Recognition Rules and Accounting-Based Compensation

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Abstract

I develop a moral hazard model in which asymmetric recognition rules for gains and losses are capable of rationalizing optimal contracts with floors, caps, and hurdle bonuses. When an agent has intricate control over the stochastic value of a firm's assets, accounting rules that err heavily on the side of recognition (false positives) or on the side of non-recognition (false negatives) can lead to the optimality of S-shaped contracts. Moreover, S-shaped contracts driven by false positive errors necessarily promote actions with less upside risk and more downside risk, whereas S-shaped contracts driven by false negative errors need not imply any productive inefficiency. In addition to these effects, a rule that errs more heavily towards non-recognition of gains (recognition of losses) also promotes a hurdle bonus and upwards (downwards) real activities manipulation at zero. I propose market-based measures of false positive, false negative, and asymmetric recognition rules based on the model that can be used to empirically falsify the theory.

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1 Introduction

Research suggests that nonlinearities in accounting-based compensation contracts impose counterproductive incentives on managers (Healy (1985), Holmstrom and Milgrom (1987), Murphy and Jensen (2011)). Yet nonlinearities continue to be pervasive in executive compensation packages—bonuses tend to be positive only if performance exceeds a minimum threshold, they increase with performance within some range, and are often capped when performance exceeds a maximum threshold (Murphy (1999), Murphy (2013)). Given the apparent costs of features such as bonus caps, floors, and hurdles (i.e., discontinuities in compensation at a threshold), why do firms largely continue to offer contracts with these features?

In this paper I offer an accounting-based explanation for the optimal use of caps, floors, and hurdles in accounting-based compensation contracts. Specifically, I develop a model in which an agent has intricate control over the stochastic value of a firm's assets and is therefore capable of "gaming" nonlinearities in the compensation structure. I show that precisely when this is possible, an accounting system that errs heavily on the side of recognition (it prefers false positive or type I errors) or heavily on the side of non-recognition (it prefers false negative or type II errors) of gains or losses promotes the optimality of S-shaped contracts and even literal caps and floors in some cases. Moreover, an accounting system that errs more heavily toward recognition of losses or non-recognition of gains promotes the optimality of a hurdle bonus at zero in addition to an S-shaped contract.

Given the productive disincentives attributed to these nonlinearities, I also examine their effects on the agent's equilibrium action under these alternative recognition rules. Consistent with the standard criticisms, bonus caps and floors that optimally arise from false positive recognition rules indeed destroy the agent's incentives when performance lies well below the minimum threshold or above the maximum threshold. By contrast, no productive inefficiency necessarily results from bonus caps or floors when the accounting system errs toward non-recognition; since non-recognition pushes the performance measure toward the incentive zone, the agent's incentives are preserved even when fundamental performance lies far below the minimum or above the maximum threshold.

Also consistent with standard criticisms, hurdle bonuses lead to an optimal action that sacrifices expected fundamental performance in order to increase the probability of achieving the hurdle, a phenomenon sometimes referred to as "real activities manipulation" (Roychowdhury (2006)), provided they are driven by a recognition rule that errs heavily toward non-recognition of gains. By contrast, hurdle bonuses need not imply any upwards real activities manipulation—and can even result in *downward* manipulation when the bonuses aren't sufficiently large—if they are driven by a recognition rule that errs heavily toward recognition of losses. This is because the tendency to recognize losses when none exist dampens the manager's incentive to avoid a fundamental loss in the first place, thereby creating a demand for a hurdle bonus at zero to restore efficient incentives. In sum, the productive effects of bonus caps, floors, and hurdles depend crucially on the properties of the recognition rules that generate them.

I draw my conclusions from a single-period moral hazard model in which an agent has intricate control over unobservable fundamentals, which in turn are mapped to a contractible performance measure by a flexible accounting technology. Specifically, I leverage an assumption introduced by Holmstrom and Milgrom (1987) that gives the agent independent control of all moments of the production function by letting him directly choose the probability of each outcome. In more applied terms, the agent has power over the firm's strategy, projects, product mix, risk profile, growth rate, and indeed its most fundamental characteristics.[1] This is in sharp contrast to the bulk of the literature which allows the agent independent control of the

[1] While Holmstrom and Milgrom point out that their approach offers opportunities to better understand the selection of these characteristics, their focus lies primarily on identifying a setting that produces tractable contracts. As they push their model to its continous time limit, the agent's intricate control over the distribution of firm output disappears and he is left controlling only the mean of a normal distribution.

mean, and sometimes the variance, of firm output.[2] By expanding the agent's control over production, it turns out that the moral hazard problem actually becomes *easier* to solve: since the agent chooses an independent probability for every contractual payment, there is a unique incentive compatible contract that binds the individual rationality constraint. Substituting this unique contract into the principal's objective function transforms the constrained maximization problem over the contract and action into an unconstrained maximization problem over the action alone.

I complement the agent's unrestricted action space with a general performance measurement system that is able to capture properties of many accounting rules applied in practice. In this paper I focus on three classes of rules, which I refer to as *false negative*, *false positive*, and *asymmetric recognition rules.* A false negative recognition rule errs on the side of non-recognition of fundamental gains and losses when the change in value is uncertain. For example, a firm that manufactures but does not sell a new product during the contracting period will likely achieve a fundamental gain or loss as a result of production, but the realization rule prohibits the recognition of the expected revenues and associated costs until the product is sold and delivered. That is, when future cash flows are uncertain, the accounting system errs on the side of non-recognition. The realization rule and historical cost accounting reflect a preference for type II errors and are examples of what I call false negative recognition.

By contrast, a false positive recognition rule errs on the side of recognition of fundamental gains and losses when the change in value is uncertain. For example, if the firm holds a portfolio of bonds and classifies them as trading securities, then any fluctuations in market price flow through earnings regardless of whether or not the bonds are sold before maturity. If they are

[2]Very few moral hazard studies allow the rich set of distributions afforded by Holmstrom and Milgrom's non-parametric action assumption. Two notable exceptions are Hellwig (2007) and Bertomeu (2008). A common feature of these models is that the principal trades off actions that are productively efficient (i.e., efficient in a first-best world) with actions that expose the agent to less risk. Since risk exposure depends on variation in both the contractual payments *and* the performance measure, an important lever that the principal would like to pull is missing in models where actions cannot affect the higher moments of the distribution of performance.

not, then fluctuations in the market price have no impact on the firm's cash flows, and mark-to-market accounting results in the recognition of gains or losses that never materialize. In this case, the accounting system errs on the side of recognition in the face of uncertainty. Mark-to-market, fair value, timeliness, and the inclusion of transitory items in earnings all reflect a relative preference for type I errors and are examples of what I call false positive recognition.[3]

Finally, an asymmetric recognition rule errs more heavily toward non-recognition of gains than it does losses or more heavily toward recognition of losses than it does gains. For example, increases in the value of long-term assets are generally deferred until the assets are sold, which induces a bias toward non-recognition of fundamental gains. By contrast, decreases in the value of long-term assets are promptly recognized via an impairment charge even though an asset's market price could recover (continue to fall) before it is sold, which induces a bias toward (reduces the bias against) recognition of fundamental losses. Conditional conservatism, timely loss recognition, and the lower-of-cost-or-market rule are all examples of what I call asymmetric recognition in this paper.

I show that both false negative and false positive recognition rules can render S-shaped contracts optimal, but for different reasons. When the accounting system errs toward non-recognition, even extreme fundamental performance could go unrecognized. This puts the agent at risk of receiving less than deserved when fundamental performance is high and more than deserved when fundamental performance is low, all else equal. To preserve incentives, the principal must award the agent a larger payment or penalty when fundamental gains or losses are respectively recognized. Moreover, the magnitude of this required increase in rewards and penalties is inversely proportional to the probability of recognition; if this probability is increasing in the magnitude of a fundamental gain or loss, the agent's incentives are preserved even when the contract has caps and floors. Specifically, when fundamental performance lies

[3]The aggregation of large transitory items with more moderate but persistent performance components tends to result type I error, since large performance realizations are not typically indicative of equally large changes in fundamental value.

below the minimum threshold, an increase in performance increases the probability that the loss will go unrecognized and that a positive bonus will be awarded. Symmetrically, when fundamental performance lies above the maximum threshold, a further increase in performance decreases the probability that the gain will go unrecognized and the maximum bonus foregone. In this case, bonus floors and caps need not lead to dysfunctional production decisions because the incentives are embedded in the recognition rule rather than the contract per se.

By contrast, a false positive recognition rule that promotes an S-shaped contract necessarily destroys incentives. If false positive errors tend to be large, then the probability of an extreme realization of the performance measure is relatively high, and the agent is exposed to substantial error-driven compensation risk when faced with an increasing contract. In response, the principal can reduce the agent's risk premium by decreasing the highest payments and increasing the lowest payments, i.e., by offering an S-shaped contract.[4] However, this comes at the cost of productive inefficiency in the tails of the distribution, since the agent receives less when fundamentals are actually very high and more when fundamentals are actually very low. The principal removes risk from the contract to the extent that the risk-sharing benefits outweigh the cost of productive inefficiency in the tails. In this case, bonus caps and floors are indicative of productive inefficiency below the minimum and above the maximum threshold, but are optimal nonetheless because of the risk-sharing benefits they generate.

Finally, asymmetric recognition is simply a special case of false positive or false negative recognition, and it therefore promotes S-shaped contracts under the same conditions for the same reasons. However, the asymmetry in the rule is also capable of explaining a hurdle bonus at zero, and this hurdle may or may not lead to real activities manipulation around the threshold. To see why, first consider an asymmetric false negative recognition rule, which errs more heavily

[4]This result formalizes Gibbs' (2012) argument that bonus caps and floors are indicative of performance measures that "do not always measure performance, particularly for extreme values ... [that] are more likely to reflect good or bad luck, measurement error, flaws in the method of measurement or judgment, or manipulation of the metric by the employee."

toward non-recognition of gains than it does non-recognition of losses. In this case, the agent needs relatively strong incentives to increase the probability of a fundamental gain, since he is only rewarded for such gains in the unlikely event that they are recognized. Strong incentives, however, expose the agent to excessive compensation risk, causing him to charge an excessive risk premium. The principal can reduce this risk premium while maintaining the incentive to produce fundamental gains by decreasing the payment when such gains are recognized and increasing the payment when they are not. As the principal squeezes risk out of the contract by increasing this payment, two things happen: (i) a hurdle bonus develops at zero, and (ii) the agent has a heightened incentive to engage in upward real activities manipulation at zero. In equilibrium, the principal increases the size of the hurdle bonus until the associated risk-sharing benefits no longer outweigh the productive cost of upward real activities manipulation.

By contrast, asymmetric false positive recognition can induce an optimal hurdle bonus at zero without any productive ramifications, and when the agent is risk averse these bonuses tend to be *too small* to incentivize otherwise-efficient production. Specifically, an asymmetric false positive rule errs more heavily toward the recognition of losses than it does gains, implying that when no change in value occurs, the accounting system tends to recognize a loss. When faced with an increasing contract written on such a performance measure, the agent has a disincentive to take actions that lead to no change in value, since this puts him in danger of actualizing a reported loss and receiving a penalty. That is, the agent has an incentive to engage in *downward* real activities manipulation (i.e., to shirk) around zero, since he is likely to incur a penalty whether or not fundamental performance is negative.

To counteract this disincentive, the principal introduces a hurdle bonus to the contract at zero, thereby offsetting the penalties invoked when losses are errantly recognized with a larger reward when they are not. If the agent is risk neutral, the optimal hurdle bonus is large enough to completely offset the disincentives created by the asymmetric recognition rule. By contrast, the principal can reduce the risk premium charged by a risk-averse agent by reducing the hurdle bonus, and she does so until the associated risk-sharing benefits no longer outweigh the productive cost of downward real activities manipulation. Overall, while asymmetric recognition rules tend to promote a hurdle bonus at zero, the productive implications of such hurdles depend crucially on the properties of the recognition asymmetries that generate them.

All of the recognition rules studied in this paper govern the mapping between fundamental performance and the performance measure, a relationship that has been of significant interest in the empirical financial accounting literature. Using insights from the model, I link the form of optimal contracts with the shape of two common empirical measures. First, I show that the shape of any contract written on a false negative performance measure is closely linked to the shape of the relation between fundamentals and conditional expected earnings, a relation first examined by Basu (1997). Specifically, Basu's conservatism construct is equivalent to asymmetric false negative recognition in my model, and should therefore be predictive of hurdle bonuses in any contract written on earnings. Furthermore, I show that the conditions on the false negative rule required to generate an S-shaped contract also imply an inverse S-shaped Basu relation.

Second, I show that any false positive recognition rule that leads to an S-shaped contract should also lead to an S-shaped relation between earnings and announcement-window returns. Freeman and Tse (1992) provide an accounting-based rationale for the S-shaped relation based on the existence of large transitory items in extreme earnings. Since shareholders weight persistent components more heavily than transitory components in their value assessments, extreme earnings that tend to be driven by transitory items elicit a smaller price reaction, thereby leading to the S-shaped relation. In other words, shareholders treat extreme earnings realizations as if they are driven by false positive errors. If false positive errors have a disproportionate impact on the probability of extreme earnings, this paper demonstrates that optimal contracts are also S-shaped. Therefore, to the extent that S-shaped contracts are driven by false positive recognition rules, I predict a link between the propensity for S-shaped contracts written on earnings and the propensity for an S-shaped earnings-returns relation.

The above predictions concerning the association between recognition rules and contractual form can be extended to make additional predictions about the conditional effect of contractual form on production. Specifically, I predict that S-shaped contracts associated with an S-shaped earnings-returns relation should lead to greater productive inefficiencies in the tails of the fundamental performance distribution than do S-shaped contracts associated with an inverse S-shaped Basu relation. Moreover, I predict that hurdle bonuses associated with asymmetries in the Basu (earnings-returns) relation should also be associated with upwards (downwards or neutral) real activities manipulation at the hurdle threshold.

This paper's primary contribution is that it provides an accounting-based rationale for the optimal use of bonus caps, floors, and hurdles in accounting-based compensation contracts. Murphy (1999) documents that seventy percent of the bonus plans in his sample have these features, and empirical investigations of these bonus plans tend to focus on their more dysfunctional properties. For example, Healy (1985) provides evidence that managers manipulate earnings downward when earnings are above the maximum threshold and far below the minimum threshold, since downward manipulation reverses in future periods and increases the chances of obtaining future bonuses.[5]

Perhaps more troubling, Murphy (2013) argues that bonus caps incentivize managers to stop producing once they achieve the maximum threshold or if they are far from attaining the minimum threshold, particularly if they are unable to transfer performance results to later periods via reporting manipulation. He also points out that hurdle bonuses provide incentives to do whatever is necessary to achieve the minimum threshold, including sacrificing long-term value to achieve short-term performance (i.e., real activities manipulation). While I do not directly speak to reporting manipulation in this paper, I do provide conditions under which the

[5]Holthausen et al. (1995) find evidence for downward manipulation when earnings are above the maximum threshold but not when they are below the minimum threshold.

productive inefficiencies associated with bonus caps, floors, and hurdles are either eliminated by the properties of the performance measure or are at least justified by the risk-sharing benefits they induce.

