Stock-Based Compensation and CEO (Dis)Incentives*

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Abstract

Stock-based compensation is the standard solution to agency problems between shareholders and managers. In a dynamic rational expectations equilibrium model with asymmetric information we show that although stock-based compensation causes managers to work harder, it also induces them to hide any worsening of the firm's investment opportunities by following largely sub-optimal investment policies. This problem is especially severe for growth firms, whose stock prices then become overvalued while managers hide the bad news to shareholders. We find that a firm-specific compensation package based on both stock and earnings performance instead induces a combination of high effort, truth revelation and optimal investments. The model produces numerous predictions that are consistent with the empirical evidence.

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I Introduction

Compensation based on the stock price is a very important feature of executive contracts of publicly traded firms in the US and elsewhere (Hall and Liebman 1998, Murhpy 1999, Murphy 2003). While a large theoretical literature views stock-based compensation as a solution to an agency problem between shareholders and managers,[1] there is also a growing body of empirical evidence that shows that it leads to earnings management, misreporting, and outright fraudulent accounting. Does stock-based compensation amplify the tension between the incentives of managers and shareholders instead of aligning them? In this paper we analyze a dynamic rational expectations equilibrium model, and identify conditions under which stock-based executive compensation leads to misreporting, suboptimal investment, run-up and a subsequent sharp decline in equity prices. We show that the problem is especially severe for high growth companies. Ironically, high growth firms are exactly the firms that rely more heavily on stock-based compensation (Murphy (2003)). In contrast, our model shows that for these firms, a compensation that is based on a combination of dividends and stock performance dominates a stock-based compensation.[2]

More specifically, we study a hidden action model of a firm that is run by a CEO, whose compensation is linked to either the stock price or the dividends. The firm initially experiences high growth in investment opportunities and the CEO must invest intensively to exploit the growth options. The key feature of our model is that at some point in time the firm "matures" and the rate of growth of its investment opportunities slows down. The CEO is able to affect the time at which the firm matures by exercising costly effort. But when the investment opportunities growth does inevitably slow down, the investment policy of the firm should change appropriately. We assume asymmetric information: while the CEO privately observes the decrease in the growth rate, shareholders are oblivious to it. Moreover, they do not observe investments, but base their valuation only on dividends. When investment opportunities decline, the CEO has two options: telling the truth, or behaving as if nothing had happened. Telling the truth leads to an immediate decline in the stock price. If the CEO chooses not to report the change in the business environment of the firm, the stock price does not fall, as the outside investors have no way of deducing this event. In the latter case the equity becomes overvalued.

Behaving as if nothing has changed means that the CEO must follow a sub-optimal investment strategy to maintain the pretense. We assume that as long as the reported dividends over time are consistent with the high growth rate, the CEO keeps his job. However, the first deviation from the high growth profile elicits an immediate audit from the shareholders, which reveals the investment strategy followed by the CEO. If the investment strategy was suboptimal, the CEO is fired. These assumptions collapse this part of the dynamic game into a static reporting game, in which the CEO chooses whether to tell the truth or to conceal, and the stock is priced accordingly over time.

[1] See for example Holmstrom (1979) and Holmstrom and Tirole (1993).

[2] We use the term "dividends" throughout to be consistent with the model. However, the same logic would hold for earnings or cash flows. Indeed, it applies to any aggregate measure that is harder for the manager to manipulate than its components.

We start by analyzing the equilibrium at the time the rate of growth of the investment opportunities slows down. First, we show that when the CEO compensation is based on his reported dividends, the only pure strategy Nash equilibrium is separating, in which the CEO tells the truth and follows the optimal investment policy. In sharp contrast, we find that whenever the CEO has a large stock-based component in his compensation, and the range of possible growth rates is large, there is a pooling Nash equilibrium for most parameter values. In this equilibrium, the CEO of a maturing firm follows a suboptimal investment policy in order to maintain the pretense that investment opportunities are still strong. Since the CEO is interested in keeping a high growth profile for as long as possible, initially he invests in negative NPV projects as storage of cash, and later on foregoes positive NPV projects in order to meet rapidly-growing demand for dividends. In short, he destroys value.

These predictions of our model are consistent with a growing evidence that stock-based executive compensation is associated with earnings management, misreporting and restatements of financial reports, and outright fraudulent accounting. Healy (1985) shows that executives manage accruals to maximize bonus payments. Likewise, Beneish (1999) finds that managers exercise stock options and are net equity sellers in periods of overstated earnings. Bergstresser and Philippon (2006) and Ke (2005), show that the use of discretionary accruals and earning management are more pronounced at firms where the CEO's compensation is closely tied to the value of the stock price. Similarly, Burns and Kedia (2006) find that CEOs with option package that is more sensitive to the stock price are more likely to misreport, and find that restatements are more prevalent in faster growing industries. Finally, Kedia and Philippon (2006) find that fraudulent accounting is associated with excessive investment and that manager exercise options during periods of suspicious accounting.

We then solve for the ex-ante incentive for the CEO to exert costly effort and prolong the high growth options period of the firm. Consistent with previous literature, our model shows that stock-based compensation is more effective than dividends-based compensation to provide the managers with the incentive to exercise costly effort, and thus increase the firms' investment opportunities. Dividends-based compensation tends to induce a low managerial effort, unless the growth prospects of the firm are unusually high, or the manager is particularly patient. Intuitively, very high-growth prospects imply large investments, which yield low current dividends. It follows that only managers with low discount rates are willing to wait a long time to be compensated for their current effort. In contrast, stock-based contracts induce the manager to increase the growth option of the firm because the latter are capitalized in the stock price. In equilibrium, however, the stock price discounts the fact that when the firm matures in the future, the manager will conceal this fact for a while and invest suboptimally.

Indeed, shareholders in our model face a difficult decision. On the one hand, the common wisdom of hidden action models is to align the manager's incentive with those of investors by tying his compensation to the stock price. On the other hand, stock-price-based compensation may lead the manager to invest suboptimally and destroy value. The trade off is made apparent by the fact that for reasonable parameter values, and especially for medium to high growth companies, we find that dividends-based compensation induces a low-effort/reveal equilibrium, while the stock-based compensation induces a high-effort/conceal equilibrium. That is, the cost of inducing high managerial effort ex-ante comes from the suboptimal investment policy after the slowdown in investment opportunities.[3]

We show that this double incentive problem (i.e. induce high effort and truth telling) can be overcome by a combined compensation package: by appropriately choosing a combination of dividends and stock-based compensation, it is possible to shift the equilibrium to a high effort / reveal equilibrium and obtain the first best for shareholders. Most important, we show that different types of firms need to put in place a different composition of dividends and stocks in the compensation package. Specifically, we find that the CEO's compensation package of growth firms, that is, those with high investment opportunities growth and high return on capital, should have only little stock-price sensitivity, but large dividend sensitivity to induce first best. Vice versa, firms with medium-low investment growth and low return on capital should rely more heavily on a high price sensitivity in the CEO's compensation packages. Intuitively, the (forward looking) stock price of growth firms increases sharply over time so long the CEO can prove that such investment opportunities are available. Thus, the CEO has a strong incentive to conceal the change in investment opportunities when the time comes. Putting a large weight on dividends-based incentives in the full package in this case is more appropriate. However, if the compensation is not sensitive to the stock price at all, then the CEO may not exert costly effort, which is also a bad outcome for shareholders. Thus, a composition of both incentives schemes may achieve the first best. We calculate numerical values for the maximum and minimum weight on the stock component of the compensation packages and find that not only they differ across firms, but that for most firms the weight on stock is strictly between zero and one.

Our paper is related to the literature on managerial "short-termism" and myopic corporate behavior (Stein (1989), Bebchuk and Stole (1993)). We present a picture of a firm, that is temporarily overvalued, and destroys value to maintain this appearance. Jensen (2005) argues that the agency costs of overvalued equity are the main causes of corporate fraud in recent years. Likewise, Aghion and Stein (2007) develop a model in which a firm can devote effort either to increasing growth, or to improving profit margins. They show that if the firm's manager cares about the current stock price, he will favor the growth strategy when there is a sentiment for growth firms. While we pursue a similar line of inquiry, we do so in a context of a rational expectations model that endogenizes overvaluation. Thus, differently from Jensen (2005) and Aghion and Stein (2007) our results do not rely on behavioral biases, and apply to a wider range of firms. In terms of assumptions, our paper bears some similarities to Miller and Rock (1985) who study the effects of dividends announcements on the value of the firm. Similar to Eisfeldt and Rampini (2007) and Inderst and Mueller (2006), we assume that the CEO has a significant informational advantage over investors, and that investors observe neither the true earnings nor the true investment, but must rely on aggregate announcements for their valuation. However, the emphasis of our paper is quite different in two dimensions: we focus on the beliefs about future growth rates, rather than

[3]Different shareholders of the firm may not agree on the optimal equilibrium. Those investors holding stock for shorter periods are interested in high effort and do not much care about the subsequent suboptimal investments. Long-term investors have the reverse preferences.

about current dividends levels; and we link them to the incentives of the managers.

Our paper is also related to Bolton, Scheinkman and Xiong (2006), Goldman and Slezak (2006), and Kumar and Langberg (2007). Bolton, Scheinkman, and Xiong (2006) study an agency model of stock-based executive compensation in a speculative stock market and show that optimal compensation contracts may emphasize short-term stock performance, at the expense of long-run fundamental value. Goldman and Slezak (2006) develop an agency model in which stock-based compensation induces managers to exert effort but also to divert firm resources. Kumar and Langberg (2007) present a static model in which manipulation and overinvestment coexist in a rational expectations equilibrium. In their framework managers derive private benefits from larger investments, and have informational advantage, thus they can misrepresent their true performance and prospects. The driving force behind the overinvestment is, therefore, the inability of the shareholders to precommit to ex post inefficient investments, which significantly restricts the set of feasible contracts. Overall, these papers complement each other, and conclude that contrary to the traditional prescriptions, providing managers with high-powered short-run incentives based on the stock price may be dangerous, because the stock price accumulates the beliefs about the uncertain future. The manager can use deceptive or even fraudulent practices that destroy value to maintain the pretense of a bright tomorrow.

Our model also yields additional predictions, that are consistent with the extant empirical evidence. In particular, the model predicts that meeting earnings forecasts is a key objective for CEOs who are concealing the worsening of investment opportunity growth, and in fact CEOs would forego positive NPV projects to achieve this goal. This implication is consistent with the evidence in Graham et al. (2005), who present the results of an extensive survey among the CFOs. Most CEOs state that they would forego a positive NPV project if it causes them to miss the earnings target; high tech firms are much more likely to do so. They also are much more likely to cut R&D and other discretionary spending to meet the target. High tech firms believe more strongly that missing earnings target introduces uncertainty and raises red flags about the company. All of these findings are consistent with the assumptions and predictions of the model. Similarly, our model also predicts that for high growth companies, failing to meet earnings forecasts would elicit a large drop in price, as it reveals that firm had a worsening of investment opportunities in the past, and that capital has been eaten up by suboptimal investments by managers who were instead keeping up the pretense to be a growth company. This prediction is consistent with the empirical findings of Skinner and Sloan (2002), who show that a decline in the firm value following a failure to meet the analysts' forecasts is much more pronounced in high growth firms. Likewise, Barth, Elliott and Finn (1999) show that firms with patterns of increasing earnings have higher price-earnings multiples.

The rest of the paper is organized as follows. Section II presents the model setup and analyzes the benchmark case of full information. Section III presents the case with asymmetric information and solves for the equilibrium. Section IV contains the results about managerial effort and optimal compensation. Section V concludes.

II The Model

We consider a publicly traded firm that is run by a CEO who chooses the firm's investment policy. Our model is dynamic and has three key ingredients: (a) the available investment opportunities and the investments of the firm at any point in time are private information of the CEO; (b) the CEO's compensation depends on the firm's performance; (c) the CEO can affect the set of available investment opportunities by exercising costly effort. We begin the setup with a description of the technology and discuss the first best solution as a benchmark for the subsequent analysis. Section III introduces asymmetric information and describes the CEO's preferences, and exogenous compensation schemes. We then solve the model by assuming a given level of the managerial effort and derive the two equilibria under different compensation regimes. Only then, in Section IV, we endogenize the managerial effort and discuss optimal compensation.

II.A Technology

The firm investment opportunities are described by the following technology: Given a level of capital K_t, firm's operating profits Y_t are given by

$$Y_t = \begin{cases} zK_t & \text{if} \quad K_t \leq J_t \\ zJ_t & \text{if} \quad K_t > J_t \end{cases} \tag{1}$$

where z is the rate of return on capital, and J_t defines an upper bound on the amount of productive capital that depends on the characteristics of the technology itself, operating costs, demand and so on. The Leontief technology specification (1) implies constant return to scale up to the upper bound J_t, and then zero return for $K_t > J_t$. This simple specification of a decreasing return to scale technology allows us to conveniently model the growth rate in profitable investment opportunities, whose different dynamics across firms will be the driving force of our model.

We assume that the upper bound J_t in (1) grows according to

$$\frac{dJ_t}{dt} = \tilde{g} J_t \tag{2}$$

where $\tilde{g}$ is a constant. The combination of (1) and (2) captures our idea of growing investment opportunities. Indeed, since the technology displays constant returns to scale up to J_t, it is optimal to keep the capital at the level J_t as long as the investments are profitable, which we assume throughout the paper. Figure 1 illustrates the growth rate in investment opportunities.

Finally, we assume that to remain productive, the firm must maintain a minimum level of capital $K_t \geq \underline{K}_t$, where $\underline{K}_t$ is exogenously specified. We assume for simplicity that

$$K_t \geq \underline{K}_t = \xi J_t \quad \text{for} \quad 0 \leq \xi < 1 \tag{3}$$

where J_t is defined in (2). This, as we will see later, is a technical assumption and ξ is a free parameter.

The firm does not retain earnings, so that the annual dividend rate equals its operating profits Y_t derived from its stock of capital, K_t, less the investment it chooses to make, I_t. Formally, given the technology in (1), dividends are

$$D_t = z \min(K_t, J_t) - I_t. \quad (4)$$

Finally, the existing stock of capital depreciates at the rate of δ.

II.B Firm Life Cycle

The firm is born as a high growth firm. At some random time τ^* the firm matures, and the growth rate in investment opportunities slows down. Formally

$$\tilde{g} = \begin{cases} G & \text{for} \quad t < \tau^* \\ g & \text{for} \quad t \geq \tau^* \end{cases}, \quad (5)$$

where $G > g$. The time at which the firm reaches maturity, i.e. shifts to a lower growth, is random. For simplicity, we assume that τ^* is exponentially distributed, with probability density function given by

$$f(\tau^*) = \lambda e^{-\lambda \tau^*}$$

That is, in every instant dt there is a constant probability λ that a shift from G to g occurs.

II.C Benchmark case: Symmetric Information

Consider first the benchmark case in which the manager and shareholders share the same information. To maximize the firm value the manager must invest to its fullest potential, that is, to keep $K_t = J_t$ for all t. To find the optimal investment policy, notice first that the capital evolution equation is given by:

$$\frac{dK_t}{dt} = I_t - \delta K_t. \quad (6)$$

From (2), the target level of capital, J_t, is given by

$$J_t = \begin{cases} e^{Gt} & \text{for} \quad t < \tau^* \\ e^{G\tau^* + g(t-\tau^*)} & \text{for} \quad t \geq \tau^* \end{cases}. \quad (7)$$

Imposing $K_t = J_t$ for every t and using (6) we find that the optimal investment policy is

$$I_t = \begin{cases} (G+\delta)e^{Gt} & \text{for} \quad t < \tau^* \\ (g+\delta)e^{G\tau^* + g(t-\tau^*)} & \text{for} \quad t \geq \tau^* \end{cases}. \quad (8)$$

The dividend stream of a firm that fully invests (see (4)) is given by:

$$D_t = zK_t - I_t = \begin{cases} D_t^G = (z - G - \delta)e^{Gt} & \text{for} \quad t < \tau^* \\ D_t^g = (z - g - \delta)e^{G\tau^* + g(t-\tau^*)} & \text{for} \quad t \geq \tau^* \end{cases} . \tag{9}$$

The top panel of Figure 2 plots the dynamics of the optimal dividend path for a firm with a high growth in investment opportunities until τ^*, and a low growth afterwards. As the figure shows, the slowdown in the investment opportunities requires a decline in the investment rate, which initially increases the dividend payout rate: $D_{\tau^*}^g - D_{\tau^*}^G = (G-g)e^{G\tau^*}$.

Given the assumptions below, the dividends are always positive. Notice that in (9) the rate of increase in dividends equals the rate of increase in the investment opportunities, $\tilde{g}$.

To calculate the value of the firm we assume that investors apply a constant discount rate r to future cash flows. In particular, the discount rate does not depend on the growth rate of the firm, as the shift from high growth to low growth is idiosyncratic to the firm. To ensure a finite value of the firm stock price, we assume

$$r > G - \lambda \quad \text{and} \quad r > g.$$

In addition, we assume that

$$z > r + \delta,$$

that is, the return on capital is sufficiently high to compensate for the cost of capital r and depreciation δ. This assumption implies that it is economically optimal for investors to provide capital to the company and invest up to its fullest potential, as determined by the Leontief technology described in (1).

