performs best when applied to homogeneous product industries, e.g., soda, entertainment, consumer goods, construction, electrical equipment, automobiles and trucks, petroleum and natural gas, communication, business services and insurance.

The paper is organized as follows. The following section reviews the related literature. Section 3 presents the basic setting of the model. Section 4 discusses the properties of the equilibrium. Section 5 analyzes separately the comparative statics with an exogenous worker's wage and an endogenous workers' wage. Section 6 proposes empirical predictions. Section 7 states empirical results and Section 8 concludes. The Appendix includes model extensions, proofs, figures and tables.

2 Related Literature

There is a growing literature theoretically examining the observed pattern in CEO compensation over the last couple of decades which has its roots in Rosen (1982) and Lucas (1978). In a recent important contribution, Murphy and Zabojnik (2004a,b) argue that the increase in managerial compensation in recent years is due to the increased importance of general managerial ability relative to firm-specific managerial capital. As a result, firms rely more on outside hiring. The increased competition in the labor market for the scarce CEO talent increases the average CEO compensation, especially for the highest-ability CEOs. Similarly to our paper, they consider an equilibrium with free entry and a competitive output market. They, however, focus on the propensity of firms to hire outside CEOs and do not offer implications on incentive compensation. It is also worth noting that, while their empirical implications are qualitative, we derive and test specific functional relationships between firm size, sales, CEO total compensation, and CEO incentive pay.

In another closely related paper, Gabaix and Landier (2006) match managerial talents to the exogenously given firm sizes. In market equilibrium, CEO pay increases one for one with the firm size. As a result, Gabaix and Landier explain the six fold increase in the CEO pay by the six fold increase in market capitalization of large US corporations between 1980 and 2003. Gabaix and Landier, however, leave open the question of whether the observed distribution of firm sizes is generated by profit-maximizing owners or powerful managers seeking rents. Additionally, they do not examine managerial incentive compensation.

Himmelberg and Hubbard (2000) link both increased CEO compensation and the raise in stock-market valuations to positive shocks in the economy. The supply of the highly talented CEOs who are able to manage the largest firms is assumed to be inelastic. This dramatically increases the marginal value of highly talented CEOs in a good economy. The importance of participation constraints for determining the top management' compensation is also stressed in Oyer (2004). The main focus of Oyer (2004), however, is to provide an explanation for the wide-spread use of option grants. In particular, Oyer argues that option grants to employees below the highest executive ranks help firms meet the participation constraint by automatically increasing (decreasing) compensation in good (bad) years without incurring the costs of adjusting contracts. Similar to Himmelberg and Hubbard (2000) and Oyer (2004), our model predicts that with positive shocks in the economy, firms expand and talented CEOs get paid more than proportionally than their peers. Our closed-form solutions and the comparative static analysis enable us to directly study the influence of exogenous shocks in product demand, profit uncertainty, and managerial risk aversion on the number of firms, firms' sizes and profits, and executive compensation.

Another strand of literature studies the influence of product market competition on executive compensation. Raith (2003) studies the relationship between managerial incentives and product market competition in an oligopolistic industry with free entry and exit. Raith shows that when the market becomes more competitive, there are fewer firms in the market, while existing firms become larger and provide stronger incentives to their managers. Aggarwal and Samwick (1999) show that strategic interactions among firms in an imperfectly competitive market generate an optimal contract lack of relative performance measure. A positive weight on both own and rival performance serves to soften detrimental competition in the industry. Observe that neither of the two product market papers models managerial talents or the link between managerial talents and managerial compensation.

The relationship between incentive pay and ability, considered in our paper, is also studied in Milbourn (2003). Milbourn (2003) argues that incentive pay is positively related to CEO ability when firm size is exogenous. The empirical analysis in Milbourn (2003) shows that, controlling for firm size (among other things), various proxies for ability such as CEO tenure, number of published articles, and CEO performance record are positively related to the CEO's incentive pay.

The idea that the observed earnings patterns can be driven by the type of production technology prevalent in the industry is also emphasized in a related paper Garicano and Hubbard (2005). They illustrate how their model implications can be used to learn about the shape of the production function from the observed data on earnings distribution and matching of workers into teams.

Empirically, Kaplan and Rauh (2006) show that the raise in CEO pay in non-financial firms from 1994 to 2004 is not higher than those of their equal talents in investment banks, hedge funds, private equity funds, and mutual funds (Wall Street); corporate lawyers; and professional athletes and celebrities. They argue the evidence is most consistent with theories of superstars, skill biased technological change, greater scale and their interaction. Bebchuk and Grinstein (2006), on the other hand, show that the expansion of firm sizes is associated with increases in subsequent CEO compensation. This evidence, however, is also in line with our model where the firm size is determined in equilibrium: highly talented managers run large firms and get paid substantially more than their less talented peers.

3 Model

3.1 Firms and agents

There are two kinds of active players in the model, "firms" and "agents", and a fixed continuum of each. Firms are all identical, and the set of firms has Lebesgue measure. Each agent has a fixed ability level, $a \in [0, \infty)$. The atomless measure μ describes the distribution of a across agents.

Each firm may elect to produce output in a market where the unit price of output (fixed, as far as the firm is concerned) is P. Nonparticipation yields zero payoff. The firm hires a single manager and chooses how many identical divisions to operate. Running these divisions requires the manager to exert effort. Specifically, if the firm hires a manager of ability a, and operates n divisions, each of which receives effort e from its manager, total output is given by

$$n(\sqrt{ae} + \varepsilon),$$

where $\varepsilon \sim N(0, \sigma^2)$ is a firm-specific random shock common to all divisions within a firm.[4] These shocks are independent across firms.

The firm chooses a salary level for the manager, s_0, and a profit share s_1. Each division requires one worker. Workers earn a wage of w. Thus, for any a, n, e, s_0, and s_1, the firm's payoff (expected profit, assuming firms are risk neutral) is

$$(1 - s_1)n(P\sqrt{ae} - w) - s_0.$$

[4]Formally, the firm operates a continuum of divisions whose measure is n.

Each agent can elect to be a worker or a manager. For simplicity, a worker's effort is normalized to zero. Managerial effort has a *unit* cost of $\frac{1}{2}c, c > 0$. Thus, a manager of ability a, employed by an n -division firm, receiving salary s_0 and profit share s_1 earns income net of effort costs equal to

$$s_0 + s_1 n \left[P(\sqrt{ae} + \varepsilon) - w\right] - \frac{1}{2}cne$$

Observe that given managerial ability and effort, a greater share of profits, more divisions, and a higher product price all expose the manager to more income risk.

For simplicity, we assume agents' utility is negative exponential with a constant absolute risk aversion $\gamma > 0$. Given the normality of the shocks to firm output, the expected utility of a manager of ability a is then

$$-\exp\left[-\gamma\left(s_0 + s_1 n(P\sqrt{ae} - w) - \frac{1}{2}cne - \frac{1}{2}\gamma s_1^2 n^2 P^2 \sigma^2\right)\right],$$

in which case the agent's choice of whether to manage or work hinges on a comparison of w with the certainty equivalent

$$s_0 + s_1 n\left(P\sqrt{ae} - w\right) - \frac{1}{2}cne - \frac{1}{2}\gamma s_1^2 n^2 P^2 \sigma^2. \tag{1}$$

The information and timing assumptions are as follows. Each agent's ability, a, is known to all at the outset. Firms decide whether to operate. Each operating firm decides, if its manager is of ability a, what salary, $s_0(a)$, and profit share, $s_1(a)$, it will offer, as well as the number of divisions it will operate, $n(a)$. Given a salary of $s_0(a)$, a profit share $s_1(a)$, and number of divisions, $n(a)$, an agent of ability a decides whether to be a worker or manager, including, if the latter is chosen, how much effort per division, $e(a)$, to expend; this choice is the manager's private information.

3.2 Equilibrium

We study two versions of the model: one where the workers' wage w is exogenous and one where w is endogenous are exogenous. The model with exogenous w implicitly assumes that the supply of agents is sufficiently abundant to accommodate the industry's need for managers and workers at wage w. While many important economic forces are similar in both versions, some comparative statics results differ. Their comparison offers interesting insights into inter-industry differences and may potentially help explain time patterns of executive compensation. In both cases we assume that firms earn zero profit, i.e., the model is one of free entry and perfect competition.

Assuming exogenous w, we have the following definition of equilibrium (in the definition, M is the set of agents choosing to be managers, and $u^m(a)$ is income net of effort costs for an agent of ability a).

Definition 1. (***Exogenous wage***)**.** *An* equilibrium *is a set* $M \subset [0, \infty)$*, a price* P*, and functions* $u^m(a)$*,* $e^*(a, s_1, n)$*,* $s_0^*(a)$*,* $s_1^*(a)$*, and* $n^*(a)$*, satisfying:*

1. *Managers optimize effort*: For any s_1, n, and $a \in M$,

$$e^*(a, s_1, n) = \arg\max_e \left\{ s_1 n P \sqrt{ae} - \frac{1}{2} cne \right\}; \tag{2}$$

2. *Agents optimize whether to work or manage*:

$$M = \{a \in [0, \infty) \mid w \le u^m(a)\};$$

3. *Firms optimize incentives*: for all a,

$$\begin{aligned} (s_0^*(a), s_1^*(a), n^*(a)) &= \arg\max_{s_0, s_1, n} \left\{ (1 - s_1) n \left[P\sqrt{ae^*(a, s_1, n)} - w \right] - s_0 \right\} \\ &\quad \text{subject to} \\ u^m(a) &\le s_0 + s_1 n \left[P\sqrt{ae^*(a, s_1, n)} - w \right] - \frac{1}{2} cne^*(a, s_1, n) - \frac{1}{2}\gamma s_1^2 n^2 P^2 \sigma^2; \end{aligned}$$

4. *All surplus goes to agents*: for all a,

$$u^m(a) = n^*(a) \left[P\sqrt{ae^*(a)} - w \right] - \frac{1}{2} cn^*(a) e^*(a) - \frac{1}{2}\gamma s_1^{*2}(a) n^{*2}(a) P^2 \sigma^2, \tag{3}$$

where $e^*(a) \equiv e^*(a, s_1^*(a), n^*(a))$; and

5. *Output market clears*: the output price P satisfies

$$\alpha - \beta P = \int_{a \in M} n^*(a) P \sqrt{a e^*(a)} d\mu(a). \tag{4}$$

Assuming an endogenous w, we have the following definition of equilibrium.

Definition 2. (***Endogenous wage***)**.** *An* equilibrium *is a set $M \subset [0, \infty)$, a price P, a wage w, and functions $u^m(a)$, $e^*(a, s_1, n)$, $s_0^*(a)$, $s_1^*(a)$, and $n^*(a)$, satisfying conditions 1 – 5 of Definition 1 and additionally satisfying:*

6. *Labor market clears:*

$$\int_{a \in M} a d\mu(a) = \int_{a \in M} n^*(a) d\mu(a), \tag{5}$$

where μ is the atomless measure that describes the distribution of a across agents.

Some things about Definitions 1 and 2 should be noted. First, managers will choose effort to maximize (1). However, changes in effort influence (1) only by altering expected incentive pay and effort costs. Thus, given that an agent has chosen to be a manager, optimal effort does not depend on salary, and the arguments of e^* are just a, s_1 and n. Second, in #3, a firm can consider offering a manager any salary, profit share and division responsibility, but must offer a combination that will attract a manager in equilibrium. Since $u^m(a)$ is required in #4 to be the equilibrium certainty equivalent for managers, #3 requires a firm's choice to be at least as attractive as $u^m(a)$. Finally, since firms earn zero profits, it follows that in equilibrium, managers receive, through a combination of salary and a share of profits, all profit. Thus, a manager's equilibrium certainty equivalent is comprised of firm profits, less the costs of effort and risk premium.

4 Characterization of Equilibrium

We first solve the choice problem of a manager with ability a and find the optimal effort per division $e^*(a)$. Next, we solve the choice problem of the firm that hires this manager, which gives us $s_0^*(a)$, $s_1^*(a)$, and $n^*(a)$. Then we show that there an ability level $\bar{a}$, such that any agent with ability $a > \bar{a}$ chooses to be a manager, any agent with ability $a < \bar{a}$ chooses to be a worker, and an agent with ability $\bar{a}$ is indifferent between becoming a worker and a manager. Although there is no closed-form solution for the threshold ability $\bar{a}$, this result allows us to obtain a detailed characterization of the equilibrium conditions of firm sizes and managerial compensation, as well as to conduct comparative statics analysis with respect to all model parameters.

