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Public Listing Choice with Persistent Hidden Information

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Abstract

How much does firm intangibility amplify CEOs' persistent private information and reduce firms' public listing propensity? We develop a model of competing public and private investors financing firms heterogeneously exposed to persistent private cash flows. Equilibrium financing is driven by information rent differentials in CEO compensation. We validate and structurally estimate the model using firm listing and CEO compensation data. We find private (intangible) cash flows exhibit 63% higher persistence than their tangible counterparts. Further, if firm intangibility levels returned to those of 1980, mean listing propensities would increase 5 percentage points while mean CEO variable pay growth would decrease by 61%.

Keywords: intangible capital, public listings, persistent private information, CEO compensation, private equity premium, assignment model, structural estimation

JEL: C78, D86, E22, G32, M12, O33

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1 Introduction

"Our problem – which we can't solve by studying up – is that we have no insights into which participants in the tech field possess a truly durable competitive advantage...Predicting the long-term economics of companies that operate in fast-changing industries is simply far beyond our perimeter."

-Warren Buffett, 1999 Berkshire Hathaway shareholder letter

Many significant firm events, such as legal settlement agreements, new trade secrets or proprietary consumer data, are associated with a firm's intangible assets. Such events have persistent effects on firm cash flows, yet are often not observed by outside investors. Furthermore, even when there is full public disclosure as in the case of newly granted patents, little consensus exists on how investors can appropriately value these individual developments. Firm insiders' persistent private information not only magnifies the lifetime impact of a cash flow shock, but also drastically alters the types of incentives needed for truthful reports. The opacity of intangible assets and the challenges in identifying their resulting cash flows may amplify the persistent private information of insiders. Spurred by the information communication technology (ICT) revolution, firms' aggregate accumulation of intangible assets may have induced a rise in public CEO compensation level and performance sensitivity. To the extent that private investors avoid these information frictions via their expertise and interaction with firm insiders, such increased compensation costs for public financiers may have reduced the net benefit of being publicly listed on stock markets and caused a listing fall.[1]

In this paper, we quantify how much rising intangibility has amplified public CEOs' persistent private information and contributed to the trends in public listings and CEO compensation. Identifying the substantive drivers of these joint patterns is crucial to evaluate the role of potential policy inter-

[1] Intangibles are knowledge-based assets like patents, software and human capital which have proliferated over the past four decades (see Brynjolfsson and McElheran (2016), Kogan et al. (2017), Bessen et al. (2018), Kini et al. (2021) and Crouzet et al. (2022)). They are fundamentally non-rival, imperfectly excludable goods, and so are at risk of imitation (Crouzet et al. (2022)). Thus, intangible investments like research and development (R&D) are often not publicly divulged to investors with the same specificity as physical investments (see Arrow (1962), Bhattacharya and Ritter (1983), and Hall et al. (2010)). This renders intangible firms generally more opaque and uncertain for investors as found by Aboody and Lev (2000), Kothari et al. (2002), Flannery et al. (2004), Gu and Wang (2005), Eisfeldt and Papanikolaou (2013), Dell'Ariccia et al. (2020) and Wu and Lai (2020). An exception, noted by Stulz (2020), Aghamolla and Thakor (2022) and Ewens and Farre-Mensa (2022), are private equity investors with whom firms don't face the same imitation risks when disclosing information.

ventions. However, the latent nature of private information precludes direct measurement. To tackle this problem, we build, empirically validate, and structurally estimate a market equilibrium model of firm financing and CEO compensation where CEOs have persistent private information over cash flows generated by intangible assets.

Our model generates variations in firm public listing decisions and CEO compensation packages through heterogeneous firms' exposure to private cash flows. Crucially, the distortions caused by these private cash flows are magnified by their persistence. To test our theory of a common information friction driving both firm listing and CEO pay decisions, we validate our model predictions proxying the exposure to private cash flows using measures of firm intangibility and primarily relying on a large dataset of public and private US firms and their CEOs. Furthermore, we separately structurally estimate the private cash flow persistence parameter identified using non-overlapping moments (and disjoint data) of firm listing choice and CEO compensation and compare their values. We then use our estimates to quantify the amplification effect the ICT revolution had on persistent private information by evaluating the counterfactual where firm intangibility remained at the levels observed in 1980, the approximate start of the technological transformation.