Proposition 1: *Under perfect information:*

(a) The value of the firm at time $t \geq \tau^*$ is:

$$P_{fi,t}^{after} = \int_t^\infty e^{-r(s-t)} D_s^g ds = \left(\frac{z - g - \delta}{r - g} \right) e^{G\tau^* + g(t-\tau^*)}. \tag{10}$$

(b) The value of the firm at time $t < \tau^*$ is:

$$P_{fi,t}^{before} = E_t \left[\int_t^{\tau^*} e^{-r(s-t)} D_s^G ds + e^{-r(\tau^*-t)} P_{fi,\tau^*}^{after} \right] \tag{11}$$

$$= e^{Gt} A_\lambda^{fi} \tag{12}$$

where

$$A_\lambda^{fi} = \frac{(z - G - \delta)}{r + \lambda - G} + \lambda \left(\frac{z - g - \delta}{(r-g)(r + \lambda - G)} \right) \tag{13}$$

In the pricing functions (10) and (12) the subscript "fi" stands for "full information" and superscript "$after$" is for $t \geq \tau^*$ and "$before$" is for $t < \tau^*$.

Under full information, the price of stock necessarily drops at time τ^*. The size of the drop is

$$P_{fi,\tau^*}^{after} - P_{fi,\tau^*}^{before} = -\frac{e^{G\tau^*}}{(r-g)(r+\lambda-G)}(z-r-\delta)(G-g).$$

The bottom panel of Figure 2 plots the price path in the benchmark case corresponding to the dividend path in top panel.

III Asymmetric Information

Clearly the manager has much more information than the investors regarding the future growth opportunities of the firm, as well as about its actual investments. We assume that the decline in investment opportunities is private information of the manager and cannot be observed by investors. Neither can they observe the investment activity of the firm. This assumption is realistic in many industries, and especially the rapidly growing new industries, as the market does not know how to distinguish between investments and costs. Indeed, in many industries, and in particular in high tech, almost all the R&D investments are expensed rather than capitalized; and are labeled as such at the discretion of the management. In mature R&D-intensive industries rules of thumb had been established over time that yield reasonable estimates of investments in R&D; in newly developing industries such rules had not yet crystallized. We assume, therefore, that investors have to base the valuation of the firm's prospects only on the dividend stream D_t.[4]

Shareholders know that at $t = 0$ the firm has a given K_0 of capital and high growth rate G of investment opportunities. Since it is costly to monitor the investment strategy of the firm every t, shareholders use dividends to assess whether the firm at any later time t is a G firm or a g firm. As long as the firm is of type G, they expect a dividend as described in (9).[5] We also assume that shareholders can monitor the manager by performing an audit. During these audits, the whole history of investments is made public. Our assumption that shareholders conduct an internal audit every time there is a change in dividend policy may appear extreme, but it matches the lack of randomness in dividend realizations in our setting. In a more realistic setting in which dividend realizations are random, the equivalent assumption is to have shareholders conduct an audit following large changes in dividend policy.

[4]While this seems like a strong assumption, it counterbalances other two assumptions that we make, namely, deterministic production function and deterministic return on capital. Relaxing all of these assumptions to a more realistic situation in which the revenues are imperfectly observed sometime after the investment, and return on capital and the production function are subject to stochastic shocks makes it impossible to solve for the fixed point in the dynamic rational expectations equilibrium model. We have no reason to believe that our results would not hold under this modification.

[5]The assumption that dividends can be used to reduce agency costs and monitor managers has been suggested by Easterbrook (1984).

At time τ^* the CEO has to choose whether to reveal the decline in the growth rate of investment opportunities, or conceal it. In the case the CEO reveals the shift in fundamentals, the price drops to P^{after}_{fi,τ^*} as shown above. If the CEO decides to conceal the truth, he must devise an investment strategy that enables the firm to continue paying the dividend stream D^G_t in (9), otherwise the shareholders perform an audit and find out the truth. Intuitively, this strategy cannot be supported forever, as investment opportunities are not growing fast enough. At some point in the future the firm will have to default on its dividend payment, in the sense that its dividend D_t will not meet expectations that are consistent with high growth. We denote that time by T^{**} .

III.A Manager's Preferences

The manager receives a performance-based compensation w_t for every t he/she is at the firm. To keep the analysis simple, we assume linear preferences

$$U_t = E_t\left[\int_t^T e^{-\beta(u-t)} w_u du\right], \tag{14}$$

where T is the time the manager leaves the firm and β is the discount rate of the manager. Potentially, $T = \infty$. However, the departing date T may occur earlier, as the manager may be fired if the shareholders learn that he has followed a suboptimal investment strategy. Note that the utility specification (14) does not depend on the cost of effort. We introduce explicitly a description of managerial effort and its utility costs in Section IV. This is to simplify the exposition and clarify the intuition of the model.

After τ^* the manager faces no uncertainty, thus

$$U_{\tau^*} = \begin{cases} \int_{\tau^*}^{\infty} e^{-\beta(s-t)} w_s ds & \text{if} \quad \text{reveal strategy} \\ \\ \int_{\tau^*}^{T^{**}} e^{-\beta(s-t)} w_s ds & \text{if} \quad \text{conceal strategy} \end{cases}. \tag{15}$$

We consider two types of pure compensation schemes: stock-based and dividends-based (or earnings-based). While these polar cases are extreme, concentrating on them first enables us to better clarify the intuition behind the incentives provided by dividends or stocks. The optimal compensation scheme, as we shall see, will be in fact a combination of both. More specifically,

$$w_t = \begin{cases} \eta_p P_t & \text{for stock-based compensation} \\ \\ \eta_d D_t & \text{for dividends-based compensation} \end{cases},$$

where η_p and η_d are two positive constants. Their levels are not important for now, but in Subsection IV.F we will choose them to make the equilibrium present value of payments to the manager the same under the two compensation schemes. We further assume that $\beta > G$, which is required to keep the total utility of the manager finite.

Finally, we assume that the manager's decision about the investment policy is firm-specific, and thus it does not affect the systematic risk of the stock and the associated cost of capital, r.

III.B Conceal Strategy

The reveal equilibrium is very similar to the one analyzed in the benchmark case. We, therefore, focus on the conceal equilibrium.

Recall that the optimal investment policy is uniquely determined by the decision at time τ^*, when the firm matures. Let K_{τ^*} be the amount of capital available at time τ^*. The choice of the manager at this point is to either change the dividend payout to reflect the new status of the firm, or keep dividends that are consistent with a high growth of the company. In the former case the optimal investment policy is to invest to the fullest potential. The following Lemma characterizes the optimal policy rule in case the manager conceals the true state of the firm.

Lemma 1: *Conditional on the decision to conceal the true state at τ^*, the manager's optimal investment policy is to maximize the time to "default", T^{**}.*

The proof is intuitive, and it is instructive to make it explicit here. Conditional on his decision to conceal the true state g, the manager must provide a dividend stream that is consistent with the G state. The first time the firm does not deliver the promised dividends, shareholders perform an audit and the manager is fired, because he did not invest optimally. However, as long as he delivers the promised dividends, the price of the stock simply reflects the present value of future cash flows conditional on the dividend, and is independent of anything else. Thus, after τ^* the manager's utility is known as long as he holds the job. Since he loses his job in case of "default", and earns rents relative to his outside options, he would like to delay the "default" as much as he can.

Given Lemma 1, we must then calculate the optimal investment strategy that leads to the longest possible pretense.

III.B.1 Time to "default"

Lemma 1 shows that once the manager chooses to conceal the true state g at τ^*, he will maintain the same dividend growth as before for as long as he can, since this will delay the reprisal and allow him to get a flow of payments for longer. Thus the time to "default", which is the maximal time the manager can maintain the appearances without violating any constraints, is the main driving force in this model. The following Proposition characterizes the maximal time to "default", T^{**}, and the associated optimal investment strategy:

Proposition 2: *Let K_{τ^*-} be the capital accumulated in the firm by time τ^*. A manager* *of the firm with the rate of growth g, but pretending to be G, chooses to employ all of his initial capital stock, i.e.* $K_{\tau^*} = K_{\tau^*-}$. *For* $t > \tau^*$ *the optimal investment, given by*

$$I_t = z \ \min\left(K_t, e^{G\tau^* + g(t-\tau^*)}\right) - (z - G - \delta)\, e^{Gt}, \tag{16}$$

satisfies

$$\frac{dK}{dt} = z \ \min(K_t, e^{G\tau^* + g(t-\tau^*)}) - \delta K_t - (z - G - \delta)\, e^{Gt} \tag{17}$$

The default time, T^{**}, *is determined by the condition* $K_{T^{**}} = \underline{K}_{T^{**}}$.

Next we show that the firm can maintain the pretense for the same amount of time regardless of the timing of the decline in investment opportunities.

Proposition 3: *The time that passes between the decline of growth options to the default time,* $h^{**} = T^{**} - \tau^*$, *is independent of the time the firm matures,* τ^*.

Figure 3 presents an illustration of the optimal investment path for a g-firm pretending to be a G firm after τ^*, and it compares it to the optimal investment policy under full information for a specific set of parameters. The top panel presents the capital dynamics, while the investment dynamics are in the bottom panel.

A few comments are in order: First, the optimal capital stock initially exceeds the upper bound on the employable capital, $K_t > J_t$. This implies that the pretending firm must initially invest in *negative* NPV projects: because of the Leontief technology (1) the capital stock $K_t - J_t$ has a zero return, yet depreciates at the rate δ. Note that the g firm has to pay lower dividends when it pretends to be G than under its own optimal investment dynamics (see Figure 2). The extra cash is invested in negative NPV projects as a storage of value that extends the default time T^{**} as much as possible.

Second, as the time goes by the pretending firm engages in disinvestment to raise cash for the larger dividends of the growing firm. The firm can do this as long as its capital K_t is above the minimal capital $\underline{K}_t$. Therefore the technical assumption of a minimal capital stock in equation (3) essentially captures the time at which the firm can no longer conceal the decline in its stock of capital.

Finally, for the realistic parameter values in our example T^{**} turns out to be very large, around 14 years. While this time to "default" is clearly too high, one must recall that it is based on a very restrictive assumption that the market cannot distinguish at all between the investments and costs. If the market can partially distinguish between the two – e.g. the market can spot the large disinvestment required by the g firm – the time to "default" is likely to become significantly lower.

In a conceal Nash equilibrium, rational investors anticipate the behavior of the managers, and price the stock accordingly. We derive the pricing function under asymmetric information next.

III.B.2 Pricing functions

For simplicity of presentation we assume that manager's compensation is external to the firm, thus does not affect its value directly - we assume that these contracts are settled elsewhere.[6] This is a simplifying assumption that does not alter the basic intuition of the model, but makes the pricing functions much more transparent.

We have shown that the pretending firm defaults at T^{**}. The post-default valuation of the firm is straightforward since the information is complete. The difference is that the firm does not have sufficient capital to employ to its full potential, thus needs to borrow. Recall that default occurs when $K_{T^{**}} = \underline{K}_{T^{**}}$, the minimum capital required to operate. The optimal capital is $J_{T^{**}}$ given in equation (7), thus, the firm must borrow $J_{T^{**}} - \underline{K}_{T^{**}}$. From assumption (3), $\underline{K}_{T^{**}} = \xi J_{T^{**}}$, which yields the pricing function:

$$P^L_{ai,T^{**}} = \int_{T^{**}}^{\infty} D^g_t e^{-r(t-T^{**})} dt - J_{T^{**}}(1-\xi) = e^{G\tau^* + g(T^{**}-\tau^*)} \left(\frac{z - r - \delta + (r-g)\xi}{r-g} \right) \quad (18)$$

The pricing formula for $t < T^{**}$ is then

$$P_{ai,t} = E_t \left[\int_t^{T^{**}} e^{-r(s-t)} D^G_s ds + e^{-r(T^{**}-t)} P^L_{ai,T^{**}} \right] \quad (19)$$

The subscript "ai" in (19) stands for "Asymmetric Information".[7] Expression (19) can be compared with the analogous pricing formula under full information (11): the only difference is that the switch time τ^* is replaced by the (later) T^{**}, and the price P^{after}_{fi,τ^*} is replaced with the much lower price $P^L_{ai,T^{**}}$. We are able to obtain an analytical solutions:

Proposition 4: *Under asymmetric information and conceal strategy equilibrium:*

(a) for $t \geq h^{**}$, the value of the stock is

$$P_{ai,t} \;=\; e^{Gt} A^{ai}_{\lambda} \quad (20)$$

where

$$A^{ai}_{\lambda} = \frac{(z - G - \delta)}{(r + \lambda - G)} + \lambda e^{-(G-g)h^{**}} \left(\frac{z - r - \delta + (r-g)\xi}{(r-g)(r+\lambda-G)} \right) \quad (21)$$

(b) for $t < h^{**}$ the value of the stock is:[8]

$$P_{ai,t} = (z - G - \delta) e^{Gt} \frac{1 - e^{-(r-G)(h^{**}-t)}}{(r-G)} + e^{rt} e^{(G-r)h^{**}} A^{ai}_{\lambda} \quad (22)$$

[6] For example, the manager may be selling his shares to outside investors, which has no direct impact on firm valuation. Including compensation into the firm valuation would not change our results qualitatively, but would significantly complicate the exposition.

[7] Differently from the case with perfect information, we do not need to specify an "$after$" and "$before$" pricing function, because at τ^* there is no revelation of the state.

[8] The case of $t < h^{**}$ does not yield additional intuition relative to the case of $t \geq h^{**}$, yet it is much more complex to analyze. For this reason in the rest of the paper, we mainly discuss the case $t \geq h^{**}$.

Comparing the pricing formulas under asymmetric and symmetric information, (20) and (12), we observe that the first term in the constants A_λ^{fi} and A_λ^{ai} is identical. However, the second term is smaller in the case of asymmetric information: the reason is that under asymmetric information, rational investors take into account two additional effects. First, even if default has not been declared yet, it may be possible that it has already taken place and the true investment opportunities are growing at a lower rate g for a while (up to h^{**}). The adjustment $e^{-(G-g)h^{**}} < 1$ takes into account this possibility. Secondly, upon default, the firm must borrow capital to resume operations, which is manifested in the smaller numerator of the second term, compared to the equivalent expression in (12).

Next corollary shows that a longer expected time to maturity, τ^* implies a higher stock price, everything else equal. The result is important, as the dynamics of the stock price induces the manager with stock-based compensation to conceal the true growth rate of the firm's investment opportunities.

Corollary 1. *The pricing function $P_{ai,t}$ in (20) and (22) is decreasing in λ (and thus increasing in $E[\tau^*] = 1/\lambda$) if*

$$\frac{z - r - \delta + (r-g)\,\xi}{z - G - \delta} < \left| \frac{r-g}{r-G} \right| e^{(G-g)h^{*}*} \tag{23}$$

Condition (23) is satisfied for most parameter values. The absolute value on the right hand side represents the fact that $r - G$ is not necessarily positive in our framework. The important restriction is that $r + \lambda - G > 0$, that is, the probability of a shift to a lower growth is sufficiently high, so that the value of the firm is finite.

The top panel of Figure 4 illustrates the welfare loss associated with the conceal strategy. Since the manager‘s compensation is not coming out of the firm‘s funds, the welfare loss is equal to the loss of the shareholders that hold the stock for a long-haul relative to what they would have got under the reveal strategy (full information). For these shareholders, intermediate prices are of no interest, since they do not intend to sell. These costs can be measured by the present value (as of τ^*) of the difference in the dividends paid out to the shareholders under the two equilibria. Relative to reveal strategy, the conceal strategy pays lower dividends for a while, as the manager pretends to actively invest, and then must pay higher dividends, that arise from allegedly high cash flow. These higher dividend payouts come at the expense of investment, thus are essentially borrowed from the future dividends. The longer the firm is able to maintain the pretense, the bigger is the loan. At the time of default the firm must borrow to return to the optimal investment path, thus will pay lower dividends forever. The extent of borrowing is increasing in time to default.

Recall, that given the optimal investment strategy, the time to default is determined by the free parameter ξ, which captures the degree to which the firm can hide disinvestment before it is discovered (see equation (3)). By varying this parameter we can change the relative size of the two regions: raising ξ increases the time to default for the concealing firm, but also forces it to borrow more after the default. Clearly, a decline in ξ increases welfare losses due to higher disinvestment.

III.B.3 Asymmetric Information and Equilibrium Stock Prices

How does asymmetric information affect the level of prices? The bottom panel of Figure 4 plots the price dynamics under the conceal equilibrium and compares it to prices under the reveal equilibrium. As it can be seen, rational investors decrease the value of prices in the conceal equilibrium, as they correctly anticipate the suboptimal investment behavior of the manager post τ^*.

The behavior of prices at T^{**} is quite common in the market. Specifically, a failure to meet dividend (earnings) expectations, even by a small amount, results in a large decrease in the stock price. A large literature documents this phenomenon. Although our model is very stylized, the basic intuition that meeting expectations / failing to meet expectations is a strong signal of the true type of the firm is likely to hold in a more general model.

III.C Equilibrium at τ^*

We finally consider the manager's incentive at time τ^* to conceal or reveal the true state. We begin with the simpler case in which the manager's compensation is tied to dividends. Under dividends-based compensation the manager's utility is given by:

$$U_{Div,\tau^*}^{reveal} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_d D_t^g)dt$$

$$U_{Div,\tau^*}^{conceal} = \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}(\eta_d D_t^G)dt$$

Recall that after τ^* there is no longer any uncertainty for the manager (even T^{**} is known), and thus these two utility levels can be computed exactly.

