Recognition Rules
and Accounting-Based Compensation

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December 16, 2019
Motivation

Agent

- Performance Measure
- Fundamental Performance

Measurement Rule

Contract

Action

Monotone Likelihood Ratio Condition (MLRC)
How does measurement affect the relationship between contracts and production?
Motivation

- **Bonus Cap**
- **Target Bonus**
- **Hurdle Bonus**

**Performance Measure**

- Lower Performance Threshold
- Performance Target
- Upper Performance Threshold

**The "Incentive Zone"**

**Pay-Performance Relation**
Standard agency models rarely produce optimal contracts with caps, floors, or hurdle bonuses.
- Arya, Glover, and Mittendorf (2007)
- Arnaiz and Salas-Fumas (2008)
- Hemmer (2012)

These nonlinearities seem to create incentive problems.
- Productive inefficiency: Holmstrom and Milgrom (1987), Murphy (2013)

Given the apparent costs, why are caps, floors, and hurdles so pervasive in practice?
Accounting rules that err heavily on the side of recognition (false positives) or on the side of non-recognition (false negatives) can lead to the optimality of S-shaped contracts.

- False positive $\implies$ shirking below floor and above cap
- False negative $\nRightarrow$ shirking below floor and above cap

Asymmetry in the recognition rules for gains versus losses can promote hurdle bonuses at zero.

- False negative $\implies$ upward real activities manipulation at zero
- False positive $\implies$ weak *downward* real activities manipulation at zero
Outline

1 Setting up the model
   • Modeling production
   • Modeling measurement
   • Modeling contracts

2 S-shaped contracts with hurdles
   • False negative recognition
   • False positive recognition
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   - False negative recognition
   - False positive recognition
Fundamental earnings are denoted $\pi$.

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Fundamental earnings are denoted $\pi_i \in \{\pi_1, \ldots, \pi_N\}$.

The agent chooses $\vec{p} = \{p_1, \ldots, p_N\}$ at personal cost $c(\vec{p})$. 
Fundamental earnings are denoted $\pi_i \in \{\pi_1, \ldots, \pi_N\}$.

The agent chooses $\bar{p} = \{p_1, \ldots, p_N\}$ at personal cost $c(\bar{p})$. 
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   - **Modeling measurement**
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Modeling measurement

\[
\pi_1 \quad \pi_2 \quad \ldots \quad \pi_i \quad \ldots \quad \pi_{N-1} \quad \pi_N
\]

\[
\lambda_{i1} \quad \lambda_{i2} \quad \ldots \quad \lambda_{ii} \quad \lambda_{i,N-1} \quad \lambda_{iN}
\]

\[
X_1 \quad X_2 \quad \ldots \quad X_i \quad \ldots \quad X_{N-1} \quad X_N
\]
Modeling measurement

\[ \pi_1 \quad \pi_2 \quad \ldots \quad \pi_i \quad \ldots \quad \pi_{N-1} \quad \pi_N \]

\[ X_1 \quad X_2 \quad \ldots \quad X_i \quad \ldots \quad X_{N-1} \quad X_N \]

\[ \begin{bmatrix}
\lambda_{11} & \lambda_{12} & \ldots & \lambda_{1,N-1} & \lambda_{1N} \\
\lambda_{21} & \lambda_{22} & \ldots & \lambda_{2,N-1} & \lambda_{2N} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\lambda_{N-1,1} & \lambda_{N-1,2} & \ldots & \lambda_{N-1,N-1} & \lambda_{N-1,N} \\
\lambda_{N1} & \lambda_{N2} & \ldots & \lambda_{N,N-1} & \lambda_{NN} \\
\end{bmatrix} \]
Perfect measurement