Lemma 1. *In equilibrium, the manager of ability a exerts effort $e^*(a)$, works for the firm of size $n^*(a)$, receives incentive pay $s_1^*(a)$ and salary $s_0^*(a)$ that are given by*

$$e^*(a) = \frac{2w}{c}, \tag{6}$$

$$n^*(a) = \frac{a}{c\gamma\sigma^2}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right), \tag{7}$$

$$s_1^*(a) = \sqrt{\frac{2cw}{aP^2}}, \tag{8}$$

$$s_0^*(a) = \frac{aw}{c\gamma\sigma^2}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)^2\left(2 - \sqrt{\frac{2cw}{aP^2}}\right). \tag{9}$$

Proof. Using the manager's participation constraint (#4 in Definition 1), we can rewrite the firm's expected profit as

$$\pi^f(a) = Pn^*(\sqrt{ae^*} - w) - \frac{1}{2}cn^*e^* - \frac{1}{2}\gamma s_1^{*2}P^2n^{*2}\sigma^2 - u_0^m(a). \tag{10}$$

In the above expression for the firm's profit, the first term is the revenue net of labor costs, the second term is the cost of the total effort and the third term is the risk premium. The last (fourth) term in (10) is a constant which does not affect the firm's maximization problem. The firm chooses the firm size n^* and the manager's incentive pay s_1^* that trade off the marginal benefits with the marginal costs. The marginal benefit of increasing s_1^* arises from higher effort which leads to larger revenue. The marginal costs consist of an increase in the cost of effort and an increase in the required risk premium. Similarly, the marginal benefit of increasing n^* is the increase in the revenue generated by the firm. The marginal costs consist of an increase in labor expenditures (total workers' compensation), an increase in the cost of total effort and an increase in the required risk premium. The derivation details are in the Appendix. □

Using the results in Lemma 1, we further obtain closed form solutions for managerial salary, dollar incentive pay, and utility.

Lemma 2. *In equilibrium, total incentive pay* $ts_1^*(a) \equiv s_1^*(a)n^*(a)(P\sqrt{ae^*(a)} - w)$*, total compensation* $tc^*(a) \equiv s_0^*(a) + ts_1^*(a)$*, and utility* $u^m(a)$ *are given by*

$$ts_1^*(a) = \frac{aw}{c\gamma\sigma^2}\left(1 - \sqrt{\frac{2cw}{aP^2}}\right)\left(\sqrt{\frac{2aP^2}{cw}} - 1\right), \tag{11}$$

$$tc^*(a) = \frac{aw}{c\gamma\sigma^2}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)\left(\sqrt{\frac{2aP^2}{cw}} - 1\right), \tag{12}$$

$$u^m(a) = \frac{aw}{c\gamma\sigma^2}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)^2. \tag{13}$$

Given the equilibrium price P, if an agent of ability a chooses to become a manager, the agent receives the expected utility $u^m(a)$ given by (13), which is increasing in a. Thus, if an agent with ability a finds it beneficial to become a manager, so does any agent with ability higher than a. Therefore, we obtain the following result:

Lemma 3. *In equilibrium, any agent with ability* $a > \bar{a}$ *is a manager and any agent with ability* $a < \bar{a}$ *is a worker:* $M = [\bar{a}, \infty)$*, where* $\bar{a}$ *is defined by*

$$w = u^m(\bar{a}) = \frac{\bar{a}w}{c\gamma\sigma^2}\left(\sqrt{\frac{\bar{a}P^2}{2cw}} - 1\right)^2. \tag{14}$$

Theorem 1. *There exists a unique equilibrium.*

Proof. See the Appendix. □

The following theorem shows that more talented managers run firms with more divisions, exert higher total efforts, receive higher salaries, more stock-based compensation, higher total pay and utilities. Additionally, all the variables above except effort per division are convex in managerial ability. The illustration is provided in Figure 1.

Theorem 2. *Firm size* $n^*(a)$*, manager's salary* $s_0^*(a)$*, total incentive pay* $ts_1^*(a)$*, total compensation* $tc^*(a)$*, and managerial utility* $u^m(a)$ *increase in* a*; managerial incentive pay* $s_1^*(a)$ *decreases in* a*; and effort per division* $e^*(a)$ *is independent of* a*. Additionally, all these variables except for* $e^*(a)$ *are strictly convex in* a*.*

Proof. These results follow directly from (6) - (13). □

The results in Theorem 2 imply the distribution of firm sizes and managerial compensation is skewed even when there is no skewness in the distribution of managerial ability. The following section discusses how the distribution of managerial compensation and firm characteristics change with model parameters.

5 Comparative Statics

In this section, we examine how do model parameters – demand parameters α and β, managerial risk aversion γ, profit uncertainty σ^2 and wage w (if exogenous)– affect product price P, marginal ability $\bar{a}$, number of divisions $n(a)$, managerial effort $e(a)$, managerial compensation variables $s_1(a)$, $s_0(a)$, $tc(a)$, utility $u^m(a)$, and wage w (if endogenous). For brevity, we present the analysis for the cost of effort c in the Appendix because it closely resembles that for $\gamma\sigma^2$. A convenient summary of the results for both exogenous and endogenous w is offered in Table 1.[5]

The positive value of the results developed in this section is threefold. First, they provide an insights concerning pay-for-luck by analyzing the effects of observable exogenous shocks on equilibrium managerial compensation. Second, thy provide a guidance for selecting model parameters that could potentially generate managerial compensation and firm characteristics whose distribution resembles that observed in the data. Third, they lead to testable empirical predictions discussed in Section 6. In particular, the results in this section imply that our variables of interest are sensitive to the elasticity of the demand in the product market. Thus, we obtain predictions contrasting industries with different demand elasticities. Additionally, the differences in the results for exogenous and endogenous wage alow us to obtain predictions comparing industries with different labor supply elasticities or comparting different stages of the economic cycle.

5.1 Exogenous Wage

In this section, we treat the workers's wage as exogenous. Thus, all the results in this section use the equilibrium definition given by Definition 1.

[5]Given that increasing the intercept α and decreasing the slope β of the demand function affect equilibrium solutions in the same way, the table combines α and $(1/\beta)$. Additionally, because variables s_0, tc, and u^m respond to changes in parameters in a very similar fashion, the table also combines these three variables.

Lemma 4. (***Output Demand Shock***) *When the intercept of the demand function α increases (the slope β decreases), product price P, firm size n^*, (managerial salary s_0^*, total compensation tc^*, and utility u^m) increase, incentive pay s_1^* and marginal ability $\bar{a}$ decrease, while effort per division e^* does not change.*

Proof. The formal proof is in the Appendix. Not surprisingly, an upward shift (or the reduction in the slope of the demand function) in the demand for the output product is good news for firms: the output price increases, and so does the firm size. The increase in firm size leads to a reduction in managerial incentive pay (the dollar value of incentive compensation increases). The increase in output price has a positive effect on effort, while the reduction in incentive pay has a negative effect. These two effects cancel out, so that the equilibrium effort per division remains unchanged, while the total managerial efforts go up. Firm profits increase, so do managerial salary, total pay and managerial utility. Given a fixed workers' wage, more workers choose to become managers: the division ability $\bar{a}$ goes down, and new firms enter the economy. $\square$

Lemma 4 implies that the equilibrium CEO compensation has elements of both compensation for luck and compensation for performance. If the economy is good (a positive shock in the demand), all CEOs get higher compensation and higher utility, which is typically considered to indicate compensation for luck. In our model, however, part (although not all) of the increase is due to the increase in the unobservable total effort exerted by each CEO. Because the most talented CEOs increase their efforts more than their less talented peers and thus make their firms a lot more profitable, they also receive substantially higher increases in compensation.

Lemma 5. (***Workers' Wage Shock***) *When the workers' wage w increases, product price P, marginal ability $\bar{a}$, effort per division e^* and incentive pay s_1^* increase, firm size n decreases, and managerial salary s_0^*, total compensation tc^*, and utility u^m may either increase or decrease depending on the other model parameters.*

Proof. The formal proof is in the Appendix. Intuitively, when the workers' wage w increases, production becomes more costly, firms reduce their size, and some small firms exit the market ($\bar{a}$ increases), total outputs go down. Both of these effects put an upward pressure on the output price. The decrease in size leads to an increase in incentive pay. The higher incentive pay and output price both induce managers to exert higher efforts. The net effect on managerial salary, total compensation, and utility depends on the relative strength of the following two effects: the increase in revenue due to a higher effort, and the increase in cost due to a higher workers' wage. □

Lemma 6. (***Output Volatility Shock***) *When $\gamma\sigma^2$ increases, product price P increases, effort per division e^* remains unchanged, incentive pay s_1^* decreases, while marginal ability $\bar{a}$, number of divisions n^*, s_0^*, tc^*, u^m, may either increase or decrease depending on other model parameters.*

Proof. The formal proof is in the Appendix. Intuitively, if managers become more risk averse or uncertainty in output increases, managers demand more risk premium. Therefore, incentive pay s_1 becomes less effective in inducing effort: the marginal benefit of the incentive pay s_1, arising from its propensity to increase effort and thus the output, does not change; while the marginal cost of s_1 increases with the increase in the required a higher risk premium. Similarly, the increase in $\gamma\sigma^2$ reduces the net marginal benefits of the number of division n, as can be seen from the firm's objective (10). Therefore, everything else equal, firms respond to an increase in $\gamma\sigma^2$ by reducing s_1 and n.

These reductions decrease the total supply of the output product. The reduction in the supply leads to an excess demand, which puts an upward pressure on the output price P. Thus, price P increases until the excess demand disappears. The higher output price increases the marginal benefits of increasing the number of divisions n, offsetting the original desire of firms to reduce n. The net effect is negative for large firms while it might be positive for small firms. Because the net marginal benefit of incentive pay s_1 decreases with firm size, the increase in the size of large firms further decreases the incentive pay of their managers. For the small firms whose size decreases, the optimal managerial incentive pay increases. We find that this increase, however, is small and does not fully reverse the original reduction in s_1. Thus, for all firms, the equilibrium incentive pay s_1 decreases in $\gamma\sigma^2$. The equilibrium effort e does not change because the negative effect of reduced incentive s_1 is exactly offset by the positive effect of an increased output price P. With an inelastic demand function (smaller β), the increase in price may be quite substantial. The substantial increase in the product price and the increase in total output result in an increase in the total revenue of each firm. The total managerial compensation, which equals to the firm's revenue less workers' wages (fixed), thus goes up. Managerial utility goes up as well when the increase in profit dominates the increase in risk premium. The increase in managerial utility attracts some relatively high ability workers to become managers and thus pushes $\bar{a}$ downward. $\square$

5.2 Endogenous Wage

In this section, we treat the workers' wage as endogenous. Thus, all the results in this section use equilibrium definition given by Definition 2.

Lemma 7. (***Output Demand Shock***) *When the intercept of the demand function α increases (slope β decreases), price P and workers' wage w increase so that $\frac{P^2}{w}$, marginal ability $\bar{a}$, firm size n^*, and incentive pay s_1^* do not change; effort per division e^*, managerial salary s_0^*, total compensation tc^*, and utility u^m increase.*

Proof. The formal proof is in the Appendix. Intuitively, when the demand curve shifts upwards (or becomes flatter), the output price P increases. Some of the gains from the improved market conditions are captured by workers: workers' wage w also increases. For firms, the positive effect from the increase in price and the negative effect from the increase in wage cancel out: the number of firms (determined by $\bar{a}$), firm sizes n^*, and managerial ownership s_1^* remain unchanged. Due to the increase in the output price, managers have stronger incentives to exert higher total efforts, which results in an increase in managerial salary s_0^*, total compensation tc^*, and utility u^m. $\square$

The result in Lemma 7 can be interpreted as the net effect of Lemma 4 and Lemma 5. Lemma 4 discussed the effect of a positive demand shock when workers' wages are fixed. It shows that, in response to a positive demand shock, firms expand, new firms enter, and and managerial compensation increases. When wages are endogenous, however, the increased demand for labor puts an upward pressure on the workers' wage. According to Lemma 5, an increase in workers' wage leads to a decrease in firm sizes, some firms exit, and the compensation of some managers falls. Combining the effects described in these two lemmas, Lemma 7 finds that the increase in wage offsets the increase in output price so that, in equilibrium, the existing firms do not change their size and no firms either enter or exit. Nevertheless, each firm's total output and profit increase, and both managerial compensation and utility, as well as workers' wage, increase.

Lemma 8. (***Output Volatility Shock***) *When $\gamma\sigma^2$ increases, product price P increases, marginal ability $\bar{a}$, and incentive pay s_1^* decrease, managerial salary s_0^*, total compensation tc^*, and utility u^m, effort per division e^*, and workers' wage w may increase or decrease depending on the parameters. Firm size n^* decreases for high ability a (large firms), and may either decrease or increase for ability levels close to $\bar{a}$.*

Proof. The formal proof is in the Appendix. Intuitively, an increase in $\gamma\sigma^2$ reduces total outputs and increases the product price P. When the demand function is inelastic, the price effect dominates the quantity effect and firms' profits, workers' wage, managerial salary, total compensation, and utility all increase. When the demand function is elastic, the quantity effect dominates the price effect and the results reverse. The change in w has a significant effect on the equilibrium effort e^*, marginal ability $\bar{a}$, and incentive pay s_1^*. An increase in $\gamma\sigma^2$ decreases both s_1^* and $\bar{a}$ if w is endogenous while the effect is indeterminate when w is exogenous. On the other hand, the effect of increasing $\gamma\sigma^2$ on e^* depends on other parameters if w is endogenous, in contrast to a decreasing effort e^* in the case of exogenous wage w. □

When the workers' wage w is endogenously determined in the labor market, an exogenous shock in labor supply changes the workers' wage. If the change in labor supply does not affect the distribution of ability among the initial managers, then the change in labor supply is equivalent to an exogenous change in the wage that clears the labor market, as discussed in Lemma 5. Thus, we obtain the following result.