Despite their empirical ubiquity, very few theoretical studies generate optimal S-shaped contracts. One exception is Arnaiz and Salas-Fumás (2008), who model the agent's production function using a Symmetric Variance-Gamma distribution characterized by semi-heavy tails and an S-shaped likelihood ratio. Since optimal contracts are monotone transformations of the likelihood ratio in the standard principal-agent framework, S-shaped contracts arise optimally under this parametric structure. Relatedly, Hemmer (2012) models the agent's production function using an Approximate Laplace distribution, which satisfies the first order approach and also has a bounded likelihood ratio. Under this approach, the optimal contract is independent of the agent's utility function and has a literal cap and floor. Both of these exceptions require specific parametric assumptions to generate bounded likelihood ratios, since most standard parametric production functions do not satisfy this property. By contrast, a nonparametric production function yields a unique incentive compatible contract, and likelihood ratios play no role in its specification. Instead, the S-shaped contracts in my setting arise due to the properties of the accounting system rather than the properties of the production function.[6]

Finally, this paper contributes to the literature by suggesting empirical falsification tests of the theory based on large-sample estimates of false negative, false positive, and asymmetric recognition rules. Specifically, I predict that S-shaped contracts should be positively associated with an inverse S-shaped Basu (1997) relation under false negative recognition and an S-shaped

[6] There are at least two other exceptions that are somewhat less related to this paper. First, Levin (2003) and the subsequent literature on relational contracts are characterized by "bang-bang" contracts that can be interpreted as possessing a floor, a cap, and a hurdle bonus, but no incentive zone. These features arise because relational contracts are written on unverifiable performance measures, and therefore must be self-enforcing in equilibrium. Given the significant institutional costs expended on ensuring that accounting-based performance measures are objective and verifiable, relational contracts seem an unlikely explanation for these nonlinearities in accounting-based compensation packages. Second, Arya et al. (2007) also provide a rationalization of bonus caps in settings characterized by organizational hierarchies, a feature I do not exploit in this paper.

earnings-returns relation under false positive recognition, and that hurdle bonuses should be positively associated with asymmetries in these relations. Moreover, the productive impact of nonlinearities in the contract can also be linked to the relative strength of the above associations. These predictions have not yet been examined and are therefore open to empirical falsification.

The remainder of the paper proceeds as follows. In Section 2 I develop the model. In Section 3 I characterize the optimal action and its unique associated incentive compatible, individually rational contract for a generic performance measure. In Section 4.1, 4.2, and 4.3 I respectively analyze the effects of false negative, false positive, and asymmetric recognition rules on the optimal action and associated contract. In Section 5 I develop empirical predictions, and in Section 6 I conclude.

2 Model

I consider a single period principal-agent model in which an agent has control over some asset owned by a risk-neutral principal. The asset generates an uncertain return referred to as *fundamental earnings* and denoted by $\pi \in \{\pi_1, ..., \pi_N\}$, where π_i is strictly increasing in i and $N \geq 5$ is sufficiently large to allow for the possibility of caps, floors, hurdles, and an incentive zone in the compensation contract. As in Holmstrom and Milgrom (1987), I allow the agent direct control over each probability $p_i \equiv \Pr(\pi_i)$, where $\mathbf{p} \equiv (p_1, \ldots, p_N)$.[7] The agent's utility is additively separable in his wealth and his action, where the nonpecuniary cost of the action is denoted $c(\mathbf{p})$ and is strictly convex in p_i for all i. I assume that $c(\mathbf{p})$ is twice continuously differentiable and additively separable in the components of $\mathbf{p}$, where $c_i \equiv \frac{\partial c(\mathbf{p})}{\partial p_i}$, $\mathbf{c} \equiv (c_1, \ldots, c_N)$, and $c_{ij} \equiv \frac{\partial^2 c(\mathbf{p})}{\partial p_i \partial p_j} \geq 0$, with equality if and only if $i \neq j$ for all i and j.[8] For

[7] The key assumption is that the agent has intricate control over the distribution of fundamental earnings over a bounded domain, not that the fundamental earnings space is necessarily bounded. I discuss some of the implications of this assumption in the conclusion.

[8] Additive separability is a strong assumption, but it is made in the spirit of allowing the agent maximal control over production. I discuss some of the assumption's implications in the conclusion and leave its relaxation to future research.

example, the cost function $c(\mathbf{p}) = \sum_{i=1}^{N} \frac{\beta_i}{2} p_i^2$ with $\beta_i > 0$ for all i satisfies the above restrictions, since $c_i = \beta_i p_i$ is increasing in p_i, $c_{ii} = \beta_i > 0$, and $c_{ij} = 0$ for all $i \neq j$.

The agent's intricate control over production is a distinctive and crucial ingredient in this model, as it allows the agent to game nonlinearities in the compensation structure. As Holmstrom and Milgrom point out, even if the agent chooses a one-dimensional action such as effort, the ability to condition this action on private information received after the contract is signed expands the agent's control over the unconditional distribution. For example, consider an agent who exerts effort throughout the contracting period and who periodically receives information about fundamental earnings. Given a compensation contract with caps, floors, and hurdles, such an agent may choose not to exert any effort until the probability of exceeding the minimum threshold is sufficiently large, and may stop exerting effort once the maximum threshold is achieved. This contingent strategy involves shirking below the minimum and above the maximum threshold, and therefore maps into a prior unconditional distribution characterized by a relatively thick lower tail and thin upper tail. Such a distribution can be easily selected by the agent in this reduced-form, single-period model.[9]

Let $x \in \{x_1, \ldots, x_N\}$ denote a contractible performance measure, and to avoid artificial biases assume that x_i and π_i take the same value for each i. A performance measurement system is characterized by a set of conditional probabilities $\lambda_{ij} \equiv \Pr(x_j|\pi_i)$ for all $i, j \in \{1, \ldots, N\}$, where I denote $\Lambda \equiv \{\lambda_{ij}\}$. In principle, x can represent reported earnings, stock returns, or any other performance measure provided that Λ is appropriately selected. For example, under the assumption that the stock price is an unbiased measure of fundamental value, x can represent the change in price provided that Λ satisfies $\mathbb{E}[\pi|x, \Lambda, \mathbf{p}] = x$ for all x. On the other hand, if x is to represent reported earnings, Λ must capture the properties of the financial reporting

[9] More formally, this distribution is characterized by $c_1 = \cdots = c_{\underline{\kappa}} < c_{\underline{\kappa}+1} < \cdots < c_{\bar{\kappa}-1} < c_{\bar{\kappa}} = \cdots = c_N$ for some $\underline{\kappa}$ and $\bar{\kappa}$ respectively determined by the minimum and maximum thresholds. That is, the agent works harder to achieve higher fundamental performance only when doing so results in a higher wage, implying that bonus floors and caps result in marginal costs that are constant below the minimum and above the maximum thresholds.

system. In this paper I focus on accounting rules that govern the recognition of gains and losses whose existence and magnitudes are uncertain. I explicitly formalize these rules in terms of the components of Λ later in the paper. Figure 1 illustrates a generic performance measurement system for the case in which $N = 4$.

Since my objective is to investigate the effects of recognition rules on production and contractual form, I place no exogenous restrictions on the shape of the contract. I denote the agent's wage in utiles conditional on earnings realization x_i by v_i. The agent's reservation utility is denoted $\bar{v}$ and the inverse utility function is denoted $h(v)$, which is strictly increasing and convex in $\mathbf{v} \equiv (v_1, \ldots, v_N)$.

3 Analysis

Given the structure in the prior section, the probability distribution over x is given by $\Pr(x_i) = \sum_{j=1}^{N} p_j \lambda_{ji}$, which is linear in the components of $\mathbf{p}$. Fixing the performance measurement system Λ and any individually rational contract $\mathbf{v}$, the agent chooses an action $\mathbf{p}$ to solve the following program:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{N} v_i \sum_{j=1}^{N} p_j \lambda_{ji} - c(\mathbf{p}) \\ \text{s.t.} \quad & 1 = \sum_{i=1}^{N} p_i. \end{aligned} \tag{1}$$

The objective function is clearly concave in the components of $\mathbf{p}$, so the first order conditions are necessary and sufficient for an interior solution. Letting ν denote the Lagrange multiplier on the constraint, these first order conditions take the following form:

$$\sum_{i=1}^{N} (v_i - \nu)\lambda_{ji} \quad = \quad c_j \quad \text{for all } j \in \{1, \ldots, N\}. \tag{2}$$

Since N is finite, (2) can be rewritten as follows:

$$\begin{bmatrix} \lambda_{11} & \lambda_{12} & \cdots & \lambda_{1,N-1} & \lambda_{1N} \\ \lambda_{21} & \lambda_{22} & \cdots & \lambda_{2,N-1} & \lambda_{2N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda_{N-1,1} & \lambda_{N-1,2} & \cdots & \lambda_{N-1,N-1} & \lambda_{N-1,N} \\ \lambda_{N1} & \lambda_{N2} & \cdots & \lambda_{N,N-1} & \lambda_{NN} \end{bmatrix} \begin{bmatrix} v_1 - \nu \\ v_2 - \nu \\ \vdots \\ v_{N-1} - \nu \\ v_N - \nu \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_{N-1} \\ c_N \end{bmatrix}, \tag{3}$$

or $\Lambda(\mathbf{v}-\nu) = \mathbf{c}$, where ν ensures that the probabilities sum to one. Note that incentive compatibility only requires appropriate variation in v_i; it does not depend on the level of compensation, which is determined instead by the individual rationality constraint $\sum_{i=1}^{N} v_i \Pr(x_i) - c(\mathbf{p}) \geq \bar{v}$. Binding this constraint and combining with (2) yields

$$\nu \;=\; \bar{v} + c(\mathbf{p}) - \sum_{i=1}^{N} c_i p_i. \tag{4}$$

As long as Λ is nonsingular, it follows that there is a unique incentive compatible contract that binds the individual rationality constraint for any feasible interior action. This unique contract is characterized by $\mathbf{v} - \nu = \Lambda^{-1}\mathbf{c}$, where ν satisfies (4).[10] Letting $\tilde{\lambda}_{ij}$ denote the ij^{th} element of the matrix Λ^{-1}, this unique contract can now be written

$$\hat{v}_i \;\equiv\; \bar{v} + c(\mathbf{p}) + \sum_{j=1}^{N} (\tilde{\lambda}_{ij} - p_j) c_j \quad \text{for all } i \in \{1, \ldots, N\}. \tag{5}$$

Given the uniqueness of an incentive compatible contract that binds the individual rationality constraint, the principal simply chooses an action $\mathbf{p}$ and the corresponding contract given

[10] Theorem 3 of Holmstrom and Milgrom (1987) establishes the uniqueness of the implementing contract when the agent's utility is multiplicatively separable in his wealth and his action. Uniqueness follows from the nonparametric nature of the agent's action space, and it significantly enhances the tractability of this approach. By contrast, models with a parametric action space tend to produce many incentive compatible contracts, and substantial analysis is required to characterize the most efficient contract in this set.

by (5) to maximize her expected residual:

$$\begin{aligned} \max_{\mathbf{p}} \quad & \sum_{i=1}^{N} \pi_i p_i - \sum_{i=1}^{N} h(\hat{v}_i) \Pr(x_i) \\ \text{s.t.} \quad 1 \ &= \ \sum_{i=1}^{N} p_i. \end{aligned} \tag{6}$$

Letting ρ denote the Lagrange multiplier on the constraint, (6) can be analyzed to obtain the first-best (FB), second-best (SB), and what I will call the third-best (TB) actions, where the first-best is attained when the agent is risk neutral ($h(v) = v$), the second-best is attained when the agent is risk averse but measurement error is absent ($\Lambda = \Lambda^{-1} = \mathbb{1}$), and the third-best is attained when the agent is risk averse and measurement error is present. These actions are characterized by the following first order conditions:

$$\pi_k - \rho \ = \ \sum_{i=1}^{N} \hat{v}_i \lambda_{ki} \ = \ \nu + c_k \tag{FB$_k$}$$

$$\pi_k - \rho \ = \ h(\hat{v}_k) + \left(h'(\hat{v}_k) - \sum_{i=1}^{N} h'(\hat{v}_i) p_i \right) p_k c_{kk} \tag{SB$_k$}$$

$$\pi_k - \rho \ = \ \sum_{i=1}^{N} h(\hat{v}_i)\lambda_{ki} + \sum_{i=1}^{N} h'(\hat{v}_i) \Pr(x_i)(\tilde{\lambda}_{ik} - p_k) c_{kk}, \tag{TB$_k$}$$

where ν is specified by (4) and the second equality in the first best solution follows from (4) and (5). The left-hand side of all three conditions is the marginal benefit to the principal of a relative increase in p_k, whereas the right-hand side is the marginal cost. I consider each of the above conditions in turn.

The first-best solution specifies that the principal's marginal benefit and the agent's marginal cost of increasing p_k covary perfectly with k regardless of the measurement rule employed, implying that c_k is increasing in k. Moreover, the right-hand side of (FB$_k$) is increasing in c_k and therefore in p_k by the convexity of $c(\mathbf{p})$. Taken together, these observations imply that the agent works harder at increasing the probability of better outcomes (or reducing the probability of worse outcomes) in equilibrium. Furthermore, if the recognition rule is perfect such that $\Lambda = \mathbb{1}$, then (FB$_k$) specifies that $\hat{v}_k = \pi_k - \rho$, which is also increasing in k. Now, since (SB$_k$) and (TB$_k$) converge to (FB$_k$) as the agent becomes risk neutral, it follows immediately from continuity that the above observations also hold for the second- and third-best actions and contracts provided the agent is not "too" risk averse.

Observation 1. *For $h''(\cdot)$ in a neighborhood of zero, (i) the optimal actions specified by* (SB$_k$) *and* (TB$_k$) *satisfy c_k increasing in k, (ii) the right-hand sides of* (SB$_k$) *and* (TB$_k$) *are increasing in p_k, and (iii) the optimal contracts satisfying* (SB$_k$) *and* (TB$_k$) *also satisfy $\hat{v}_k$ increasing in k for λ_{kk} in a neighborhood of one.*

Observation 1 provides useful regularity conditions on the agent's utility and on the measurement system that ensure monotonicity of the optimal contract and action. This observation does not, however, provide much insight into how the second- or third-best contracts and actions depart from the first best. Consider, then, the right-hand side of (SB$_k$), which represents the marginal cost to the principal of increasing p_k when the agent is risk averse. The first term, $h(\hat{v}_k)$, represents the change in the distribution of dollar wages resulting from a marginal increase in p_k. This term is convex in $\hat{v}_k = \nu + c_k$ (the equality follows from (4) and (5)) and thus promotes an action with c_k concave in k. This, given the additive separability of $c(\mathbf{p})$, promotes thinner tails under (SB$_k$) than under (FB$_k$), consistent with the principal inducing a less risky action when the agent is more risk averse.