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]
False negative recognition

\[
\begin{bmatrix}
\lambda_{11} & 0 & \lambda_{13} & 0 & 0 \\
0 & \lambda_{22} & 0 & \lambda_{23} & 0 \\
0 & 0 & \lambda_{33} & 0 & 0 \\
0 & 0 & \lambda_{43} & \lambda_{44} & 0 \\
0 & 0 & \lambda_{53} & 0 & \lambda_{55}
\end{bmatrix}
\]
False negative recognition: Asymmetric

\[
\begin{bmatrix}
\lambda_{11} & 0 & 0 & 0 & 0 \\
0 & \lambda_{22} & 0 & 0 & 0 \\
0 & 0 & \lambda_{33} & 0 & 0 \\
0 & 0 & 0 & \lambda_{44} & 0 \\
0 & 0 & 0 & 0 & \lambda_{55}
\end{bmatrix}
\]
False positive recognition: Asymmetric

\[
\begin{bmatrix}
\lambda_{11} & 0 & 0 & 0 & 0 \\
0 & \lambda_{22} & 0 & 0 & 0 \\
\lambda_{31} & \lambda_{32} & \lambda_{33} & \lambda_{34} & \lambda_{35} \\
0 & 0 & 0 & \lambda_{44} & 0 \\
0 & 0 & 0 & 0 & \lambda_{55}
\end{bmatrix}
\]
Modeling measurement

- A **False Negative Recognition Rule** errs on the side of non-recognition in the face of uncertainty.
  - Historical cost accounting

- A **False Positive Recognition Rule** errs on the side of recognition in the face of uncertainty.
  - Mark-to-market accounting

- An **Asymmetric Recognition Rule** errs more heavily on the side of non-recognition of gains than it does losses in the face of uncertainty.
  - Historical cost accounting with impairments
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Modeling contracts

\[ \max_{a, v(\cdot)} E[\pi] - E[h(\nu)] \]

s.t. \( v(\cdot) \) is individually rational

\( a \) is incentive compatible

\[ \max_{p, v} E[\pi] - E[h(\nu)] \]

s.t. \( v \) is individually rational

\( p \) is incentive compatible
Modeling contracts

\[
\max_{a, \nu(\cdot)} E[\pi] - E[h(\nu)] \\
\text{s.t. } \nu(\cdot) \text{ is individually rational}
\]

\[
\frac{dE[\nu(\pi)]}{da} - c'(a) = 0
\]

\[
\max_{p, \nu} E[\pi] - E[h(\nu)] \\
\text{s.t. } \nu \text{ is individually rational}
\]

\[
\begin{bmatrix}
\nu_1 \\
\vdots \\
\nu_N
\end{bmatrix} =
\begin{bmatrix}
c_1 \\
\vdots \\
c_N
\end{bmatrix}
\]
Modeling contracts

\[
\max_{a, \nu(\cdot)} \quad E[\pi] - E[h(\nu)]
\]

s.t. \( \nu(\cdot) \) is individually rational

\[
\frac{dE[\nu(\pi)]}{da} - c'(a) = 0
\]

\[
\max_{\mathbf{\pi}, \mathbf{\nu}} \quad E[\pi] - E[h(\nu)]
\]

s.t. \( \mathbf{\nu} \) is individually rational

\[
\begin{bmatrix}
\nu_1 \\
\vdots \\
\nu_N
\end{bmatrix} =
\begin{bmatrix}
c_1 \\
\vdots \\
c_N
\end{bmatrix}
\]
Modeling contracts

\[ \max_{\pi, v(\cdot)} E[\pi] - E[h(v)] \]
\[ \text{s.t. } v(\cdot) \text{ is individually rational} \]
\[ \frac{dE[v(\pi)]}{da} - c'(a) = 0 \]

\[ \max_{p, v} E[\pi] - E[h(v)] \]
\[ \text{s.t. } v \text{ is individually rational} \]
\[ \begin{bmatrix} v_1 \\ \vdots \\ v_N \end{bmatrix} = \Lambda^{-1} \begin{bmatrix} c_1 \\ \vdots \\ c_N \end{bmatrix} \]
Modeling contracts

\[
\max_{a, v(\cdot)} \quad E[\pi] - E[h(v)]
\]

s.t. \( v(\cdot) \) is individually rational

\[
\frac{dE[v(\pi)]}{da} - c'(a) = 0
\]
Modeling contracts

\[
\max_{\pi, v(\cdot)} E[\pi] - E[h(v)] \\
\text{s.t. } v(\cdot) \text{ is individually rational} \\
\frac{dE[v(\pi)]}{da} - c'(a) = 0
\]