Lemma 9. (***Labor Supply Shock***) *When the supply of workers decreases, the workers' wage w increases, product price P, marginal ability $\bar{a}$, effort per division e^* and incentive pay s_1^* increase, firm size n^* decreases, and managerial salary s_0^*, total compensation tc^*, and utility u^m may either increase or decrease depending on the other model parameters.*

Proof. Follows from Lemma 5. □

6 Empirical Predictions

Our model assumes that firms have the same technology, face the same output price, profit volatility, and workers' wages, and employ managers with similar risk aversion and cost of effort. Thus, it is reasonable to think of our empirical predictions as describing firms within one fairly homogeneous industry. Additionally our model assumes a perfectly competitive product market, and is therefore less applicable to concentrated industries.

6.1 Distribution of CEO Compensation, Firm Size, and Sales

The most direct test of our model is a comparison between managerial compensation and firm sizes generated by the model and those observed in the data. The variable distributions generated by the theoretical model, however, can be fairly easily manipulated using different distributions of managerial talent. For example, to match the high skewness of CEO compensation observed in the data, we could just assume a high skewness of the underlying managerial ability. The validity of such an assumption is difficult to verify because ability is hard to measure in practice. Thus, a more interesting test of our model would show whether the model can match the observed data when the distribution of ability has no distinctive properties, as for example, uniform. Thus, our main hypothesis is

Hypothesis H1. *Given a uniform distribution of ability, our model is able to generate firm size, CEO salary, dollar incentive pay, incentive pay, and total pay with distributions similar to those observed in the data.*

Comparative statics analysis described in Section 5 can be used to guide the choice of parameters that can generate distributions with desirable characteristics. It is probably straightforward that this analysis can be used to adjust the first moments (levels) of the distributions because it tells us how the levels of firms sizes, sales, or managerial compensation packages are affected by model parameters. It is less straightforward, but still possible, to use this analysis to adjust second and third moments of the distributions as we discuss next. To that end, we first show the following lemma (that holds for both exogenous and endogenous w).

Lemma 10. *Given a uniform ability distribution $a \sim U[0,1]$, changes in model parameters that decreases $\bar{a}$ increase the skewness of managerial incentive pay $skew(s_1^*)$.*

Proof. The formal proof is in the Appendix. The proof shows that the equilibrium value of the skewness of s_1^* can be expressed as a decreasing function of $\bar{a}$, independent of other model parameters. Intuitively, when $\bar{a}$ increases, low-ability managers who earn high incentive pay s_1^* choose to become workers, which implies that small firms exit. Thus, the distribution of s_1^* becomes less skewed. □

Comparative statics results in Lemmas 4 - 8 indicate how the marginal ability $\bar{a}$ changes when firm parameters change. Combining these results with Lemma 10, we obtain the following corollary.

Corrolary 1. *Suppose that wage w is exogenous. Then, the skewness of $s_1^*(a)$ increases when the demand for output increases (α increases or β decreases), and workers' wage w decreases. When the demand is inelastic, the skewness also increases with the cost of effort c, the risk aversion γ, and the volatility σ^2. When the demand is elastic, however, the skewness decreases with c and $\gamma\sigma^2$.*

It is difficult to derive closed-form expressions for the skewness of the rest of the compensation variables, as well as firm size and sales. Similar intuition, however, applies to these variables, as our numerical simulations confirm. Specifically, we find that parameter changes that lead to lower $\bar{a}$ also tend to increase the skewness of the variables of interest.

6.2 Linking Compensation, Firm Size, and Sales to Incentive Pay

The driving force of our model, managerial ability, is difficult to measure empirically. Instead, we use the closed-form equilibrium solution to derive testable predictions on variables that are available: firm size, sales, managerial salary, incentive pay, and total compensation. Specifically, we express all other variables in terms of managerial incentive pay s_1^*. To that end we introduce the following new notation. Let

$$R^*(a) \equiv n^*(a)P\sqrt{ae^*(a)}$$

denote the total sales (revenue) of the firm that hires a manager of ability a, and let $\bar{s}_1 = s_1^*(\bar{a})$ denote the incentive pay of the marginal manager with ability $\bar{a}$. Using the new notation and the equilibrium results (7) - (12), we can express the number of divisions $n^*(a)$, sales $R^*(a)$, managerial salary $s_0^*(a)$ and total compensation $tc^*(a)$ as[6]

$$\begin{aligned} n^*(a) &= \frac{\bar{s}_1^4}{(1-\bar{s}_1)^2}\left[\frac{1-s_1^*(a)}{s_1^{*3}(a)}\right] \\ R^*(a) &= \frac{P^2\gamma\sigma^2\bar{s}_1^8}{(1-\bar{s}_1)^4}\left[\frac{1-s_1^*(a)}{s_1^{*4}(a)}\right] \\ s_0^*(a) &= \frac{P^2\gamma\sigma^2\bar{s}_1^8}{2(1-\bar{s}_1)^4}\left[\frac{(1-s_1^*(a))^2(2-s_1^*(a))}{s_1^*(a)^4}\right] \\ tc^*(a) &= \frac{P^2\gamma\sigma^2\bar{s}_1^8}{2(1-\bar{s}_1)^4}\left[\frac{(1-s_1^*(a))(2-s_1^*(a))}{s_1^*(a)^4}\right] \end{aligned}$$

The above expressions allow us to develop the following set of hypotheses.

Hypothesis H2. *Firm size n^* is proportional to $\frac{1-s_1^*}{s^{*3}_1}$, where s_1^* is CEO incentive pay.*

Hypothesis H3. *Sales R^* are proportional to $\frac{1-s_1^*}{s^{*4}_1}$.*

Hypothesis H4. *CEO salary s_0^* is proportional to $\frac{(1-s_1^*)^2(2-s_1^*)}{s_1^{*4}}$.*

Hypothesis H5. *Total CEO compensation tc^* is proportional to $\frac{(1-s_1^*)(2-s_1^*)}{s_1^{*4}}$.*

6.3 Inter-Industry and Time Series Predictions

The comparative statics analysis in Section 5 offers a number of empirical implications that apply to industries with different elasticities of product demand and labor supply, as well as to different stages of business cycle. Because testing these predictions requires detailed product and labor market information, we leave empirical tests of these predictions for future research.

Lemmas 6 and 8 indicate that an increase in output volatility increases all components of managerial compensation (s_0, ts_1, and tc) in industries with an inelastic product demand; it decreases all components of managerial compensation otherwise. Lemmas 6 and 8 also indicate that an increase in output volatility reduces the number of firms ($\bar{a}$ increases and thus some firms exit) in industries with an elastic product demand and a perfectly elastic labor supply (exogenous w). This will in turn decrease the skewness of managerial compensation according to our numerical simulations.

[6] Here are the detailed steps to derive $n^*(a)$. Let $\bar{s}_1 = \sqrt{\frac{2cw}{\bar{a}P^2}}$. By (8), we have $\frac{a}{c} = \frac{2w}{s_1^2P^2}$. Thus, we have $\frac{2w}{P^2\gamma\sigma^2} = \frac{\bar{s}_1^4}{(1-\bar{s}_1)^2}$ by (17). Therefore, we obtain $n^* = \frac{2w}{s_1^{*2}P^2}\frac{1}{\gamma\sigma^2}\frac{1-s_1^*}{s_1^*} = \frac{\bar{s}_1^4}{(1-\bar{s}_1)^2}\frac{1-s_1^*}{s_1^{*3}}$.

In industries with either an inelastic product demand or an inelastic labor supply, an increase in output volatility increases the number of firms and thus the skewness of managerial compensation. Lemma 8 indicates that in industries with an elastic product demand and an inelastic labor supply (endogenous w), an increase in output volatility increases worker's wage w. The result reverses otherwise.

The analysis of the distributions of firm size and managerial compensation across industries with different product demand and labor supply elasticities can be extended to different stages of business cycles. Consider, for example, an economy coming out of a recession. The improving economic conditions are likely to be first reflected in the product market: product demand increases and output price rises. Labor market, however, adjusts slower due to long-term (annual) contracts with workers and potential excess supply of workers (unemployed during the recession period). Thus, in the early stages (recovery economy), the exogenous wage model applies, while in the later stages (expansion economy), endogenous wage model applies. By Lemma 4, an increasing product demand in the recovery economy increases both firm size and managerial compensation, leaving the workers' wage unchanged. Thus, the gap between the workers' wage and average managerial compensation increases. Additionally, new firms enter, which in particular suggests that managerial compensation becomes more skewed. Lemma 7 indicates that an increasing product demand in the expansion economy has no further effect on firm sizes, but continues to increase managerial compensation. Additionally, because labor supply is less elastic, workers' wages start rising. At this stage, no firms enter or exit and skewness of managerial incentive pay does not change.

7 Empirical Results

7.1 Sample and Variable Selection

We obtain data on CEO compensation and firm characteristics from *ExecuComp* and *Compustat* databases. Our initial combined database contains approximately 20,000 firm-year observations and covers years 1992-2004. Because our model describes firms within one homogeneous industry, we create year and industry indicators using Fama and French 48 industry groups. Our testable hypotheses then require us to generate empirical measures for the following two groups of variables:

(1) CEO compensation: CEO salary s_0, incentive pay s_1, total compensation tc, and (2) firm characteristics: firm size n and sales R. We next describe what measures we use.

Our main proxy for firm size n is the number of employees in the firm[7] (*Employee*). Our choice is motivated by our theoretical model setup where firms of size n employ n workers. For robustness, however, we also perform our tests using two additional proxies for firm size n: *Book Value of Assets* and *Market Capitalization*. We use the Compustat variable *Sales* to measure firm sales R.

For CEO compensation variables, we use ExecuComp item *Salary* to measure managerial salary s_0 and *TDC1* to measure CEO total compensation tc. Our empirical measure *inc_pay* of CEO incentive pay (pay-for-performance sensitivity) s_1 is defined as CEO stock ownership plus CEO incentives due to stock options (new grants, un-exercisable options excluding new grants, and exercisable options) calculated using Core and Guay (1999) methodology.[8] According to this methodology, the incentives from options are measured as the delta of options times the number of options scaled by the number of shares outstanding. The delta on options is calculated using Black-Scholes formula, which requires information on exercise price and time to maturity. This information is available from ExecuComp only for newly granted options. Thus, for previously granted options (both exercisable and unexercisable), both the exercise price and time to maturity are estimated from the data. Specifically, the exercise price is estimated as the stock price minus the average "profit" per option: value of all options divided by the number of options. The time to maturity of unexercisable options is estimated as the time-to-maturity of the recent option grants (typically 10 years) minus one year (because the most typical vesting requirement is three years); while the time-to-maturity of exercisable options is estimated as the time-to-maturity of the recent option grants minus three years. If no options are granted in the most recent fiscal year, then the time-to-maturity of unexercisable and exercisable options are set equal to 9 and 6 years, respectively. More detail on this methodology can be found in Core and Guay (1999).

[7] Compustat reports either the yearly average or the number of employees at the year end. The item includes all employees of consolidated subsidiaries, both domestic and foreign, all part-time and seasonal employees, full-time equivalent employees, and company officers. It excludes consultants, contract workers, directors, and employees of unconsolidated subsidiaries.

[8] In unreported regressions, we conducted robustness tests using the following alternative measures of incentives: stock ownership and stock ownership plus incentives from option grants and bonus payments (scaled by firm equity). Results are qualitatively very similar to those presented in the paper.

After removing the observations without sufficient information for calculating CEO incentive pay, the sample size reduces to about 12,000 firm-year observations, covering 2,295 unique firms. We next remove the outliers by winsorizing our data using incentive pay *inc_pay* at 1% and 99%: we exclude an observation if its *inc_pay* is lower than the 1 percentile or higher than the 99 percentile of *inc_pay*.[9] Summary statistics for the final sample is reported in Table 2. According to this table, our sample contains large firms with the average (median) number of employees of about 20,000 (6,000) and the average (median) sales of about \$4.6 billion (\$1.3 billion). For these firms, the average CEO stock-based incentive pay is 2.9% (1.5%), of which 1.6% (1.1%) is due to stock ownership, and the average salary is \$630,000 (\$580,000).