Proposition 5: *At time τ^*, dividends-based compensation yields the following utility functions under "reveal" and "conceal" strategies:*

$$U_{Div,\tau^*}^{reveal} = \eta_d e^{G\tau^*}\left(\frac{z-g-\delta}{\beta-g}\right) \tag{24}$$

$$U_{Div,\tau^*}^{conceal} = \eta_d(z-G-\delta)e^{G\tau^*}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \tag{25}$$

A conceal equilibrium results if $U_{Div,\tau^*}^{reveal} < U_{Div,\tau^*}^{conceal}$. Similarly, a reveal equilibrium results if $U_{Div,\tau^*}^{reveal} > U_{Div,\tau^*}^{conceal}$.

We now turn to stock-based compensation. In this case, the rational expectations Nash equilibrium must take into account investors' beliefs about the manager strategy at time

τ^*. These beliefs in turn determine the price function. There are three intertemporal utility levels to be computed at τ^* depending on which equilibrium we are considering, conceal equilibrium versus reveal equilibrium.

1. *Reveal Equilibrium.* In a pure strategy Nash reveal equilibrium, the price function is the one supporting the equilibrium. Thus, the manager's utility is determined by $P_{fi,t}^{after}$ in equation (10) if at τ^* the manager decides to reveal. In contrast, if the manager decides to conceal, his utility is determined by the price function $P_{fi,t}^{before}$ in equation (12). The two utility levels to compare in this case are

$$U_{Stock,\tau^*}^{reveal} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_p P_{fi,t}^{after})dt \tag{26}$$

$$U_{Stock,\tau^*}^{conceal,fi} = \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}(\eta_p P_{fi,t}^{before})dt \tag{27}$$

2. *Conceal Equilibrium.* In a conceal equilibrium, if the manager follows the Nash equilibrium strategy (conceal at τ^*), then the price function must be the asymmetric information price function $P_{ai,t}$ in equation (20). The resulting utility level is

$$U_{Stock,\tau^*}^{conceal,ai} = \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}(\eta_p P_{ai,t})dt \tag{28}$$

   If instead, the manager reveals at τ^* the true state of the firm, the price function reverts back to the full information price $P_{fi,t}^{after}$ in equation (10). Thus, the utility level is still given by (26) above.

The superscripts "fi" and "ai" in the "conceal" utility functions (27) and (28) indicates the use of a full information or asymmetric information price. The utility under reveal strategy, instead, always uses the full information price, and so does not need qualification.

Proposition 6: *Let $\tau^* \geq h^{**}$. Then, the CEO utility levels under the "reveal" and "conceal" strategies in the two Nash equilibria are as follows:*[9]

$$U_{Stock,\tau^*}^{reveal} = \eta_p \frac{e^{G\tau^*}}{r-g}\left(\frac{z-g-\delta}{\beta-g}\right) \qquad \textit{in both equilibria} \tag{29}$$

$$U_{Stock,\tau^*}^{conceal,fi} = \eta_p e^{G\tau^*} A_\lambda^{fi}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \qquad \textit{in a reveal equilibrium} \tag{30}$$

$$U_{Stock,\tau^*}^{conceal,ai} = \eta_p e^{G\tau^*} A_\lambda^{ai}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \qquad \textit{in a conceal equilibrium} \tag{31}$$

Thus,

[9] If $\tau^* < h^{**}$, the utility function $U_{\tau^*}^{conceal,j}$ for $j = ai, fi$ is more complicated, and it is left to the appendix.

1. A conceal Nash equilibrium obtains if and only if

$$U^{reveal}_{Stock,\tau^*} < U^{conceal,ai}_{Stock,\tau^*},$$

2. A reveal Nash equilibrium obtains if and only if

$$U^{reveal}_{Stock,\tau^*} > U^{conceal,fi}_{Stock,\tau^*}.$$

From equations (30) and (31), $U^{conceal,fi}_{Stock,\tau^*} > U^{conceal,ai}_{Stock,\tau^*}$, as $A^{fi}_{\lambda} > A^{ai}_{\lambda}$. This implies that the two equilibria "conceal Nash" and "reveal Nash" are mutually exclusive. That is, it is not possible to find parameters for which both equilibria can exist at the same time. However, it may happen that for some parameter combination, no pure strategy Nash equilibrium exists.

III.D Equilibrium Conditions

Corollary 2: *A necessary and sufficient condition for a "reveal" equilibrium under the dividends-based compensation is:*

$$\left(\frac{z-G-\delta}{z-g-\delta}\right)\left(1-e^{-(\beta-G)h^{**}}\right) < \frac{\beta-G}{\beta-g}. \tag{32}$$

This condition is satisfied if the return on capital net of depreciation is less than or equal to the time discount: $z-\delta \leq \beta$. This latter sufficient condition is in fact too strong: even if it is violated, i.e. $z-\delta > \beta$, in our numerical calculations we were not able to find reasonable parameter values under which condition (32) is also violated. In other words, dividend compensation generates a "reveal" equilibrium, leading to the optimal investments by managers.

Exactly the opposite occurs under stock-based compensation. In this case we obtain the following necessary and sufficient condition for a conceal and reveal equilibrium:[10]

Corollary 3: *Let $\tau^* \geq h^{**}$. A necessary and sufficient condition for a "conceal" equilibrium under stock-based compensation is*

$$\frac{A^{ai}_{\lambda}(r-g)}{(z-g-\delta)}\left(1-e^{-(\beta-G)h^{**}}\right) > \frac{\beta-G}{\beta-g} \tag{33}$$

where the constant A^{ai}_{λ} is given in equation (21). Similarly, a necessary and sufficient condition for a "reveal" equilibrium under stock-based compensation is

$$\frac{A^{fi}_{\lambda}(r-g)}{(z-g-\delta)}\left(1-e^{-(\beta-G)h^{**}}\right) < \frac{\beta-G}{\beta-g} \tag{34}$$

[10]Since the formulas are complicated for the case in which $\tau^* < h^{**}$, we leave this case to the Appendix.

where the constant A_λ^{fi} is given in equation (13).

Comparing equation (33) and (32), we see that stock-based compensation is more likely to imply a conceal equilibrium than dividends-based compensation if

$$A_\lambda^{ai}(r-g) > z - G - \delta \tag{35}$$

From equation (21) we see that

$$A_\lambda^{ai}(r-g) = \left(\frac{r-g}{r+\lambda-G}\right)(z-G-\delta) + \lambda e^{-(G-g)h^{**}}\left(\frac{z-r-\delta}{r+\lambda-G}\right)$$

which implies that condition (35) is certainly satisfied whenever $G > g+\lambda$, that is, whenever the initial growth rate G is sufficiently high compared to the rate of growth of a mature firm, g. Intuitively, when G is high, the price of stock is high as well, as it reflects the higher potential growth in dividends. The higher stock price implies a higher compensation for the firm's manager, and thus generates a greater incentive for him or her to conceal the decrease in g when it happens at τ^*. Indeed, as we shall see below, we find that condition (33) is satisfied for most parameter configurations, unless G is quite close to g. In the latter case it may be not worth concealing and being fired, as the impact of G on the stock price, and thus on compensation, is small.

III.E A Numerical Example

When is it optimal to conceal the change in the growth rate of investment opportunities? Figure 5 plots the areas in which pure strategy Nash conceal or reveal equilibria obtain under *stock-based compensation*. The base numerical values of the parameters are as follows: The discount rate is $r = 10\%$, the return on capital $z = 20\%$ with a depreciation rate of $\delta = 1\%$. The free parameter regulating the minimum amount of capital $\xi = 80\%$. The subjective discount rate is $\beta = 20\%$ and the expected time of maturity $E[\tau^*] = 1/\lambda = 15$ years. Finally, at maturity, the firm moves to a growth rate $g = 0\%$.

In each panel, we let the initial growth rate of firm G range on the $x-$axis from a minimum of 3% to a maximum of 16%, which is only 1% below the maximum possible value of G that still yields a finite value of the firm ($G < r + \lambda$). In each panel, the $y-$axis represents a different variable: in the top panel these are the values of g ranging between 3% and 16%. In the middle panel, the $y-$axis represents the return on capital z, which ranges between 14% and 30%, and finally, in the bottom panel, the $y-$axis captures the expected time of maturity of the firm $E[\tau^*] = 1/\lambda$, that range between 3 and 25 years.

Starting with the top panel, we see that under stock-based compensation, even a small difference between the two possible growth rates G and g is sufficient to induce a conceal Nash equilibrium, in which the manager chooses the conceal strategy and investors rationally anticipate this behavior. As we know from the previous discussion, this equilibrium ultimately leads to a run-up in prices, culminating with a large negative reaction of prices to a dividend (earnings) report that misses a target. In striking contrast, although not reported in the figure, we could find *no* combination of G and g such that the dividends-based compensation would lead the manager to conceal the change in growth rate. That is, dividends-based compensation always generates truth telling for a wide range of parameter values. This difference in behavior under the two compensation regimes has been discussed above: the intuition, recall, is that under stock-based compensation, the manager obtains a much higher compensation if he conceals, compared to dividends-based compensation, because of the multiplier effect that is generated by the present value formula in the discount of future cash flows.

Finally, stock-based compensation does not generate a reveal Nash equilibrium for any combination of G and g. The intuition is as follows: if investors think that the manager will follow a reveal strategy, the pricing function would reflect this belief and it is then given by the perfect information pricing formula (12). However, given these high prices, it is optimal for the manager to deviate and conceal the shift in investment opportunities. Thus, for small differences between G and g no pure strategy equilibrium is supported: if investors believe that the manager follows a reveal (conceal) strategy, it is optimal for the manager to deviate and follow a conceal (reveal) strategy.

The middle panel of Figure 5 plots the areas of conceal and reveal equilibrium under stock-based compensation in the (z, G) space, where z is the instantaneous return on capital. We see that the conceal strategy choice is, once again, a pervasive equilibrium outcome. As before, even in this case, we could not find any combination of (G, z) that would yield an optimal conceal strategy under dividends-based compensation. However, in contrast with the top panel, there is a small region in which a reveal Nash equilibrium obtains under stock-based compensation. This is the area in the top-left corner, in which G is small and the return on capital z is high (note that for this case we had to increase the range of G to include very low values of G). The intuition is that if growth is low, and return on capital high, there is little gain from concealing the change in investment opportunities (G is low anyway) and the cost of future repercussion is high, as the higher profitability of investments implies higher future prices, and thus a higher utility of the manager.

Finally, the bottom panel reports the conceal and reveal strategy areas under stock-based compensation in the space $(E[\tau^*], G)$. The outcome is once again the same: the conceal equilibrium prevails for most parameters, and especially for high growth G and high expected maturity time τ^* (or low λ). As before, for all combinations of parameters reveal is optimal under dividends-based compensation.

These examples illustrate that there seems to be a broad dichotomy: the stock-based compensation induces concealing strategy, while the dividend-based compensation yields truth revelation. We now turn to endogenizing the compensation by introducing costly effort.

IV Incentives and Managerial Effort

We now expand our model to endogenize the compensation when it affects the propensity of the CEO to exert costly effort that prolongs the firm's high growth period. Formally, the CEO can choose whether to exert high effort e^H or low effort e^L. The choice affects the probability λ of a shift to lower growth, such that $\lambda^H < \lambda^L$ (or $E[\tau^*|e^H] > E[\tau^*|e^L]$). The CEO must actively search for investment opportunities, monitor markets and internal development, all of which require time and effort. In our model, these activities translate into a smaller probability to shift to the mature state. Once the shift occurs, however, we assume that there is nothing that the manager can do to put the firm back into the high growth state.

Effort is costly to exert. We let $c(e)$ denote the cost function with $c^H = c\left(e^H\right) > c\left(e^L\right) = 0$. We assume that effort is neither observable nor verifiable. An audit, for instance, would not reveal the past or present effort choices. As before, we assume the manager's utility is linear in the payoff w_u, and it is given by

$$U_t = E\left[\int_t^T e^{-\beta(u-t)}\left[w_u\left(1 - c\left(e_u\right)\right)\right] du\right], \tag{36}$$

where T is the time the manager leaves the firm. The multiplicative form, standard in dynamic models, ensures the homogeneity of the utility function with respect to the level of payoff, leading to optimal rules that are independent of the level of the payoff per se.[11]

To see the link with the previous sections, note that after the slowdown event at τ^* there is no reason to exert any effort, independently of the type of compensation or the equilibrium after τ^*. It follows that $e_u = e^L$ for $u \geq \tau^*$ and thus, $c(e_u) = 0$ for $u \geq \tau^*$ in (36). The utility function then reverts back to the one discussed in the previous section, and our earlier conclusions hold. In particular, let U_{τ^*} be the expected utility at time τ^*, conditional on the decision to reveal or conceal. The exact formulas for the various cases are contained in Propositions 5 and 6.

The expected utility for $t < \tau^*$ is then given by

$$U_t = E_t\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left[w_u\left(1 - c\left(e_u\right)\right)\right] du + e^{-\beta(\tau^*-t)} U_{\tau^*}\right]. \tag{37}$$

Note from equation (37) that λ affects the utility U_t in two distinct ways: it affects the expected time till the decline in the growth options (i.e. τ^*) as well as the size of U_{τ^*} through the price of the stock P_{τ^*}. The latter effect also determines the choice of the action (conceal or reveal) at time τ^*. As we assume that the choice of effort determines λ, it is clear that the ex ante choice of effort by the manager and the ex post choice of revealing or concealing strategy are determined jointly. This means that the equilibrium in pure strategies may not

[11] We also analyzed the linear utility $U_t = E\left[\int_t^T e^{-\beta(u-t)}\left[w_u - c\left(e_u\right)\right] du\right]$ and obtained similar insights.

exist for all parameter values. As we are interested in presenting the managerial incentives in the most intuitive way, we only focus on cases where pure strategy equilibria exist.

IV.A Stock-based compensation.

We now derive the conditions under which the stock-based compensation induces high effort, conditional on concealing being the optimal strategy at time τ^*. Since we focus on a pure strategy equilibrium, the equilibrium choice of effort is common knowledge.

Proposition 7: *Let $t \geq h^{**}$ and let λ^H be such that a conceal equilibrium obtains at τ^*. Then, high effort e^H is the equilibrium strategy if and only if*

$$\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{1 + \lambda^L H^{Stock}}{1 - c^H + \lambda^H H^{Stock}} \tag{38}$$

where

$$H^{Stock} \equiv \frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G}$$

Condition (38) has an intuitive interpretation. First, if effort is costly, but has a low impact on λ, i.e. $\lambda^H \approx \lambda^L$, then the condition is violated, and the manager never chooses high effort. Second, if exercising effort costs little, i.e. $c^H \approx 0$, then the manager always chooses high effort. In other words, if it wasn't for the fact that exercising effort is costly for the manager, he would always choose a high effort e^H with stock-based compensation. The benefit for the manager to exert a high effort stems from the longer tenure period (T^{**} is pushed forward) while enjoying earlier the long term reward of his effort as they are capitalized in the stock price.

IV.B Dividends-based compensation.

In this case, we concentrate our analysis on the reveal equilibrium at τ^*,which is by far the most common outcome for essentially any parameter configuration. We obtain the following:

Proposition 8: *Let "reveal" be the equilibrium at τ^*. Then, e^H is the equilibrium if and only if*

$$\frac{\lambda^L + \beta - G}{\lambda^H + \beta - G} > \frac{1 + \lambda^L H^{Div}}{1 - c^H + \lambda^H H^{Div}}, \tag{39}$$

where

$$H^{Div} = \frac{(z - g - \delta)}{(z - G - \delta)(\beta - g)}.$$

Condition (39) is also intuitive. First, again, $\lambda^H \approx \lambda^L$ makes the manager exert low effort. Second, differently from the stock-based compensation (with conceal equilibrium), however, even a zero cost of effort, $c_H = 0$, does not necessarily guarantee that dividends-based compensation induces the manager to exert high effort. For this to be the case, it must be the case that

$$\frac{(z-g-\delta)}{(z-G-\delta)}\frac{(\beta-G)}{(\beta-g)} < 1,$$

which is not necessarily satisfied. Indeed, this condition tends to be satisfied when the return on capital z is especially high, or the manager's discount rate β is low, relative to the growth rate G. Intuitively, recall the dividend profile under high growth and low growth discussed in Figure 2: a high growth rate implies low dividends today, but high in the future. It is worth to exert effort and have a longer-lasting high dividends growth only if the manager is particularly patient – as his payoff today is low compared to the future – or the return on capital is very high, so that dividends tend to be high as well.

IV.C High effort under stock-based compensation

The key question is whether the stock-based compensation is more likely to produce a high-effort equilibrium, which is the good outcome for investors. The conditions in Propositions 7 and 8 show that this is the case. In fact, it follows from equation (32) that

$$H^{Stock} = \frac{1-e^{-(\beta-G)h^{**}}}{\beta-G} < \frac{(z-g-\delta)}{(z-G-\delta)(\beta-g)} = H^{Div}.$$

This condition immediately yields the following corollary:

Corollary 4. *If*

$$\lambda^L/\lambda^H > 1/\left(1-c^H\right), \tag{40}$$

then there exist parameter values for which:

1. *(Low Effort, reveal at τ^*) is an equilibrium for dividends-based compensation,*
2. *(High Effort, conceal at τ^*) is an equilibrium for stock-based compensation.*

Mechanically, Corollary 4 follows from the fact that its condition implies

$$\frac{1+\lambda^L H^{Div}}{1-c^H+\lambda^H H^{Div}} > \frac{1+\lambda^L H^{Stock}}{1-c^H+\lambda^H H^{Stock}},$$

which in turn implies that there are parameter value for which

$$\frac{1+\lambda^L H^{Div}}{1-c^H+\lambda^H H^{Div}} > \frac{\lambda^L+\beta-G}{\lambda^H+\beta-G} > \frac{1+\lambda^L H^{Stock}}{1-c^H+\lambda^H H^{Stock}}. \tag{41}$$

The statement of Corollary 4 then follows from Proposition 7 and 8.