\[
\max_{p} E[\pi] - E[h(\hat{v})] \\
\text{foc: } \pi_k = \sum_i h(\hat{v}_i) \lambda_{ki} \\
+ \sum_i h'(\hat{v}_i) \Pr(x_i) \\
\cdot (\hat{\lambda}_{ik} - p_k) c_{kk}
\]
Modeling contracts

\[
\max_{a, \nu(\cdot)} E[\pi] - E[h(\nu)]
\]

s.t. \( \nu(\cdot) \) is individually rational

\[
\frac{dE[\nu(\pi)]}{da} - c'(a) = 0
\]

\[
\max_{\mathbf{\hat{\nu}}} E[\pi] - E[h(\hat{\nu})]
\]

s.t. \( \nu(\cdot) \) is individually rational

foc: \( \pi_k = \sum_i \hat{\nu}_i \lambda_{ki} = c_k \)
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False negative recognition

Under false negative recognition:

- S-shaped contracts are optimal if the probability of recognition is increasing in the magnitude of the gain or loss, even if the agent is risk neutral.

- Asymmetry and risk aversion jointly promote a hurdle bonus and upward real activities manipulation at zero.

Incentive compatibility:

\[
\begin{bmatrix}
    c_1 \\
    \vdots \\
    c_z \\
    \vdots \\
    c_N \\
\end{bmatrix} = 
\begin{bmatrix}
    \lambda_{11} & \lambda_{1z} \\
    \vdots & \ddots \\
    \lambda_{z1} & \lambda_{zz} \\
    \vdots & \ddots \\
    \lambda_{N1} & \lambda_{Nz} \\
\end{bmatrix} 
\begin{bmatrix}
    v_1 \\
    \vdots \\
    v_z \\
    \vdots \\
    v_N \\
\end{bmatrix}
\]
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  \vdots & \ddots & \ddots & \vdots \\
  \lambda_{z1} & \cdots & \lambda_{zz} & \cdots \\
  \vdots & \ddots & \ddots & \vdots \\
  \lambda_{N1} & \cdots & \lambda_{ Nz} & \lambda_{NN} 
\end{bmatrix}
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  v_z \\
  \vdots \\
  v_N 
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  \vdots & \ddots & \vdots \\
  \lambda_{zz} & \ddots & \lambda_{zz} \\
  \vdots & \ddots & \ddots \\
  \lambda_{Nz} & \ddots & \lambda_{NN}
\end{bmatrix}
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  v_1 \\
  \vdots \\
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  \vdots \\
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    \lambda_{zz} & \lambda_{zz} \\
    \vdots & \vdots \\
    \lambda_{Nz} & \lambda_{NN}
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    \vdots \\
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  \vdots \\
  c_N
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\begin{bmatrix}
  \lambda_{11} & \lambda_{1z} & \vdots & \vdots \\
  \vdots & \lambda_{zz} & \ddots & \vdots \\
  \vdots & \vdots & \ddots & \lambda_{Nz} \\
  \lambda_{Nz} & \lambda_{Nz} & \cdots & \lambda_{NN}
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  \vdots \\
  v_z \\
  \vdots \\
  v_N
\end{bmatrix}
\]
False negative recognition

Take-aways

- False negative recognition rules promote an S-shaped contract if the probability of recognition is increasing in the magnitude of a gain or loss.

- S-shaped contracts driven by false negative recognition rules arise even when the agent is risk neutral, and they therefore need not lead to any productive inefficiency.

- Asymmetric false negative recognition rules can promote a hurdle bonus and upward real activities manipulation at zero if the agent is risk averse.
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   - False negative recognition
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Under false positive recognition:

- Asymmetry promotes a hurdle bonus at zero, even under risk neutrality.
- Risk aversion promotes an S-shaped contract if errors are large.
- Asymmetry and risk aversion jointly promote *downward* real activities manipulation at zero.