The second term, $(h'(\hat{v}_k) - \sum_{i=1}^{N} h'(\hat{v}_i)p_i)p_k c_{kk}$, represents the change in wages needed to ensure that a marginal increase in p_k is incentive compatible while continuing to bind the individual rationality constraint. The effect of this term on risk is somewhat ambiguous, since $h'(\cdot)$ and $p_k c_{kk}$ could be concave or convex in c_k, but the term is clearly negative for small c_k and positive for large c_k. Since the right- and left-hand sides of (SB$_k$) must equate, it follows that the second term promotes a larger (smaller) c_k under (SB$_k$) than under (FB$_k$) when k is small (large). This can be interpreted as a downward shift in the fundamental earnings distribution. Thus the mean- and variance-reducing effects of moral hazard documented by Holmström

(1979), Rogerson (1985), and Sung (1995) seem to be preserved in this nonparametric setting without measurement error. Figure 2 illustrates these benchmarks, assuming that the convexity in the first term on the right-hand side of (SB_k) dominates any potential concavity in the second.

Turning to the third-best solution, the first term on the right-hand side of (TB_k) is the change in expected compensation that results from changing the agent's action. The second term is the cost of inducing the new action via the contract given by (5), and fully analyzing this term requires the computation of Λ^{-1}. As is well known, there is no general closed-form expression for the elements of an inverted matrix, so further analysis of (TB_k) requires that I impose additional restrictions on Λ. I select these restrictions with the aim of capturing some of the institutional features of accounting measurement. Define two subsets of $\{1, \ldots, N\}$ as follows:

$$\begin{aligned} \Psi &\equiv \{i | \lambda_{ij} > 0 \text{ for some } j \neq i\}, \text{ and} \\ \Phi &\equiv \{j | \lambda_{ij} > 0 \text{ for some } i \neq j\}. \end{aligned} \tag{7}$$

Also denote $\Pi_\Psi \equiv \{\pi_i | i \in \Psi\}$ and $\Pi_\Phi \equiv \{\pi_i | i \in \Phi\}$, and let X_Ψ and X_Φ have analogous definitions. Intuitively, one can think of Π_Ψ as the set of fundamentals that "contaminate" the reported outcomes in the set X_Φ with measurement error. It turns out that Λ^{-1} has a tractable analytical characterization under the restriction that $\Psi \cap \Phi = \emptyset$. Technically, this restriction states that if the i^{th} row of Λ has any non-zero off-diagonal entries, then the i^{th} column of Λ does not, and vice versa. More intuitively, this restriction specifies that π_i can be a "contaminator" or x_i can be "contaminated", but not both.

Lemma 1. *If $\Psi \cap \Phi = \emptyset$, then*

$$\tilde{\lambda}_{ij} = \begin{cases} \frac{1}{\lambda_{ii}} & \text{if } i = j \\ -\frac{\lambda_{ij}}{\lambda_{ii}} & \text{if } i \neq j \end{cases} \quad \text{and} \quad \hat{v}_i = \begin{cases} \nu + c_i + \frac{\sum_{j \in \Phi}(c_i - c_j)\lambda_{ij}}{\lambda_{ii}} & \text{if } i \in \Psi \\ \nu + c_i & \text{if } i \in \Phi, \end{cases} \tag{8}$$

where ν satisfies (4)*.*

All proofs are in the appendix. Substituting (8) into (TB_k) yields a characterization of the optimal action contracted for by the principal:

Lemma 2. *If $\Psi \cap \Phi = \emptyset$ then* (TB_k) *can be rewritten*

$$\pi_k - \rho = \sum_{i=1}^{N} h(\hat{v}_i)\lambda_{ki} + (h'(\hat{v}_k) - \gamma)\, p_k c_{kk} + \sum_{i=1}^{N} (h'(\hat{v}_k) - h'(\hat{v}_i)) p_i c_{kk} \lambda_{ik}, \tag{TB'_k}$$

where $\gamma \equiv \sum_{i=1}^{N} h'(\hat{v}_i)\Pr(x_i)$ is independent of k and $\hat{v}_i$ satisfies (8) *for all i.*

It turns out that false negative, false positive, and asymmetric recognition rules can all be captured by some Λ satisfying $\Psi \cap \Phi = \emptyset$ for an appropriate choice of Ψ and Φ, which allows for repeated use of (TB'_k) in the upcoming applications. Before turning to these accounting-based applications, a final result will prove useful:

Lemma 3. *If $\Psi \cap \Phi = \emptyset$, if the agent is not too risk averse, and if λ_{ii} is sufficiently close to one for all i, then $\frac{\mathrm{d}p_k}{\mathrm{d}\lambda_{jk}}$ and $\frac{\mathrm{d}\hat{v}_k}{\mathrm{d}\lambda_{jk}}$ have the same sign as $\hat{v}_j - \hat{v}_k$ for all $j \in \Psi$ and $k \in \Phi$.*

To sketch the proof, first note that if the agent is not too risk averse and if λ_{ii} is sufficiently close to one for all i, then Observation 1 applies; that is, c_i is increasing in i, the right-hand side of (TB'_k) is increasing in p_k, and the optimal contract $\hat{v}_i$ is increasing in i. Now consider increasing a single λ_{jk} for $j \in \Psi$ and $k \in \Phi$. If $\hat{v}_j$ is greater than $\hat{v}_k$, then the j^{th} term in the summation on the right-hand side of (TB'_k) is negative, and an increase in λ_{jk} decreases the right-hand side of (TB'_k). This change must be offset by an increase in p_k and therefore $\hat{v}_k$ by

(8), since the right- and left-hand sides of (18) must equate. Thus λ_{jk} and p_k (and λ_{jk} and $\hat{v}_k$) covary positively if $\hat{v}_j$ is greater than $\hat{v}_k$. A symmetric argument holds for $\hat{v}_j$ less than $\hat{v}_k$.

Lemma 3 establishes the key trade-off between productive and risk-sharing efficiency at work in this paper. To understand this trade-off, suppose the principal wants to incentivize some action with a fixed c_j for $j \in \Psi$. Equation (2) requires that $c_j = \sum_i (v_i - \nu)\lambda_{ji}$; that is, the agent sets the marginal cost of increasing p_j equal to the expected wage given π_j. When faced with an increase in λ_{jk} for some $v_k < v_j$, this conditional expected wage declines, and the principal must respond by increasing v_j to maintain the incentive compatibility of c_j. Doing so, however, exposes the agent to additional compensation risk, because this increases the spread between v_j and v_k. Lemma 3 establishes that the principal optimally chooses to squeeze some of the risk out of the contract by increasing v_k and decreasing v_j at a rate that maintains the incentive compatibility of c_j; however, $k \in \Phi$ implies that $c_k = v_k - \nu$, so an increase in v_k comes at the cost of increasing c_k above its otherwise efficient level. The principal does this until the marginal benefit of squeezing risk out of the contract equals the marginal cost of this productive inefficiency. A symmetric argument applies if $v_k > v_j$.

4 Accounting-based compensation

4.1 False negative recognition

Suppose that there is uncertainty regarding the value created or destroyed during the contracting period, and that some measurement error is inevitable in expectation. For example, consider an agent who receives an order from a customer to manufacture and deliver a specialized product. In principle, income could be recognized when the order is placed, when or as the product is manufactured, when the product is delivered, when the warranty expires, or at any point before or after these events. Moreover, the income (or loss) associated with this transaction is uncertain no matter when it is recognized: the customer could cancel the order before the product is delivered, manufacturing costs could be smaller or greater than anticipated, the customer could go bankrupt before cash is received, the product could turn out to be defective and elicit a refund, the customer could sue the agent for unanticipated damages caused by the product, and so on. How should the accounting system treat such uncertainty?

Under the realization rule or historical cost accounting, the accountant errs on the side of non-recognition of gains and losses until uncertainty is sufficiently resolved, which typically happens late in the life of the transaction. Such an accounting system is likely to understate the fundamental gains and losses generated during the reporting period, leading to relatively frequent false negative errors. I operationalize this understatement of unrealized gains and losses by partitioning the space $\{1, \ldots, N\}$ into three subsets: losses denoted $L \equiv \{1, \ldots, z-1\}$, zero denoted $Z \equiv \{z\}$, and gains denoted $G \equiv \{z+1, \ldots, N\}$ for some $2 < z < N-1$ satisfying $\pi_z \equiv 0$. Also denote $\Pi_A \equiv \{\pi_i | i \in A\}$ and $X_A \equiv \{x_i | i \in A\}$ for all $A \in \{L, Z, G\}$. Loosely speaking, Λ errs on the side of non-recognition if it tends to map $\pi \in \Pi_L \cup \Pi_G$ to $x \in X_Z$.

Definition 1. *Λ applies a **false negative recognition rule** if $\lambda_{ij} > 0$ if and only if $j \in \{i, z\}$. In matrix notation,*

$$\Lambda = \begin{bmatrix} \lambda_{11} & & \lambda_{1z} & & \\ & \ddots & \vdots & & \\ & & \lambda_{zz} & & \\ & & \vdots & \ddots & \\ & & \lambda_{Nz} & & \lambda_{NN} \end{bmatrix}, \tag{9}$$

where blank entries are equal to zero by convention.

The important feature of Definition 1 is that λ_{ij} tends to be greater than λ_{ji} for extreme i and moderate j, so that the accounting system tends to err on the side of non-recognition. Setting $\lambda_{ij} = 0$ for j between i and z is conceptually consistent with "all-or-nothing" recognition of the fundamental value arising from a single transaction. This condition also allows for the application of Lemmas 1, 2, and 3 with $\Psi = L \cup G$ and $\Phi = Z$, which considerably enhances the problem's tractability.[11]

Suppose the agent is risk neutral, so that $h(v) = v$. By (FB$_k$), the optimal action is independent of the accounting system and the optimal contract sets $\sum_{i=1}^{N} \hat{v}_i \lambda_{ki} = \nu + c_k = \pi_k - \rho$ for all k. Invoking a false negative recognition rule conforming to Definition 1, this unique incentive compatible contract that binds the individual rationality constraint is derived from (8):

$$\hat{v}_i \quad = \quad \pi_i - \rho + \tfrac{\lambda_{iz}}{\lambda_{ii}}(\pi_i - \pi_z). \tag{10}$$

Λ can now be designed to construct an optimal contract with a bonus cap and floor. While many such constructions are possible, for the purpose of illustration I assume that z is equidistant from 1 and N and I construct a symmetric recognition rule in which $\lambda_{z+i,z} = \lambda_{z-i,z}$ and $\lambda_{11} = \lambda_{NN} = 1$. Take any $\{\underline{\kappa}, \bar{\kappa}\}$ equidistant from z satisfying $1 < \underline{\kappa} < z < \bar{\kappa} < N$ and consider the following false negative recognition rule:

$$\lambda_{ii} \quad = \quad \begin{cases} \frac{\pi_z - \pi_i}{\pi_z - \pi_1} & \text{for all } i \in \{1, \ldots, \underline{\kappa}\} \\ \frac{\pi_i - \pi_z}{\bar{\pi}_i - \pi_z} & \text{for all } i \in \{\underline{\kappa}+1, \ldots, \bar{\kappa}-1\} \backslash \{z\} \\ 1 & \text{for } i = z \\ \frac{\pi_i - \pi_z}{\pi_N - \pi_z} & \text{for all } i \in \{\bar{\kappa}, \ldots, N\}, \end{cases} \tag{11}$$

where $\bar{\pi}_i \equiv \pi_N \frac{\pi_i - \pi_{\underline{\kappa}}}{\pi_{\bar{\kappa}} - \pi_{\underline{\kappa}}} + \pi_1 \frac{\pi_{\bar{\kappa}} - \pi_i}{\pi_{\bar{\kappa}} - \pi_{\underline{\kappa}}}$, which is greater than (less than) π_i if i is greater than (less than)

[11] If, instead, the change in fundamental value arises from several aggregated transactions, some individual gains and losses could be recognized and some not, leading to a less stark bias in the aggregate performance measure toward zero. This can be captured by allowing $\lambda_{ij} > 0$ for j between i and z; however, this allowance generally prevents a closed-form analytical characterization of Λ^{-1}. While I am therefore unable to definitively determine the extent to which the results in this paper hold under these alternative assumptions, my numerical investigations have not uncovered any meaningful qualitative difference the optimal contract or action. Moreover, the results are analytically robust to expanding the set Z to include small gains and losses, an assumption that preserves tractability while capturing the spirit of a less stark bias towards zero.

z. For this recognition rule, $\lambda_{ii} \in (0, 1]$ is always satisfied and the probability of recognition scales linearly with π_i for $i < \underline{\kappa}$ and $i > \bar{\kappa}$. Substituting this choice of Λ into (10) yields

$$\hat{v}_i = \begin{cases} \pi_1 - \rho & \text{for all } i \in \{1, \ldots, \underline{\kappa}\} \\ \bar{\pi}_i - \rho & \text{for all } i \in \{\underline{\kappa}+1, \ldots, \bar{\kappa}-1\} \backslash \{z\} \\ \pi_z - \rho & \text{for } i = z \\ \pi_N - \rho & \text{for all } i \in \{\bar{\kappa} \ldots, N\}. \end{cases} \tag{12}$$

Figure 3 illustrates this case. For the recognition rule specified by (11), the optimal contract has a floor when performance is below $x_{\underline{\kappa}}$, a cap when performance is above $x_{\bar{\kappa}}$, and is linear when performance lies between these two values. In other words, first-best production can be perfectly incentive compatible under a contract exhibiting caps and floors, provided that performance measures are subject to an appropriate false negative recognition rule. The above derivations constitute a proof of the following result.

Observation 2. *Let Λ apply a false negative recognition rule according to* (11)*. If the agent is risk neutral, then the optimal contract exhibits a bonus cap above $x_{\bar{\kappa}}$ and a bonus floor below $x_{\underline{\kappa}}$. Moreover, this contract implements the first-best action in equilibrium.*

First-best production with bonus caps and floors is possible because efficient incentives are embedded in the recognition rule rather than the contract per se. To see this, consider a manager who is confident that fundamental performance lies above the maximum threshold. Under perfect measurement, the manager shirks by setting $c_{\bar{\kappa}} = c_{\bar{\kappa}+1} = \cdots = c_N$. By contrast, under false negative recognition there is a possibility that high performance will go unrecognized, pushing measured performance below the maximum threshold and back into the incentive zone. By further increasing fundamental performance beyond this threshold, the manager can reduce the probability that the false negative rule will fail to recognize the gain, provided the probability of recognition is increasing the gain's magnitude. Thus efficient incentives with $c_{\bar{\kappa}} < c_{\bar{\kappa}+1} < \cdots < c_N$ are preserved even in the capped region; they are embedded in the recognition rule rather than the contract. A symmetric argument holds for fundamental performance below the minimum threshold when the contract exhibits a floor.

The literal bonus caps and floors derived in Observation 2 depend on risk neutrality and a false negative recognition rule that scales the probability of recognition linearly with the magnitude of the fundamental gain or loss. When the agent is risk averse or scaling is nonlinear, optimal contracts are S-shaped but do not generically exhibit literal caps and floors.