7.2 Distribution of CEO Compensation, Firm Size, and Sales

We test Hypothesis H1 by comparing the observed CEO compensation, firm sizes, and sales with those generated by the model in terms of the mean, the ratio of the mean to the standard deviation, and skewness. Because our theoretical model assumes that firms are homogeneous accessing the same output and labor markets, the empirical analysis in this subsection narrows the original sample to firms in the retail industry (the 9^{th} in Fama and French 12 industries) in year 2002. The final sample contains 130 firms. We choose the retail industry because, first, the industry is fairly competitive, second, we have more observations for this industry than for other industries, and third, the sample statistics for the retail industry is similar to that of the majority of the 12 Fama and French industries.

To generate managerial compensation, firm size, and sales from the theoretical model, we first assume that (the unobserved) managerial ability is uniformly distributed: $a \sim U[0, 1]$. We calibrate the model by matching the characteristics of CEO incentive pay. The values of parameters chosen for the model are: $w = 0.375$, $c = 1$, $\gamma = 2$, $\sigma^2 = 73.7$. The sample statistics for the observed and the simulated data is presented in Table 3. Note that all variables are right-skewed (in line with Theorem 2 applied to uniformly distributed ability), and firm characteristics (number of employees and sales) appear to have higher skewness than compensation variables (CEO incentive pay, salary,

[9] Making the lowest 1% *inc_pay* equal to the one percentile and the highest 1% *inc_pay* equal to the 99 percentile yield similar but somewhat weakened results.

and total compensation). As shown in Table 3, the simulation captures fairly well the mean and the skewness of the observed CEO incentive pay (s_1) and CEO salary (s_0). The standard deviation of the simulated data, however, is somewhat low for the incentive pay and somewhat high for the salary. The simulation preforms noticeably less well for total CEO pay as well as for firm size (*Employee*) and sales. For these three variables, the simulated standard deviation and skewness are considerably below those observed in the data for a wide range of parameter values. This may suggest that the managerial ability is somewhat right skewed in reality.

We next infer the distribution of ability from the data as follows. Using the closed form solution for managerial incentive pay s_1 (8), we infer the managerial ability a by

$$a \sim \frac{1}{s_1^2}.$$

This inferred distribution of ability has a high skewness of 7.23 and the ratio of mean to standard deviation of 0.19. Using this inferred ability distribution, we simulate CEO salary, total compensation, firm size, and sales (by construction, the simulated CEO incentive pay perfectly matches the observed one). The comparison of the observed and the simulated data is presented in Table 4. According to Table 4, the simulation captures fairly well the skewness of the observed firm sizes (*Employee*) and sales, somewhat overstating their relative standard deviations. The simulated data, however, significantly overstates the skewness of CEO salary and total pay.[10]

7.3 Linking Compensation, Firm Size, and Sales to Incentive Pay

7.3.1 Methodology

To test Hypotheses H2 through H5, we first estimate OLS regressions with the variable of interest as the dependent variable and the functional form of *inc_pay* (our proxy for s_1) suggested in the corresponding hypothesis as the independent variable. For brevity, we refer to this model using the name of the corresponding hypothesis. For example, to test Hypotheses H2, we regress *Employee*

[10] Note that our model is static and the closed form solutions for all variables including incentive pay describe the equilibrium result in a steady state. In the dynamic real world, on the other hand, the observed incentive pay may be below the optimal level for some newly hired CEOs who need more time to accumulate the desired ownership stake. This bias in the incentive pay data causes an upward bias in the skewness of the inferred ability, which in turn increases the skewness of simulated firm sizes and total CEO compensation. Unless this bias is very large, however, our results suggest that the observed dispersion in CEO compensation is not inconsistent with the hypothesis that CEO pay is determined by rational profit-maximizing shareholders.

(our proxy for firm size n) on $\frac{1-inc\_pay}{inc\_pay^3}$ and refer to this regression as model H2. The sign and significance of the estimated coefficients then show whether the hypotheses have explanatory power. Next, we compare the performance of these regressions to the following three benchmark models: linear in s_1 (the independent variable is s_1), quadratic (the independent variables are s_1 and s_1^2), and cubic (the independent variables are s_1, s_1^2 and s_1^3). We compare model performance using adjusted R^2 (R^2_{adj}) and Schwarz information criterion (SIC). Finally, we test test the explanatory power of the functional forms suggested in H2 through H5 using nested models. Specifically, we add the functional forms suggested in the hypotheses to each of the three benchmark cases. A significant and positive coefficient on our functional forms would indicate that they have explanatory power beyond that captured by the other terms. Note that, in contrast to typical empirical studies on CEO compensation, our regressions have firm size, sales, CEO salary or CEO total compensation on the left hand side and measures of incentive pay on the right hand side. The unusual choice of specification is determined by the functional forms derived from our theoretical model. The economic implications of the tests, however, are consistent with the standard specifications.

Because our hypotheses describe homogeneous firms, all our regressions include year and industry indicators using Fama and French 48 industry groups. We also conduct robustness tests that include controls for firm age, cash availability, volatility, governance quality (GIM index of Gompers, Ishii and Metrick 2003) and CEO tenure. Additionally, we analyze the effect of alternative winsorizing, using log transformations, separating the data set into quartiles by inc_pay, and using alternative proxies for firm size.

7.3.2 Results

Hypothesis H2. Table 5 summarizes the results of regressing the number of employees (a proxy for n) on incentive pay. The coefficients of inc_pay are statistically significant at the 1% level. Model comparison using R^2_{adj} suggests model H2 is only slightly worse than the cubic model and is superior to both the linear and the quadratic models. Schwarz Information Criterion (SIC) suggests the same ranking of the four models. Table 6 reports the results obtained by estimating the nested models: we add our variable to the linear, quadratic and cubic regressions. In all three cases, the coefficient on our variable is positive and significant and adding our variable improves the R^2_{adj} of the corresponding benchmark regression.

Hypothesis H3. Table 7 reports the results of regressing sales on incentive pay. Based on both R^2_{adj} and SIC, we conclude that, similar to the test of H2, our model H3 (that uses $\frac{1-inc\_pay}{inc\_pay^4}$) is only slightly worse than the cubic model and is superior to the linear and the quadratic models. Additionally, the nested models produce positive and significant coefficients on our variable and improve the R^2_{adj} of the corresponding benchmark regressions (results omitted for brevity).

Hypothesis H4. Table 8 reports the results of regressing CEO salary on incentive pay. In this case, our model H4 (that uses $\frac{(1-inc\_pay)^2(2-inc\_pay)}{inc\_pay^4}$) is slightly outperformed by the three alternative model specifications: linear, quadratic, and cubic. Nevertheless, the nested models produce positive and significant coefficients on our variable and improve the R^2_{adj} of the corresponding benchmark regressions (results omitted for brevity).

Hypothesis H5. Table 9 reports the results of regressing total CEO compensation on CEO incentive pay. Based on both R^2_{adj} and SIC, our model H5 (that uses $\frac{(1-inc\_pay)(2-inc\_pay)}{inc\_pay^4}$) is worse than the quadratic and the cubic models but is superior to the linear model. As in the above tests, the nested models produce positive and significant coefficients on our variable and improves the R^2_{adj} of the corresponding benchmark regressions (results omitted for brevity).

7.3.3 Robustness

Analysis within Fama and French Industries. We re-estimate our models within industry groups using both Fama and French 12 and 48 industry groups. For each of the 12 Fama and French industry groups, we obtain positive and significant coefficients on the functional form suggested by H2. Additionally, our model beats the cubic model in six out of 12 industries (these six industries are consumer durables, energy, chemical, business equipment, communication, and shops). For the 48 Fama and French industry groups, we obtain positive and significant coefficients on the functional form suggested by H2 in 40 out of 48 industries[11] and our model outperforms the cubic model in 11 out of 48 industries.[12]

[11]Some of the eight industries (agriculture, books-printing and publishing, health-care, textiles, coal, boxes-shipping containers, real estate, and finance-trading) that produce negative coefficients have very few observations (for example, five observations for the coal industry and nine for real estate).

[12]These industries are soda, entertainment, consumer goods, construction, electrical equipment, automobiles and trucks, petroleum and natural gas, communication, business services, retail, and insurance.

Analysis by *inc_pay* quartiles. We sort the data into quartiles by *inc_pay* and run regressions of the number of employees on $\frac{1-inc\_pay}{inc\_pay^3}$ for each quartile separately. Results are reported in Tables 11. All regressions are statistically significant. Interestingly, firms in the top quartile of *inc_pay* have the opposite relationship (significant at the 10% level), which implies that the number of divisions increases with incentive pay for these firms. This appears to be quite contrary to the general belief as well as our model prediction that incentive pay decreases with firm size. We further look at relationship of number of employees on CEO ownership in four *inc_pay* quartiles. The coefficients of *own* show a strikingly monotonic pattern from very negative and statistically significant to positive and statistically significant when we move from the bottom quartile to the top quartile of *inc_pay*; see Table 12.

The scatter plots of the number of employees versus different transformations of incentive pay seem to confirm the reverse relationship for the top quartile of *inc_pay*; see figure 3. While the scatter plots using all samples show a pretty clear negative correlation between the number of employees and CEO incentive pay, the the correlation reverses in the plots that use only the data in the top quartile of *inc_pay*.

Sample statistics summarized in Table 10 shows that, as expected, the firms in the top quartile of *inc_pay* are smaller (as measured by the number of employees, book value of assets, market capitalization, and sales). These firms have approximately the same amount of market capitalization per employee, lower cash availability, and similar CEO salary payments. Interestingly, CEOs are in place longer while firms are noticeably younger in this quartile than in the total sample. Possibly, these firms have founder CEOs in place.[13]

Additional control variables. Our model assumes that all firms are homogeneous. Thus, we expect the performance of our model to be improved if we control for heterogeneity present in the data. We find that our model outperforms all alternative specifications including the cubic model, when volatility, cash availability, governance index, firm age, debt ratio, and CEO tenure are added to the regression as control variables.

[13]Firm age was defined as 2005 minus the first year when the firm is recorded in the Compustat database. Compustat recorded firm data starting from 1950. If we exclude observations with CEO tenure greater than 80% of firm age (more than 50% of the observations in the top quartile), then the reverse relationship in the top quartile becomes insignificant. The CEOs excluded from the sample are more likely to be founders.

Log transformation of variables. The skewness of the firm size and CEO compensation suggests that the empirical fit may be improved by using log transformations of variables. We look at the relationship between $log(employee)$ and $log(inc\_pay)$ and the one between $log(employee)$ and $log(\frac{1-inc\_pay}{inc\_pay^3})$. Based on R^2_{adj}, the model using log(inc_pay) works similar to our model (0.3834 vs. 0.3820).

We further analyze the relationship by inc_pay quartiles; Tables 13 and 14 summarize the results. Based on R^2_{adj}, the performance of our model is very similar to that using $log(inc\_pay)$, although the coefficient on $log(\frac{1-inc\_pay}{inc\_pay^3})$ is significantly less than 1, the predicted coefficient given by our model. Notice that the top inc_pay quartile still shows the opposite relationship to the other three quartiles in both regressions.

Alternative proxies for number of divisions. In testing model prediction H1, we use the number of employees as the proxy for number of divisions n^*. We find that using market capitalization or book value of assets yield similar results, although the support for our model appears to be slightly weaker.

Furthermore, we use the number of industry segments (the number of unique primary industry SIC codes, $SSIC1$ in the Compustat/Segment database) as the proxy for the number of divisions. In unreported regressions, we use both the number of business segments and the number of all segments, the specific functional relationship provided by our model works as well as the cubic model.

8 Conclusion

This paper places executive compensation into a broader context of product market and labor market equilibrium. We consider this as the first step towards examining executive compensation in a more general equilibrium model. Standard simplifying assumptions such as a Cobb-Douglas production function (with managerial effort and ability as inputs) and a linear product demand allow us to derive closed-form solutions and obtain testable empirical implications. Our result offers a possible economic rationale for the dramatic and uneven increases in CEO compensation in the recent years.

Appendix A: Model Extensions

The model analyzed in this paper considers a competitive market with homogeneous firms that have no risk diversification across divisions within a firm. These assumptions simplify the analysis, nevertheless, our model provides a fairly good fit to the data, as we discuss in Section 7. However, the model can be fairly easily extended to accommodate a possibility of differentiation among firms and diversification, as we discuss in this section. Additionally, in order to explore the robustness of our results, this section discusses how the results are affected by changes in the assumed production function and market competition structure.

Differentiated Firms

In reality, firms may differ in growth potentials among other dimensions. Our model can be modified to allow firms to be different ex ante. In equilibrium, CEO talents match the growth opportunities of firms so that the most talented CEOs manage firms with the best growth potentials.