Intuitively, condition (40) requires that higher effort increases the expected maturity time τ^* sufficiently compared to the cost c^H. In this case, as we had already discussed after Proposition 7, a high effort - conceal equilibrium prevails under the stock-based compensation. What about the low-effort equilibrium under the dividends-based compensation? As mentioned after Proposition 8, for high enough β, the manager prefers low effort.

Figure 6 shows the partition of the parameter space of (z, G) into regions corresponding to various equilibria.[12] We start from the top-right corner: the triangle in that corner is the area where the manager chooses high effort regardless of the compensation mode. Consequently, in this region compensating the manager based only on dividends achieves the first best, as in this case he would also reveal the bad news to investors, and thus maximize firm value. This region is characterized by very high returns on investment, z, and very high growth rates. Firms with such combination of parameters do not have to use stock-based compensation to induce high effort.

The region below and to the left of the top-right triangle is where the dividends-based compensation no longer induces high effort, while the stock-based compensation does, although in a conceal equilibrium. This is indeed the most interesting region, where we observe a trade-off between the effort inducement and truth telling inducement - one cannot obtain both. It seems that the firms with reasonably high growth rates and return on capital are in that region. Finally, the region below and to the left from there is where we may no longer have a pure strategy equilibrium. This is a region, where a stock-compensated manager prefers to conceal if he chooses high effort, but would no longer choose high effort, if conceals. Part of that region corresponds to the conceal-low effort equilibrium (the worst possible scenario), whenever it exists. The existence depends on λ^L: it does not exist for high levels of λ^L. The remainder of the region corresponds to equilibria in mixed strategies. Solving for those is complicated, as the dynamic updating of investors' believes becomes very tedious. They are not likely to provide new intuitions, thus we ignore them. In fact we find the region above that, where the real trade-off takes place of most interest.

IV.D Dividends-based or stock-based compensation?

The implication of the discussion above is that unless the company is expected to have a very large growth of investment opportunities, or a very large return on capital, it appears that stock-based compensation is more likely to induce a high-effort equilibrium. The drawback is that a conceal equilibrium will follow, and with it, the suboptimal investment strategy discussed in subsection III.B The question is whether shareholders are better off with low effort and an optimal investment strategy, or high effort and a suboptimal investment strategy. Figure 7 plots hypothetical price and dividend paths under the stock-based compensation equilibrium (high effort, conceal) and the dividends-based compensation equilibrium (low effort, reveal). For comparison, it also reports the first best, featuring high effort and the optimal investment after τ^*. It appears from the figure, and follows from the earlier discussion,

[12] In fact the space is $(z, G - g)$ as we assume $g = 0$ in these numerical calculations.

that the dividends-based compensation may induce too low an effort, and this loss outweighs the benefits of the optimal investment behavior. Stock-based compensation, in contrast, gets closer to the first best, benchmark case, yet also leads to suboptimal investment behavior, which generates the bubble-like pattern in dividend growth and prices.

IV.E Inducing a high effort / reveal equilibrium

We conclude this section with a possible solution to the problem. The earlier sections showed that dividends-based compensation is successful in inducing a reveal equilibrium. The drawback is that it induces low effort. In contrast, stock-based compensation induces a conceal equilibrium, but high effort. A potential solution is in the middle. Consider a performance-based contract which combines dividends and stock prices in the appropriate proportion. Specifically, consider the following compensation scheme:

$$w_t = \omega \eta_p P_t + (1-\omega)\eta_d D_t. \tag{42}$$

The linearity of the utility function implies that for any t

$$U_{Comb,t} = \omega U_{Stock,t} + (1-\omega) U_{Div,t}$$

We want to obtain a high-effort / reveal equilibrium. We proceed as follows. Assume that indeed the manager follows a high-effort / reveal strategy, so that the equilibrium price function is (12) with $\lambda = \lambda^H$. Conditional on the price function, we can search for the ω such that reveal is optimal at τ^* and high effort is optimal before τ^*. The resulting conditions are contained in Proposition 9.

For expositional convenience, we set the constant $\eta_d = \eta_p/(r-g)$. This choice corresponds to the case in which the utility from the revealing strategy is identical under either dividend or stock compensation, as it can be seen from equations (24) and (29) in propositions 5 and 6. We obtain the following:

Proposition 9: *Let $\omega^* \in [0,1]$ be such that the following two conditions are satisfied*

$$\mathcal{L}_2 \;>\; \mathcal{L}_1(\omega^*)\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right) \tag{43}$$

$$\frac{\lambda^L+\beta-G}{\lambda^H+\beta-G} \;>\; \frac{\mathcal{L}_1(\omega^*)+\lambda^L\mathcal{L}_2}{(1-c^H)\,\mathcal{L}_1(\omega^*)+\lambda^H\mathcal{L}_2} \tag{44}$$

where

$$\mathcal{L}_1(\omega) = \omega A^{fi}_{\lambda^H} + (1-\omega)\left(\frac{z-G-\delta}{r-g}\right); \mathcal{L}_2 = \frac{z-g-\delta}{(\beta-g)(r-g)}$$

and $A^{fi}_{\lambda^H}$ is in (13). Then the combined compensation $w = \omega^*\eta_p P_t + (1-\omega^*)\,\eta_d D_t$ achieves the first best equilibrium: high-effort /reveal.

Condition (43) guarantees that reveal is optimal at time τ^*, conditional on $\lambda = \lambda^H$ in the full information pricing function (12). The second condition (43) guarantees that high effort is optimal at $t < \tau^*$, conditional on reveal being optimal at time τ^*.

Figure 8 shows the range of stock share in the compensation package, ω, that can induce the first best outcome as a function of G. That is, those ω's that satisfy both conditions (43) and (44). The top panel corresponds to the case in which the return on capital is $z = 20\%$ and the bottom panel to the case in which $z = 25\%$. In each panel, the upper line indicates the ω at which the manager is indifferent between conceal and reveal strategies under the high effort choice. For values of ω below that the manager prefers to reveal, which is what long term shareholders would like him to do. The lower line represents the level of at which the manager is indifferent between choosing high and low effort, when he is in a reveal equilibrium. For ω above that the manager prefers to exert high effort. Thus the area between the two lines represents all possible combinations of stocks and dividends in the compensation package that would support the first best.

Notice that when the lower line reaches zero, this means that dividend-based compensation alone is enough to induce high effort (recall the top-right corner in Figure 6). This does not mean that a little stock-based component would necessarily ruin the incentives to reveal, but aggressive stock compensation (or high proportion of ownership) will. This is true for other levels of G as well: there is a range of compensation packages that induce the first best. Figure 9 shows the minimum weights in stocks for the whole range of G and z.

Another way to see the effect of the optimal compensation package is by looking at Figure 10, which repeats Figure 6, while also indicating the area in which the first best is obtained. Compared with the only stock-based compensation, the combined compensation package not only increases the area in which a pure strategy Nash equilibrium is obtained, but the equilibrium is a first best equilibrium, in that it achieves high effort and revealing.

In conclusion, this section shows that the choice of a compensation package should be firm-specific and depends on the firm's characteristics. As a consequence, the exact compensation package that induces the manager to act in the interest of the shareholders in all stages of the life cycle of the firm has to be chosen carefully. In particular, excessive stock compensation or too little stock compensation are clearly suboptimal choices for most cases. This implies, for instance, that executives' bonuses that depend exclusively on either earnings or stocks performance are not advisable. For the same reasons, situations in which the CEO owns a large packet of shares will also likely lead to suboptimal investment. Our model suggests that the manager should either reduce his holdings to what would be prescribed under the optimal compensation level, or commit to holding on to his shares for a very long term, in return the company should tie a large part of his compensation to dividends.[13]

[13]Indeed, our model provides a rationale to the vesting of stock shares in compensation packages, although the vesting period that is implied by our model is much longer than the relatively standard three years (see e.g. Figure 3). While theoretically it is advisable a long-term vesting, realistically the long term performance of a firm depends on stochastic variables that are independent of the manager's actions. Standard risk aversion arguments would imply that managers would demand a large compensation in exchange for a longer vesting of shares in their compensation contract.

IV.F CEO's incentives costs to the firm

In our analysis so far we have abstracted from the costs that different incentive schemes impose on the firm itself. For instance, inducing high effort by using the combined compensation package discussed in the previous section may be too costly, and thus the firm could be better off with a low effort equilibrium. Endogenizing the costs to the firm in our dynamic model, however, is quite hard, as dividend flows have to be adjusted depending on the equilibrium, and the fixed point that sustains the Nash equlibrium is harder to obtain. However, we can approximate the size of these costs in the various equilibria by taking their present value at the cost of capital of the firm (r). We can then compare the value of the firm net of these costs across the various incentive schemes and gauge whether an incentive scheme is too honerous compared to another.

More specifically, we approximate the total costs paid to the CEO under the various cases by computing the following quantity:

$$V = E\left[\int_0^\infty e^{-rt} w_t dt\right] \tag{45}$$

where w_t is the CEO compensation in the various cases. Indeed, consider first as a benchmark the pure dividend based compensation under full revelation and low effort. This is an equilibrium for most parameters. In this case, $w_t = \eta_d D_t$ and, for given value of η_d, the present value of the total cost to the firm can be computed as

$$V_{Div}^{Reveal,L} = E\left[\int_0^\infty e^{-rt}\eta_d D_t dt\right] = \eta_d A_{\lambda^L}^{fi} \tag{46}$$

where $A_{\lambda^L}^{fi}$ is given in (13).

In a similar fashion we can compute the present value of all payments to the CEO under the stock compensation and conceal equilibrium in which the CEO exerts high effort. In this case, $w_t = \eta_p P_{ai,t}$, where $P_{ai,t}$ is given by (20) and (22). We obtain

$$V_{Stock}^{Conceal,H} = E\left[\int_0^{T^*} e^{-rt}\eta_p P_{ai,t} dt\right] \tag{47}$$

$$= \eta_p \left[(z-G-\delta)\left(\frac{1-e^{-(r-G)h^{**}}}{r-G}\right)^2 + A_{\lambda^H}^{ai} e^{-(r-G)h^{**}}\left(h^{**} + \frac{z-G-\delta}{r+\lambda-G}\right)\right] \tag{48}$$

where A_{λ}^{ai} is given in (21).

We now notice that we have enough degrees of freedom to choose η_p such that the present value of payments in the two cases above, namely (46) and (47), equal each other. That is, such that $V_{Stock}^{Conceal,H} = V_{Div}^{Reveal,L}$. Indeed, under a pure compensation scheme, the absolute level of η_d or η_p has no impact on the decision to conceal or reveal at time τ^*. This can be easily seen by comparing the utility expressions (24) and (25) for the dividend-based compensation, and expressions (29), (30) and (31) for the stock-based compensation. In all cases, the constants η_d or η_p drop out, and the CEO decision at time τ^* is really independent of this quantity.

The combined compensation, in constrast, depends on both η_d and η_p. The total cost under the full revelation / high effort equilibrium can be computed in the same fashion as above, to yield

$$V_{Comb}^{Reveal,H} = \omega \eta_p \left(A_{\lambda^H}^{fi} + \lambda^H \frac{z-g-\delta}{(r-g)^2} \right) \frac{1}{(r-G+\lambda^H)} + (1-\omega)\,\eta_d A_{\lambda^H}^{fi} \tag{49}$$

where $A_{\lambda^H}^{fi}$ is given in (13).

We now compare the costs to the firm from using these incentive schemes. We do so by computing, for each case, the quantity $P_t - V_t$, that is, the firm value net of payments to the CEO. We choose $\eta_d = 5\%$ as our benchmark value under dividend compensation and full revelation. That is, in this case the CEO's compensation equals 5% of firm value (see 46). The net firm value in this case is then $P_{fi,\lambda^L,0}^{before,fi} - V_{Div}^{reveal,H} = (1-\eta_d)A_{\lambda^L}^{fi}$.

Given this benchmark value for η_d, we compute then η_p to make $V_{Stock}^{Conceal,H} = V_{Div}^{Reveal,L}$, and compute the net value of the stock $P_{ai,0} - V_{Stock}^{Conceal,H}$. Finally, given these two values for η_d and η_p, we can compute the cost for the mixed compensation, $V_{Comb}^{Reveal,H}$ and compute the net firm value $P_{fi,0}^{before} - V_{Comb}^{Reveal,H}$. One additional choice has to be made in this case, namely, the weight ω to use in the compensation scheme: as shown in Figure 8 there is a whole range that would do. We choose the minimum ω that induces the CEO to exert costly effort: in this case, when G is high, the combined compensation boils down to a dividend-based compensation with high effort, as shown in the bottom panel of Figure 8.

Figure 11 plots the firm value net of payments to the CEO for the three cases discussed above, as a function of the high growth rate G and for two values of return on capital z. In both panels, the solid line corresponds to the mixed compensation case, which leads to a high effort/reveal equilibrium. The dotted line corresponds to the stock based, high effort/conceal equilibrium. Finally, the dashed line corresponds to the low effort/reveal equilibrium. The figure makes apparent two facts: First, inducing high effort increases the net firm value, especially for high growth companies. This is true for both the stock based compensation, which has a conceal behavior as a side effect, and the combined compensation. Second, the combined compensation leads to a higher net firm value compared to the stock based, conceal equilibrium case, although the difference is small when the return on capital is small.

This analysis, although approximate, shows that indeed, the combined optimal compensation plan discussed in the previous section achieve the first best without imposing too high a burden on the company.

V Conclusions

Our paper contributes to the debate on executive compensation.[14] On the one hand, advocates of stock-based compensation highlight the importance of aligning shareholder objectives with managers' and argue that compensating managers with stocks achieves the goal. Detractors argue that stock-based compensation instead gives managers the incentives to misreport the true state of the firm, and in fact even engage at times in outright fraudulent behavior. This paper sheds new light on the debate by analyzing both the ex ante incentive problem to induce managers to exert costly effort to maximize the firms' investment opportunities, and simultaneously to induce the manager to reveal the true state of the firm's outlook and thus follow an optimal investment rule.

We find that stock-based compensation does in fact induce the manager to conceal a weakening of the firm's investment opportunities. In order to do so, the manager must follow grossly suboptimal investment rules. This behavior destroys firm value. This problem is particularly acute for growth firms. In stark contrast, a performance based contract that is based on cash flows or earnings (but not stocks) always achieve truth telling, and thus an optimal investment strategy.

Stock-based compensation, however, is much more effective than cash flow based compensation in inducing the manager to exert costly effort and constantly seek additional investment opportunities for the firm. That is, stock-based compensation *does* in fact align the manager's objective with the shareholders'. This good outcome for the shareholders, though, is countervailed by the fact that the manager will not reveal any worsening of the investment opportunities if it happens, and will then invest suboptimally later on, as discussed above.

While we believe that the problem with stock-based compensation in growth firms is widespread in general, the 1990's Hi-Tech boom provides an interesting illustration of the mechanism discussed in our model. This period was characterized by expectations of high growth rates and high uncertainty, coupled with high-powered stock-based executive compensation. Firms with perceived high growth options were priced much higher than firms with similar operating performance, but with lower perceived growth options. We argue that because of their high-powered incentives, executives had an incentive to present theirs as high growth firms, even when the prospects of high future growth faded at the end of the 1990s. Our analysis suggests that the combination of high-powered incentives and the pretense of high growth firms will lead eventually to the firm's stock to crash. Indeed, while all the effects in our paper are purely firm-specific, an extension of our model in which the timings of a slow down in investment opportunities are correlated across firms may be able to predict market crashes as well.[15] We leave this extension to future research.

Finally, our model enables us to make a normative statement as well. In fact, we show

[14]See Hall and Murphy (2003), Bebchuk and Fried (2004), and Gabaix and Landier (2007) for recent discussions.

[15]See Zeira (1999) and Hertzberg (2005) for models of market crashes that occur when market participants suddenly realize that the unobserved aggregate state.

that a combined compensation package that uses both dividends-based performance and stock-based performance reaches the first best, inducing the manager to exert costly effort and reveal any worsening of the investment opportunities, if it happens. Firm value is then maximized in this case. Each component (dividends and stocks) in the combined compensation package serves a different purpose and thus they are both necessary "ingredients": the stock-based component increases the manager's effort to expand the growth options of the firm, while compensating managers also proportionally to reported earnings significantly reduces his incentives to engage in value destroying activities to support the inflated expectations. It is crucial to realize, though, that the weight on stocks in the combined compensation package is not identical across firms: for instance, high growth firms should not make much use of stocks in their compensation package, while the opposite is true for low growth firms. That is, there is no fixed rule that work for every type of firm. As a consequence, generalized regulatory decisions that ban stock-based compensation, for instance, or board of directors' decisions on CEO compensation that are based on some "common wisdom" are particularly dangerous, as they do not consider that each firm necessitates a different incentive scheme.