Definition 2. *A contract is **S-shaped relative to a benchmark** if the difference between the average slopes of the contract and the benchmark is relatively small (large) for extreme (moderate) realizations of the performance measure.*

For example, a contract in which $\frac{v_{\underline{\kappa}}-v_1}{x_{\underline{\kappa}}-x_1} < 1$, $\frac{v_{\bar{\kappa}}-v_{\underline{\kappa}}}{x_{\bar{\kappa}}-x_{\underline{\kappa}}} \geq 1$, and $\frac{v_N-v_{\bar{\kappa}}}{x_N-x_{\bar{\kappa}}} < 1$ is S-shaped relative to the benchmark $\{x_i\}$, whereas a contract in which $\frac{v_{\underline{\kappa}}-v_1}{x_{\underline{\kappa}}-x_1} < \frac{c_{\underline{\kappa}}-c_1}{x_{\underline{\kappa}}-x_1}$, $\frac{v_{\bar{\kappa}}-v_{\underline{\kappa}}}{x_{\bar{\kappa}}-x_{\underline{\kappa}}} \geq \frac{c_{\bar{\kappa}}-c_{\underline{\kappa}}}{x_{\bar{\kappa}}-x_{\underline{\kappa}}}$, and $\frac{v_N-v_{\bar{\kappa}}}{x_N-x_{\bar{\kappa}}} < \frac{c_N-c_{\bar{\kappa}}}{x_N-x_{\bar{\kappa}}}$ is S-shaped relative to the benchmark $\{c_i\}$. It turns out that any false negative recognition rule that results in an S-shaped contract relative to $\{x_i\}$ when the agent is risk neutral also results in an S-shaped contract relative to $\{c_i\}$ when the agent is risk averse.

Proposition 1. *Fix $1 < \underline{\kappa} < z < \bar{\kappa} < N$, and let Λ apply any false negative recognition rule conforming to Definition 1 in which $\lambda_{11} = \lambda_{NN} = 1$ and the probability of recognition is increasing in $|\pi_i - \pi_z|$ for all $i < \underline{\kappa}$ and $i > \bar{\kappa}$. Then for any action exhibiting c_i increasing in i, the optimal contract $\{\hat{v}_i\}$ is S-shaped relative to $\{c_i\}$. Moreover, if the agent is not too risk averse, then the optimal action exhibits c_i increasing in i and the optimal contract $\{\hat{v}_i\}$ is also S-shaped relative to $\{x_i\}$.*

To see the link between Observation 2 and Proposition 1, consider the unique incentive compatible contract that binds the individual rationality constraint for an arbitrary action:

$$\hat{v}_i \quad = \quad \nu + c_i + \tfrac{\lambda_{iz}}{\lambda_{ii}}(c_i - c_z). \tag{13}$$

When the principal implements the first-best action, $\nu + c_i$ is replaced with $\pi_i - \rho$ and (13) is reduced to (10). Since x and π take the same values, whether a contract that is S-shaped relative to $\{c_i\}$ is also S-shaped relative to $\{x_i\}$ depends on how drastically c_i departs from the first-best action. Provided the agent is not too risk averse, c_i does not depart too drastically from the first best, and a contract that is S-shaped relative to $\{c_i\}$ is also S-shaped relative to $\{x_i\}$.

4.2 False positive recognition

While false negative recognition can lead to S-shaped contracts even when the agent is risk neutral and can therefore be benign with respect to production, false positive recognition induces S-shaped contracts only when the agent is risk averse, and its impact on production in this case is relatively severe. To fix ideas, consider again the earlier example of an agent who receives an order from a customer to manufacture and deliver a specialized product. An accounting rule that recognizes the associated income early in the life of the transaction, say, when the order is placed or as the product is manufactured, is likely to overstate revenue given the potential for obsolete inventory and order cancellation early in the production process. Such a rule errs on the side of recognition of income in the face of uncertainty.

Alternatively, consider the example given in the introduction of an agent who manages a portfolio of bonds. If the bonds are classified as held-to-maturity, then any fluctuation in the market value of the bonds is ignored until the bonds are sold, implying that the accounting system errs on the side of non-recognition by applying the realization rule. By contrast, if the bonds are classified as trading then all unrealized fluctuations in market value flow through earnings even if the agent holds the bonds to maturity; that is, the accounting system errs on the side of recognition by reporting unrealized gains and losses in a timely manner. A similar analogy can be drawn from the delayed versus expected loan loss recognition rules for banks, and more generally from any consideration of historical cost versus mark-to-market accounting.

Finally, consider the inclusion of transitory items in earnings, in which large realizations of the performance measure are not generally indicative of a proportional change in fundamental value. Specifically, including transitory items in the performance measure causes fundamental gains and losses to be overstated relative to a more persistent benchmark, which is equivalent to a preference for type I error. All of the above examples exhibit such a preference, which can be captured by a measurement matrix Λ that tends to map $\pi \in \Pi_z$ to $x \in X_L \cup X_G$:

Definition 3. *Λ applies a **false positive recognition rule** if $\lambda_{ij} > 0$ if and only if $i \in \{j, z\}$. In matrix notation,*

$$\Lambda = \begin{bmatrix} \lambda_{11} & & & & \\ & \ddots & & & \\ \lambda_{z1} & \cdots & \lambda_{zz} & \cdots & \lambda_{zN} \\ & & & \ddots & \\ & & & & \lambda_{NN} \end{bmatrix}, \tag{14}$$

where blank entries are equal to zero by convention.

The important feature of Definition 3 is that λ_{ij} tends to be greater than λ_{ji} for moderate i and extreme j, so that the accounting system tends to err on the side of recognition. Setting $\lambda_{ij} = 0$ for i between j and z is done primarily for tractability, since it yields a straightforward analytical characterization of Λ^{-1}; specifically, these conditions allow for the application of Lemmas 1, 2, and 3 with $\Phi = L \cup G$ and $\Psi = Z$.[12] Substituting this into (TB$'_k$) yields

$$\pi_k - \rho = \begin{cases} h(\hat{v}_z) + (h'(\hat{v}_z) - \gamma)\, p_z c_{zz} + \sum_{i \neq z} (h(\hat{v}_i) - h(\hat{v}_z))\lambda_{zi} & \text{if } k = z \\ h(\hat{v}_k) + (h'(\hat{v}_k) - \gamma)\, p_k c_{kk} + (h'(\hat{v}_k) - h'(\hat{v}_z)) c_{kk} p_z \lambda_{zk} & \text{if } k \neq z, \end{cases} \tag{15}$$

[12] Again, numerical investigation reveals no meaningful differences in the optimal contract or action when $\lambda_{ij} > 0$ for i between j and z. Moreover, qualitatively similar results to those in this section are attainable if Z is redefined to include small gains and losses ($Z \equiv \{m, \ldots, z, \ldots, n\}$ for some $1 < m < z < n < N$).

where

$$\hat{v}_k \quad = \quad \nu + c_k + \frac{\sum_{i \neq z}(c_k - c_i)\lambda_{ki}}{\lambda_{kk}}. \tag{16}$$

Again, since (FB_k) sets c_k increasing in k it follows that the third-best action also sets c_k strictly increasing in k for any Λ as long as the agent is not too risk averse, which implies that $c_i < c_z < c_j$ for $i < z < j$.

Equations (15) and (16) reveal that the effect of symmetric false positive measurement error on $\hat{v}_z$ and c_z is minimal in the following sense: first, note that roughly half of the terms being summed in the numerator of (16) for $k = z$ are positive ($i < z$) and roughly half are negative ($i > z$), implying that $\hat{v}_z \approx \nu + c_z$.[13] That is, since the accounting system maps π to x in both X_L and X_G, the bidirectional effects of false positive error on $\hat{v}_k$ largely offset each other for $k \in Z$. Second, note that roughly half of the terms in the summation on the right-hand side of (15) for $k = z$ are also positive ($i > z$) while the other half are negative ($i < z$), implying that $\pi_z - \rho \approx h(\hat{v}_z) + (h'(\hat{v}_z) - \gamma)p_z c_{zz}$, which is identical to (SB_k). Again, the bidirectional effects of false positive error error on c_z largely offset each other unless the recognition rule is asymmetric, a case I return to shortly.

By contrast, for $k \neq z$ the effect of false positive measurement error on $\hat{v}_k$ and c_k is unambiguously one-directional. Specifically, setting $\lambda_{kk} = 1$ in (16) yields $\hat{v}_k = \nu + c_k$, implying that the first two terms on the right-hand side of (15) are identical to the right-hand side of (SB_k). In other words, the extent to which $\hat{v}_k$ departs from the second best depends on the extent to which the last term on the right-hand side of (15) departs from zero. This last term is unambiguously positive (negative) for $k > z$ ($k < z$) and is scaled by λ_{zk}. Since the right- and left-hand sides of (15) must equate, it follows that $\hat{v}_k$ is less than (greater than) its second best level for $k > z$ ($k < z$). Furthermore, since this difference is scaled by λ_{zk}, an S-shaped

[13]This approximation is perfect if z is equidistant from 1 and N and $\frac{\lambda_{z,1+i}}{\lambda_{z,N-i}} = \frac{c_{N-i} - c_z}{c_z - c_{1+i}}$ for all $i \in \{0, \ldots, z-1\}$. This Λ also satisfies $\mathbb{E}[x|\pi] = \pi$ if the agent takes the first-best action.

contract is optimal provided λ_{zj} is large relative to λ_{zi} for extreme j and moderate i.

Proposition 2. *Fix* $1 < \underline{\kappa} < z < \bar{\kappa} < N$, *and let* Λ *apply a false positive recognition rule conforming to Definition 3 satisfying* $\lambda_{zj} > 0$ *for* $j < \underline{\kappa}$ *and* $j > \bar{\kappa}$. *If the agent is not too risk averse and if* λ_{zi} *is sufficiently close to zero for all* $\underline{\kappa} \leq i \leq \bar{\kappa}$, *then the optimal contract* $\{\hat{v}_i\}$ *is S-shaped relative to the second-best contract and to* $\{x_i\}$.

Figure 4 illustrates this result. Intuitively, a false positive recognition rule makes extreme payments more likely for any given contract and action, which exposes the agent to excessive compensation risk. The principal responds by reducing the high payments and increasing the low payments in an attempt to squeeze risk out of the contract and reduce the risk premium charged by the agent. However, doing so requires that variation also be squeezed out of the marginal costs, resulting in productive inefficiency. The principal removes risk from the contract until the associated marginal risk-sharing benefits no longer exceed the marginal productive costs. Finally, if the false positive rule tends to result in very large errors, then the greatest risk-sharing benefits are achieved by removing variation from the very largest and the very smallest payments, thereby leading to the optimality of an S-shaped contract.

It is conceivable that literal caps and floors could arise as optimal for an appropriately-tailored false positive recognition rule, but this rule would need to depend on the properties of h and c, and I make no attempt to construct such a rule here. It is clear, however, that since $\hat{v}_k = \nu + c_k$ for $k \neq z$, any S-shaped contract arising as the result of a false positive recognition rule necessarily damages incentives in the tails of the distribution $\mathbf{p}$. To see this, simply note that if $\hat{v}_k$ is constant for $k \leq \underline{\kappa}$ then the agent chooses an action that equates the marginal costs $c_1 = \cdots = c_{\underline{\kappa}}$, and similarly for $k \geq \bar{\kappa}$. That is, the agent exerts no incremental effort to increase the probability of better outcomes within these regions. This is in sharp contrast to false negative recognition rules, which are capable of inducing first-best production under S-shaped contracts.

4.3 Asymmetric recognition

Many accounting rules condition the standard of proof for recognition of a change in value on whether that change is good or bad. For example, the lower-of-cost-or-market rule for inventory valuation delays the recognition of gains until inventory is sold while recognizing losses whenever market price falls below cost. If market prices tend to fluctuate, this rule could overstate losses on inventory holdings because inventory cannot be written back up to its market value once prices recover. By contrast, if market prices decline over time, this rule could understate losses on inventory holdings that will ultimately be sold at even lower prices. That is, the rule could err in the direction of recognition or non-recognition of fundamental losses, but in either case it errs more heavily on the side of non-recognition of gains than it does losses in the face of uncertainty. To parsimoniously capture the effects of these two types of asymmetry, I define the following asymmetric recognition rules:

Definition 4. Λ *applies an* ***asymmetric false negative recognition rule*** *if it satisfies Definition 1 and if* $\lambda_{z+i,z} \geq \lambda_{z-i,z}$ *for all* $i > 0$*, with at least one strict inequality. By contrast,* Λ *applies an* ***asymmetric false positive recognition rule*** *if it satisfies Definition 3 and if* $\lambda_{z,z-j} \geq \lambda_{z,z+j}$ *for all* $j > 0$*, with at least one strict inequality.*

To visualize these recognition rules, the limiting case of maximal asymmetry in which $\lambda_{iz} = \lambda_{zj} = 0$ for $i < z < j$ yields the following asymmetric false negative and false positive measurement matrices, respectively:

$$\Lambda = \begin{bmatrix} \lambda_{11} & & & \\ & \ddots & & \\ & & \lambda_{zz} & & \\ & & \vdots & \ddots & \\ & & \lambda_{Nz} & & \lambda_{NN} \end{bmatrix}, \quad \Lambda = \begin{bmatrix} \lambda_{11} & & & & \\ & \ddots & & & \\ \lambda_{z1} & \cdots & \lambda_{zz} & & \\ & & & \ddots & \\ & & & & \lambda_{NN} \end{bmatrix}. \tag{17}$$

Of course, for less extreme asymmetries these matrices are given by (9) and (14), and Propositions 1 and 2 continue to apply under the same conditions. While asymmetry need not, therefore, affect the optimality of S-shaped contracts, it does have an impact on the optimal contract in a neighborhood of zero.

Beginning with the asymmetric false negative case, implicitly substituting (13) into (TB'_k) yields the principal's first order conditions, which characterize the optimal action:

$$\pi_k - \rho \;=\; \begin{cases} h(\hat{v}_k) + (h'(\hat{v}_k) - \gamma)\, p_k c_{kk} + (h(\hat{v}_z) - h(\hat{v}_k))\lambda_{kz} & \text{for } k \neq z \\ h(\hat{v}_z) + (h'(\hat{v}_z) - \gamma)\, p_z c_{zz} + \sum_{i \neq z} (h'(\hat{v}_z) - h'(\hat{v}_i)) p_i c_{zz} \lambda_{iz} & \text{for } k = z. \end{cases} \tag{18}$$

With $\hat{\mathbf{v}}$ specified by (13), (18) characterizes $\hat{v}_z$ as an implicit function of λ_{iz} for all $i \neq z$. That is, since the left-hand side of (18) is constant, any change in the right-hand side caused by variation in λ_{iz} must be offset by variation in $\hat{v}_z$ (and, therefore, p_z by (13)). This joint variation is specified by Lemma 3 for $\Psi = L \cup G$ and $\Phi = Z$, which can be applied separately to each $i \neq z$ to deliver an aggregate assessment of false negative asymmetry on the $(\hat{v}_z, c_z)$ pair implemented in equilibrium. Provided $\hat{v}_i$ is increasing in i, the summation on the right-hand side of (18) can be decomposed into two terms, one positive and the other negative: $\sum_{i<z}(h'(\hat{v}_z) - h'(\hat{v}_i))p_i c_{zz}\lambda_{iz} > 0$ and $\sum_{i>z}(h'(\hat{v}_z) - h'(\hat{v}_i))p_i c_{zz}\lambda_{iz} < 0$. If the recognition rule is symmetric, then these two summations largely cancel each other out, and $(\hat{v}_z, c_z)$ is "close" to its second best level; however, if the recognition rule errs much more heavily towards non-recognition of gains than it does losses, then the second summation is larger in magnitude than the first. This causes the right-hand side of (18) to decline, which necessitates an increase in $\hat{v}_z$ (and, therefore, p_z) to maintain the equality.