The output per division is modified to $\sqrt{kae} + \varepsilon$ where k captures the difference in firms. The optimal number of divisions and incentive pay are

$$\begin{aligned} n^*(a) &= \frac{ka}{c\gamma\sigma^2}\left(\sqrt{\frac{kaP^2}{2cw}} - 1\right) \\ s_1^*(a) &= \sqrt{\frac{2cw}{kaP^2}} \end{aligned}$$

Compared with the solution in the basic model with homogeneous firms, this solution replace a with ka. Variable k serves a purpose that is similar to firm size in Gabaix and Landier (2006). Managerial utility $u^m(a)$ is[14]

$$u^m(a) = a^4\frac{p^2}{4\gamma c^2\sigma^2} - a^3\frac{P\sqrt{w}}{\gamma c\sigma^2\sqrt{2c}} + a^2\frac{w}{2\gamma c\sigma^2} - F,$$

where F is determined by the initial conditions: firms with the lowest type $\bar{k}$ earns zero profit and the agent with ability $\bar{a}$ earns the workers' wage, $u^m(\bar{a}) = w$.

[14]This expression was obtained by setting the derivative of the firm's expected profit with respect to a to zero at $k = a$:

$$u^{m'}(a) = \left[\frac{kaw}{\gamma c\sigma^2}\quad \sqrt{\frac{kaP^2}{2cw}} - 1\right)^2\Bigg]'_{a|k=a}.$$

and then integrating $u^{m'}(a)$ with respect to a.

This expression helps determine s_0^* and the remaining compensation variables. Not surprisingly, the ex-ante differences grant firms (except the firm with the lowest k) positive profits. The managerial compensation may become more convex even if k is not skewed because higher ability managers work for firms with higher k, which are large in equilibrium.[15]

Diversification Within Firms

In the basic model, all divisions within a firm are identical and they experience a common shock. If we assume some shocks have a idiosyncratic component that can be diversified across divisions within a firm, then this diversification effectively reduces the CEOs risk exposure. The demand for risk premium goes down, total compensation becomes more convex in ability a.

We assume the total risk a firm with n divisions is $((1-\rho)vn+\rho n^2)\sigma^2$, where v is the size of firm at which the total risk equals to $v^2\sigma^2$, which is the total risk of the firm in our basic model. Essentially, we reallocate some systematic risks to idiosyncratic risks for a firm with v divisions. We'll discuss the effect of ρ and v on the convexity of the number of divisions n^*, total compensation tc^* and managerial utility $u^m(a)$. We have

$$\begin{aligned}
\tilde{e}^*(a) &= e^*(a) = \frac{s_1^2 p^2 a}{c^2} \\
\tilde{s}_1^*(a) &= s_1^*(a) = \sqrt{\frac{2cw}{ap^2}} \\
\tilde{n}^*(a) &= \frac{1}{\rho} n^*(a) - \frac{(1-\rho)v}{2ps_1^*\rho}.
\end{aligned}$$

Thus,

$$\frac{d^2\tilde{n}^*(a)}{da^2} = \frac{1}{\rho}\frac{d^2n^*}{da^2} + \frac{(1-\rho)v}{8\rho\sqrt{2a^3cw}},$$

which decreases in ρ (equals zero when $\rho = 1$) and is greater than the convexity in the base without diversification.

Alternative Production Technologies

Some properties of the equilibrium solution hold when managerial talent and effort are substitute ($q = a + e$) or the production function is a more general Cobb-Douglas function ($q = a^{1-\eta}e^{\eta}$, where

[15] Although k can also be interpreted as capital available, introducing k, however, may raise the following question: why isn't k tradable across firms?

$0 < \eta < 1$). Specifically, firm size n and managerial utility u^m are increasing and convex in ability a; incentive pay s_1^* is decreasing and convex in a. Other properties of the solution depend on model spification, for example, the optimal effort per division e^* increases (decreases) in a if $\eta > 1/2$ ($\eta < 1/2$) with the general Cobb-Douglas production function; and it decreases in a if managerial ability and effort are perfect substitutes.

Upper Limit on Managerial Effort

Suppose that a manager can exert effort only up to a certain fixed level e_0 (there is a limited number of hours per day that managers can possibly devote to work). When this constraint is not binding, it has no effect on the solution. If this constraint binds, however, the solution to the firm's problem changes as follows. The optimal incentive pay is determined from

$$\frac{s_1^2 P^2 a}{c^2} = \frac{e_0}{n},$$

where the expression on the left is the effort per division exerted by a manager with incentive pay s_1 and ability a, and on the right is the maximum possible managerial effort per division. From the above, we obtain

$$s_1^2 = \frac{e_0 c^2}{P^2 n a}. \tag{15}$$

Substituting (15) into the firm's objective function and taking into account that $e = e_0/n$, we obtain

$$P\sqrt{nae_0} - nw - \frac{1}{2}ce_0 - \frac{1}{2a}\gamma\sigma^2 e_0 c^2 n.$$

The number of divisions n that maximizes the above objective is

$$n^*(a) = \frac{P^2 a^3 e_0}{(\gamma\sigma^2 e_0 c^2 + 2wa)^2}$$

substituting the above two conditions into (15), we obtain

$$s_1^*(a) = \frac{c(\gamma\sigma^2 e_0 c^2 + 2wa)}{P^2 a^2}.$$

Using the solution to the firm's problem, we find that the manager with ability a receives total compensation, incentive pay, salary, and utility given by

$$
\begin{aligned}
tc^*(a) =& \frac{P^2 a^2 e_0^2 \gamma \sigma^2 c^2}{(\gamma\sigma^2 e_0 c^2 + 2wa)^2};\\
ts_1^*(a) =& \frac{e_0^2 \gamma \sigma^2 c^3}{\gamma\sigma^2 e_0 c^2 + 2wa};\\
s_0^*(a) =& \frac{P^2 a^2 e_0^2 \gamma \sigma^2 c^2}{(\gamma\sigma^2 e_0 c^2 + 2wa)^2} - \frac{e_0^2 \gamma \sigma^2 c^3}{\gamma\sigma^2 e_0 c^2 + 2wa};\\
u^m(a) =& \frac{P^2 a^2 e_0^2 \gamma \sigma^2 c^2}{2(\gamma\sigma^2 e_0 c^2 + 2wa)^2} - \frac{1}{2} c e_0.
\end{aligned}
$$

Non-competitive Product Market

In the basic model, the product market is competitive and firms produce identical products. High-ability managers are important because more output is equivalent to higher firm profits. If firms in the product market strategically determine output and price, then they may constrain the outputs even if their highly talented managers can produce more. Therefore, the total compensation becomes less convex if the product market is less competitive. Notice that in monopolistic or oligopolistic markets, modelling the effect of managerial talent on the output quantity alone may miss some important factors in the game.

Appendix B: Proofs

Many of the proofs in the appendix use the equilibrium conditions of the output market (4) and the indifference condition of the marginal agent (14). Thus, we first rewrite these conditions using the equilibrium values for firm size, managerial compensation, and effort:

$$
H \equiv -\alpha + \beta P + \int_{\bar{a}}^{\infty} \frac{a}{c\gamma\sigma^2} \left(\sqrt{\frac{aP^2}{2cw}} - 1 \right) \sqrt{\frac{2wa}{c}} d\mu(a) = 0, \tag{16}
$$

$$
G \equiv \frac{\bar{a}}{c\gamma\sigma^2} \left(\sqrt{\frac{\bar{a}P^2}{2cw}} - 1 \right)^2 - 1 = 0, \tag{17}
$$

where G was obtained by dividing (14) by w.

Proof of Theorem 1.

Proof. When the workers' wage is exogenous, we have shown that, given product price P and marginal ability level $\bar{a}$, there is a unique solution for all other variables: n^*, s_0^*, s_1^*, and e^*. Claims

1 and 2 proven in the Appendix show that there is a unique pair of $\bar{a}$ and P that solve the clearance condition of the product market (16) and the indifference condition of the agent with ability $\bar{a}$ (17). Thus, the equilibrium is unique.

It is left to show that Theorem 1 holds when the workers' wage is endogenous. From (8)-(13), there is a unique solution for n, s_0, s_1, e, u^m, and tc given w, P, and $\bar{a}$. Therefore, it remains to show that there is unique $(w, P, \bar{a})$ that satisfies the product market clearing condition (16), the reservation utility of the marginal manager (17), and the labor market clearing condition (5).

Suppose there are multiple equilibria in this model. Consider two distinct equilibria characterized by (w_1, P_1, $\bar{a}_1$) and (w_2, P_2, $\bar{a}_2$). Without loss of generality, we assume that $w_2 > w_1$. Lemmas 1 and 2 show that, given w, there is a unique pair of $\bar{a}$ and P that solve (16) and (17). Lemma 5 implies that $\bar{a}_2 > \bar{a}_1$ and thus $\int_0^{\bar{a}_2} d\mu(a) > \int_0^{\bar{a}_1} d\mu(a)$. Because $\bar{a}_2 > \bar{a}_1$, (17) implies that $\frac{P_2^2}{w_2} < \frac{P_1^2}{w_1}$, and thus, $\int_{\bar{a}_2}^{\infty} n(a)d\mu(a) < \int_{\bar{a}_1}^{\infty} n(a)d\mu(a)$. By (5), we have $\int_{\bar{a}}^{\infty} n(a)d\mu(a) = \int_0^{\bar{a}} d\mu(a)$. Therefore, either ($w_1$, P_1, $\bar{a}_1$) or (w_2, P_2, $\bar{a}_2$) violates the equilibrium condition (16). □

Claim 1. *For each $\bar{a}$ there is a unique output price P that solves (16). This price increases in $\bar{a}$.*

Proof. From (16), note that when $P = 0$ we have $F < 0$, and when $P = +\infty$ we have $F = +\infty$. Because $\frac{\partial H}{\partial P} > 0$, there exists a unique solution for P given $\bar{a}$. Because H decreases with $\bar{a}$: $\frac{\partial H}{\partial \bar{a}} < 0$, the output price P that solves (16) increases in $\bar{a}$. □

Claim 2. *For each P there is a unique $\bar{a}$ that solves (17). This $\bar{a}$ decreases in P.*

Proof. From (17), note that when $\bar{a} = 0$ we have $G = -1 < 0$, and when $\bar{a} = +\infty$ we have $G = +\infty$. Because $\frac{\partial G}{\partial \bar{a}} > 0$, there exists a unique solution for $\bar{a}$ given P. Additionally, because G also increases with P: $\frac{\partial G}{\partial P} > 0$, the value of $\bar{a}$ that solves (17) decreases in P. □

Proof of Theorem 2.

Proof. The first order condition of (2) with respect to e yields

$$e^*(a) = \frac{as_1^2P^2}{c^2}. \tag{18}$$

Plugging the above expression for e^* into (2) and using the binding participation constraint of the manager (3), we have the firm's expected profit as

$$\pi^f(a) = -u^m(a) - nw + \frac{nas_1P^2}{c} - \frac{nas_1^2P^2}{2c} - \frac{1}{2}\gamma\sigma^2 s_1^2 n^2 P^2.$$

The first-order conditions with respect to s_1 and n are

$$\begin{aligned} \frac{\partial \pi^f(a)}{\partial s_1} &= \frac{naP^2}{c} - \frac{nas_1P^2}{c} - \gamma\sigma^2 s_1 n^2 P^2 \\ \frac{\partial \pi^f(a)}{\partial n} &= \frac{as_1P^2}{c} - w - \frac{1}{2}\frac{as_1^2P^2}{c} - \gamma\sigma^2 s_1^2 n P^2, \end{aligned} \tag{19}$$

Solving the above two equations for the equilibrium n^* and s_1^* shows that these variables satisfy (7) (8) correspondingly. Substituting the above expressions for s_1^* and n^* into effort (18), we obtain e^* as in (6). The firm's zero-profit condition (3) yields the managerial salary s_0^* as in (9). □

Proof of Lemma 4.

Proof. Using (16) and (17), the derivatives $\frac{dP}{d\alpha}$ and $\frac{d\bar{a}}{d\alpha}$ can be found from the following system of equations:

$$\begin{aligned} \frac{\partial H}{\partial \alpha} + \frac{\partial H}{\partial \bar{a}}\frac{d\bar{a}}{d\alpha} + \frac{\partial H}{\partial P}\frac{dP}{d\alpha} &= 0 \\ \frac{\partial G}{\partial \alpha} + \frac{\partial G}{\partial \bar{a}}\frac{d\bar{a}}{d\alpha} + \frac{\partial G}{\partial P}\frac{dP}{d\alpha} &= 0 \end{aligned}$$

Because the determinant

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial P} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial P} \end{pmatrix} < 0,$$

the sign of $\frac{dP}{d\alpha}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial \alpha} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial \alpha} \end{pmatrix} > 0,$$

and the sign $\frac{d\bar{a}}{d\alpha}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \alpha} & \frac{\partial H}{\partial P} \\ \frac{\partial G}{\partial \alpha} & \frac{\partial G}{\partial P} \end{pmatrix} < 0.$$

Similarly, the sign of $\frac{d\bar{P}}{d\beta}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial \beta} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial \beta} \end{pmatrix} < 0,$$

and the sign $\frac{d\bar{a}}{d\beta}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \beta} & \frac{\partial H}{\partial P} \\ \frac{\partial G}{\partial \beta} & \frac{\partial G}{\partial P} \end{pmatrix} > 0.$$

The direction of the change in n, s_1, s_0, tc, u^m, and e follow directly from the equilibrium expressions (8)-(13) because price P decreases with β and everything else in these expressions remains constant.