Appendix

Proof of Proposition 1. For any $t > \tau^*$ we have

$$P_{fi,t}^{after} = \int_t^\infty D_t e^{-r(s-t)} ds = (z-g-\delta) e^{G\tau^* + g(t-\tau^*)} \int_t^\infty e^{-(r-g)(s-t)} ds$$
$$= \left(\frac{z-g-\delta}{r-g}\right) e^{G\tau^* + g(t-\tau^*)}. \tag{50}$$

In particular, at $t = \tau^*$ we have

$$P_{fi,\tau^*}^{after} = \left(\frac{z-g-\delta}{r-g}\right) e^{G\tau^*}. \tag{51}$$

Thus, for $t < \tau^*$, we have

$$P_{fi,t}^{before} = E_t \left[\int_t^{\tau^*} e^{-r(s-t)} D_s ds + e^{-r(\tau^*-t)} P_{fi,\tau^*}^{after} \right] \tag{52}$$

where $D_s = (z - G - \delta)e^{Gs}$. By using integration by parts, we find

$$P_{fi,t}^{before} = \int_t^\infty e^{-(r+\lambda)(\tau^*-t)} D_{\tau^*} + \lambda e^{-(r+\lambda)(\tau^*-t)} P_{fi,\tau^*}^{after} d\tau^* \tag{53}$$
$$= \int_t^\infty (z-G-\delta) e^{-(r+\lambda)(\tau^*-t)} e^{G\tau^*} + \lambda \left(\frac{z-g-\delta}{r-g}\right) e^{-(r+\lambda)(\tau^*-t)} e^{G\tau^*} d\tau^* \tag{54}$$
$$= \frac{(z-G-\delta)}{r+\lambda-G} e^{Gt} + \lambda \left(\frac{z-g-\delta}{(r-g)(r+\lambda-G)}\right) e^{Gt} \tag{55}$$
$$\tag{56}$$

Q.E.D.

Proof of Proposition 2: The dividend stream corresponding to a G firm is:

$$D_t^G = (z - G - \delta)e^{Gt}. \tag{57}$$

After τ^*, the manager of the g firm must reproduce this pattern for as long as possible, i.e. as long as $K_t > \underline{K}_t$. Recall that for an arbitrary investment policy the dividend is:

$$D_t = z \min(K_t, J_t) - I_t, \tag{58}$$

where $J_t = e^{G\tau^* + g(t-\tau^*)}$, as given in (7). Since the capital evolution is given by the differential equation

$$dK_t/dt = I_t - \delta K_t,$$

it follows that the investment policy I_t that generates the required dividend stream D_t^G in (57) subject to (58) for at least some time, has to solve (17). Given the continuity of the solution of an ODE in its initial condition, $K_t = f(K_{\tau^*})$, ODE (17) implies that the amount of capital available at time t, K_t, is monotonically increasing in its starting value K_{τ^*}. Since T^{**} is determined by the condition $K_{T^{**}} = \underline{K}_{T^{**}} = hJ_{T^{**}}$, where the latter is exogenously specified, but monotonically increasing, it follows from Lemma 1 that the optimal choice of the manager of a g firm is to invest as much as possible in the technology at τ^*. Given a maximum amount $K_{\tau^*} = K_{\tau^*-}$ of capital available, this amount is the optimal solution. The path for investments follows from $I_t = z\min(K_t, J_t) - dK_t/dt$ and equation (17). **Q.E.D.**.

Proof of Proposition 3: The amount of capital at time τ^* is $K_{\tau^*} = J_{\tau^*} = e^{G\tau^*}$. As it can be seen, this amount enters the optimal solution (16) and (17) in a multiplicative fashion. Indeed, we can rewrite (16) and (17) by pulling $e^{G\tau^*}$ as a common factor on the right hand sides, obtaining

$$I_t = e^{G\tau^*}\left(z\ \min\left(K_t/e^{G\tau^*}, e^{g(t-\tau^*)}\right) - (z-G-\delta)\, e^{G(t-\tau^*)}\right),$$
$$\frac{dK}{dt} = e^{G\tau^*}\left(z\ \min(K_t/e^{G\tau^*}, e^{g(t-\tau^*)}) - \delta K_t/e^{G\tau^*} - (z-G-\delta)\, e^{G(t-\tau^*)}\right)$$

Since the initial condition of the ODE is $K_{\tau^*} = e^{G\tau^*}$, we can define new variables $\tilde{K}_t = K_t/e^{G\tau^*}$ and $\tilde{I}_t = I_t/e^{G\tau^*}$. We then have that these latter two variables satisfy

$$\tilde{I}_t = z\ \min\left(\tilde{K}_t, e^{g(t-\tau^*)}\right) - (z-G-\delta)\, e^{G(t-\tau^*)},$$
$$\frac{d\tilde{K}}{dt} = z\ \min(\tilde{K}_t, e^{g(t-\tau^*)}) - \delta\tilde{K}_t - (z-G-\delta)\, e^{G(t-\tau^*)}$$

with initial condition $\tilde{K}_{\tau^*} = 1$. This $\tilde{K}_{T^{**}} = he^{g(T^{**}-\tau^*)}$ if and only if $K_{T^{**}} = hJ_{T^{**}} = he^{G\tau^*+g(T^{**}-\tau^*)}$. That is, the distance between T^{**} and τ^* is independent of the capital accumulated up to τ^*, and thus default time is independent of τ^* itself. **Q.E.D.**

Proof of Proposition 4: Consider first the case in which $t > h^{**}$. If default has not been observed at time t, then a shift cannot have occurred before $t-h^{**}$. In other words, the conditioning event is that $\tau^* > t-h^{**}$. Recalling that τ^* has an exponential distribution, we have that default time $T^{**} = \tau^* + h^{**}$ conditional on no default by time t has the following conditional distribution:

$$\begin{aligned}F_{T^{**}}(t'|\tau^* > t-h^{**}) &= Pr\left(T^{**} < t'|\tau^* > t-h^{**}\right)\\ &= Pr\left(\tau^* < t'-h^{**}|\tau^* > t-h^{**}\right)\\ &= \frac{e^{-\lambda(t-h^{**})} - e^{-\lambda(t'-h^{**})}}{e^{-\lambda(t-h^{**})}}\\ &= 1 - e^{-\lambda(t'-t)}\end{aligned}$$

That is, the default time T^{**} has still the exponential distribution

$$f(T^{**}|\text{no default by } t) = \lambda e^{-\lambda(T^{**}-t)}$$

The value of the firm at time t is then equal to the present value of dividends D_t^G until default (at time T^{**}), plus the present value of $P^L_{ai,T^{**}}$ at default. That is

$$\begin{aligned}
P_{ai,t} &= E_t\left[\int_t^{T^{**}} e^{-r(s-t)} D_s^G ds + e^{-r(T^{**}-t)} P^L_{ai,T^{**}} | \text{no default by } t\right] \\
\text{[Use Integration by Parts]} &= \int_t^{\infty} e^{-(r+\lambda)(T^{**}-t)} D^G_{T^{**}} + \lambda e^{-(r+\lambda)(T^{**}-t)} P^L_{ai,T^{**}} dT^{**} \\
\left[\text{Substitute } D^G_{T^{**}}\right] &= \int_t^{\infty} e^{-(r+\lambda)(T^{**}-t)} (z-G-\delta) e^{GT^{**}} dT^{**} \\
\left[\text{Use } P^L_{ai,T^{**}} \text{ and } h^{**} = T^{**} - \tau^*\right] &+ \int_t^{\infty} \lambda e^{-(r+\lambda)(T^{**}-t)} e^{G(T^{**}-h^{**})+gh^{**}} \left(\frac{z-r-\delta+\xi(r-g)}{r-g}\right) dT^{**} \\
&= e^{Gt}\frac{(z-G-\delta)}{(r+\lambda-G)} + \lambda e^{G(t-h^{**})+gh^{**}} \left(\frac{z-r-\delta+\xi(r-g)}{(r-g)(r+\lambda-G)}\right) \\
&= e^{Gt} A^{ai}_{\lambda},
\end{aligned}$$

where

$$A^{ai}_{\lambda} \equiv \frac{(z-G-\delta)}{(r+\lambda-G)} + \lambda e^{-(G-g)h^{**}} \left(\frac{z-r-\delta+\xi(r-g)}{(r-g)(r+\lambda-G)}\right).$$

If $t < h^{**}$, then the conditional distribution of T^{**} is zero in the range $[t, h^{**}]$: Indeed, even if a shift occurred at time 0, there would be no default before h^{**}. Thus, we have

$$f(T^{**}) = \lambda e^{-\lambda(T^{**}-h^{**})} 1_{T^{**}>h^{**})}$$

We then obtain

$$\begin{aligned}
P_{ai,t} &= E_t\left[\int_t^{T^{**}} e^{-r(s-t)} D_s^G ds + e^{-r(T^{**}-t)} P^L_{ai,T^{**}} | \text{no default by } t\right] \\
\text{[No default before } h^{**}] &= \int_t^{h^{**}} e^{-r(s-t)} D_s^G ds \\
&+ e^{-r(h^{**}-t)} E_t\left[\int_{h^{**}}^{T^{**}} e^{-r(s-h^{**})} D_s^G ds + e^{-r(T^{**}-h^{**})} P^L_{ai,T^{**}} | \text{no default by } h^{**}\right] \\
\text{[Use } P_{ai,t} \text{ above for } t = h^{**}] &= \int_t^{h^{**}} e^{-r(s-t)} D_s^G ds + e^{-r(h^{**}-t)} e^{Gh^{**}} A^{ai}_{\lambda} \\
&= \int_t^{h^{**}} e^{-r(s-t)} (z-G-\delta) e^{Gs} ds + e^{rt} e^{(G-r)h^{**}} A^{ai}_{\lambda} \\
&= (z-G-\delta) e^{Gt} \frac{1-e^{-(r-G)(h^{**}-t)}}{(r-G)} + e^{rt} e^{(G-r)h^{**}} A^{ai}_{\lambda}
\end{aligned}$$

Q.E.D

Proof of Proposition 5: At time τ^* the manager must decide whether he/she wants to reveal the change in growth rate or not. Given that any change in dividend policy results in an audit, we know from Lemma 1 that if the manager decides to conceal the new growth rate, he/she will choose an investment strategy to maximize the default time T^{**} given in Proposition 2. In this case, the dividend path is D_t^G until T^{**}. The behavior after T^{**} does not matter. If instead the manager decides to reveal the new state, the dividend path after τ^* is given by D_t^g. Thus, we must simply compare two utilities.

$$
\begin{aligned}
U_{Div,\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_d D_t^g)dt \\
U_{Div,\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta_d D_t^G)dt
\end{aligned}
$$

We can compute the value of the two utilities exactly

$$
\begin{aligned}
U_{Div,\tau^*}^{reveal} &= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_d D_t^g)dt \\
&= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}\left(\eta_d(z-g-\delta)\right) e^{G\tau^*+g(t-\tau^*)}dt \\
&= (\eta_d(z-g-\delta))e^{G\tau^*}\int_{\tau^*}^{\infty} e^{-(\beta-g)(t-\tau^*)}dt \\
&= \frac{(\eta_d(z-g-\delta))}{(\beta-g)}e^{G\tau^*}
\end{aligned}
$$

Similarly, the utility under the "conceal" strategy is

$$
\begin{aligned}
U_{Div,\tau^*}^{conceal} &= \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta_d D_t^G)dt \\
&= \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta_d(z-G-\delta)e^{Gt}dt \\
&= (\eta_d(z-G-\delta)e^{G\tau^*}\int_{\tau^*}^{T^*} e^{-(\beta-G)(t-\tau^*)}dt \\
&= \frac{(\eta_d(z-G-\delta)}{(\beta-G)}e^{G\tau^*}\left(1-e^{-(\beta-G)h^{**}}\right) \\
&= (\eta_d(z-G-\delta))e^{G\tau^*}\left(\frac{\left(1-e^{-(\beta-G)h^{**}}\right)}{(\beta-G)}\right)
\end{aligned}
$$

Proof of Proposition 6: At time τ^* the manager must decide whether he/she wants to reveal the change in growth rate or not. Given that any change in dividend policy results in an audit, we know from Lemma 1 that if the manager decides to conceal the new growth rate, he/she will choose an investment strategy to maximize the default time T^{**} given in Proposition 2. In this case, the price path will be given by the one described in equations (22) and (20) until T^{**}. The behavior of prices after T^{**} does not matter. If instead the manager decides to reveal the new state, the price path after τ^* is given by $P_{fi,t}^{after}$ in equation

(12). Thus, we must simply compare three utilities (two for each equilibrium).

$$U^{reveal}_{Stock,\tau^*} = \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta P^{after}_{fi,t})dt$$
$$U^{conceal,ai}_{Stock,\tau^*} = \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta P_{ai,t})dt$$
$$U^{conceal,fi}_{Stock,\tau^*} = \int_{\tau^*}^{T^*} e^{-\beta(t-\tau^*)}(\eta P^{before}_{fi,t})dt$$

We concentrate on the conceal Nash equilibrium. The computations for the reveal equilibrium are identical. Note that after τ^* there is no longer any uncertainty on the price path, as discussed in Lemma 1. We can compute the value of the three utilities exactly:

$$\begin{aligned}
U^{reveal}_{Stock,\tau^*} &= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}(\eta_p P^{after}_{fi,t})dt \\
&= \int_{\tau^*}^{\infty} e^{-\beta(t-\tau^*)}\left(\eta_p \frac{z-g-\delta}{r-g}\right)e^{G\tau^*+g(t-\tau^*)}dt \\
&= \left(\eta_p \frac{(z-g-\delta)}{(r-g)}\right)e^{G\tau^*}\int_{\tau^*}^{\infty} e^{-(\beta-g)(t-\tau^*)}dt \\
&= \frac{e^{G\tau^*}}{(\beta-g)}\left(\frac{\eta_p(z-g-\delta)}{(r-g)}\right)
\end{aligned}$$

Similarly, assume that $\tau^* > h^{**}$, so that for this calculation only price (20) applies. Then

$$\begin{aligned}
U^{conceal,ai}_{Stock,\tau^*} &= \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}(\eta_p P_{ai,t})dt \\
&= \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}e^{Gt}(\eta_p A^{ai}_{\lambda})dt \\
&= (\eta_p A^{ai}_{\lambda})e^{G\tau^*}\int_{\tau^*}^{T^{**}} e^{-(\beta-G)(t-\tau^*)}dt \\
&= (\eta_p A^{ai}_{\lambda})e^{G\tau^*}\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right)
\end{aligned}$$

Note that the same calculation also yields the formula for the utility under a conceal strategy but in a reveal equilibrium $U^{conceal,fi}_{Stock,\tau^*}$. The only difference is that A^{ai}_{λ} has to be substituted for A^{fi}_{λ}. This formula holds for every τ^*.

If $\tau^* < h^{**}$, then the utility function uses the pricing function (22), yielding

$$\begin{aligned}
U^{conceal,ai}_{Stock,\tau^*} &= \int_{\tau^*}^{T^{**}} e^{-\beta(t-\tau^*)}\eta_p P_{ai,t}dt \\
&= e^{\beta\tau^*}\int_{\tau^*}^{h^{**}} e^{-\beta t}\eta_p P_{ai,t}dt + e^{\beta\tau^*}\int_{h^{**}}^{T^{**}} e^{-\beta t}\eta_p P_{ai,t}dt
\end{aligned}$$

Using the two formulas for $P_{ai,t}$ before and after h^{**}, and taking the integral, tedious calculations show:

$$U^{conceal,ai}_{Stock,\tau^*} = e^{\beta\tau^*}\eta_p\frac{(z-G-\delta)}{r-G}\left(\frac{e^{-(\beta-G)\tau^*}-e^{-(\beta-G)h^{**}}}{\beta-G} - e^{-(r-G)h^{**}}\frac{e^{-(\beta-r)\tau^*}-e^{-(\beta-r)h^{**}}}{\beta-r}\right)$$
$$+\eta_p e^{\beta\tau^*}e^{-(r-G)h^{**}}A^{ai}_{\lambda}\frac{e^{-(\beta-r)\tau^*}-e^{-(\beta-r)h^{**}}}{\beta-r} + e^{\beta\tau^*}\eta_p A^{ai}_{\lambda}e^{-(\beta-G)h^{**}}\frac{1-e^{-(\beta-G)\tau^*}}{\beta-G}$$

Note that at $\tau^* = h^{**}$ we have

$$U^{conceal}_{\tau^*} = e^{\beta\tau^{**}}\eta_p A^{ai}_{\lambda}e^{-(\beta-G)h^{**}}\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}$$
$$= \eta_p A^{ai}_{\lambda}e^{Gh^{**}}\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}$$

which equal the one above for $\tau^* \geq h^{**}$, so that it converges there. **Q.E.D.**

Proof of Proposition 7: If e^H is chosen, then λ^H is the probability of switching. Let $U^{conceal,ai}_{Stock,\tau^*}$ be the utility at τ^* from the high effort conceal equilibrium (i.e. using λ^H). Its formula is in Proposition 6 for the two cases in which $\tau^* < h^{**}$ and $\tau^* > h^{**}$. We now compute the utility of the agent conditional to either e^H or e^L, and check that the utility from high effort is larger than from low effort: $U\left(e^H\right) > U\left(e^L\right)$. For the case in which $t \geq h^{**}$, we have

$$U_{Stock,t} = E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(w_u\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}U^{conceal,ai}_{Stock,\tau^*}\right]$$
$$= \int_t^{\infty} e^{-(\beta+\lambda(e_u))(u-t)}\left(\eta_p P_{ai,u}\left(1-c\left(e_u\right)\right)\right) + \lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}U^{conceal,ai}_{Stock,u}du$$

Given that $t > h^{**}$, then also $\tau^* > h^{**}$, which implies that equations (20) with $\lambda = \lambda^H$ applies

$$U_{Stock,t} = \int_t^{\infty} e^{-(\beta+\lambda(e_u))(u-t)}\left(\eta_p e^{Gu}A^{ai}_{\lambda^H}\left(1-c\left(e_u\right)\right)\right) + \lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}\eta_p A^{ai}_{\lambda^H}e^{Gu}H^{Stock}du$$
$$= e^{Gt}\eta_p A^{ai}_{\lambda^H}\int_t^{\infty} e^{-(\beta+\lambda(e_u)-G)(u-t)}\left[\left(1-c\left(e_u\right)\right)+\lambda\left(e_u\right)H^{Stock}\right]du$$

where

$$H^{Stock} = \frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}$$

Note that the choice e^H versus e^L only affects the integral, which is time independent. Thus, if either e^H or e^L is optimal at t, it is optimal at any other time, as the objective function is the same. Thus, we obtain

$$U_{Stock,t} = \int_t^{\infty} e^{-(\beta+\lambda(e_u))(u-t)}\left(\eta_p e^{Gu}A^{ai}_{\lambda^H}\left(1-c\left(e_u\right)\right)\right) + \lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}\eta_p A^{ai}_{\lambda^H}e^{Gu}H^{Stock}du$$
$$= e^{Gt}\eta_p A^{ai}_{\lambda^H}\frac{\left[1-c\left(e\right)+\lambda\left(e\right)H^{Stock}\right]}{\beta+\lambda\left(e\right)-G}$$

We obtain

$$U\left(e^{H}\right)>U\left(e^{L}\right) \text{ iff } \frac{\left[1-c^{H}+\lambda^{H}H^{Stock}\right]}{\lambda^{H}+\beta-G}>\frac{\left[1-c^{L}+\lambda^{L}H^{Stock}\right]}{\lambda^{L}+\beta-G}$$

which leads to the condition in the statement of the Proposition.