Proposition 3. *Let Λ apply any asymmetric false negative recognition rule conforming to Definition 4. If the conditions in Lemma 3 apply for $\Psi = L \cup G$ and $\Phi = Z$, then p_z is increasing, $\hat{v}_z$ is increasing, and $\hat{v}_{z-1}$ is decreasing in $\lambda_{z+i,z} - \lambda_{z-i,z}$ for all $i > 1$. That is, an* *asymmetric false negative recognition rule leads to a hurdle bonus and upward real activities manipulation at zero.*

The intuition behind the proof is as follows. When fundamental gains frequently go unrecognized, the agent is exposed to excessive compensation risk whenever he attains a fundamental gain. The principal can reduce the associated risk premium by squeezing risk out of the contractual payments conditional on fundamental gains, i.e., by increasing $\hat{v}_z$. Again, this comes at the cost of increasing c_z above its otherwise efficient level, and the principal squeezes risk from the contract until the marginal risk-sharing benefit ceases to exceed the marginal productive cost. Of course, a symmetric argument applies when fundamental losses also go unrecognized, and the principal can achieve risk-sharing benefits by *reducing* $\hat{v}_z$ to squeeze risk from the contractual payments conditional on fundamental losses. When the false negative recognition rule is symmetric, this latter effect largely offsets the former, and $\hat{v}_z$ is roughly independent of the severity of measurement error. However, when the measurement rule is asymmetric, the former effect overpowers the latter, and the greatest risk-sharing benefits are achieved by squeezing risk out of the contract over gains, i.e., by increasing $\hat{v}_z$ and, consequently, p_z. Finally, c_{z-1} is simply a linear combination of $\hat{v}_{z-1} - \nu$ and $\hat{v}_z - \nu$, with weights that are independent of λ_{iz} for $i \neq z-1$. It follows that an asymmetry-driven increase in $\hat{v}_z$ causes c_{z-1} to increase unless the principal reduces $\hat{v}_{z-1}$. If the agent is not too risk averse, the principal will always reduce $\hat{v}_{z-1}$ in this way to prevent excessive distortion in c_{z-1}.

In sum, asymmetric false negative recognition increases the spread between $\hat{v}_z$ and $\hat{v}_{z-1}$, which can be interpreted as a hurdle bonus. Moreover, any increase in p_z requires a decrease in p_i for all $i \neq z$, which is captured by the Lagrange multiplier ρ on the constraint that the probabilities sum to one. In other words, asymmetry causes the agent to sacrifice p_i for $i \neq z$ in order to achieve a larger probability p_z of achieving the hurdle. To the extent that the second-best action leads to an expected gain, this can be interpreted as a sacrifice in expected fundamentals in order to increase the probability of just achieving the hurdle bonus, which is the essence of real activities manipulation.

While the hurdle bonus derived in Proposition 3 necessarily leads to productive distortion around zero, hurdles driven by asymmetric false positive recognition rules need not imply any productive inefficiency. Suppose the agent is risk neutral, so that the first best action in (FB_k) is implemented for any recognition rule. The unique incentive compatible contract that binds the individual rationality constraint is given by (16), and it satisfies

$$\hat{v}_k = \begin{cases} \pi_k - \rho & \text{if } k \neq z \\ \pi_z - \rho + \frac{\sum_{i<z}(\pi_z - \pi_i)\lambda_{zi}}{\lambda_{zz}} - \frac{\sum_{i>z}(\pi_i - \pi_z)\lambda_{zi}}{\lambda_{zz}} & \text{if } k = z. \end{cases} \tag{19}$$

Note that $\hat{v}_z$ is the only contractual payment that depends on the measurement rule, and that $\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}(\lambda_{z,z-j} - \lambda_{z,z+j})} = \frac{\pi_{z,z+j} - \pi_{z,z-j}}{\lambda_{zz}} > 0$. That is, asymmetry increases the spread between $\hat{v}_z$ and $\hat{v}_{z-1}$, which can be interpreted as a hurdle bonus at zero. Conceptually, the agent sets the marginal cost c_z equal to the expected wage conditional on π_z. If the false positive recognition rule is asymmetric, then the agent is likely to receive a wage below $\hat{v}_z$. To maintain the incentive compatibility of c_z, the principal responds by increasing $\hat{v}_z$. In other words, if the accounting system errs towards recognition of a loss when the change in fundamental value is uncertain, the principal must pay the agent more when the accounting system does not recognize a loss in order to elicit the same action.

Not only does this example illustrate that hurdle bonuses do not necessarily imply upward real activities manipulation around the threshold, but it can actually be extended to show that the opposite can be true. Suppose the agent is risk averse and that the false positive recognition rule is asymmetric, so that $\hat{v}_z > \nu + c_z$. Because the hurdle exposes the agent to measurement-driven compensation risk, the principal can reduce the risk premium by reducing $\hat{v}_z$ and shrinking the magnitude of the hurdle bonus, thereby squeezing risk out of the contractual payments conditional on π_z. This comes at the cost of *reducing* c_z below its otherwise efficient level; that is, the magnitude of the hurdle bonus is too small to incentivize an otherwise efficient p_z, causing the agent to engage in "downward" real activities manipulation.

Proposition 4. *Let Λ apply any asymmetric false positive recognition rule conforming to Definition 4. If the agent is risk neutral, then $\hat{v}_z$ is increasing and $\hat{v}_i$ is constant in λ_{zj} for all $j < z$ and $i \neq z$. Moreover, this contract implements the first-best action in equilibrium. As the agent becomes risk averse, p_z decreases in λ_{zj} for all $j < z$. That is, an asymmetric false positive recognition rule leads to a positive hurdle bonus and downward real activities manipulation at zero.*

This case is illustrative of a larger point, namely that the incentives embedded in the contract depend crucially on the properties of the performance measure. Even in this case where a large hurdle bonus would seem to indicate strong incentives to manipulate fundamental performance upward, a tendency to recognize losses when none exist can render the bonus necessary to maintain incentives. In fact, the equilibrium hurdle bonus with a risk-averse agent is not large enough to fully prevent downward real activities manipulation (i.e., shirking) around the threshold, a conclusion that directly opposes that obtained when the properties of the performance measure are ignored. Figure 5 illustrates representative contracts associated with Propositions 3 and 4.

5 Empirical Predictions

I conclude by suggesting two market-based measures of false negative, false positive, and asymmetric recognition rules based on the model that are related to measures widely used in the accounting literature. To the extent that these measures can be adequately estimated, they can be used to falsify the predictions in this paper that link recognition rules to the use of caps, floors, and hurdles in accounting-based compensation contracts.

The first measure is based on Basu (1997), whose conservatism construct is closely related to the asymmetric recognition rules studied in this paper. Basu's measure is based on the observation that conservative accounting practices are more reluctant to recognize good news than bad, which causes the relation between news and earnings to be steeper over losses than over gains. In the context of this paper, "news" is the non-contractible fundamental realization π_i, and the extent to which π_i is reflected in the accounting report is equal to

$$\hat{x}_i \equiv \mathbb{E}[x|\pi_i] = \sum_j \lambda_{ij}\pi_j \forall i \iff \hat{\mathbf{x}} = \Lambda\boldsymbol{\pi}, \tag{20}$$

where $\boldsymbol{\pi} \equiv \{\pi_1, \dots, \pi_N\}$. Since incentive compatibility specifies that $\mathbf{v} - \nu = \Lambda^{-1}\mathbf{c}$, and since the optimal action sets $\mathbf{c}$ equal to $\boldsymbol{\pi}$ plus a constant when the agent is risk neutral, the first-best contract and the Basu relation are closely linked. This link is particularly relevant for S-shaped contracts driven by false negative recognition rules, as these arise independent of the agent's risk aversion under the condition that the probability of recognition is increasing in the magnitude of the fundamental gain or loss. In fact, this condition is sufficient for the Basu relation to be inverse S-shaped.

Proposition 5. *Suppose the agent is risk neutral and let the accounting system satisfy* (11)*, so that the optimal contract given by* (12) *exhibits a floor below $x_{\underline{\kappa}}$ and a cap above $x_{\bar{\kappa}}$. Then the Basu relation given by* (20) *satisfies $\hat{x}_i$ increasing (decreasing) quadratically in $|i-z|$ for i greater than $\bar{\kappa}$ (less than $\underline{\kappa}$). Moreover, for any false positive recognition rule satisfying Definition 3 the Basu relation satisfies $\hat{x}_i = \pi_i = \rho + \hat{v}_i$ for all $i \neq z$.*

Proposition 5 indicates that S-shaped contracts are associated with an inverse S-shaped Basu relation only if the recognition rule is false negative. A different measure is therefore required to examine the link between false positive recognition rules and S-shaped contracts. I base this measure on the rationalization of nonlinear earnings response coefficients put forward by Freeman and Tse (1992), who provide evidence that this S-shaped relation is at least partially driven by the non-persistent nature of special items that lead to extreme earnings. These items are not indicative of the same fundamental value as are the more typical moderate but persistent components of earnings, and shareholders therefore weight extreme earnings less heavily in their value assessments. In other words, shareholders price earnings as if extreme realizations are the result of large false positive errors. Conceptually, these are the same types of errors that lead to S-shaped contracts in this paper. More formally, the price conditional on x_i can be calculated using Bayes' rule:

$$\hat{\pi}_i \equiv \mathbb{E}[\pi|x_i] = \sum_j \pi_j \Pr(\pi_j|x_i) = \frac{\sum_j \pi_j p_j \lambda_{ji}}{\sum_j p_j \lambda_{ji}}. \tag{21}$$

It is straightforward to show that $\hat{\pi}_i$ is S-shaped under the same conditions that render $\hat{v}_i$ S-shaped given a false positive recognition rule.

Proposition 6. *Given the conditions on Λ in Proposition 2, $\{\hat{\pi}_i\}$ is S-shaped relative to $\{x_i\}$. Moreover, for any false negative recognition rule satisfying Definition 1, $\hat{\pi}_i = \pi_i$ for all $i \neq z$.*

Figure 6 illustrates the predictions from Propositions 5 and 6. Specifically, an inverse S-shaped Basu relation and an S-shaped earnings-returns relation should be predictive of S-shaped contracts, as the former captures false negative recognition rules in which the probability of recognition is increasing in the magnitude of a gain or loss (see Proposition 1), and the latter is indicative of false positive recognition rules whose errors predominantly lead to extreme earnings realizations (see Proposition 2). Moreover, S-shaped contracts associated with an S-shaped earnings-returns relation should be accompanied by less upside risk and more downside risk in the fundamental earnings distribution, whereas S-shaped contracts associated with an inverse S-shaped Basu relation need not result in any productive inefficiency.

Finally, it is easily shown that Basu's measure of conservatism captures asymmetric false negative recognition rules conforming to Definition 1. For this class of rules, $\hat{x}_i = \pi_z + (\pi_i - \pi_z)\lambda_{ii}$, implying that $\hat{x}_i$ is closer to the x-axis when λ_{ii} is small. Since an asymmetric false negative rule sets $\lambda_{z+i,z+i} < \lambda_{z-i,z-i}$ for $i > 0$, it follows that $\hat{x}_{z+i}$ is closer to the x-axis than $\hat{x}_{z-i}$. That is, the slope of the Basu relation is steeper over losses than over gains. Combining this insight with Proposition 3 suggests that Basu's measure of conservatism should be predictive of hurdle bonuses in accounting-based compensation schemes, and that such hurdle bonuses should be accompanied by upward real activities manipulation at the hurdle threshold.

Similarly, it is easily demonstrated that an asymmetric false positive recognition rule leads to an earnings-returns relation that is steeper over gains than losses; that is, the price reaction to large reported losses is less pronounced than is the reaction to large reported gains. Combining this insight with Proposition 4 suggests that such asymmetry in the earnings-returns relation should also be predictive of hurdle bonuses in accounting-based compensation schemes, and that such hurdle bonuses should be accompanied by downward real activities manipulation at the hurdle threshold.

6 Conclusion

Accounting-based recognition rules predominantly exhibit built-in preferences for false positive or false negative errors, depending on the type or directional implication of the measured transaction or event. Recognition of fundamental gains is almost exclusively subject to false negative error imposed by the realization rule or historical cost accounting, which err on the side of non-recognition if the existence or magnitude of the gain is uncertain. By contrast, recognition of fundamental losses tends to be subject to either less severe false negative error or to false positive error due to timely write-downs that may or may not materialize in the firm's cash flows. In this paper I show that when the accounting system errs heavily toward false positive or false negative recognition, S-shaped contracts naturally arise in equilibrium. Moreover, if recognition rules err more heavily toward non-recognition of gains than they do losses, or if they err more heavily toward recognition of losses than they do gains, the optimal contract develops a hurdle bonus at zero.

The model renders several empirical predictions. Specifically, I propose two market-based measures used extensively in the empirical accounting literature to proxy for false positive, false negative, and asymmetric recognition rules. I predict that the properties of these measures are closely linked to the shape of the optimal contract: S-shaped contracts driven by false negative errors should be associated with an inverse S-shaped Basu relation, whereas S-shaped contracts driven by false positive errors should be associated with an S-shaped earnings-returns relation. Moreover, I predict that asymmetry in these relations is associated with the inclusion of hurdle bonuses in earnings-based contracts.

Second, I predict that S-shaped contracts associated with S-shaped earnings-returns relations promote fundamental earnings distributions with thicker lower tails and thinner upper tails, whereas S-shaped contracts associated with inverse S-shaped Basu relations need not result in any productive inefficiency. Prior archival studies suggest that the observed direction of asymmetry in the distribution of earnings depends heavily on the empirical specification, while others have documented cross-sectional variation in earnings skewness.[14] This paper postulates accounting recognition rules as potential drivers of this variation.

Finally, I predict that the direction of real activities manipulation around the hurdle threshold depends on whether the hurdle is associated with asymmetry in the Basu or the earnings-returns relation. Specifically, asymmetry in the Basu relation is indicative of asymmetric false negative error, which leads to upward real activities manipulation at the hurdle threshold. By contrast, asymmetry in the earnings-returns relation is indicative of asymmetric false positive error, which weakly leads to downward real activities manipulation at the hurdle threshold.