In the model, public investors design optimal compensation contracts to dynamically induce truth-telling akin to Williams (2011) and Bloedel et al. (2020).[2] Despite the manager having no influence over the actual cash flow process, to preclude cash flow diversion the optimal public CEO pay is performance sensitive, with the level of sensitivity increasing in the lifetime size of the private information. The risk built into the contract to incentivize truthtelling is compensated with positive expected growth in pay over time. Private investors have access to a costly monitoring technology which allows them to design first-best efficient contracts. Competition in financing between public and private investors then generates a private equity (PE) premium as the foregone information rents net monitoring costs.[3] Capacity constraints

[2]Bloedel et al. (2020) demonstrate with a counterexample that the contract in Williams (2011) is not optimal amongst a general class of contracts, but only amongst stationary contracts satisfying first-order incentive compatibility. In light of this, we implicitly restrict the space of contracts to those ensuring no hidden savings and leverage recent advancements in the stochastic maximum principle literature to assure optimality of the contract amongst all incentive compatible contracts (under mild regularity restrictions).

[3]This PE premium is correlated to the equity share of public CEO compensation, with higher private information persistence mapping to higher equity-based pay, and hence, given the continuously priced equity in public markets, a higher pay-performance sensitivity. While a

for individual private investors together with competition against the public investors induce a selection effect whereby more intangible firms are more likely to be privately financed, and, conditional on being publicly financed, typically have higher CEO pay level and performance sensitivity. Aggregate private investment is tied to the average PE premium and in (general) equilibrium is an increasing function of average firm intangibility. Then, while an increase in intangibility amongst public firms will lead to higher compensation, an increase in aggregate intangibility can in fact lower average public CEO compensation due to a selection effect with rising PE funds.

Our mechanism combined with a relaxation of PE funds by the National Securities Market Improvement Act (NSMIA) in the late 1990s can jointly rationalize the broad historical patterns in aggregate US public listing, CEO pay, PE premium, and new business formation and hence economic growth. Moreover, through the lens of our model, the observed patterns suggest that the increased disclosure requirements of the Sarbanes-Oxley Act (SOX) may not have been effective in increasing transparency in public markets.

We obtain additional support for our theory in supplemental historical data by estimating time-varying elasticities of CEO pay to intangibility. These elasticity estimates rationalize the inflection point in aggregate US public CEO pay trends around 1980 and overall patterns in the past half a century.[4] Furthermore, we use aggregate US time-trends and international data to validate the differential implications of our theory of rising intangibility.

Our estimates of private information persistence across the two structural estimations are statistically indistinguishable from each other. The estimation suggests a 63% higher persistence in private information cash flows than persistence in the tangible cash flows implied by physical investment. The inferred aggregate effects of a secular increase in firm intangibility is large. If US firm intangibility had remained at its 1980 level, listing propensities would be 5 percentage points higher, while the annual growth in average CEO pay would be reduced by 61%. To the extent fears that the "growing lack of transparency in capital markets will lead once again to the misallocation of capital that we saw at the inception of the federal securities laws," (Lee

positive PE premium is generated with permanent shocks (i.e. if cash flows follow a Brownian motion), the gain in theoretical simplicity is diluted by empirical difficulties implied by non-stationary cash flow and compensation processes. Moreover, our empirical work suggests reported earnings and compensation dynamics are better captured by persistent cash flows.

[4]In particular, we find that the time-series variation in aggregate firm levels of intangibility helps rationalize the patterns of average public CEO compensation observed over the second half of the 20th century both in level and use of equity pay. This is important as according to Edmans et al. (2017) "[t]he reasons for this evolution are not fully understood."

(2021)) our results suggest public listing reforms mitigating the informational advantages for intangible private firms may be critical.