Conditional on e^H being chosen by the manager, then λ^H applies, and a conceal equilibrium obtains at time τ^*. The price function P_t then is given by equation (20), concluding the proof.

Formula for $t < h^{**}$ In the case $t < h^{**}$, it is useful to denote by $\overline{U}_{h^{**}}$ the utility computed in the previous part when $t = h^{**}$. We obtain

$$\begin{aligned}
U_{Stock,t} &= \int_t^{\infty} e^{-(\beta+\lambda(e_u))(u-t)}\left(\eta_p P_{ai,u}\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}U^{conceal,ai}_{Stock,u}du \\
&= \int_t^{h^{**}} e^{-(\beta+\lambda(e_u))(u-t)}\left(\eta_p P_{ai,u}\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}U^{conceal,ai}_{Stock,u}du \\
&+ e^{-(\beta+\lambda(e_u))(h^{**}-t)}\int_{h^{**}}^{\infty} e^{-(\beta+\lambda(e_u))(u-h^{**})}\left(\eta_p P_{ai,u}\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e_u\right)e^{-(\beta+\lambda(e_u))(u-t)}U^{conceal,ai}_{Stock,u}du \\
&= \int_t^{h^{**}} e^{-(\beta+\lambda(e_u))(u-t)}\left[\left(\eta_p P_{ai,u}\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e_u\right)U^{conceal,ai}_{Stock,u}\right]du+e^{-(\beta+\lambda(e_u))(h^{**}-t)}\overline{U}_{h^{**}}
\end{aligned}$$

We solve this numerically, and check whether $U_t(e^H) > U_t(e^L)$. **Q.E.D.**

Proof of Proposition 8. In this case, the reveal equilibrium is the prevailing outcome for most parameter configurations, and we focus on this case. Thus, ex ante, the utility of the entrepreneur (for any t)

$$U_{Div,t}=E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(\eta_d D_u\left(1-c\left(e_u\right)\right)\right)du+e^{-\beta(\tau^*-t)}U^{reveal}_{Div,\tau^*}\right]$$

We then have

$$\begin{aligned}
U_{Div,t} &= \int_t^{\infty} e^{-(\beta+\lambda(e))(u-t)}\left(\eta_d D^G_u\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e\right)e^{-(\beta+\lambda)(u-t)}U^{reveral}_{Div,u}du \\
&= \int_t^{\infty} e^{-(\beta+\lambda(e))(u-t)}\left(\eta_d\left(z-G-\delta\right)e^{Gu}\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e\right)\left(e^{Gu}\eta_d\frac{(z-g-\delta)}{\beta-g}\right)du \\
&= e^{Gt}\int_t^{\infty} e^{-(\beta+\lambda(e)-G)(u-t)}\left[\left(\eta_d\left(z-G-\delta\right)\left(1-c\left(e_u\right)\right)\right)+\lambda\left(e\right)\eta_d\frac{(z-g-\delta)}{\beta-g}\right]du \\
&= e^{Gt}\eta_d\left(z-G-\delta\right)\left[1-c\left(e_u\right)+\lambda\left(e\right)\frac{(z-g-\delta)}{(z-G-\delta)(\beta-g)}\right]\frac{1}{\beta+\lambda-G}
\end{aligned}$$

Let

$$H^{Div}=\frac{(z-g-\delta)}{(z-G-\delta)(\beta-g)}$$

we obtain the condition

$$U(e_H) > U(e_L) \text{ iff } \frac{\left[1 - c^H + \lambda^H H^{Div}\right]}{\beta + \lambda^H - G} > \frac{\left[1 - c^L + \lambda^L H^{Div}\right]}{\beta + \lambda^L - G}$$

or e^H is the equilibrium outcome if and only if

$$\frac{\beta + \lambda^L - G}{\beta + \lambda^H - G} > \frac{\left[1 - c^L + \lambda^L H^{Div}\right]}{\left[1 - c^H + \lambda^H H^{Div}\right]}$$

Since in this case there is no feedback effect of prices on the decision of the manager, and a reveal equilibrium obtains at τ^* independently of his/her decision, the statement of the proposition follows. **Q.E.D.**

Proof of Corollary 4: The function

$$f(x) = \frac{1 + \lambda^L x}{1 - c^H + \lambda^H x}$$

is increasing if and only if

$$\begin{aligned} f'(x) &= \frac{\lambda^L \left(1 - c^H + \lambda^H x\right) - \left(1 + \lambda^L x\right) \lambda^H}{\left(1 - c^H + \lambda^H x\right)^2} \\ &= \frac{\lambda^L \left(1 - c^H\right) - \lambda^H}{\left(1 - c^H + \lambda^H x\right)^2} > 0 \end{aligned}$$

This is satisfied if and only if

$$\frac{\lambda^L}{\lambda^H} > \frac{1}{\left(1 - c^H\right)}$$

Q.E.D.

Proof of Proposition 9: First, we need to compute the condition that guarantees a reveal strategy at time τ^*. Using the linearity of the utility function and the previous computed utility values, we obtain

$$\begin{aligned} U^{reveal}_{comb,\tau^*} &= \omega \eta_p \frac{e^{G\tau^*}}{r - g} \left(\frac{z - g - \delta}{\beta - g}\right) + (1 - \omega)\, \eta_d e^{G\tau^*} \left(\frac{z - g - \delta}{\beta - g}\right) \\ &= \left(\omega \eta_p \frac{1}{r - g} + (1 - \omega)\, \eta_d\right) e^{G\tau^*} \left(\frac{z - g - \delta}{\beta - g}\right) \\ U^{conceal}_{comb,\tau^*} &= \omega \left(\eta_p A^{fi}_\lambda e^{G\tau^*} \left(\frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G}\right)\right) + (1 - \omega)\, \eta_d \left(z - G - \delta\right) e^{G\tau^*} \left(\frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G}\right) \\ &= \left(\omega \eta_p A^{fi}_\lambda + (1 - \omega)\, \eta_d \left(z - G - \delta\right)\right) e^{G\tau^*} \left(\frac{1 - e^{-(\beta - G)h^{**}}}{\beta - G}\right) \end{aligned}$$

Thus, at τ^*, we must impose the condition for a reveal equilibrium

$$U^{reveal}_{comb,\tau^*} > U^{conceal}_{comb,\tau^*}$$

or

$$\left(\omega\eta_p \frac{1}{r-g} + (1-\omega)\,\eta_d\right)\left(\frac{z-g-\delta}{\beta-g}\right) > \left(\omega\eta_p A^{fi}_{\lambda} + (1-\omega)\,\eta_d\,(z-G-\delta)\right)\left(\frac{1-e^{-(\beta-G)h^{**}}}{\beta-G}\right)$$

Defining $\eta_d = \eta_p/(r-g)$, and dividing through by η_p, we obtain condition (43).

Before τ^* the equilibrium must support high effort. So, suppose that in equilibrium λ^H is the true λ, so that the above inequality must hold with $A^{fi}_{\lambda^H}$. Before τ^* the utility using the combined package depends on its two components, one that depends only on dividends, and one that depends only on stocks. We can compute the two components separately, as follows:

- Dividend Component: the utility under dividend compensation (and reveal at τ^*) is

$$\begin{aligned} U_{Div,t} &= E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(w_u\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}U^{reveal}_{Div,\tau^*}\right] \\ &= E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(\eta_d D^G_u\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}\eta_d e^{G\tau^*}\left(\frac{z-g-\delta}{\beta-g}\right)\right] \end{aligned}$$

  Integration by parts and taking the integral yield

$$U_{Div,t} = \frac{e^{Gt}}{\beta+\lambda(e)-G}\eta_d\left(\left(z-G-\delta\right)\left(1-c\left(e_u\right)\right) + \lambda\left(e_u\right)\left(\frac{z-g-\delta}{\beta-g}\right)\right)$$

- Stock Component: Similarly, the utility under stock compensation (and reveal at τ^*) is

$$\begin{aligned} U_{Stock,t} &= E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(w_u\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}U^{reveal}_{Stock,\tau^*}\right] \\ &= E\left[\int_t^{\tau^*} e^{-\beta(u-t)}\left(\eta_p P^{before}_{fi,u}\left(1-c\left(e_u\right)\right)\right)du + e^{-\beta(\tau^*-t)}\eta_p\frac{e^{G\tau^*}}{r-g}\left(\frac{z-g-\delta}{\beta-g}\right)\right] \end{aligned}$$

  Note that we use the full information price function, as it is common knowledge in a Nash reveal equilibrium. Integration by parts and taking the integral yield

$$U_{Stock,t} = \frac{e^{Gt}}{\beta+\lambda(e)-G}\eta_p\left(A^{fi}_{\lambda^H}\left(1-c\left(e_u\right)\right) + \frac{\lambda(e)}{r-g}\left(\frac{z-g-\delta}{\beta-g}\right)\right)$$

Thus, the total combined utility before τ^*

$$\begin{aligned} U_{Comb,t}(e) &= \omega U_{Stock,t} + (1-\omega) U_{Div,t} \\ &= \frac{e^{Gt}}{(\beta+\lambda(e)-G)} \times \begin{pmatrix} \omega\eta_p \left(A^{fi}_{\lambda^H} (1-c(e_u)) + \frac{\lambda(e)}{r-g} \left(\frac{z-g-\delta}{\beta-g} \right) \right) \\ +(1-\omega)\eta_d \left((z-G-\delta)(1-c(e)) + \lambda(e) \left(\frac{z-g-\delta}{\beta-g} \right) \right) \end{pmatrix} \end{aligned}$$

Tedious computations show that the Nash condition

$$U_{Comb}\left(e^H\right) > U_{Comb}\left(e^L\right)$$

holds if and only if condition (44) is satisfied. **Q.E.D.**

Proof of Formulas (47)

The present value of total payment from the firm is

$$\begin{aligned} V^{Concealing,H}_{Stock} &= E\left[\int_0^{T^*} e^{-rt}\eta_p P_{ai,t}\right] \\ &= \int_0^{h^{**}} \eta_p e^{-rt} P_{ai,t} dt + E\left[\int_{h^{**}}^{T^{**}} e^{-rt}\eta_p P_{ai,t} dt\right] \end{aligned}$$

where the first integral stems from the fact that h^{**} is the period during no revelation will certainly occur. The first term is then given by

$$\begin{aligned} \int_0^{h^{**}} \eta_p e^{-rt} P_{ai,t} dt &= \int_0^{h^{**}} \eta_p e^{-(r-G)t} \left((z-G-\delta) \frac{1-e^{-(r-G)h^{**}}}{r-G} \right) dt + \int_0^{h^{**}} \eta_p e^{-rt} A^{ai}_{\lambda} e^{rt} e^{(G-r)h^{**}} dt \\ &= \eta_p \left[(z-G-\delta) \left(\frac{1-e^{-(r-G)h^{**}}}{r-G} \right)^2 + A^{ai}_{\lambda} e^{-(r-G)h^{**}} h^{**} \right] \end{aligned} \tag{59}$$

The second term is instead computed as follows: recall that by definition of h^{**} we have $T^{**} = \tau^{**} + h^{**}$

$$\begin{aligned} E\left[\int_{h^{**}}^{T^{**}} e^{-rt}\eta_p P_{ai,t} dt\right] &= E\left[\int_{h^{**}}^{h^{**}+\tau^{**}} e^{-rt}\eta_p P_{ai,t} dt\right] \\ &= \eta_p A^{ai}_{\lambda} E\left[\int_{h^{**}}^{h^{**}+\tau^{**}} e^{-rt} D^G_t dt\right] \\ &= \eta_p A^{ai}_{\lambda} (z-G-\delta) E\left[\int_{h^{**}}^{h^{**}+\tau^{**}} e^{-(r-G)t} dt\right] \\ &= \eta_p A^{ai}_{\lambda} (z-G-\delta) e^{-(r-G)h^{**}} E\left[\int_{h^{**}}^{h^{**}+\tau^{**}} e^{-(r-G)(t-h^{**})} dt\right] \\ &= \eta_p A^{ai}_{\lambda} (z-G-\delta) e^{-(r-G)h^{**}} \frac{1}{r+\lambda-G} \end{aligned}$$

where the last equality stems from the fact that τ^{**} has an exponential distribution and the use of integration by parts. Using this last result together with (59) yields (47). Q.E.D.

Proof of Formulas (49) From the definition of $V_{Comb}^{Reveal,H}$ and the linearity of the payment structure, we obtain

$$\begin{aligned} V_{Comb}^{e^H} &= E\left[\int_0^\infty e^{-rt}\left(\omega\eta_p P_{fi,t} + (1-\omega)\,\eta_d D_t\right)dt\right] \\ &= \omega\eta_p E\left[\int_0^{\tau^*} e^{-rt} P_{fi,t}^{before} dt + e^{-r\tau^*}\int_{\tau^*}^\infty e^{-r(t-\tau^*)} P_{fi,t}^{after} dt\right] \\ &\quad + (1-\omega)\,\eta_d P_{fi,0}^{before} \end{aligned}$$

where we used the fact that, by definition, $P_{fi,0}^{before} = E\left[\int_0^\infty e^{-rt} D_t dt\right]$. We now use the closed form formulas to compute the two quantities. In particular, note that

$$\begin{aligned} \int_{\tau^*}^\infty e^{-r(t-\tau^*)} P_{fi,t}^{after} dt &= \frac{z-g-\delta}{r-g} e^{G\tau^*} \int_{\tau^*}^\infty e^{-(r-g)(t-\tau^*)} dt \\ &= \frac{z-g-\delta}{(r-g)^2} e^{G\tau^*} \end{aligned}$$

Thus, we need to compute

$$E\left[\int_0^{\tau^*} e^{-rt} P_{fi,t}^{before} dt + e^{-r\tau^*}\int_{\tau^*}^\infty e^{-r(t-\tau^*)} P_{fi,t}^{after} dt\right] = E^{\lambda^H}\left[\int_0^{\tau^*} e^{-rt} P_{fi,t}^{before} dt + e^{-(r-G)\tau^*}\frac{z-g-\delta}{(r-g)^2} dt\right]$$

Integration by parts yields

$$\begin{aligned} & E\left[\int_0^{\tau^*} e^{-rt} P_{fi,t}^{before} dt + e^{-(r-G)\tau^*}\frac{z-g-\delta}{(r-g)^2} dt\right] \\ =& \int_0^\infty e^{-\left(r+\lambda^H\right)t} P_{fi,t}^{before} dt + \lambda^H \int_0^\infty e^{-\left(r+\lambda^H-G\right)t}\frac{z-g-\delta}{(r-g)^2} dt \\ =& \left(A_{\lambda^H}^{fi} + \lambda^H \frac{z-g-\delta}{(r-g)^2}\right)\int_0^\infty e^{-\left(r-G+\lambda^H\right)t} dt \\ =& \left(A_{\lambda^H}^{fi} + \lambda^H \frac{z-g-\delta}{(r-g)^2}\right)\frac{1}{\left(r-G+\lambda^H\right)} \end{aligned}$$

Thus, the total cost to the firm in equation (49) follows. Q.E.D.