□

Proof of Lemma 5.

Proof. As in the proof of Lemma 4, using (16) and (17) we obtain that the sign of $\frac{dP}{dw}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial w} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial w} \end{pmatrix} > 0,$$

and the sign $\frac{d\bar{a}}{dw}$ is the same as the sign of

$$\det\begin{pmatrix} \frac{\partial H}{\partial w} & \frac{\partial H}{\partial P} \\ \frac{\partial G}{\partial w} & \frac{\partial G}{\partial P} \end{pmatrix}.$$

Substituting for the partial derivatives from (16) and (17), we obtain that the above determinant is positive, and thus $\frac{d\bar{a}}{dw} > 0$. The positive effect of w on e follows (6). Because $\bar{a}$ increases in w, (17) implies that P^2/w decreases in w. Thus, s_1 increases in w by (8) while n decreases in w by (7).

Assume $\alpha = 1$; $a \sim U[0,1]$; $\gamma = 10$; and $\sigma^2 = 0.25$. When $c = 1$ and $\beta = 0.25$, increasing wage w from 0.1 to 0.11 increases $\bar{a}$ from 0.3434 to 0.3609, decreases the salary $s_0(1)$ from 2.0775 to 2.0548 and $s_0(0.3609)$ (the new $\bar{a}$) from 0.1953 to 0.1897. When $c = 0.1$ and $\beta = 0.25$, increasing wage w from 0.1 to 0.11 increases $\bar{a}$ from 0.7229 to 0.7342, increases $s_0(1)$ from 0.4411 to 0.4618 and $s_0(0.7342)$ (the new $\bar{a}$) from 0.1456 to 0.1505. □

Proof of Lemma 6.

Proof. Let $g = \gamma\sigma^2$. Using (16) and (17), the derivative $\frac{dP}{dg}$ can be derived from the following system of equations:

$$\frac{\partial H}{\partial g} + \frac{\partial H}{\partial \bar{a}}\frac{d\bar{a}}{dg} + \frac{\partial H}{\partial P}\frac{dP}{dg} = 0$$
$$\frac{\partial G}{\partial g} + \frac{\partial G}{\partial \bar{a}}\frac{d\bar{a}}{dg} + \frac{\partial G}{\partial P}\frac{dP}{dg} = 0$$

Because the determinant

$$\det \begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial P} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial P} \end{pmatrix} < 0,$$

the sign of $\frac{dP}{dg}$ is the same as the sign of

$$\det \begin{pmatrix} \frac{\partial H}{\partial \bar{a}} & \frac{\partial H}{\partial g} \\ \frac{\partial G}{\partial \bar{a}} & \frac{\partial G}{\partial g} \end{pmatrix} > 0.$$

Effort e is independent of g from (6). Incentive pay s_1 decreases with γ by (8) because price P increases while everything else does not change. The rest of the variables may either increase or decrease, as illustrated in the following numerical examples.

Assume $c = 1$; $w = 0.1$; $\alpha = 1$; $a \sim U[0, 1]$; $\sigma^2 = 0.25$. When $\beta = 0.05$, increasing γ from 10 to 11 decreases $\bar{a}$ from 0.1504 to 0.1465, decreases $n(1)$ from 4.8377 to 4.7032 and $n(0.1504)$ (the initial $\bar{a}$) from 0.2452 to 0.2408; it increases $s_0(1)$ from 11.2547 to 11.7295 and $s_0(0.1504)$ from 0.1803 to 0.1925.

When $\beta = 0.25$, increasing γ from 10 to 11 increases $\bar{a}$ from 0.3434 to 0.347, decreases $n(1)$ from 2.1243 to 1.9913 and $n(0.347)$ (the new $\bar{a}$) from 0.3772 to 0.3552; it decreases $s_0(1)$ from 2.0775 to 2.0125 and $s_0(0.347)$ from 0.1774 to 0.1738.

In the numerical examples above, firm size $n(a)$ decreases with γ. This, however, is not always the case. When $\beta = 0.001$, increasing γ from 10 to 11 reduces $\bar{a}$ from 0.1069 to 0.1014; it decreases $n(1)$ from 6.7416 to 6.7284 while increasing $n(0.1069)$ (the initial $\bar{a}$) from 0.2068 to 0.2071. $\square$

Proof of Lemma 7.

Proof. Suppose that, given α_0, the equilibrium price is P_0, wage is w_0 and marginal ability is $\bar{a}_0$. Consider $\alpha_1 > \alpha_0$ and let $\sqrt{w_1} = \frac{\alpha_1}{\alpha_0}\sqrt{w_0}$, $P_1 = \frac{\alpha_1}{\alpha_0}P_0$, and $\bar{a}_1 = \bar{a}_0$. We will show that price P_1, wage w_1 and marginal ability $\bar{a}_1$ form an equilibrium given α_1 by checking that these variables satisfy the output market and labor market equilibrium conditions (4) and (5), as well as the marginal manager's reservation wage condition (14). The labor market equilibrium (5) follows from $\bar{a}_1 = \bar{a}_0$ and $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$. The equality $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$ also implies that the reservation wage constraint (14) holds. The output market equilibrium condition can be verified as follows. Divide (4) by $\sqrt{w}$ and observe that $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$ and $\frac{\alpha_1}{\sqrt{w_1}} = \frac{\alpha_0}{\sqrt{w_0}}$.

Suppose that, given β_0, the equilibrium price is P_0, wage is w_0 and marginal ability is $\bar{a}_0$. Consider $\beta_1 > \beta_0$. Define w_1 as follows:

$$\alpha\left(\frac{1}{\sqrt{w_1}} - \frac{1}{\sqrt{w_0}}\right) = \frac{\beta_1 P_1}{\sqrt{w_1}} - \frac{\beta_0 P_0}{\sqrt{w_0}}, \tag{20}$$

and let $P_1 = P_0\frac{\sqrt{w_1}}{\sqrt{w_0}}$, and $\bar{a}_1 = \bar{a}_0$. We will show that price P_1, wage w_1 and marginal ability $\bar{a}_1$ form an equilibrium given β_1 by checking equilibrium conditions (16), (17), and (5). The labor market equilibrium (5) follows from $\bar{a}_1 = \bar{a}_0$ and $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$. The equality $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$ also implies that the reservation wage constraint (17) holds. The output market equilibrium condition can be verified by dividing (16) by $\sqrt{w}$ and using (20) along with $\frac{P_1^2}{w_1} = \frac{P_0^2}{w_0}$.

The direction of the change in n, s_1, s_0, tc, u^m, and e follow directly from the equilibrium expressions (8)-(13) because price P increases with α (decreases with β) and everything else in these expressions remains constant. $\square$

Proof of Lemma 8.

Proof. Let $g = \gamma\sigma^2$ and $x \equiv \sqrt{\frac{P^2}{w}}$. Then the equilibrium conditions (5), (16), and (17) can be rewritten in terms of $w, x, \bar{a}$ as follows:

$$L_1 \equiv -\mu(\bar{a}) + \int_{\bar{a}}^{\infty} \frac{a}{cg}\left(\sqrt{\frac{a}{2c}}x - 1\right) d\mu(a) = 0 \tag{21}$$

$$H_1 \equiv -\frac{\alpha}{\sqrt{w}} + \beta x + \int_{\bar{a}}^{\infty} \frac{a}{cg}\left(\sqrt{\frac{a}{2c}}x - 1\right)\sqrt{\frac{2a}{c}} d\mu(a) = 0 \tag{22}$$

$$G_1 \equiv \frac{\bar{a}}{cg}\left(\sqrt{\frac{\bar{a}}{2c}}x - 1\right)^2 - 1 = 0. \tag{23}$$

The derivatives $\frac{d\bar{a}}{dg}$ and $\frac{dx}{dg}$ can be found by solving the following system of equations:

$$\begin{aligned}
\frac{\partial H_1}{\partial g} + \frac{\partial H_1}{\partial \bar{a}}\frac{d\bar{a}}{dg} + \frac{\partial H_1}{\partial x}\frac{dx}{dg} + \frac{\partial H_1}{\partial w}\frac{dw}{dg} &= 0\\
\frac{\partial G_1}{\partial g} + \frac{\partial G_1}{\partial \bar{a}}\frac{d\bar{a}}{dg} + \frac{\partial G_1}{\partial x}\frac{dx}{dg} + \frac{\partial G_1}{\partial w}\frac{dw}{dg} &= 0\\
\frac{\partial L_1}{\partial g} + \frac{\partial L_1}{\partial \bar{a}}\frac{d\bar{a}}{dg} + \frac{\partial L_1}{\partial x}\frac{dx}{dg} + \frac{\partial L_1}{\partial w}\frac{dw}{dg} &= 0
\end{aligned}$$

Note that $\frac{\partial G_1}{\partial w} = \frac{\partial L_1}{\partial w} = 0$. Therefore, the determinant

$$\det\begin{pmatrix} \frac{\partial H_1}{\partial \bar{a}} & \frac{\partial H_1}{\partial x} & \frac{\partial H_1}{\partial w} \\ \frac{\partial G_1}{\partial \bar{a}} & \frac{\partial G_1}{\partial x} & \frac{\partial G_1}{\partial w} \\ \frac{\partial L_1}{\partial \bar{a}} & \frac{\partial L_1}{\partial x} & \frac{\partial L_1}{\partial w} \end{pmatrix} = \frac{\partial H_1}{\partial w}\det\begin{pmatrix} \frac{\partial G_1}{\partial \bar{a}} & \frac{\partial G_1}{\partial x} \\ \frac{\partial L_1}{\partial \bar{a}} & \frac{\partial L_1}{\partial x} \end{pmatrix} > 0.$$

The sign of $\frac{d\bar{a}}{dg}$ is the same as the sign of

$$-\det\begin{pmatrix} \frac{\partial H_1}{\partial g} & \frac{\partial H_1}{\partial x} & \frac{\partial H_1}{\partial w} \\ \frac{\partial G_1}{\partial g} & \frac{\partial G_1}{\partial x} & \frac{\partial G_1}{\partial w} \\ \frac{\partial L_1}{\partial g} & \frac{\partial L_1}{\partial x} & \frac{\partial L_1}{\partial w} \end{pmatrix} = \frac{\partial H_1}{\partial w}\det\begin{pmatrix} \frac{\partial G_1}{\partial x} & \frac{\partial G_1}{\partial g} \\ \frac{\partial L_1}{\partial x} & \frac{\partial L_1}{\partial g} \end{pmatrix},$$

The sign $\frac{d\bar{a}}{dg} < 0$ follows from

$$\frac{\partial G_1}{\partial x}\frac{\partial L_1}{\partial g} - \frac{\partial G_1}{\partial g}\frac{\partial L_1}{\partial x} \le -\frac{\bar{a}\sqrt{\bar{a}}}{\sqrt{2c}c^2g^3}\left(\sqrt{\frac{\bar{a}}{2c}}x - 1\right)\int_{\bar{a}}^{\infty} a\left(\sqrt{\frac{a}{2c}}x - 1\right)d\mu(a) < 0. \tag{24}$$

Similarly, $\frac{dx}{dg} > 0$ follows from

$$\det\begin{pmatrix} \frac{\partial G_1}{\partial g} & \frac{\partial G_1}{\partial \bar{a}} \\ \frac{\partial L_1}{\partial g} & \frac{\partial L_1}{\partial \bar{a}} \end{pmatrix} > 0.$$

If w increases in g, then $\frac{dx}{dg} > 0$ implies that $\frac{dP}{dg} > 0$. It remains to show that $\frac{dP}{dg} > 0$ if w decreases in g. Taking the derivative of (22) with respect to g, we obtain

$$\begin{aligned} \frac{d}{dg}\left(\frac{\alpha - \beta P}{w}\right) =& \frac{d}{dg}\left[\int_{\bar{a}}^{\infty}\frac{a}{cg}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)\sqrt{\frac{2a}{c}}d\mu(a)\right] \\ =& -\frac{\bar{a}}{cg}\left(\sqrt{\frac{\bar{a}P^2}{2cw}} - 1\right)\sqrt{\frac{2\bar{a}}{c}}\frac{d\mu(\bar{a})}{dg} + \int_{\bar{a}}^{\infty}\frac{d}{dg}\left[\frac{a}{cg}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)\right]\sqrt{\frac{2a}{c}}d\mu(a) \\ \le& \sqrt{\frac{2\bar{a}}{c}}\left[-\frac{\bar{a}}{cg}\left(\sqrt{\frac{\bar{a}P^2}{2cw}} - 1\right)\frac{d\mu(\bar{a})}{dg} + \int_{\bar{a}}^{\infty}\frac{d}{dg}\left[\frac{a}{cg}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)\right]d\mu(a)\right] \\ =& \sqrt{\frac{2\bar{a}}{c}}\left[\frac{d}{dg}\left(\int_{\bar{a}}^{\infty}\frac{a}{cg}\left(\sqrt{\frac{aP^2}{2cw}} - 1\right)d\mu(a)\right)\right] \\ =& \sqrt{\frac{2\bar{a}}{c}}\left[\frac{d\mu(\bar{a})}{dg}\right] \\ <& 0. \end{aligned} \tag{25}$$

Because $\frac{d}{dg}\left(\frac{\alpha-\beta P}{w}\right) < 0$, $\alpha - \beta P > 0$, and $\frac{dw}{dg} < 0$, we obtain $\frac{dP}{dg} > 0$.