Related Literature: We quantify a new technological channel driving the decline in publicly listed US firms since 1997 documented by Gao et al. (2013b) and Doidge et al. (2017). Other explanations for the decline have largely focused on US regulatory and institutional changes.[5] However, this phenomenon has recently become apparent across other advanced economies. Our proposed driver of rising information frictions generates joint predictions on public listings, CEO compensation, PE premia, and business dynamism which are consistent with established US and international evidence.[6]

Our theory of a firm's decision to go public is based on lower intangible cash flow risk and involves a competitive matching equilibrium between heterogeneous firms and investors. Classical theories highlight the benefits of going public from a lower cost of capital.[7] Other studies focus on heterogeneity in investment opportunities and costs of going public.[8] These models largely favor sorting in terms of older, bigger, and more productive firms, whereas Doidge et al. (2017) find that listing propensities have declined across all sizes and industries. This suggests that neither the amount of capital nor the type of investments undertaken across industries are core to the decline.

Due to limited data on private firms, empirical examinations of firm listing decisions have been scarce.[9] We contribute to the empirical evidence of firm listing decisions using Capital IQ data on both public and private firms.[10]

We contribute to the theory and measurement of the underlying drivers of PE premia both in the cross-section and time-series. To our knowledge the

[5]For example, Leuz (2007) and Iliev (2010) discuss the increase of compliance costs of being public due to SOX, Ewens and Farre-Mensa (2020) and Kwon et al. (2020) the relaxation of private equity funding due to NSMIA, and Davydiuk et al. (2020) a combination of the two.

[6]Other works have examined these trends in isolation. See Kahle and Stulz (2017), Caskurlu (2020), Frydman and Saks (2010), Edmans et al. (2017), Moskowitz and Vissing-Jorgensen (2002), Kartashova (2014), Bloom et al. (2020), Pellegrino (2021) and Eckbo and Lithell (2022).

[7]This reduction arises due to (i) a broader pool of financiers in Merton (1987) and Rajan (1992), (ii) diversification of insiders' risk in Levine (1991) and Pagano (1993), (iii) improved monitoring in Holmstrom and Tirole (1993) and Pagano and Roell (1998), or (iv) better guidance from market experts in Maug (2001).

[8]For example, Clementi (2002), Ferreira et al. (2012), and Spiegel and Tookes (2013) focus on firms' productivity, while Ritter (1987) and Gupta and Rust (2017) study regulatory costs, Campbell (1979), Yosha (1995), Maksimovic and Pichler (2001) analyze costs associated with the potential loss of confidentiality, Jensen and Meckling (1976), Leland and Pyle (1977), Jensen (1989), and Chemmanur and Fulghieri (1999) examine asymmetric information costs.

[9]A few exceptions include Lerner (1994), Pagano et al. (1998), and Chemmanur et al. (2010) who have data only on certain industries.

[10]Other studies, such as Gao et al. (2013a), Gao and Li (2015) and Acharya and Xu (2017), have used our main data source, Capital IQ, to study differences between public and private firms, but, to our knowledge, none examine listing status and CEO pay in relation to intangibility.

paper is the first to connect the average level or distribution of PE premia to contracting frictions and imperfect competition. Moreover, our theory and empirical exercises speak to the open question posed by the recent survey Ewens and Farre-Mensa (2022) "[w]hat are the real consequences of the decline in U.S. listings and the growth of the private equity market?"[11]

Our paper contributes to the literature examining the determinants of CEO compensation. Leading theories on the rise in the level of CEO compensation have tended to focus on size-driven mechanisms like Tervio (2008) and Gabaix and Landier (2008) or size and market power-driven like Bao et al. (2022). Our work complements these stories by examining the role of firm intangibility which we find not only to have similar levels of explanatory power in the cross-section but also to help rationalize the evolution of US CEO compensation level and composition over the post-war sample documented by Frydman and Saks (2010).[12]