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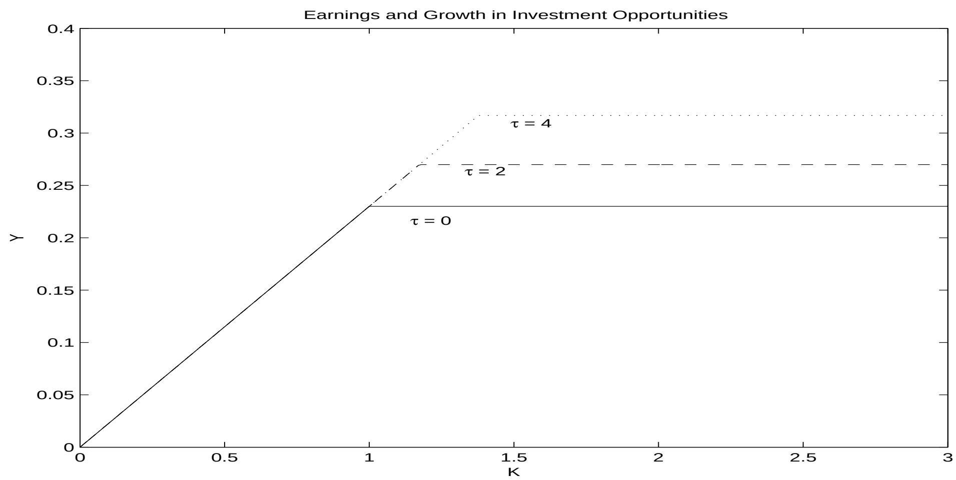

Figure 1: Growth in Investment Opportunities. This figure reproduces the earnings profile Y_t as a function of capital K_t, for three different time periods $t = 0$, $t = 1$ and $t = 2$.

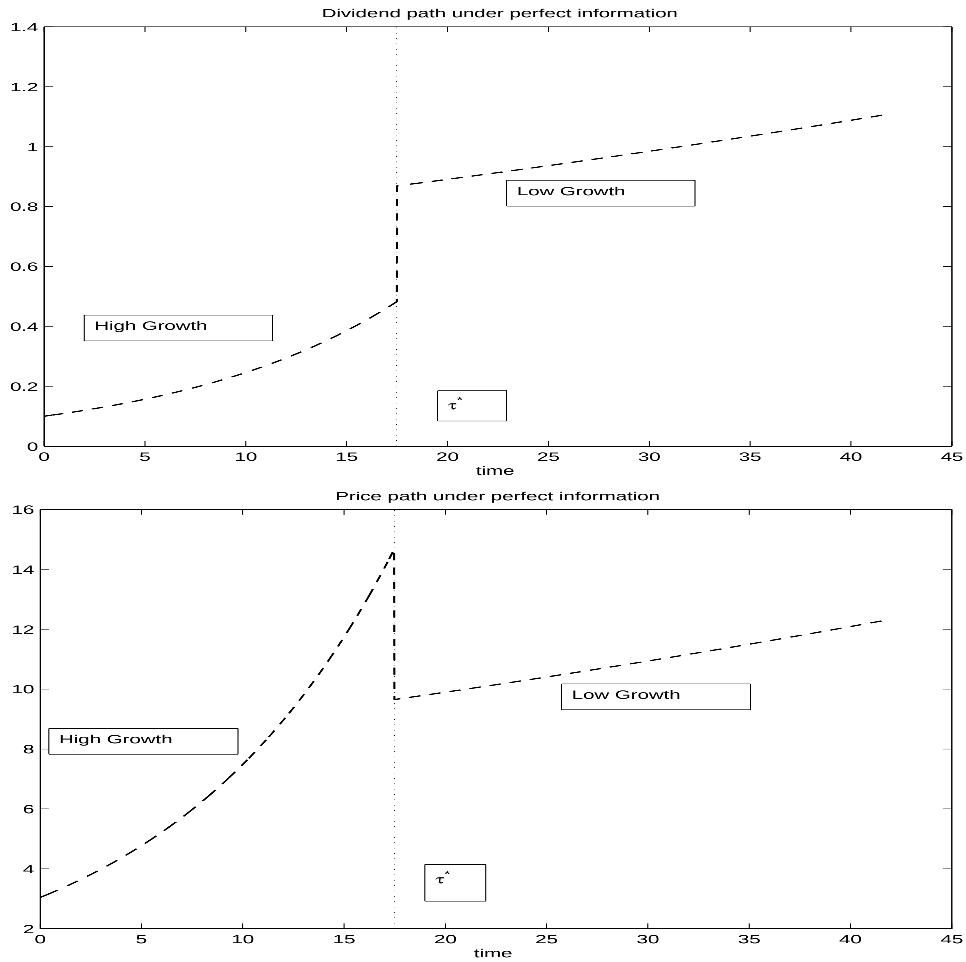

Figure 2: A dividend path (top panel) and a price path (bottom panel) under perfect information. We use the following parameters: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$, $\delta = 1\%$.

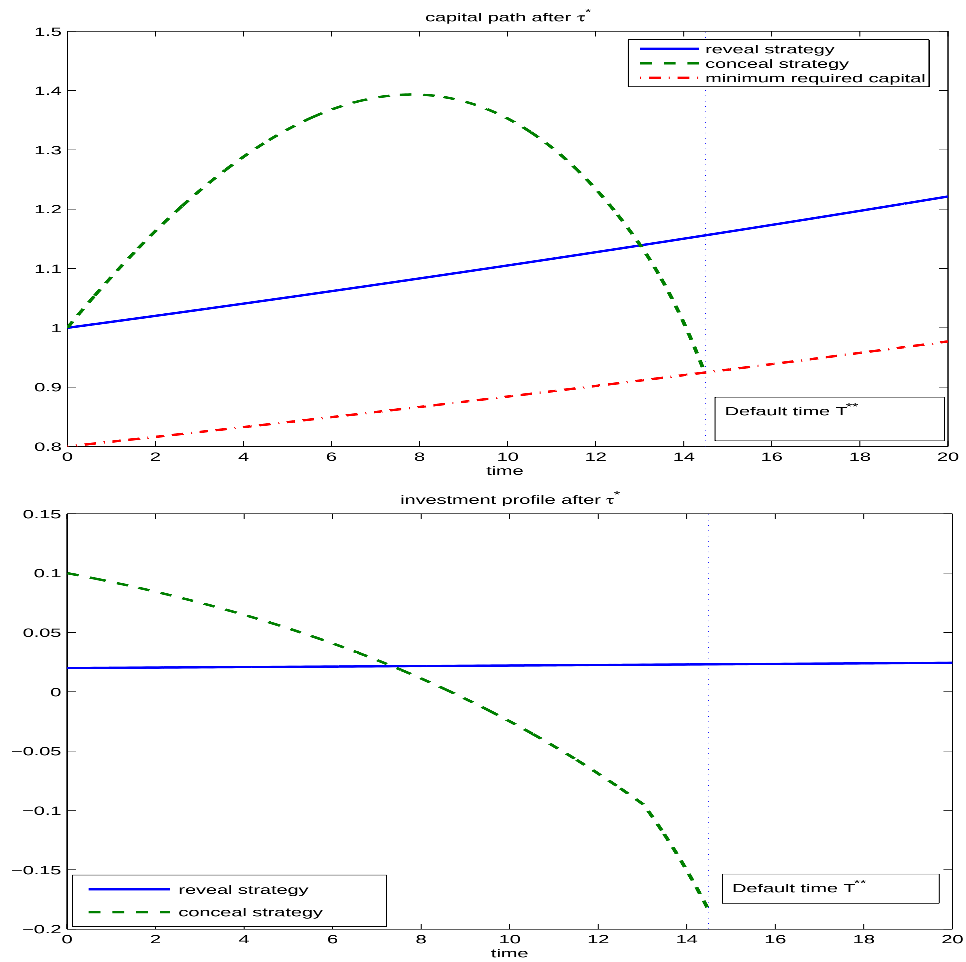

Figure 3: The dynamics of capital and investments under reveal and conceal equilibrium after τ^*. This figure shows the capital dynamics (top panel) and investment dynamics (bottom panel) for a g firm pretending to be a G firm (dashed line), relative to the revealing strategy (solid line). The vertical dotted line denotes "default" time T^{**}. The following parameters are used: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$,$\delta = 1\%$, $\lambda = 1/15$, $\xi = .8$.

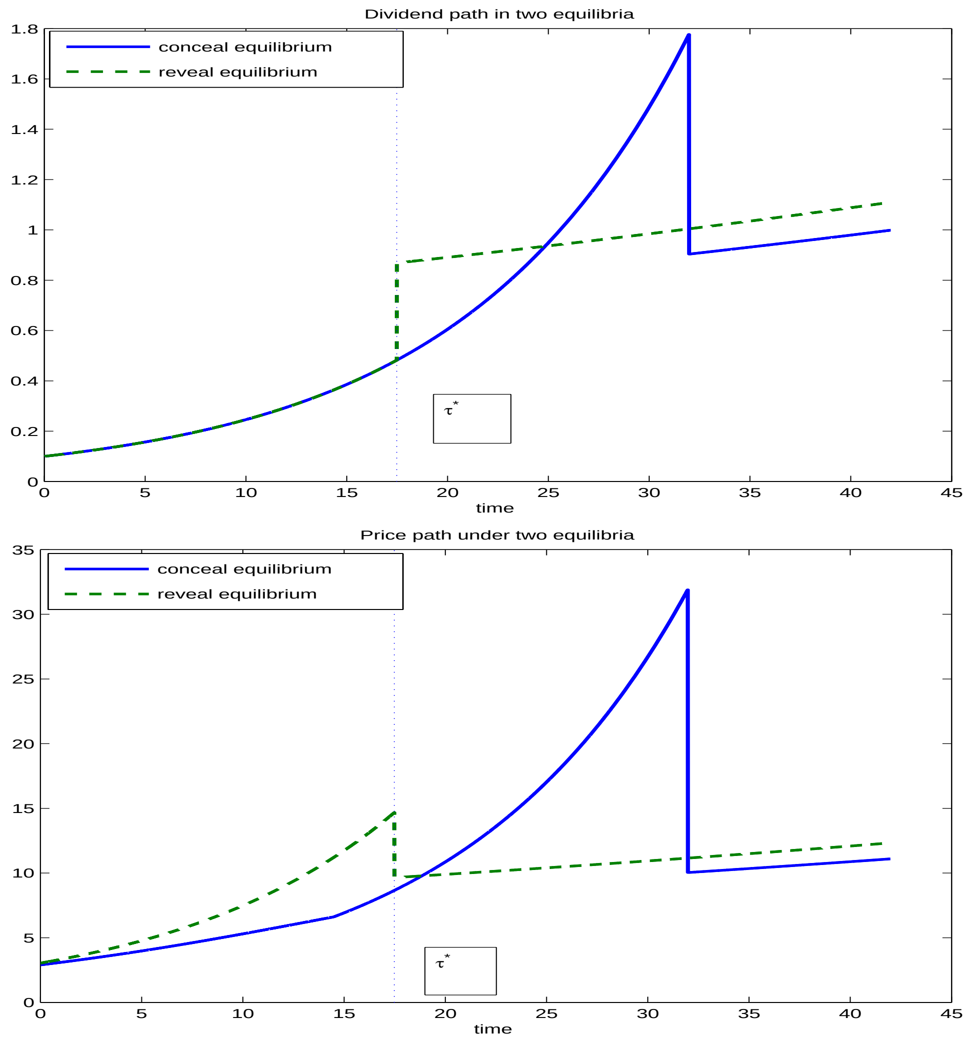

Figure 4: Dividend dynamics and price dynamics in reveal and conceal equilibria. The vertical dotted line denotes time τ^* of the growth change from G to g. The following parameters are used: $r = 10\%$, $z = 20\%$, $g = 1\%$, $G = 9\%$, $\delta = 1\%$, $\xi = .8$, $\lambda = 1/15$.

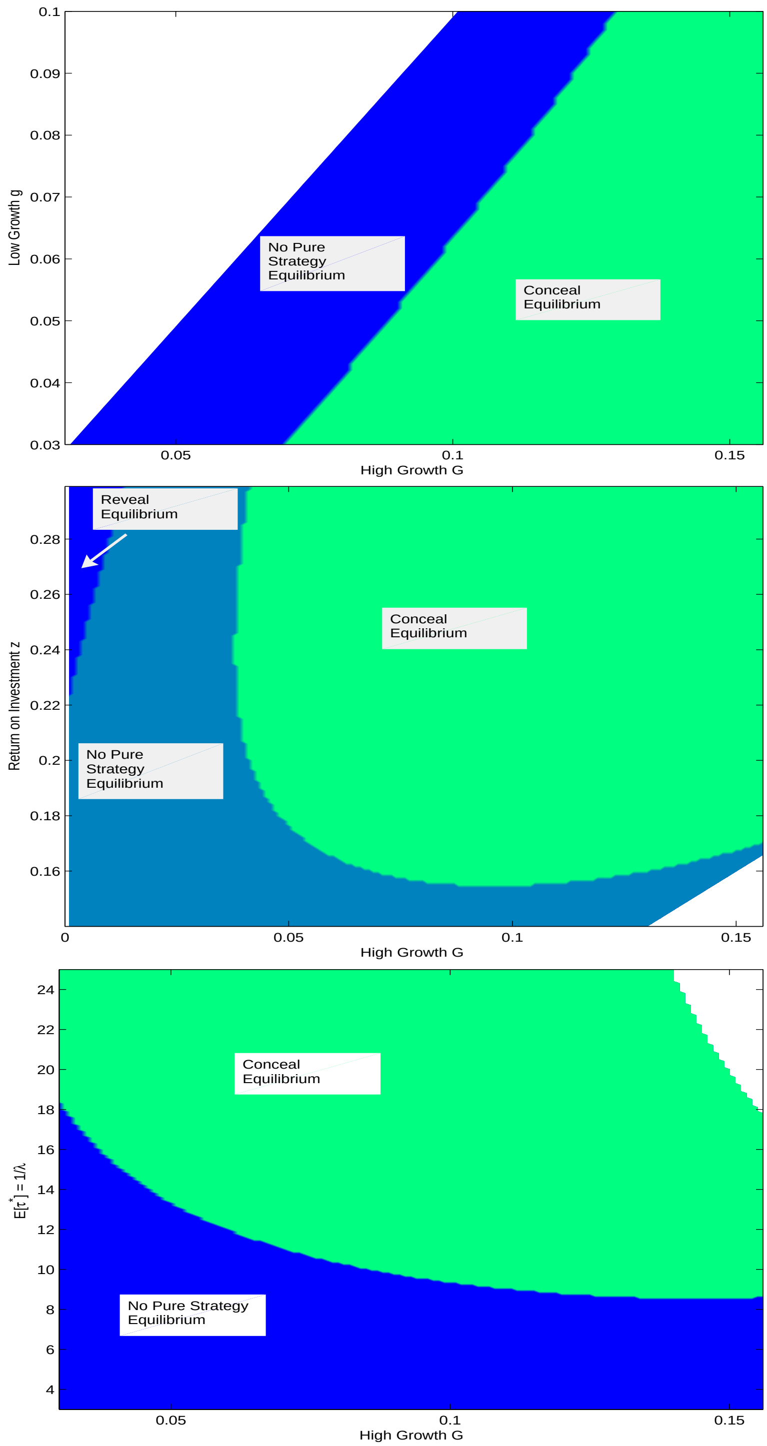

Figure 5: Conceal equilibrium under stock compensation The figure reports the conceal and reveal equilibria areas under stock compensation. In all figures, the $x-$axis reports the initial high growth G, which ranges between 3% and 16%. In the top panel, the $y-$axis is the maturity g, which also ranges between 3% and 16%. In the middle panel, the $y-$axis is the return on capital z, which ranges between 12% and 30%. In the bottom panel, the $y-$axis is given by the expected time to maturity $E[\tau^*] = 1/\lambda$, which ranges between 3% and 30%. The base parameters for the numerical computatations are as follows: $r = 10\%$, $g = 0$, $z = 20\%$, $\delta = 1\%$, $\lambda = 1/15$, $\beta = 20\%$,$\xi = .8$.

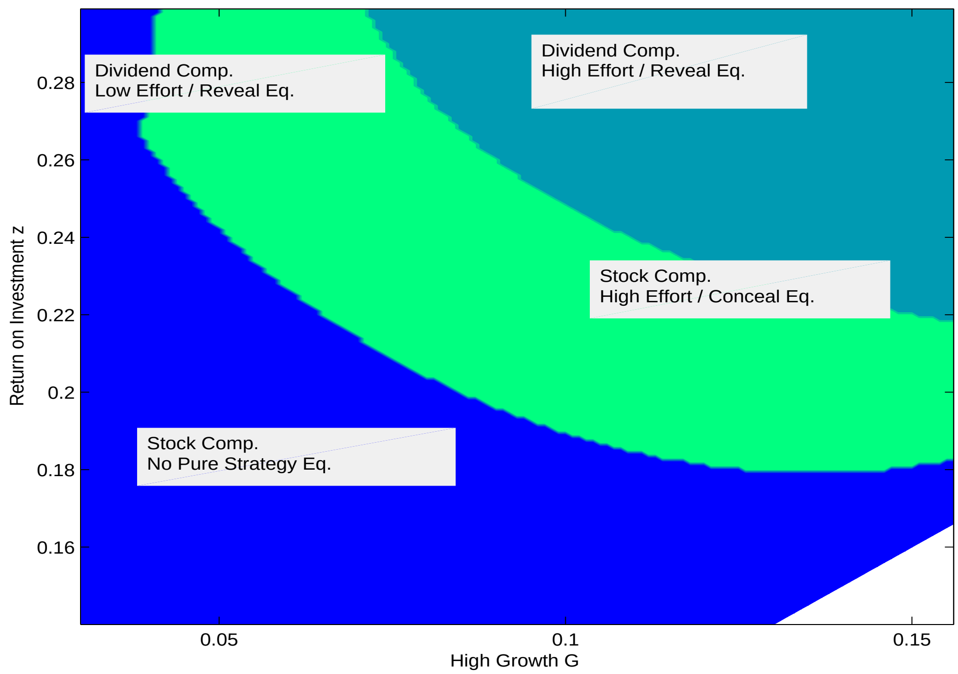

Figure 6: Equilibrium Areas under Stock Compensation and Dividend Compensation. In the (z, G) space, the figure shows the areas in which the following equilibria are defined: (a) the high effort / revealing equilibrium under dividends-based compensation; (b) the low effort / revealing equilibrium under dividends based compensation; (c) the high effort / conceal equilibrium under stock-based compensation. For all combination of parameters, dividend compensation generates a reveal equilibrium. z ranges between 12% and 30%, while G ranges between 6% and 16%. The remaining parameters are as follows: $r = 10\%$, $g = 0\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.