I conclude with a brief discussion of several important limitations of this study and opportunities for future research. First, an analytical characterization of the optimal contract and action requires that I invert the measurement matrix in (3). For the sake of tractability, I impose certain restrictions on the measurement matrix that allow me to analytically characterize

[14]For example, the direction of skewness appears to change across Panels A and B of Figure 1 in Durtschi and Easton (2005) and Figures 2 and 4 in Beaver et al. (2007), whereas Gu and Wu (2003) documents cross-sectional variation in skewness.

its inverse (namely, that $\Psi \cap \Phi = \emptyset$). Future research could relax or replace these restrictions to assess the robustness of the results in this paper.

Second, the model assumes that the agent's domain of control over production is bounded above and below. If, instead, the agent can intricately control the distribution of fundamentals over an unbounded domain, then optimal contracts are no longer bounded under either recognition rule, although they may still be S-shaped. In the case of false negative recognition rules, literal caps and floors arise when the probability of recognition is increasing in the magnitude of the fundamental gain or loss at a sufficiently fast rate; given an unbounded domain, this rate forces the probability of recognition to one at some finite point, beyond which the contract is again increasing in performance.

In the case of false positive recognition rules, the risk-sharing benefit of reducing high payments and increasing low payments is proportional to the probability that a particular extreme report reflects errantly-measured moderate fundamentals and is therefore finite (see (15)). By contrast, if the agent controls production over an unbounded domain then a bounded contract would result in an infinite degree of productive inefficiency, which could not be justified by any finite risk-sharing benefit. In either case, a bounded domain of control is critical for literal caps and floors, although it is not necessary for hurdle bonuses or unbounded S-shaped contracts.

Finally, I restrict the cost function $c(\mathbf{p})$ to be additively separable in the components of $\mathbf{p}$. This assumption implies that the cost of a marginal increase in p_j is independent of p_i for all $i \neq j$. To see how this assumption affects the analysis in this paper, suppose that the cost of decreasing p_i from .2 to .1 is prohibitively costly unless p_i is simultaneously increased from .3 to .35 and p_k is simultaneously increased from .2 to .25 for some particular $i \neq j \neq k$. In this case, the agent is effectively choosing a single parameter that affects all three probabilities. In other words, imposing these types of restrictions on the agent's cost function reduces his span of control over production, which works against the motivation for modeling a nonparametric production function in the first place. Indeed, it is straightforward to construct a cost function such that $c(\mathbf{p})$ is prohibitively large unless $\mathbf{p}$ follows a particular parametric distribution such as the Poisson or Uniform. It therefore seems reasonable to invoke the assumption of additive separability as the limiting case in which the agent has maximal control over production. That being said, future research could explore the extent to which the results in this paper are robust to a less extreme degree of managerial control.

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A Proofs

Proof of Lemma 1: I prove the lemma directly. Let $\tilde{\Lambda}$ denote the matrix of elements characterized by (8). I must show that if $\Psi \cap \Phi = \emptyset$, then $\tilde{\Lambda}\Lambda = \mathbb{1}$.

Going row by row, first suppose that $i \in \Psi$. This implies that for $j \neq i$, $\lambda_{ij}\lambda_{jk} > 0$ only if $j \in \Phi$ (else $\lambda_{ij} = 0$) and $k = j$ (else $\lambda_{jk} = 0$). Using (8), the ii^{th} element of the matrix $\tilde{\Lambda}\Lambda$ is given by

$$\sum_{j=1}^{N} \tilde{\lambda}_{ij}\lambda_{ji} \quad = \quad \tfrac{1}{\lambda_{ii}}\left(\lambda_{ii} - \sum_{j\neq i}\lambda_{ij}\lambda_{ji}\right) \quad = \quad \tfrac{\lambda_{ii}}{\lambda_{ii}} \quad = \quad 1.$$

Moreover, the ik^{th} element of the matrix $\tilde{\Lambda}\Lambda$ for some $k \neq i$ is given by

$$\sum_{j=1}^{N} \tilde{\lambda}_{ij}\lambda_{jk} \quad = \quad \tfrac{1}{\lambda_{ii}}\left(\lambda_{ik} - \sum_{j\neq i}\lambda_{ij}\lambda_{jk}\right) \quad = \quad \tfrac{1}{\lambda_{ii}}\left(\lambda_{ik} - \lambda_{ik}\right) \quad = \quad 0.$$

Now, suppose that $i \in \Phi$, which implies that $\lambda_{ij} = \mathbb{1}_{j=i}$. Using (8), the ik^{th} element of the matrix $\tilde{\Lambda}\Lambda$ is given by

$$\sum_{j=1}^{N} \tilde{\lambda}_{ij}\lambda_{jk} \quad = \quad \tfrac{1}{\lambda_{ii}}\left(\lambda_{ik} - \sum_{j\neq i}\lambda_{ij}\lambda_{jk}\right) \quad = \quad \tfrac{1}{\lambda_{ii}}\left(\lambda_{ik} - \sum_{j\neq i} 0\cdot\lambda_{jk}\right) \quad = \quad \mathbb{1}_{k=i}.$$

Since $\Psi \cup \Phi = \{1, \ldots, N\}$, I have characterized the ij^{th} entry of $\tilde{\Lambda}\Lambda$ for all $i, j \in \{1, \ldots, N\}$, and shown that the entry is equal to $\mathbb{1}_{i=j}$. It follows that $\tilde{\Lambda}\Lambda = \mathbb{1}$, which implies that $\tilde{\Lambda} = \Lambda^{-1}$.

Now, by (4), (5), and (8), if $\Psi \cap \Phi = \emptyset$ then

$$\tilde{v}_i \quad = \quad \nu + \textstyle\sum_{l=1}^{N}\tilde{\lambda}_{il}c_i \quad = \quad \nu + \tfrac{1}{\lambda_{ii}}c_i - \displaystyle\sum_{j\in\Phi}\tfrac{\lambda_{ij}}{\lambda_{ii}}c_j \quad = \quad \nu + c_i + \frac{\sum_{j\in\Phi}(c_i - c_j)\lambda_{ij}}{\lambda_{ii}}.$$

Noting that $\lambda_{ij} = \mathbb{1}_{j=i}$ if $i \in \Phi$ yields (8). $\square$

Proof of Lemma 2: Substituting (8) into (TB_k) and invoking $\Psi \cap \Phi = \emptyset$ yields

$$
\begin{aligned}
\pi_k - \rho &= \sum_{i=1}^{N} h(\tilde{v}_i)\lambda_{ki} + \sum_{i=1}^{N} h'(\tilde{v}_i)\Pr(x_i)(\tilde{\lambda}_{ik} - p_k)c_{kk} \\
&= \sum_{i=1}^{N} h(\tilde{v}_i)\lambda_{ki} - \sum_{i=1}^{N} h'(\tilde{v}_i)\Pr(x_i)p_k c_{kk} \\
&\quad + h'(\tilde{v}_k)\Pr(x_k)\tfrac{1}{\lambda_{kk}}c_{kk} - \sum_{i\neq k} h'(\tilde{v}_i)\Pr(x_i)\tfrac{\lambda_{ik}}{\lambda_{ii}}c_{kk} \\
&= h(\tilde{v}_k) + \left(h'(\tilde{v}_k) - \sum_{i=1}^{N} h'(\tilde{v}_i)\Pr(x_i)\right)p_k c_{kk} \\
&\quad + \sum_{i\in\Phi}\left(h(\tilde{v}_i) - h(\tilde{v}_k)\right)\lambda_{ki} \\
&\quad + \sum_{i\in\Psi\setminus\{k\}}\left(\tfrac{h'(\tilde{v}_k)}{\lambda_{kk}} - h'(\tilde{v}_i)\right)p_i\lambda_{ik}c_{kk} \\
&= \begin{cases}
h(\tilde{v}_k) + \left(h'(\tilde{v}_k) - \sum_{i=1}^{N} h'(\tilde{v}_i)\Pr(x_i)\right)p_k c_{kk} & \\
+ \sum_{i\in\Phi}\left(h(\tilde{v}_i) - h(\tilde{v}_k)\right)\lambda_{ki} & \text{if } k \in \Psi \\
& \\
h(\tilde{v}_k) + \left(h'(\tilde{v}_k) - \sum_{i=1}^{N} h'(\tilde{v}_i)\Pr(x_i)\right)p_k c_{kk} & \\
+ \sum_{i\in\Psi}(h'(\tilde{v}_k) - h'(\tilde{v}_i))p_i\lambda_{ik}c_{kk} & \text{if } k \in \Phi.
\end{cases}
\end{aligned}
$$

The last equality holds because $\lambda_{ik} = 0$ for all $i \neq k$ if $k \in \Psi$, and $\lambda_{ki} = \mathbb{1}_{i=k}$ if $k \in \Phi$. Noting that the first lines are equivalent whether $k \in \Psi$ or $k \in \Phi$ and that the second lines are zero if k is in the opposite subset yields the result. $\square$

Proof of Lemma 3: First, note from (FB_k) that the right-hand side of (TB'_k) converges to $\nu + c_k$ as $h(v)$ converges to v. Continuity immediately implies that if the agent is not too risk averse, then (i) the third-best action specifies c_k is increasing in k, and (ii) the right-hand side of (TB'_k) is increasing in c_k (or equivalently, p_k) for all k. Moreover, combining (i) with Lemma 1 reveals that $\hat{v}_k$ is also increasing in k provided λ_{kk} is sufficiently close to one, since $\hat{v}_k \to \nu + c_k$ as $\lambda_{kk} \to 1$.

Substitute $\gamma = \sum_{i=1}^{N} h'(\hat{v}_i) \Pr(x_i)$, $\lambda_{ii} = 1 - \sum_{j\in\Phi} \lambda_{ij}$, and (8) into ($\mathrm{TB}'_k$). Differentiating the right-hand side of (TB'_k) with respect to λ_{jk} for some $k \in \Phi$ yields

$$\frac{\mathrm{dr.h.s.}(\mathrm{TB}'_k)}{\mathrm{d}\lambda_{jk}} = \left[\frac{h'(\hat{v}_k)-h'(\hat{v}_j)}{c_k-c_j}(1-p_k) + h''(\hat{v}_j)\frac{\lambda_{jk}+\lambda_{jj}p_k}{\lambda_{jj}^2}\right](c_k - c_j)p_j c_{kk}.$$

Since $h'(\cdot)$ is increasing, $\hat{v}_k - \hat{v}_j$ has the same sign as $c_k - c_j$, and $h''(\cdot) > 0$, it follows that the bracketed term is strictly positive provided the agent is risk averse. Thus the derivative has the same sign as $c_k - c_j$.

Combining the above two comparative statics, it follows that the right-hand side of (TB'_k) for $k \in \Phi$ is increasing in p_k and increasing (decreasing) in λ_{jk} for c_j less than (greater than) c_k. Since the right-hand side of (TB'_k) must equal the left-hand side ($\pi_k - \rho$), it follows that in equilibrium

$$\frac{\mathrm{d}p_k}{\mathrm{d}\lambda_{jk}} \begin{cases} > 0 & \text{if } c_j > c_k \\ < 0 & \text{if } c_j < c_k. \end{cases}$$

That is, $\frac{\mathrm{d}p_k}{\mathrm{d}\lambda_{jk}}$ has the same sign as $c_j - c_k$, which has the same sign as $\hat{v}_j - \hat{v}_k$ for λ_{jj} sufficiently close to one. Finally, $\hat{v}_k = \nu + c_k$ for $k \in \Phi$ by (8), which implies that $\frac{\mathrm{d}p_k}{\mathrm{d}\lambda_{jk}}$ and $\frac{\mathrm{d}\hat{v}_k}{\mathrm{d}\lambda_{jk}}$ have the same sign.

□

Proof of Proposition 1: Fix any action $\mathbf{p}$ satisfying c_i increasing in i and consider the corresponding incentive compatible contract specified by (8) with $\Psi = L \cup G$ and $\Phi = Z$. Specifically,

$$\begin{aligned}
\hat{v}_1 &= \nu + c_1 \\
\hat{v}_{\underline{\kappa}} &= \nu + c_{\underline{\kappa}} - \tfrac{(c_z - c_{\underline{\kappa}})\lambda_{\underline{\kappa}z}}{\lambda_{\underline{\kappa}\underline{\kappa}}} \quad < \quad \nu + c_{\underline{\kappa}} \\
\hat{v}_{\bar{\kappa}} &= \nu + c_{\bar{\kappa}} + \tfrac{(c_{\bar{\kappa}} - c_z)\lambda_{\bar{\kappa}z}}{\lambda_{\bar{\kappa}\bar{\kappa}}} \quad > \quad \nu + c_{\bar{\kappa}} \\
\hat{v}_N &= \nu + c_N.
\end{aligned}$$

Now, consider the slope of the contract from x_1 to $x_{\underline{\kappa}}$, from $x_{\underline{\kappa}}$ to $x_{\bar{\kappa}}$, and from $x_{\bar{\kappa}}$ to x_N:

$$\begin{aligned}
\tfrac{\hat{v}_{\underline{\kappa}} - \hat{v}_1}{x_{\underline{\kappa}} - x_1} &< \tfrac{\nu + c_{\underline{\kappa}} - \nu - c_1}{x_{\underline{\kappa}} - x_1} = \tfrac{c_{\underline{\kappa}} - c_1}{x_{\underline{\kappa}} - x_1} \\
\tfrac{\hat{v}_{\bar{\kappa}} - \hat{v}_{\underline{\kappa}}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} &> \tfrac{\nu + c_{\bar{\kappa}} - \nu - c_{\underline{\kappa}}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} = \tfrac{c_{\bar{\kappa}} - c_{\underline{\kappa}}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} \\
\tfrac{\hat{v}_N - \hat{v}_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}} &< \tfrac{\nu + c_N - \nu - c_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}} = \tfrac{c_N - c_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}}.
\end{aligned}$$

That is, the average slope of the contract is less than (greater than) the average slope of the marginal costs for extreme (moderate) i. It follows that the contract is S-shaped relative to the marginal costs.

Now, since (TB$_k$) converges to (FB$_k$) as $h(v) \to v$, and since the first-best action satisfies c_i increasing in i, the third-best action also satisfies c_i increasing in i for any accounting system as long as $h(\cdot)$ is not too convex. Moreover, note that any contract that is S-shaped relative to the first best marginal costs is also S-shaped relative to $\{x_i\}$ by (FB$_k$), since the first best action specifies $\nu + c_k = \pi_k - \rho \equiv x_k - \rho$. By the convergence of (TB$_k$) to (FB$_k$) as $h(v) \to v$, it follows that any contract that is S-shaped relative to the third best marginal costs is also S-shaped relative to $\{x_i\}$ provided the agent is not too risk averse. $\square$

Proof of Proposition 2: Let c_k^{sb} and c_k^{tb} denote the second- and third-best actions characterized by (SB_k) and (15), respectively. Also denote $\hat{v}_k^{sb} = \nu + c_k^{sb}$ and $\hat{v}_k^{tb} = \nu + c_k^{tb}$ for $k \neq z$.

Since (SB_k) and (TB_k) converge to (FB_k) as $h(v) \to v$, and since the first-best action satisfies c_k increasing in k, the second- and third-best actions also satisfy c_k increasing in k for any accounting system as long as $h(\cdot)$ is not too convex. Moreover, the right-hand sides of (SB_k) and (15) converge to c_k as $h(v) \to v$, implying that they are both increasing in c_k as long as $h(\cdot)$ is not too convex. Accordingly, fix some level of risk-aversion such that (i) c_k^{sb} is strictly increasing in k, and (ii) the right-hand side of (SB_k) is increasing in c_k for all k.