Other papers have examined different facets of exogenous technological change driving compensation trends. Much of this literature focuses on human capital as a facet of intangible assets, which induces increased restricted equity compensation for retention. To the extent that executive human capital is embedded in the private information of the firm, the reduced form predictions on the level of CEO pay are the same. However, in our context, performance sensitivity is an intentional response to, rather than a by-product of, evolving outside options, and rationalizes the initial popularity in the 1980s of option-based compensation rather than simply deferred stock grants.[13]

Beyond the impact on compensation, our paper also examines the implications of contracts on firm dynamics.[14] The closest paper in this literature is Ward (2022) who studies the effect of an intangibility driven agency friction on public firms' investment and market valuation dynamics. The "pure moral hazard" agency friction in his model is tied to hidden effort on intangible

[11]Cross-sectional and time-series patterns in private equity premia are documented by Harris et al. (2021) and Kartashova (2014). Other theories for PE premia focus on its illiquidity and diversification issues such as Angeletos (2007), Ang et al. (2014), and Abudy et al. (2016).

[12]In perhaps the closest paper empirically examining firm agency-inducing characteristics in the finance industry, Cheng et al. (2015) shows that persistent firm-specific risk induces higher levels of CEO pay to compensate the CEO for the magnified pay sensitivity risk faced.

[13]See Lustig et al. (2011), Sun and Xiaolan (2019), Frydman and Papanikolaou (2018) and Kline et al. (2019). See Murphy (2013) for a survey on explanations about these trends.

[14]Continuous time contracting frameworks have become increasingly popular due to their tractability, beginning with DeMarzo and Sannikov (2006), Biais et al. (2007), and Sannikov (2008). In contrast to our paper, these works largely focus on agency issues with independent and identically distributed private information. Garrett and Pavan (2012) extend this framework to have persistent private information through unknown initial conditions. Bolton et al. (2019) examine how human capital affects debt capacity and CEOs' risk exposure.

asset accumulation while we study a "hybrid moral hazard" framework.[15] As noted by Edmans et al. (2017) small differences in information frictions generally lead to substantively different economic implications. For instance, the "pure moral hazard" friction predicts a positive correlation between intangibility and performance while the "hybrid moral hazard" does not require any type of association.[16] Other quantitative papers, such as Ai et al. (2022), Gayle and Miller (2009) and Gayle et al. (2015), focus on "pure moral hazard" models. In these papers, the estimated size of the private information shock is found to be relatively small and dwarfed by effects based on firm size.[17] To our knowledge, our paper is the first to quantify economic effects of persistent private information arising from intangible assets, as well as provide a credible estimate for the level of persistent private information.

The remainder of the paper is structured as follows. Section 2 presents the model and testable implications. Section 3 describes our data. Section 4 uses proxies of firms' private information cash flow characteristics to empirically validate our cross-sectional and time-series predictions. Section 5 structurally estimates the model and conducts counterfactual experiments to infer the quantitative importance of persistent private information to firm listing and CEO compensation decisions. We conclude in Section 6.

2 Theory

The model adapts the principal-agent contracting environment of Williams (2011) to a corporate finance setting with cash flows driven by a mixture of publicly and privately observed cash flows, referred to as tangible and intangible cash flows respectively. Taking into account the discussion in Bloedel et al. (2020) we allow the agent to privately save and borrow directly at the risk-free

[15] Gayle and Miller (2015) define "pure moral hazard" models as those with hidden actions and "hybrid moral hazard" models as those with hidden information and actions. Our choice to study a "hybrid moral hazard" friction is in part motivated by their empirical work which tests the two model classes on data of CEO compensation and performance, and find evidence that the former class is rejected in the data, while the latter class cannot be.

[16] Controlling for selection is important for understanding the fundamental information frictions at play in our dataset, as we find that firm intangibility has a slightly negative correlation with productivity when pooling across public and private firms, as opposed to the positive correlation found in Ward (2022) based on public firms.