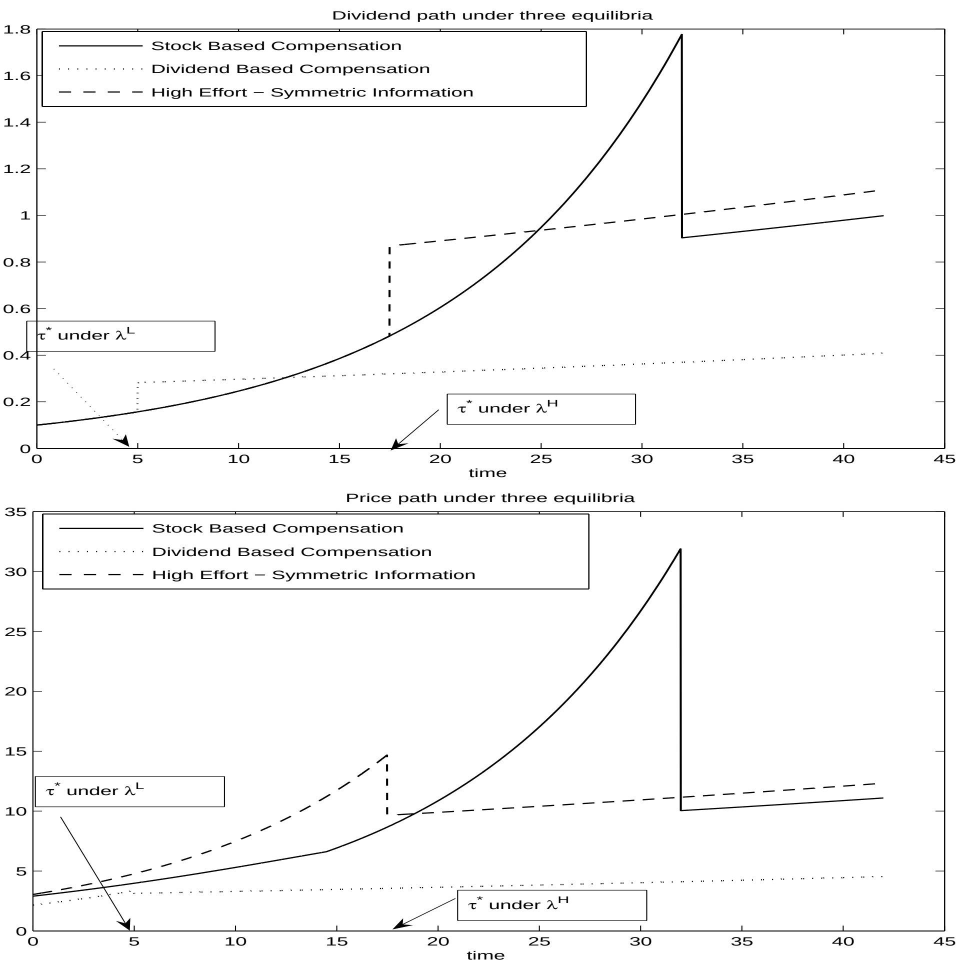

Figure 7: Dividend and Price Paths in Three Equilibria. The Figure plots hypothetical dividend (top panel) and price (bottom panel) paths under the case of "Stock-Based Compensation" (solid line); "dividends-based Compensation" (dotted line); and the first best Benchmark Case with Symmetric Information and Optimal Invemstment (dashed line). Parameters used are $r = 10\%$, $z = 20\%$, $G = 9\%$, $g = 1\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.

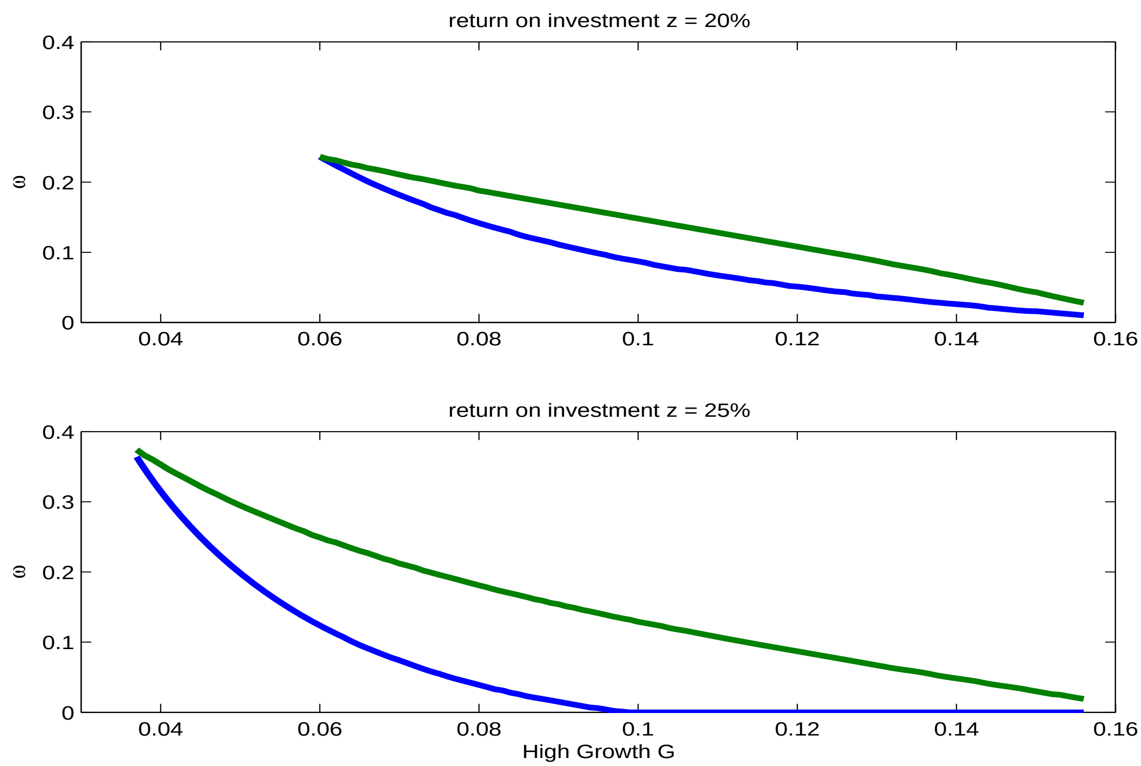

Figure 8: Optimal Weight ω on Stocks in Compensation Package. This figure reports the range of weights on the stock component of the combined compensation package that induces the first best for shareholders, that is, the high effort / reveal equilibrium. In each panel, which only differ for the level of return on capital z, the top line is the maximum ω that still induces the manager to reveal the shift in investment opportunuties, while the bottom line is the minimum ω that induces the manager to exert high effort. The remaining parameters are as follows: $r = 10\%$, $g = 0\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.

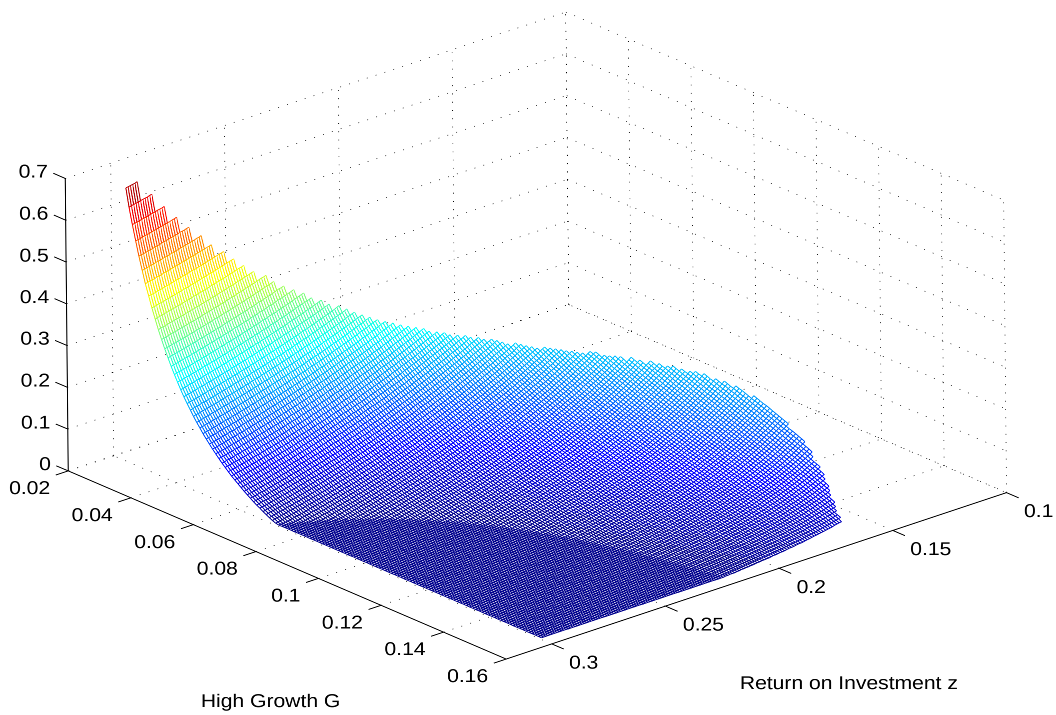

Figure 9: Minimum Weight ω on Stocks in Compensation Package in a High Effort / Reveal Equilibrium. This figure reports the minimum weight on the stock component of the combined compensation package that induces the first best equilibrium, that is, high effort / reveal equilibrium. Return on capital z ranges between 12% and 30%, while high growth G ranges between 2% and 16%. The remaining parameters are as follows: $r = 10\%$, $g = 0\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.

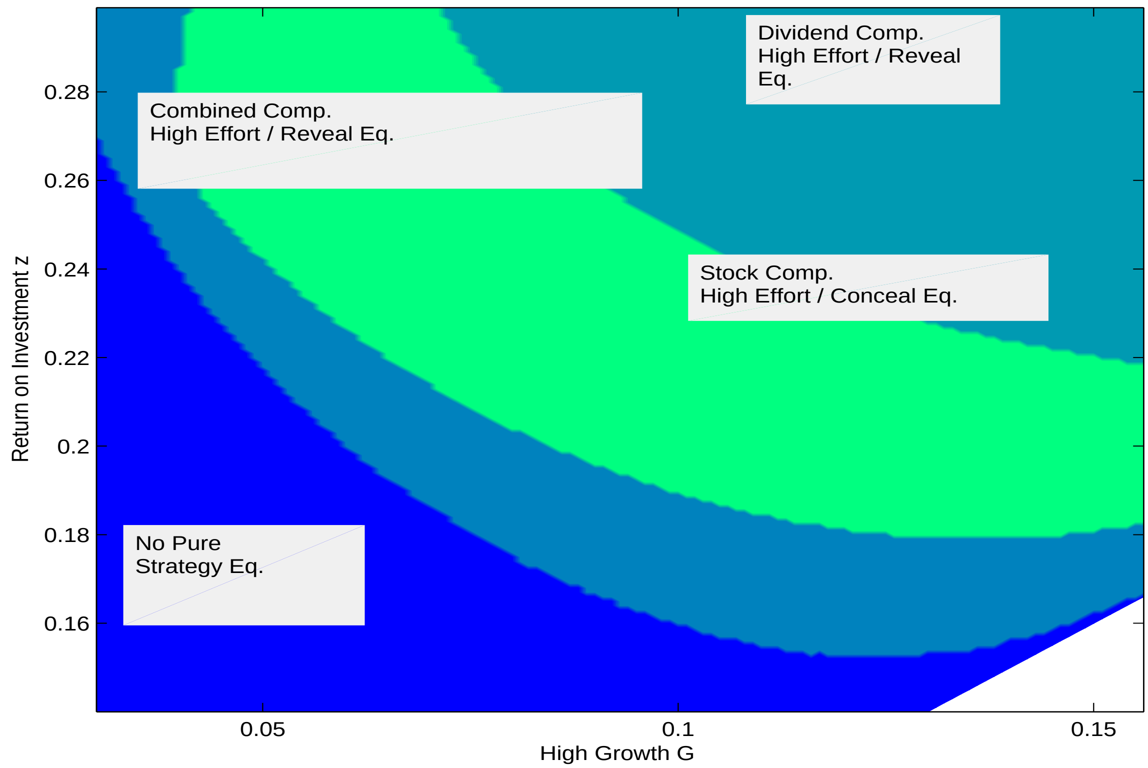

Figure 10: Equilibrium Areas under Combined Stock and Dividends Compensation. In the (z, G) space, in addition to the areas in Figure 6, this figure shows the equilibrium are for the optimal combined compensation package. For all combination of parameters, dividend compensation generates a reveal equilibrium. z ranges between 12% and 30%, while G ranges between 6% and 16%. The remaining parameters are as follows: $r = 10\%$, $g = 0\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.

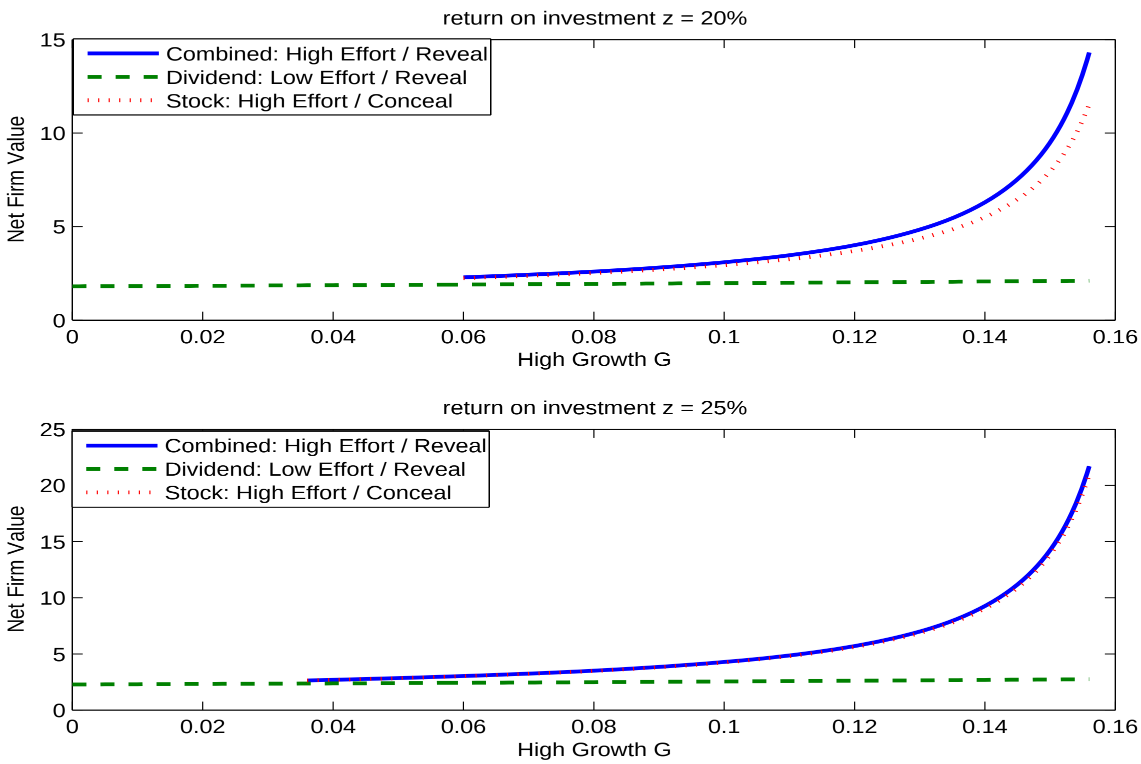

Figure 11: Firm Value Net of CEO's Incentive Contract Cost. This figure compares the firm value net of the CEO incentive contract costs in the first best equlibrium under the combined compensation package (solid line) to the firm value under (a) dividend compensation when CEO exerts low effort (dashed line), and (b) stock based compensation when CEO exerts high effort but conceals the worsening of investment opportunities at τ^* (dotted line). Each panel corresponds to a different return on capital z. The combined package in each panel is the one corresponding to the minimum weight ω to stock that still induces the CEO to exerts high effort. $\eta_d = 5\%$ while for each panel η_p is chosen so that the cost to the firm under case (a) and (b) is the same, and thus differens across G and z cases. The remaining parameters are as follows: $r = 10\%$, $g = 0\%$, $\delta = 1\%$, $\lambda^H = 1/15$, $\lambda^L = 1/2$, $c^H = 10\%$, $\beta = 20\%$, $\xi = .8$.