Invoking Lemma 3 with $j = z$ implies that $\frac{\mathrm{d}\hat{v}_k^{tb}}{\mathrm{d}\lambda_{zk}}$ has the same sign as $\hat{v}_z - \hat{v}_k$, which is positive (negative) for k less than (greater than) z. I can therefore write $\hat{v}_k^{tb} \equiv \hat{v}_k^{tb}(\lambda_{zk})$ as a continuous, increasing (decreasing) function of λ_{zk} for k less than (greater than) z, where $\hat{v}_k^{tb}(0) = \hat{v}_k^{sb}$. Take any $1 < \underline{\kappa} < z < \bar{\kappa} < N$ and fix $\lambda_{zj} > 0$ for j below $\underline{\kappa}$ and above $\bar{\kappa}$. Consider the average slope of the contract between x_1 and $x_{\underline{\kappa}}$, $x_{\underline{\kappa}}$ and $x_{\bar{\kappa}}$, and $x_{\bar{\kappa}}$ and x_N as λ_{zi} approaches zero for all $i \in \{\underline{\kappa}, \ldots, \bar{\kappa}\}$:

$$\frac{\hat{v}_{\underline{\kappa}}^{tb}(\lambda_{z\underline{\kappa}}) - \hat{v}_1^{tb}(\lambda_{z1})}{x_{\underline{\kappa}} - x_1} \xrightarrow[\lambda_{z\underline{\kappa}} \to 0]{} \frac{\hat{v}_{\underline{\kappa}}^{sb} - \hat{v}_1^{tb}(\lambda_{z1})}{x_{\underline{\kappa}} - x_1} < \frac{\hat{v}_{\bar{\kappa}}^{sb} - \hat{v}_1^{sb}}{x_{\bar{\kappa}} - x_1}$$

$$\frac{\hat{v}_{\bar{\kappa}}^{tb}(\lambda_{z\bar{\kappa}}) - \hat{v}_{\underline{\kappa}}^{tb}((\lambda_{z\underline{\kappa}}))}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} \xrightarrow[\lambda_{z\underline{\kappa}} \to 0]{\lambda_{z\bar{\kappa}} \to 0} \frac{\hat{v}_{\bar{\kappa}}^{sb} - \hat{v}_{\underline{\kappa}}^{sb}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} = \frac{\hat{v}_{\bar{\kappa}}^{sb} - \hat{v}_{\underline{\kappa}}^{sb}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}}$$

$$\frac{\hat{v}_N^{tb}(\lambda_{zN}) - \hat{v}_{\bar{\kappa}}^{tb}(\lambda_{z\bar{\kappa}})}{x_N - x_{\bar{\kappa}}} \xrightarrow{\lambda_{z\bar{\kappa}} \to 0} \frac{\hat{v}_N^{tb}(\lambda_{zN}) - \hat{v}_{\bar{\kappa}}^{sb}}{x_N - x_{\bar{\kappa}}} < \frac{\hat{v}_N^{sb} - \hat{v}_{\bar{\kappa}}^{sb}}{x_N - x_{\bar{\kappa}}}.$$

That is, the contract $\{v_k^{tb}\}$ is S-shaped relative to the contract $\{v_k^{sb}\}$ provided λ_{zi} is sufficiently close to zero for $i \in \{\underline{\kappa}, \ldots, \bar{\kappa}\}$. Moreover, the contract $\{v_k^{tb}\}$ is also S-shaped relative to $\{x_i\}$ if the agent is not too risk averse by the convergence of (SB_k) to (FB_k) as $h(v) \to v$. $\square$

Proof of Proposition 3: If Lemma 3 applies, it follows immediately that p_z and $\hat{v}_z$ are increasing in $\lambda_{z+i,z}$ and decreasing in $\lambda_{z-i,z}$ for all $i > 0$.

Now, note that (13) can be rewritten

$$\hat{v}_k \quad = \quad \nu + c_k + \tfrac{\lambda_{kz}}{\lambda_{kk}}(c_k - c_z) \quad = \quad (\nu + c_k)\left(1 + \tfrac{\lambda_{kz}}{\lambda_{kk}}\right) - \tfrac{\lambda_{kz}}{\lambda_{kk}}\hat{v}_z,$$

implying that $\frac{\mathrm{d}\hat{v}_k}{\mathrm{d}\hat{v}_z} = -\frac{\lambda_{kz}}{\lambda_{kk}}$ for any fixed c_k. If the agent is risk neutral, (FB_k) establishes that c_k is invariant to the measurement rule. In this case, for any $i \neq z-1$,

$$\frac{\mathrm{d}\hat{v}_{z-1}}{\mathrm{d}\lambda_{iz}} \quad = \quad \frac{\mathrm{d}\hat{v}_{z-1}}{\mathrm{d}\hat{v}_z}\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}\lambda_{iz}} \quad = \quad -\frac{\lambda_{z-1,z}}{\lambda_{z-1,z-1}}\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}\lambda_{iz}},$$

implying that $\frac{\mathrm{d}\hat{v}_{z-1}}{\mathrm{d}\lambda_{iz}}$ and $\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}\lambda_{iz}}$ have opposite signs if the agent is not too risk averse, a condition required for the application of Lemma 3.

$\square$

Proof of Proposition 4: If the agent is risk neutral, (FB_k) ensures that the first-best action is implemented for any invertible Λ, implying that $\nu + c_i = \pi_i - \rho$ for all i. It follows that $\hat{v}_z$ is given by (19) and $\frac{\mathrm{d}\hat{v}_z}{\mathrm{d}(\lambda_{z,z-j}-\lambda_{z,z+j})} = \frac{\pi_{z,z+j}-\pi_{z,z-j}}{\lambda_{zz}} > 0$, whereas $\hat{v}_i$ is independent of the recognition rule for all $i \neq z$.

Suppose, then, that the agent is risk averse. Since the recognition rule is false positive, the principal designs a contract satisfying (16) to incentivize an action satisfying (15), which are rewritten below for $k = z$ and $i \neq z$:

$$\begin{aligned}
\hat{v}_i &= \nu + c_i \\
\hat{v}_z &= \nu + c_z + \tfrac{\sum_{i\neq z}(c_z - c_i)\lambda_{zi}}{\lambda_{zz}} \\
\pi_z - \rho &= h(\hat{v}_z) + \left(h'(\hat{v}_z) - \gamma\right) p_z c_{zz} + \sum_{i\neq z}(h(\hat{v}_i) - h(\hat{v}_z))\lambda_{zi}.
\end{aligned}$$

Implicitly substituting $\gamma = \sum_i h'(\hat{v}_i)\Pr(x_i)$ and $\hat{v}_z$ into the last equation and differentiating the right-hand side with respect to λ_{zj} for some $j < z$ yields

$$\begin{aligned}
\tfrac{\mathrm{d}(\mathrm{r.h.s.}(15))}{\mathrm{d}\lambda_{zj}} &= \left(\tfrac{h'(\hat{v}_z)}{\lambda_{zz}} - \tfrac{h(\hat{v}_z)-h(\hat{v}_j)}{c_z - c_j}\right)(c_z - c_j) + (h'(\hat{v}_z) - h'(\hat{v}_j))p_z^2 c_{zz} \\
&> \left(h'(\hat{v}_z) - \tfrac{h(\hat{v}_z)-h(\hat{v}_j)}{c_z - c_j}\right)(c_z - c_j) \\
&> 0.
\end{aligned}$$

If the agent is not too risk averse, then the right-hand side of (15) is increasing in p_k by the convergence of (15) to (FB_k) as $h(v) \to v$. Since the right- and left-hand sides must equate, it follows that $\frac{\mathrm{d}p_z}{\mathrm{d}\lambda_{zj}} < 0$. □

Proof of Proposition 5: The first best contract satisfies $\pi_j - \rho = \sum_k \hat{v}_k \lambda_{jk}$ for all j. Substituting this into (20) yields $\hat{x}_i = \sum_j \lambda_{ij}(\rho + \sum_k \hat{v}_k \lambda_{jk})$. Moreover, any false negative recognition rule satisfying Definition 1 satisfies $\lambda_{ij} > 0$ if and only if $j \in \{i, z\}$. It follows that

$$\hat{x}_i - \rho \;=\; \sum_j \sum_k \lambda_{ij}\lambda_{jk}\hat{v}_k \;=\; \hat{v}_z + (\hat{v}_i - \hat{v}_z)\lambda_{ii}^2.$$

Furthermore, note that (i) $\hat{v}_i - \hat{v}_z$ is constant for $i \geq \bar{\kappa}$ and $i \leq \underline{\kappa}$, and (ii) λ_{ii} specified by (11) is increasing linearly in $|i - z|$ for $i \geq \bar{\kappa}$ and $i \leq \underline{\kappa}$. It follows that $\hat{x}_i$ is increasing quadratically in $|i - z|$ for $i > \bar{\kappa}$ and is decreasing quadratically in $|i - z|$ for $i < \underline{\kappa}$.

Now, any false positive recognition rule satisfying Definition 3 satisfies $\lambda_{ij} > 0$ if and only if $i \in \{j, z\}$. It follows that for all $i \neq z$,

$$\hat{x}_i - \rho \;=\; \sum_j \sum_k \lambda_{ij}\lambda_{jk}\hat{v}_k \;=\; \lambda_{ii}\lambda_{ii}\hat{v}_i \;=\; \hat{v}_i \;=\; \nu + c_i \;=\; \pi_i - \rho.$$

Thus $\hat{x}_i$ and $\hat{v}_i + \rho$ are both linear in i for all $i \neq z$ for any symmetric or asymmetric false positive recognition rule satsifying Definition 3. $\square$

Proof of Proposition 6: Any false positive recognition rule satisfying Definition 3 satisfies $\lambda_{ji} > 0$ if and only if $j \in \{i, z\}$. Consistent with the conditions in Proposition 2, let $\lambda_{zi} > 0$ for $i < \underline{\kappa}$ and $i > \bar{\kappa}$, and let $\lambda_{zi} = 0$ for $\underline{\kappa} \leq i \leq \bar{\kappa}$. Substituting these into (21) yields

$$\hat{\pi}_i \quad = \quad \frac{\sum_j \pi_j p_j \lambda_{ji}}{\sum_j p_j \lambda_{ji}} \quad = \quad \frac{\pi_i p_i \lambda_{ii} + \pi_z p_z \lambda_{zi}}{p_i \lambda_{ii} + p_z \lambda_{zi}} \begin{cases} > \pi_i & \text{if } i \in \{1, \ldots, \underline{\kappa} - 1\} \\ = \pi_i & \text{if } i \in \{\underline{\kappa}, \ldots, \bar{\kappa}\} \\ < \pi_i & \text{if } i \in \{\bar{\kappa} + 1, \ldots, N\}. \end{cases}$$

It follows that

$$\begin{aligned} \frac{\hat{\pi}_{\underline{\kappa}} - \hat{\pi}_1}{x_{\underline{\kappa}} - x_1} \quad &= \quad \frac{\pi_{\underline{\kappa}} - \hat{\pi}_1}{x_{\underline{\kappa}} - x_1} \quad < \quad \frac{\pi_{\underline{\kappa}} - \pi_1}{x_{\underline{\kappa}} - x_1} \\ \frac{\hat{\pi}_{\bar{\kappa}} - \hat{\pi}_{\underline{\kappa}}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} \quad &= \quad \frac{\pi_{\bar{\kappa}} - \pi_{\underline{\kappa}}}{x_{\bar{\kappa}} - x_{\underline{\kappa}}} \\ \frac{\hat{\pi}_N - \hat{\pi}_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}} \quad &= \quad \frac{\hat{\pi}_N - \pi_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}} \quad < \quad \frac{\pi_N - \pi_{\bar{\kappa}}}{x_N - x_{\bar{\kappa}}}. \end{aligned}$$

That is, $\{\hat{\pi}_i\}$ is S-shaped relative to $\{x_i\}$ whenever a false positive recognition rule causes $\{\hat{v}_i\}$ to be S-shaped relative to $\{x_i\}$.

Now, any false negative recognition rule satisfying Definition 1 satisfies $\lambda_{ji} > 0$ if and only if $i \in \{j, z\}$. It follows that for all $i \neq z$,

$$\hat{\pi}_i \quad = \quad \frac{\sum_j \pi_j p_j \lambda_{ji}}{\sum_j p_j \lambda_{ji}} \quad = \quad \frac{\pi_i p_i \lambda_{ii}}{p_i \lambda_{ii}} \quad = \quad \pi_i.$$

Thus $\hat{\pi}_i$ is linear in i for all $i \neq z$ given any symmetric or asymmetric false negative recognition rule satisfying Definition 1. $\square$

B Figures

Figure 1. Illustration of the production and performance measurement process with $N = 4$.

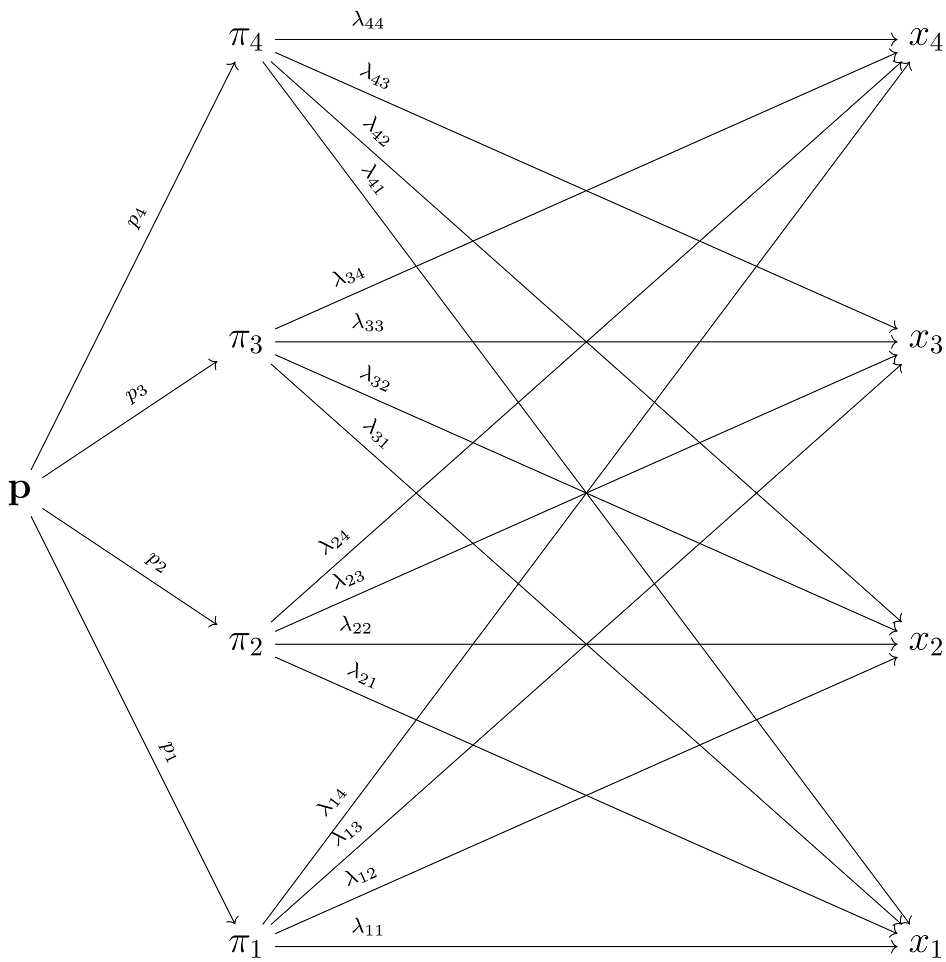


Figure 2. First best and hypothetical second best (no measurement error) fundamental performance distributions and contracts.