[17] In a "hybrid moral hazard" model like ours, for a given level of private information shocks, persistence magnifies the aggregate size of private information and introduces a mixture of moral hazard and adverse selection considerations within the contracting environment which substantially alters the firm financing and compensation structures.

rate.[18] Our characterization of optimal contracts in this setting improves on Williams (2011) and Bloedel et al. (2020) by establishing global incentive compatibility, rather than restricting to contracts which satisfy first-order incentive compatibility.[19] We do so by introducing a stochastic maximum principle (SMP) new to the this contracting setting and demonstrating its applicability subject to mild technical restrictions to the contract space.[20]

To study the potential equilibrium effects of intangibility-induced private information, we embed the optimal contracting problem into a market equilibrium setting with two competing principals, representing the public and private equity markets respectively. The principal representing the public investors is unable to directly observe the intangibility-driven cash flows, but has deep pockets. The other principal has a costly monitoring technology which allows to avoid these intangibility-induced information frictions, but has limited funds.

The remainder of the section is as follows. We begin by describing the contracting problem and the partial equilibrium model environment where the private principal is assumed to be financially constrained. We then endogenize the supply of funds and the PE premium in a general equilibrium, competitive matching model extension. Finally, we conduct comparative static analysis to evaluate the impact of changing regulation and technology on the public listing and compensation patterns and other economic aggregates. A formal description of the problem and all the proofs are relegated to Appendix A.

2.1 Environment

Time is continuous and infinite. All parties share a time discount rate ρ. There is a unit mass of firms, each owned by a risk-averse agent (also called "CEO" or "entrepreneur") with constant absolute risk-aversion (CARA) utility function of consumption c equal to $u(c) = -e^{-\psi c}$, where ψ is the risk-aversion

[18] Bloedel et al. (2020) show that, if the agent has the option to engage in private saving, the Williams (2011) contracts are optimal amongst the class of first-order incentive compatible contracts implementing no hidden savings. Further discussion is provided in Appendix A.1.

[19] Characterizations of sufficient conditions to appeal to the first-order approach in static settings are provided by Mirrlees (1971), Rogerson (1985) and Jewitt (1988). For a discussion of the applicability of the first-order approach in dynamic settings with persistent private information (though in a slightly different environment) see Battaglini and Lamba (2019).

[20] To apply an appropriate SMP we draw on the results in Maslowski and Veverka (2014), Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019). Details on the SMP introduced here and the establishment that its application is amenable to this setting are given in Appendix A.1. Nakajima (2021) draws from some of these same advances in a different setting with independent and identically distributed private information.

coefficient. At time $t = 0$ an entrepreneur needs one unit of funds to get the firm off the ground. If funding is not obtained, the agent has an outside option of $\underline{q} \in \mathbb{R}_{-}$. If funding is obtained, after the initial financing, the agent, unobserved by the principal, can privately save and borrow at a risk-free rate r.[21] Then, for $t > 0$ a firm of size K produces cash flows $Y_t = y_t K$ where y_t is the firm profitability rate. Since the channel we wish to highlight does not depend on size, we will abstract from size effects and set $K = 1$ throughout the model. Firm profitability y_t can be decomposed into a tangible component x_t and an intangible component z_t, with share $\tau \in [0, 1]$ of tangible profitability dependence, so that

$$y_t = \tau x_t + (1 - \tau) z_t.$$

Both x_t and z_t follow an Ornstein-Uhlenbeck process, which is the continuous time equivalent of an autoregressive of order 1 (AR(1)) process, with persistence λ^i, drift μ^i, volatility σ^i, and initial condition i_0 which is fixed at the steady state long-run average, for $i \in \{x, z\}$, that is

$$di_t = \left(\mu^i - \frac{i_t}{\lambda^i}\right) dt + \sigma^i dW_t^i, \qquad i_0 = \lambda^i \mu^i, \qquad i \in \{x, z\}. \tag{1}$$

We assume that the tangible component x_t is publicly observable while the intangible component z_t is observable only by the agent. As such, the agent has persistent private information on the firm profitability y_t. This induces hybrid moral hazard wherein the agent has private information on the current state and can adjust their decisions to misreport accordingly. We take firms to draw their cashflow characteristics $\theta = (\{\mu^i, \lambda^i, \sigma^i\}_{i=\{x,z\}}, \tau)$ from a distribution with cumulative distribution function (CDF) $G_\theta(\cdot)$.