**Incentive compatibility:**

$$
\begin{bmatrix}
    c_1 \\
    \vdots \\
    c_z \\
    \vdots \\
    c_N \\
\end{bmatrix} =
\begin{bmatrix}
    \lambda_{11} \\
    \vdots \\
    \lambda_{zz} \\
    \vdots \\
    \lambda_{NN} \\
\end{bmatrix}
\begin{bmatrix}
    v_1 \\
    \vdots \\
    v_z \\
    \vdots \\
    v_N \\
\end{bmatrix}
$$
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Incentive compatibility:

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\begin{bmatrix}
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c_z \\
\vdots \\
c_N \\
\end{bmatrix} =
\begin{bmatrix}
\lambda_{11} & \cdots & \lambda_{z1} \\
\vdots & \ddots & \vdots \\
\lambda_{z1} & \cdots & \lambda_{zz} \\
\vdots & \ddots & \vdots \\
\lambda_{zN} & \cdots & \lambda_{NN} \\
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\vdots \\
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\vdots \\
v_N \\
\end{bmatrix}
\]
Under false positive recognition:

- Asymmetry promotes a hurdle bonus at zero, even under risk neutrality.

- Risk aversion promotes an S-shaped contract if errors are large.

- Asymmetry and risk aversion jointly promote downward real activities manipulation at zero.

Incentive compatibility:

\[
\begin{bmatrix}
  c_1 \\
  \vdots \\
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  \vdots \\
  c_N \\
\end{bmatrix} =
\begin{bmatrix}
  \lambda_{11} & \cdots & \cdots & \lambda_{z1} \\
  \vdots & \ddots & \ddots & \vdots \\
  \lambda_{z1} & \cdots & \lambda_{zz} & \cdots & \lambda_{zN} \\
  \vdots & \ddots & \ddots & \ddots & \vdots \\
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\end{bmatrix}
\begin{bmatrix}
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\end{bmatrix}
\]
False positive recognition

Take-aways

- False positive recognition rules promote an S-shaped contract if errors tend to be large and if the agent is risk averse.

- S-shaped contracts driven by false positive recognition rules necessarily lead to actions with less upside and more downside risk.

- Asymmetric false positive recognition rules can promote a hurdle bonus and \textit{downward} real activities manipulation at zero.
Results:

- Accounting rules that err heavily on the side of recognition or on the side of non-recognition can lead to the optimality of S-shaped contracts.

- Asymmetry in the recognition rules can promote a hurdle bonus at zero.

- The productive impact of these nonlinearities depends on the properties of the recognition rules that generate them.

- Techniques following Basu (1997) can be used to measure false positive, false negative, and asymmetric recognition rules, and can therefore be used to falsify the theory.
Summary

Caveats:

- Strict caps and floors depend on a bounded domain of control.
- The link between contractual form and the distribution of fundamentals depends on the cost function $c(p)$, which I assume is additively separable.
- Optimal contracts and actions are analytically derived under restrictions that render the measurement matrix invertible.
Thank you
Empirical predictions

Basu Relation

\[ R_{it} \approx \pi_i \quad \text{and} \quad X_{it} = x_i \]

\[ \{E[x|\pi_i]\}_i \equiv \hat{x} = \Lambda \pi \]

Earnings-Returns Relation

\[ \text{Earnings Surprise} \approx x_i \quad \text{and} \quad \text{Abnormal Return} \approx E[\pi|x_i] \]

\[ \{E[\pi|x_i]\}_i \equiv \hat{\pi} \]
Empirical predictions

Basu (1997) relation:
- False negative recognition $\iff$ slope less than one.
- Asymmetry $\implies$ steeper slope over losses than over gains.
- $\lambda_{ii}$ increasing in $|i - z|$ $\implies$ inverse S-shaped relation.

Earnings-returns relation:
- False positive recognition $\iff$ slope less than one.
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Earnings-returns relation:
- False positive recognition $\iff$ slope less than one.
- Asymmetry $\implies$ steeper slope over gains than over losses.
- $\lambda_{zi}$ increasing in $|i - z|$ $\implies$ S-shaped relation.
Empirical predictions

Take-aways

- Optimal contracts are more likely to be S-shaped when $\hat{x}$ is inverse S-shaped or when $\hat{\pi}$ is S-shaped.

- Hurdle bonuses are more likely when $\hat{x}$ is steeper over losses than over gains and $\hat{\pi}$ is steeper over gains than over losses.

- S-shaped contracts lead to thinner upper tails and thicker lower tails when they are associated with S-shaped $\hat{\pi}$. Hurdles lead to upwards (downwards) real activities manipulation when they are associated with asymmetry in $\hat{x}$ ($\hat{\pi}$).
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