Incentive pay $s_1(a)$ decreases with g from (8) because $\frac{dx}{dg} > 0$. We next consider $\frac{dn}{dg}$. From (7),

$$\frac{dn(a)}{dg} = \frac{a}{cg^2} + \frac{a\sqrt{a}}{cg\sqrt{2c}}\left(\frac{d}{dg}\left[\frac{P}{\sqrt{w}}\right] - \frac{P}{g\sqrt{w}}\right). \tag{26}$$

We have $\frac{d}{dg}\left(\frac{P}{\sqrt{w}}\right) - \frac{P}{g\sqrt{w}} < 0$; otherwise (26) would be positive for any a, which violates (23). Therefore, $\frac{dn}{dg}$ decreases in a and there exists a_1 such that $\frac{dn(a)}{dg} < 0$ for $a > a_1$ and $\frac{dn(a)}{dg} > 0$ for $\bar{a} < a < a_1$. Using straight forward but tedious calculations, we can show that for a sufficiently large, firm size $n(a)$ decreases with $\gamma\sigma^2$; for a close to $\bar{a}$, firm size $n(a)$ may either increase or decrease.

The following numerical examples show that the equilibrium wage w, effort per division e, and managerial wage s_0 may increase or decrease with g depending on the other parameters of the model. By (6), effort e changes in the same direction as w. Assume $\alpha = 1$, $a \sim U[0, 1]$, $c = 1$, and $\sigma^2 = 0.25$. When $\beta = 5$, w, e, and s_0 decrease with γ: if $\gamma = 2$ then $w = 0.0033$, $e = 0.0067$, and $s_0(0.6502) = 0.0049$ (at the initial $\bar{a}$); if $\gamma = 2.2$ then $w = 0.0031$, $e = 0.0063$, and $s_0(0.6502) = 0.00488$. When $\beta = 0.01$, w, e, and s_0 increase with γ: if $\gamma = 2$ then $w = 1.2954$, $e = 2.5908$, and $s_0(0.6502) = 1.9006$; if $\gamma = 2.2$ then $w = 1.3366$, $e = 2.6731$ and $s_0(0.6502) = 2.0804$.

□

Proof of Lemma 10.

Proof. When the distribution of ability is uniform, for $s_1^*(1) \leq s_1^* \leq s_1^*(\bar{a})$, we can obtain the cumulative distribution function for the managerial incentive pay using (8):

$$Pr\{s_1^*(a) \leq s_1^* | a \geq \bar{a}\} = Pr\left\{\sqrt{\frac{2cw}{aP^2}} \leq s_1^*\right\} \frac{1}{1-\bar{a}} = \left(1 - \frac{2cw}{P^2 s_1^{*2}}\right) \frac{1}{1-\bar{a}}.$$

The skewness of the above distribution can be expressed as

$$\text{skew}(s_1^*) = \frac{\sqrt{2}(1-\sqrt{\bar{a}})\left[(1-\bar{a})^2 + 3\sqrt{\bar{a}}(1-\bar{a})\ln\bar{a} + 8\sqrt{\bar{a}}(1-\sqrt{\bar{a}})^2\right]}{\sqrt{\bar{a}}[(\bar{a}-1)\ln\bar{a} - 4(1-\sqrt{\bar{a}})^2]^{3/2}}.$$

This skewness monotonically decreases with $\bar{a}$: from infinity when $\bar{a} = 0$ to zero when $\bar{a} = 1$. □

Lemma 11. *With an exogenous workers' wage, when the unit cost of effort c increases, price P increases, effort per division e^* decreases, while the rest of the variables, $\bar{a}$, n^*, s_1^*, s_0^*, tc^*, u^m, may either increase or decrease depending on the values of other model parameters.*

The proof and economic intuitions (similar to those of Lemma 6) are omitted but available upon request.

Lemma 12. *With an endogenous workers' wage, when the unit cost of effort c increases, price P increases, marginal ability $\bar{a}$ and incentive pay s_1^* decrease, managerial salary s_0^*, total compensation tc^*, utility u^m, effort per division e^*, and workers' wage w may increase or decrease depending on the parameters. Firm size n^* decreases for sufficiently high ability levels a, and may either decrease or increase for ability levels close to $\bar{a}$.*

The proof and economic intuitions (similar to those of Lemma 8) are omitted here but available upon request.

Appendix C: Figures and Tables

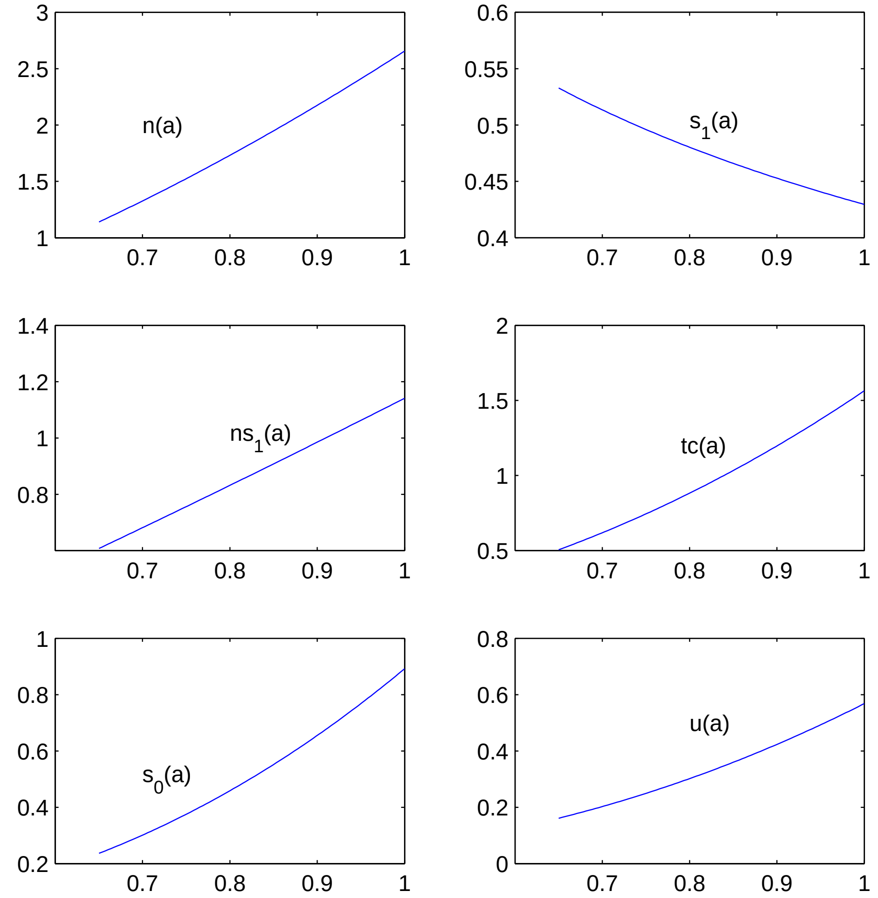
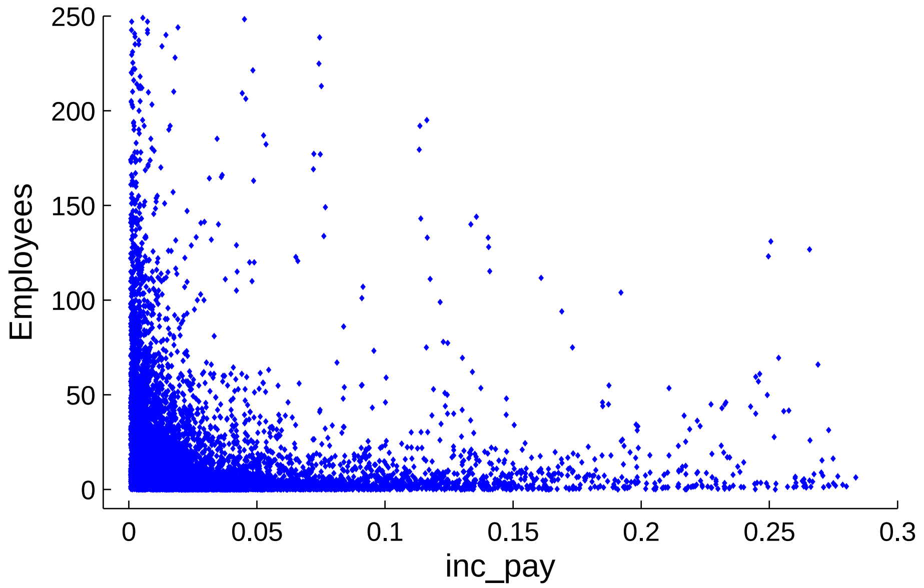


Figure 1: The horizontal axis depicts the managerial ability while the vertical axis depicts the number of divisions n, incentive pay s_1, ns_1, total compensation tc, managerial salary s_0, and managerial utility u^m. The graphs assume $c = 1$, $\alpha = 1$, $\beta = 0.5$, $\gamma = 2$, $\sigma^2 = 0.25$, and $a \sim U[0, 1]$. In equilibrium, $\bar{a} = 0.65$, $w = 0.16$, and $P = 1.32$.



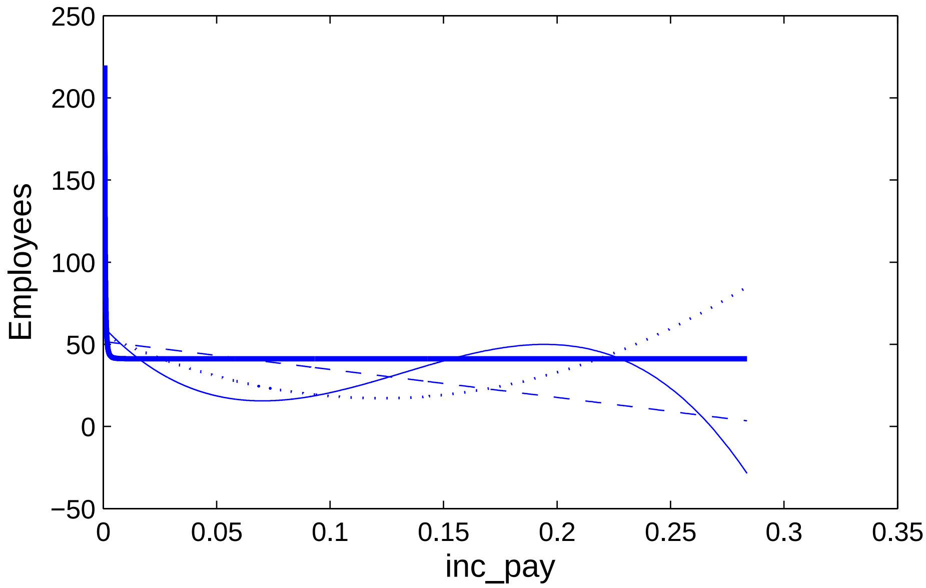


Figure 2: Predictions of polynomial models and our model.
The top figure depicts the observed scatter plot of the number of employees and *inc_pay* (only observations with the number of employees under 250 thousand are displayed for ease of comparison). The lower figure shows the prediction of the four models (linear-dashes, quadratic-dotted, cubic-solid, and ours-thick solid) based on the estimates reported in Table 5. Notice that the regressions reported in Table 5 also contain industry and year indicators. This figure, however, uses only the estimated intercept and the coefficient on variables related to *inc_pay*.