There are two representative, risk-neutral principals (also called "financiers" or "investors") who compete using compensation contracts to fund the pool of heterogeneous firms. The representative public investor, P, has unlimited funds but cannot observe the intangible cash flows z_t, and so restricts the offered contracts to those that induce truthful reporting. The representative private investor, S, is a specialist and so uses a monitoring technology connected to their expertise to observe the intangible component z_t, avoiding the associated information frictions. This monitoring comes at a lifetime cost ν. This financier is however financially constrained by a budget $B < \infty$.

[21] We implicitly assume that there are borrowing limits which preclude the agent from self-financing the project through risk-free borrowing. This assumption is consistent with borrowing limits due to exogenous elements, such as lack of reputation or collateral, that could preclude the agent the access to the saving technology.

2.2 Optimal Compensation Contracts

We restrict our analysis to subgame perfect Nash equilibria (SPNE) which assures that the compensation contracts offered by the representative investors at time 0 are optimally designed at the firm-investor level.

Denote $\omega_t^f(q_0,\theta)$ as the optimal compensation contract offered by a financier of type $f \in \{S,P\}$ to a CEO of a type θ firm with initial promised utility level q_0. A type f investor's time 0 expected return from financing a CEO of a firm of type θ offering q_0 initial promised utility is

$$R^f(q_0,\theta) = \mathbb{E}\left[\int_0^\infty e^{-\rho t}\left(y_t - \omega_t^f(q_0,\theta)\right)dt\middle|\theta\right] - \nu\mathbf{1}_{\{f=S\}} - 1.$$

Different exposures to persistent private information generate a firm-specific premium between the private and public investor's returns. Denote $\pi(q_0,\theta)$ as the information premium, that is the expected information cost of the public investor financing a firm of type θ offering promised utility q_0. Since the private investor avoids these information frictions at cost ν, for fixed initial promised utility q_0 and firm type θ, the PE premium takes the form

$$R^S(q_0,\theta) - R^P(q_0,\theta) = \pi(q_0,\theta) - \nu. \tag{2}$$

Theorem 1 presents the optimal compensation contract offered by the two financiers and establishes that the above premium is independent of the level of initial promised utility.

Theorem 1. *Under mild technical restrictions, the optimal contract between the public principal and an agent owning a type θ firm yields an information premium*

$$\pi(\theta) = \frac{\psi}{2}\left(\frac{(1-\tau)\sigma^z}{\rho + \frac{1}{\lambda^z}}\right)^2, \tag{3}$$

with total compensation to the agent financed by a public investor given by

$$\omega_t^P(q_0,\theta) = \omega_{fix}(q_0) + \omega_{gro}(\theta)t + \omega_{per}(\theta)W_t^z, \tag{4}$$

where the first term is a fixed salary compensation component with $\omega_{fix}(q_0) = -\frac{1}{\psi}\log(-\rho q_0)$, which is equal to the total compensation offered by a private investor $\omega_t^S(q_0,\theta) = \omega^S(q_0)$, the second term is a growth compensation component with $\omega_{gro}(\theta) = \rho^2\pi(\theta)$, and the third term is a performance-based compensation component with $\omega_{per}(\theta) = \rho\sqrt{\frac{2}{\psi}\pi(\theta)}$.