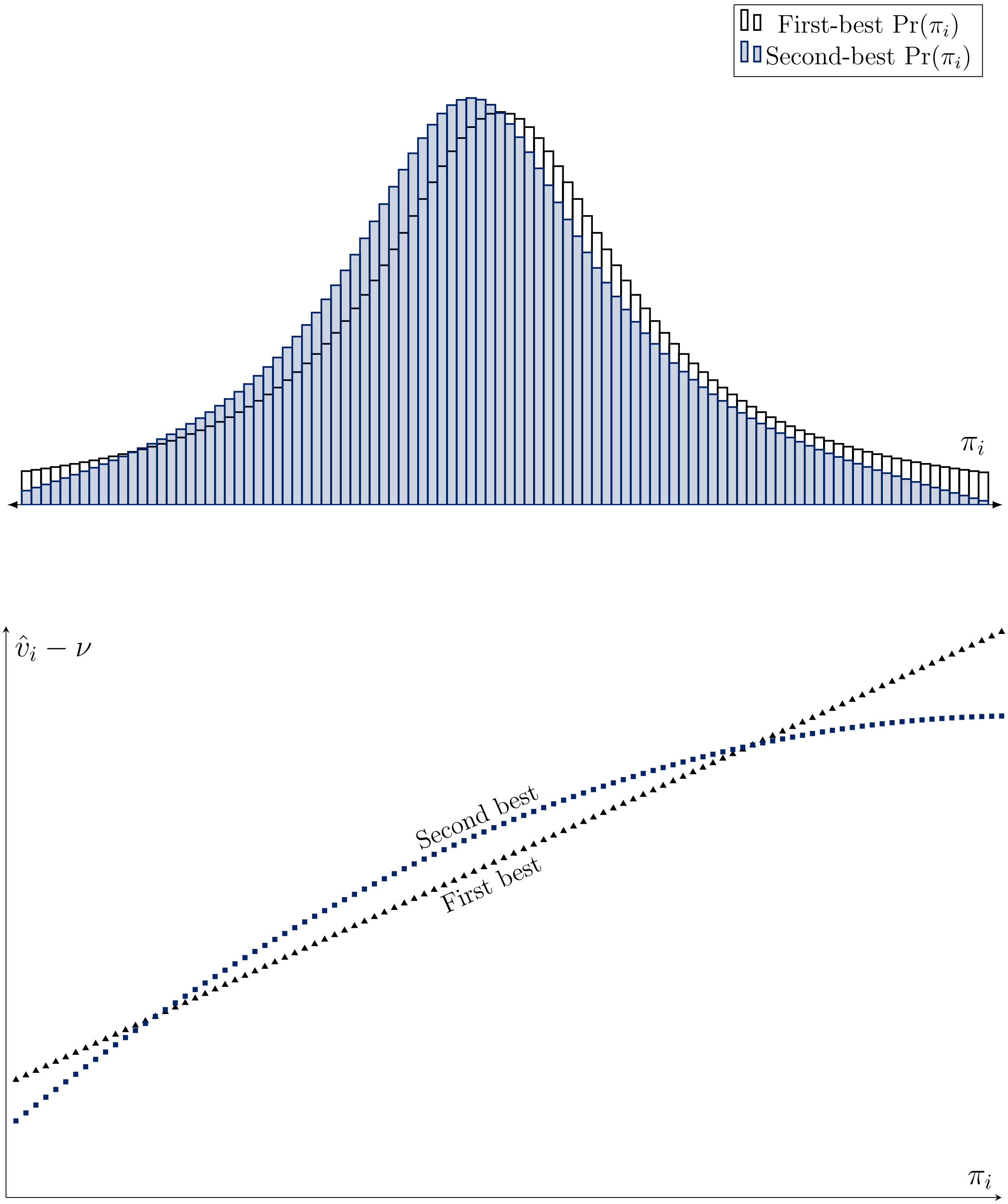


Figure 3. The impact of false negative recognition rules on optimal contracts when the agent is risk neutral, $Z = \{z\}$, and λ_{ii} satisfies (11).

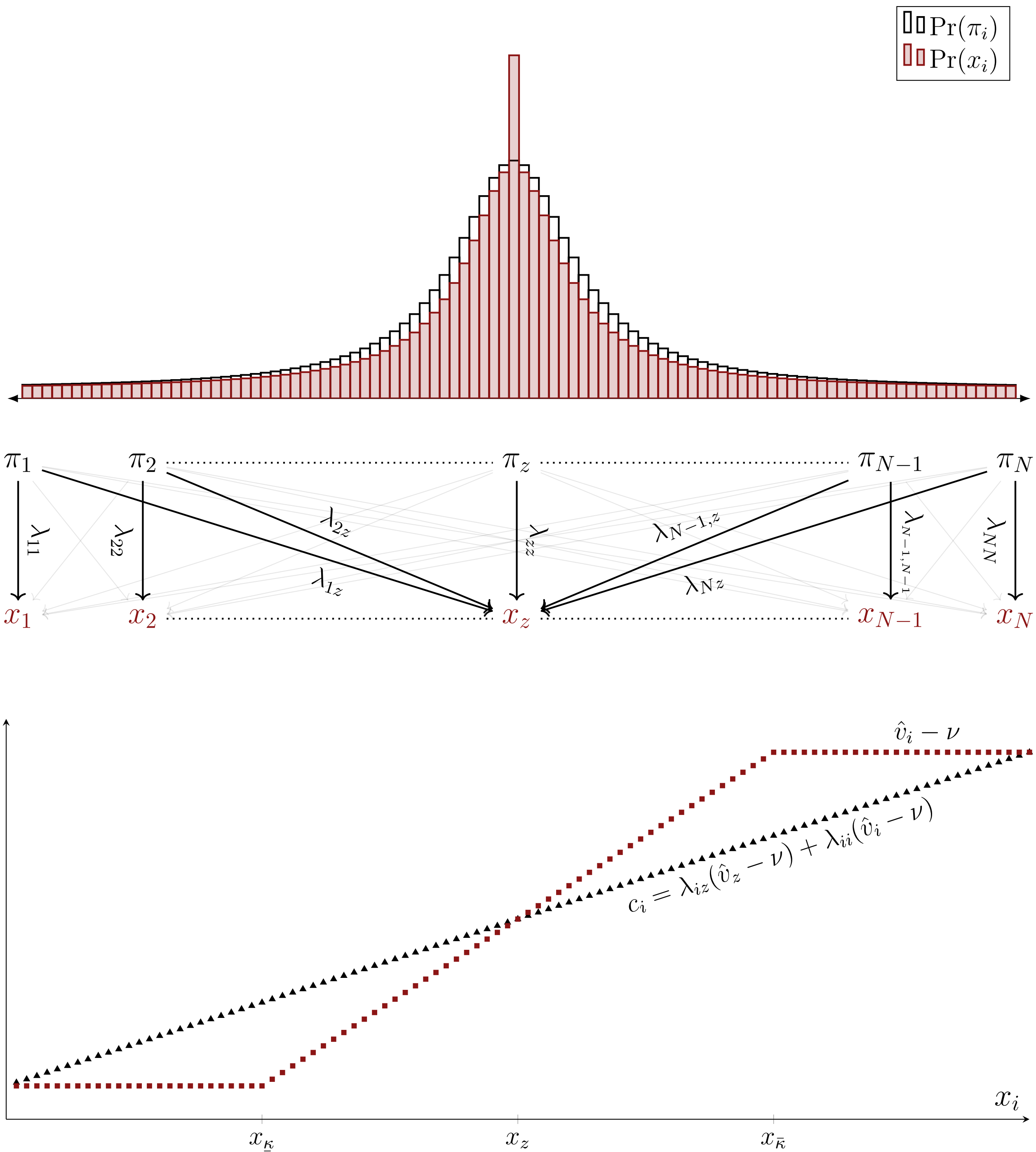


Figure 4. The impact of false positive recognition rules on optimal contracts and actions when the agent is risk averse, where $\hat{v}_i^{sb}$ denotes the second-best contract.

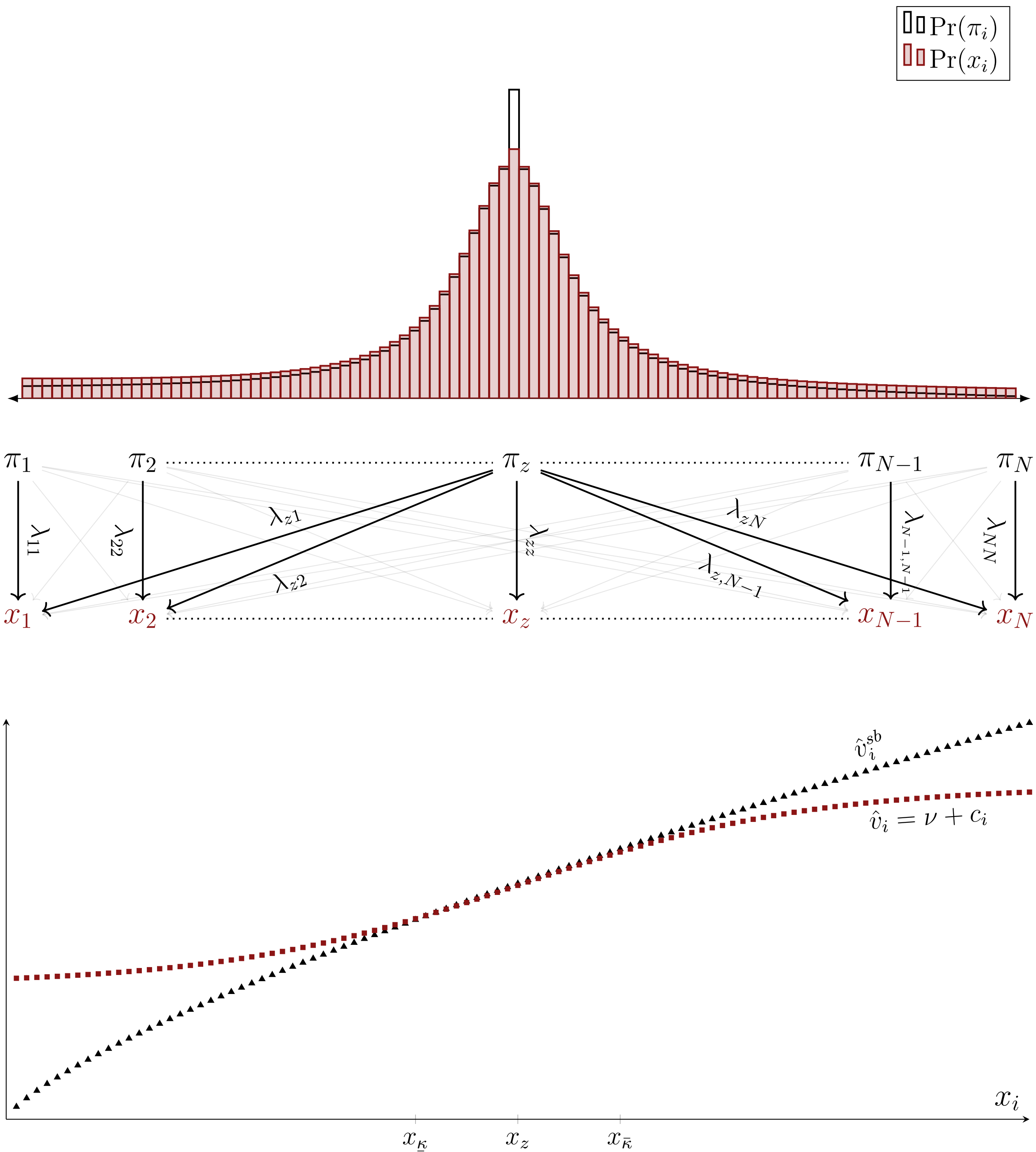
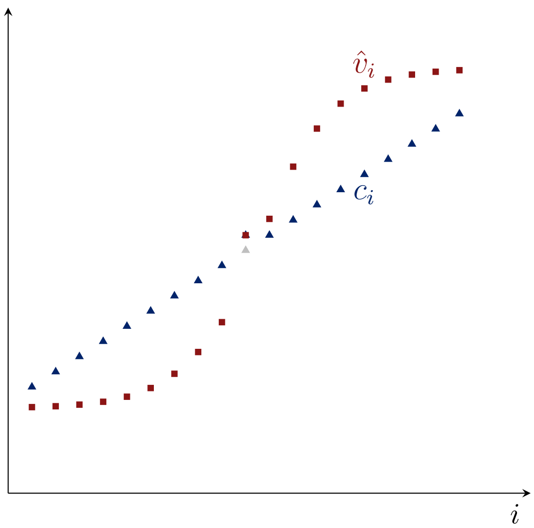


Figure 5. Left: The impact of an asymmetric false negative recognition rule on the optimal contract $\hat{v}_i$ and action c_i. Right: The impact of an asymmetric false positive recognition rule on the optimal contract $\hat{v}_i$ and action c_i.



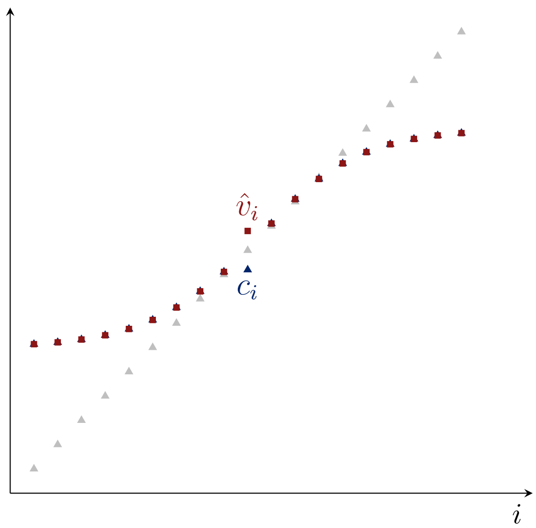


Figure 6. Left: the Basu relation $\hat{x}_i \equiv \mathbb{E}[x|\pi_i]$ for the false negative recognition rule given by (11) and any arbitrary false positive recognition rule conforming to Definition 3. Right: the earnings-returns relation $\hat{\pi}_i \equiv \mathbb{E}[\pi|x_i]$ for any false positive recognition rule leading to an optimal S-shaped contract and any arbitrary false negative recognition rule conforming to Definition 1.