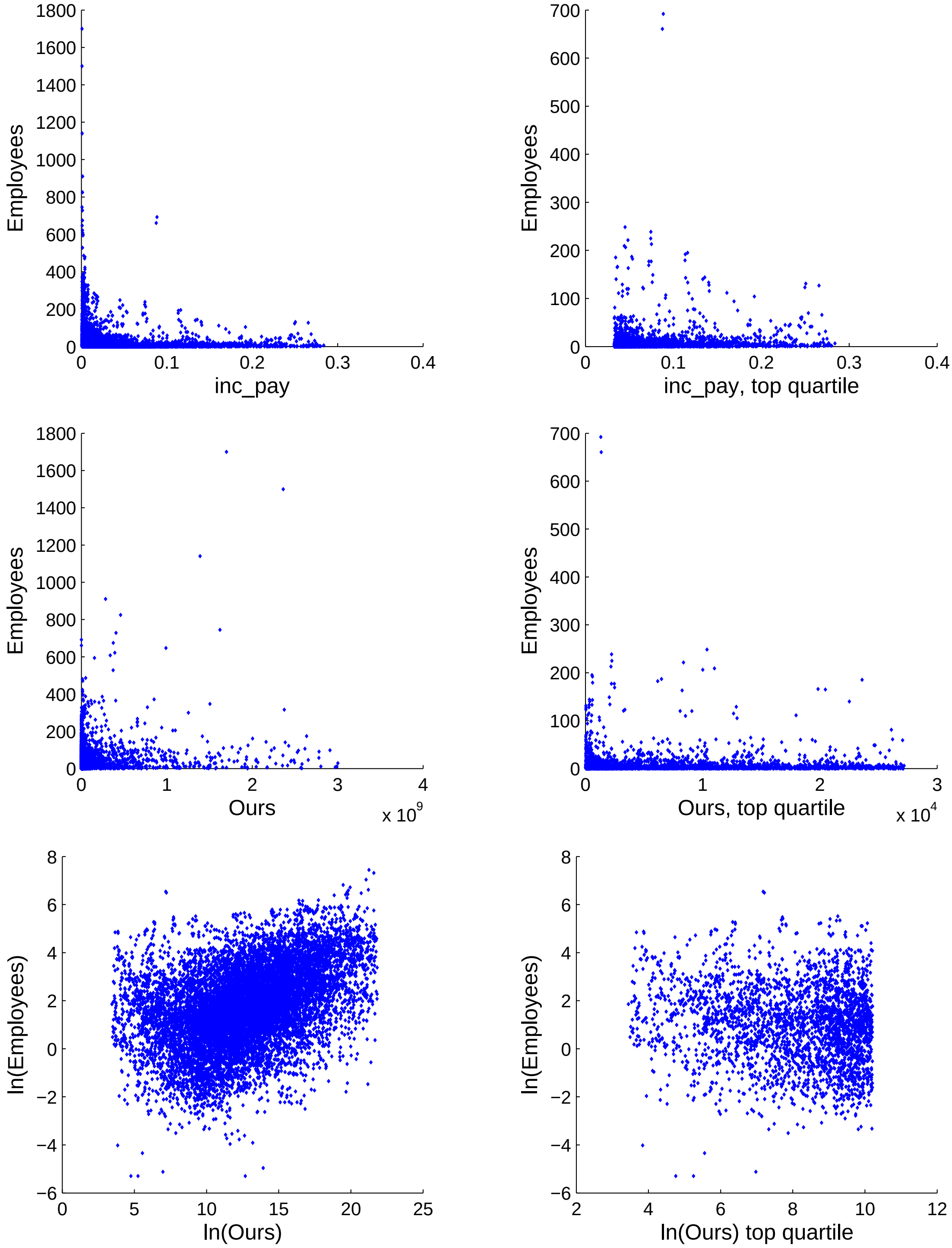


Figure 3: Scatter plots of the number of employees (vertical axes) vs. the incentive pay (horizontal axes). The top two plots show the number of employees vs. *inc_pay*, the next two show the number of employees vs. our variable $\frac{1-inc\_pay}{inc\_pay^3}$, and the bottom two show the natural log of the number of employees vs. the natural log of our variable. The three plots on the left use all observations in the sample, while the three plots on the right use only the observations in the top quartile of *inc_pay*.

Table 1: Comparative Statics Summary.
Arrows pointing upwards (↑) or downwards (↓) indicate whether the variables are increasing or decreasing in the corresponding parameters; 0 means that the variable does not change with the corresponding parameter. Star * indicates that, according to our numerical examples, the direction of the change holds for an inelastic demand but reverses if the demand is elastic; † means that the direction holds for low ability levels a, but may reverse for high ability levels. tc_1 is the total compensation. The lemmas in section **??** provide more detail and the Appendix offers proofs and numerical examples. Note that we have listed effort e under economy characteristics for two reasons: first, in our model, e is the same across all firms; second, all the variables listed as firm-specific are measurable, while those listed as economy characteristics are difficult to measure. c: unit cost of effort, γ: risk aversion, σ^2: uncertainty in profit, α and β: intercept and slope of the demand function. n: number of divisions, s_1: incentive pay, s_0: salary, ts_1: dollar value of total incentive compensation, u: managerial utility, P: product price, $\bar{a}$: marginal ability, e: effort per division, w: workers' wage.

	wage exogenous				wage endogenous		
Firm-specific Variables (a)	$\alpha(\frac{1}{\beta})$	w	$\gamma\sigma^2$	c	$\alpha(\frac{1}{\beta})$	$\gamma\sigma^2$	c
n	↑	↓	↑†	↑†	0	↑†	↑†
s_1	↓	↑	↓	↓*	0	↓	↓
s_0, tc_1, u	↑	↑*	↑*	↑*	↑	↑*	↑*
Economy Characteristics							
P	↑	↑	↑	↑	↑	↑	↑
$\bar{a}$	↓	↑	↓*	↓*	0	↓	↓
e	0	↑	0	↓	↑	↑*	↑*
w	n/a	n/a	n/a	n/a	↑	↑*	↑*

Table 2: Summary Statistics

Variable	Mean	S.D.	Skewness	1%	25%	Median	75%	99%
Employee	19.63	49.45	11.65	0.128	2.100	6.035	17.56	216.0
Sales	4,598	12,073	9.028	17.77	451.9	1,264	3,869	50,100
Inc_Pay	0.0292	0.0399	2.973	0.001	0.0066	0.0154	0.0337	0.2103
Ownership	0.0159	0.0352	3.740	0.000	0.0009	0.0029	0.0114	0.1919
Salary	630.4	323.5	1.951	110.0	400.0	580.0	800.0	1,685
Total_Pay	5,121	11,085	21.16	401.2	1,405	2,630	5,308	37,235

Table 3: Real vs. Simulated Data: Uniform Ability

	Mean		Mean/St. Dev.		Skewness	
Variable	Real	Simulated	Real	Simulated	Real	Simulated
inc_pay (s_1)	0.0286	0.0262	0.9280	1.7066	2.3690	2.6490
Salary (s_0)	759.44	764.03	2.3389	1.1230	0.7927	0.6324
Total Pay (tc)	4,770	778	0.9350	1.1264	2.4119	0.6285
Employee (n)	50.05	18.46	0.3546	1.3487	8.5982	0.3560
Sales (R)	9,233	785	0.3612	1.1281	7.4260	0.6266

Table 4: Real vs. Simulated Data: Inferred Ability

	Mean		Mean/St. Dev.		Skewness	
Variable	Real	Simulated	Real	Simulated	Real	Simulated
inc_pay (s_1)	0.0286	0.0286	0.9280	0.9280	2.3690	2.3690
Salary (s_0)	759.44	764.05	2.3389	0.1304	0.7927	9.2370
Total Pay (tc)	4,770	765	0.9350	0.1305	2.4119	9.2363
Employee (n)	50.05	26.05	0.3546	0.1456	8.5982	8.3126
Sales (R)	9,233	765	0.3612	0.1305	7.4260	9.2359

Table 5: *Employee* and *inc_pay*.

	Linear	Quadratic	Cubic	Ours
intercept	51.74	55.8	60.92	41.18
inc_pay	-170.4	-633.2	-1,468	
inc_pay^2		2,594	14,225	
inc_pay^3			-35,822	
$\frac{1-inc\_payy}{inc\_pay^3}$				5.946×10^{-8}
R^2_{adj}	0.1174	0.1404	0.1641	0.1591
SIC	88,269	87,976	87,661	87,704

Table 6: *Employee* and *Inc_Pay*: nested models.

	Linear	Quadratic	Cubic
intercept	108.5	113.6	119.2
inc_pay	-120.1	-493.7	-1,123
inc_pay^2		2,089	10,777
inc_pay^3			-26,676
$\frac{1-inc\_pay}{inc\_pay^3}$	5.51×10^{-8}	5.11×10^{-8}	4.66×10^{-8}
R^2_{adj}	0.1698	0.1847	0.1997
SIC	87,560	87,361	87,157

Table 7: *Sales* and *inc_pay*.

Model	Linear	Quadratic	Cubic	Ours
$intercept$	10,704	11,763	13,064	8,624
inc_pay	-43,312	-163,394	-373,817	
inc_pay^2		700,537	3,607,502	
inc_pay^3			-9,036,341	
$\frac{1-inc\_pay}{inc\_pay^4}$				1.23×10^{-8}
R^2_{adj}	0.0841	0.1102	0.1355	0.1297
SIC	213,980	213,660	213,330	213,400

Table 8: *Salary* and *inc_pay*.

	Linear	Quadratic	Cubic	Ours
$intercept$	729	758	795	701
inc_pay	-940	-4,239	-10,080	
inc_pay^2		18,514	99,943	
inc_pay^3			-250,819	
$\frac{(1-inc\_pay)^2(2-inc\_pay)}{inc\_pay^4}$				6.485×10^{-11}
R^2_{adj}	0.141	0.1684	0.1956	0.1388
SIC	130,750	130,390	130,030	130,790

Table 9: *Total_Compensation* and *Inc_Pay*.

	Linear	Quadratic	Cubic	Ours
$intercept$	4,386	4,724	5,062	4,308
inc_pay	4,139	-341,130	-88,790	
inc_pay^2		214,747	976,816	
inc_pay^3			2,347,347	
$\frac{(1-inc\_pay)(2-inc\_pay)}{inc\_pay^4}$				5.314×10^{-10}
R^2_{adj}	0.0549	0.0579	0.0597	0.0553
SIC	213,184	213,160	213,150	213,180

Table 10: Summary Statistics on All Samples and the Top Quartile of Inc_Pay

Financial Variable	Top quartile of inc_pay Mean	Median	Std	All sample Mean	Median	Std
$Employee$	9.702	2.955	28.29	19.63	6.035	49.45
$Sales$	1,530	504.9	3,399	4,599	1,265	12,074
MV	2,285	650.2	7,575	6,540	1,516	19,833
$MV/empl$	692.1	265.9	1,540	706.1	281.6	1,632
$Cash$	440.2	92.64	2,563	1,287	193.1	7,591
$Firm\_age$	25.54	19.00	13.38	32.45	34.00	14.84
$Debt\_ratio$	0.6661	0.1580	11.16	0.6711	0.2222	6.549
Inc_pay	0.0799	0.0590	0.0516	0.0288	0.0151	0.0394
$Ownership$	0.0538	0.0324	0.0557	0.0159	0.0029	0.0352
$Salary$	541.5	463.4	352.1	630.5	580.0	323.5
CEO_tenure	16.72	15.68	8.981	13.32	12.01	7.471

Table 11: $Employee$ and $\frac{1-Inc\_Pay}{Inc\_Pay^3}$ by Inc_Pay Quartile.

Quartile	$[0, 99\%]$	$[0, 25\%]$	$[25\%, 50\%]$	$[50\%, 75\%]$	$[75\%, 99\%]$
$Intercept$	41.18	84.04	24.76	0.653	6.357
$\frac{1-Inc\_Pay}{Inc\_Pay^3}$	5.946×10^{-8}	4.222×10^{-8}	4.250×10^{-6}	2.536×10^{-5}	$-1.273 \times 10^{-4} (P = 0.088)$
R^2_{adj}	0.1591	0.3212	0.2633	0.2240	0.0767

Table 12: $Employee$ and Own by Inc_Pay Quartile.

Quartile	$[0, 99\%]$	$[0, 25\%]$	$[25\%, 50\%]$	$[50\%, 75\%]$	$[75\%, 99\%]$
$Intercept$	50.50	98.32	32.06	3.234	2.309
Own	-113.5	-10,722	-245.7(P=0.228)	14.90(P=0.787)	41.71
R^2_{adj}	0.1065	0.2956	0.2430	0.2156	0.0818

Table 13: $Log(Employee)$ and $log(Inc\_Pay)$ by Inc_Pay Quartile.

Quartile	$[0, 99\%]$	$[0, 25\%]$	$[25\%, 50\%]$	$[50\%, 75\%]$	$[75\%, 99\%]$
$Intercept$	-0.320	-0.743	-2.532	-2.106	1.938
$log(Inc\_Pay)$	-0.652	-0.805	-1.069	-0.848	0.206
R^2_{adj}	0.3834	0.3995	0.3203	0.3478	0.2734

Table 14: $Log(Employee)$ and $log(\frac{1-Inc\_Pay}{Inc\_Pay^3})$ by Inc_Pay Quartile.

Quartile	$[0, 99\%]$	$[0, 25\%]$	$[25\%, 50\%]$	$[50\%, 75\%]$	$[75\%, 99\%]$
$Intercept$	-0.272	-0.738	-2.512	-2.074	1.917
$log(\frac{1-Inc\_Pay}{Inc\_Pay^3})$	0.2146	0.2680	0.3550	0.2804	-0.0669
R^2_{adj}	0.3820	0.3995	0.3203	0.3478	0.2734

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