By inspection, the optimal contracts offered by the public and private investors generate stark differences in the compensation level, growth rate, and risk. The performance-based compensation component of equation (4) suggests that, in order to incentivize truthful revelation of the entire cash flows, the public investor rewards (penalizes) unexpectedly high (low) reported profitability. Since agents are risk-averse, they must be compensated for this additional risk exposure. As the possibility for the agent to privately save precludes more complex, non-stationary contracts (including those which result in immiseration), this risk requires the public investor to steadily increase the expected compensation over time, resulting in a positive growth component.[22] Consequently, for a given level of promised utility, the compensation proposed by the public investor offers positive average compensation growth, but is performance sensitive. In contrast to the model of Ai et al. (2022), the optimal compensation with the uniformed public principal only depends on the realizations of the unobserved cash flows, fully insuring the observable cash flows.[23]

Of course, the complete stabilization of the CEO compensation with private financing is a highly stylized result and abstracts from the illiquid equity tied to private CEO contracts. Consider an extension of the model with a dynamic hidden effort choice (pure moral hazard friction) that shapes the long-run level of the cash flows. Directly applying the results of He (2011), which are amenable to our setting, the CEO is optimally paid a salary and deferred equity compensation scheme similar to Holmstrom and Milgrom (1987).[24] As is standard, the optimal contract results in an effort level below that of the efficient level which widens by the level of uncontrolled volatility

[22]This feature implies that, over a sufficiently long time horizon, this investor may have to actually subsidize the CEO compensation from own funds. This is easily precluded with the introduction of a stochastic Poisson firm destruction rate, η, which yields identical results with an appropriate modification of the discount rate from ρ to $\rho + \eta$.

[23]Ai et al. (2022) study an environment where the agent has a hidden investment activity which influences the long-run level of the project cash flows, as opposed to the splitting of the cash flows in our setting. This feature combined with a constant relative risk-averse agent, which implies non-zero wealth effects, result in an optimal compensation contract which depends on both the realizations of the observable and unobservable stochastic cash flow processes. See Appendix A.1 for details on our results.

[24]In particular, take the redefined cash flows $Y_t = y_t - \mathbb{E}_0[y_t] + \delta_t$ where $\delta_0 = \mathbb{E}_0[y_t] = y_0$ and δ_t governs the stochastic long-run level of the cash flows which itself is a function of the agent's hidden effort a_t, evolving according to $d\delta_t = [\mu_0^\delta + \mu_1^\delta a_t]dt + \sigma^\delta dW_t^\delta$ with dW_t^δ independent of the private cash flow evolution dW_t^z. Note given the independence of our optimal contract subject to hidden information in z from the tangible process x, we can for simplicity set $W_t^\delta = W_t^x$. Taking the agent's utility to be $e^{-\psi c - g(a_t,\delta) - \rho t}$, where $g(a_t, \delta) = \frac{\theta}{2} a_t^2$ is the cost of the hidden action, then the optimal effort under the contract can be obtained directly from applying the arguments of He (2011).

of the long-run level outcome process. In particular, this agency friction is eliminated when the volatility in this process is zero.[25]

We connect the size of this pure moral hazard friction to the amount of public disclosure and outside scrutiny of the firm and CEOs actions. That is, we assume that for a market transaction of the firm's equity to occur, for a fixed window around the transaction, the CEOs actions must be disclosed and are heavily scrutinized (as is the case for M&A transactions or other large private placements), so that over this window the moral hazard friction is exogenously precluded. Thus, for liquid public equity markets with a diverse shareholder base subject to idiosyncratic (Poisson) sell shocks, the expected horizon to the next market transaction is zero, and hence is effectively unexposed to this friction. In contrast, private firms with concentrated ownership and infrequently traded equity will suffer from these pure moral hazard distortions, with lower on average long-run performance (consistent with Larrain et al. (2021)).[26] Observe that with this extension, when the intangibility induced information premium is small, public listing provides net economic benefits associated to having large dispersed ownership, but is inefficient when intangibility amplified information frictions are large relative to the pure moral hazard friction.

Note also that our results extend to private investors who are merely relatively better informed about the intangible cash flows. For instance, assuming the private investor suffers from less persistence in the private cash flows results in same qualitative contracts as public but just smaller performance components. With either adaptation, the testable predictions are as follows.

CEO Pay Testable Prediction. *All else equal, higher total pay and performance sensitive pay share are associated with (i) higher levels of firm intangibility, (ii) higher intangible cash flow volatility, and (iii) being publicly listed.*

[25]Recall that Gayle and Miller (2015) differentially tests the predictions of moral hazard versus hidden income and finds the latter carries more empirical content. With our dynamic continuous time contracts here, a key difference in contracts is how the average sensitivity of compensation is tied to the magnitude of a firm's drift, and its association with volatility. For instance, with the parameterization of g and μ as above, as volatility increases effort decreases and so does their performance sensitivity. In contrast, with persistent information, performance sensitivity in the contract increases rather than decreases.

[26]For concreteness, take N as the number of investors retaining equity in the firm who each receive independent Poisson shocks η forcing them to sell on the open market, in public markets with diffuse ownership the expected time arrival to the next transaction $\frac{1}{\eta N} \to 0$, while for private firms the horizon this duration is strictly positive. Upon the sale, for a fixed window of time say length $\tau > 0$, $\sigma^{\delta} = 0$, and hence in this interval, first best effort will occur.

2.3 The Market for Firm Financing and Listing Choice

The private and public investors compete to finance the heterogeneous pool of entrepreneurs using the optimal contracts from above. CEOs' preferences over financing are summarized by the expected lifetime utility offered under the contract. Hence, their financier type selection rule (also referred to as their "listing choice") is, in a SPNE, given by an indicator function $\mathbb{I}^f\left(q_0^f, q_0^{-f}\right) \in \{0,1\}$ which is non-zero for at most one financier and only if q_0^f, the initial promised utility offered by financier f, is (weakly) greater than q_0^{-f}, the rival financier's promised utility, and the resulting compensation gives a (weakly) greater utility than $\underline{q}$, the CEOs' value of forgoing financing.

Investors maximize their total expected profits from contracting with the various firm types by selecting the firms they will finance through bids of initial promised utility subject to their budget constraint and taking into account the agents' outside option and their own financing type. That is, taking as given both the bidding strategy of the rival financier, $q_0^{-f}(\theta)$ and the financier selection strategy of the agent conditional on the bids, $\mathbb{I}^f\left(q_0^f(\theta), q_0^{-f}(\theta)\right)$, an investor of type $f \in \{P, S\}$ solves the following firm bidding problem:

$$\max_{q_0(\theta)} \int_\theta R^f(q_0(\theta), \theta)\mathbb{I}^f\left(q_0(\theta), q_0^{-f}(\theta)\right) dG_\theta(\theta) \tag{5}$$

s.t.

$$\int_\theta \mathbb{I}^f\left(q_0(\theta), q_0^{-f}(\theta)\right) dG_\theta(\theta) \leq B^f,$$

where the budgets of the private and public investor are $B^S = B < \infty$ and $B^P \to \infty$, respectively.

Using the results from Theorem 1, each representative investor's payoff from financing a given firm with information premium $\pi = \pi(\theta)$ and expected cash flow return $\mu = \mu(\theta) := \mathbb{E}[y_t|\theta]$ with a bid of initial promised utility q_0 can be decomposed as follows

$$R^f(q_0, \theta) = \Upsilon(\mu) - \Lambda^f(\pi) - X(q_0) \tag{6}$$

where $\Upsilon(\mu) := \frac{\mu - \omega^S(\underline{q})}{\rho} - 1$ is the net present value (NPV) absent information frictions and paying the agent's outside option, that is the net payoff for an investor with all the market power and full information on the projects cash flows; $\Lambda^f(\pi)$ is a financier type f-firm type θ specific information cost with

$\Lambda^S(\pi) = \nu$ and $\Lambda^P(\pi) = \pi$; and $X(q_0) := \frac{\omega^S(q_0) - \omega^S(\underline{q})}{\rho}$ is a financier's fixed compensation cost in excess of paying the agent's outside option.

Given the different monitoring capabilities and optimal contracts, the set of firms which are individually rational for the private and public financiers to finance are distinct. The minimal bid an agent will accept is $\underline{q}$, so for it to be individually rational for a type f financier to finance a type θ firm, the expected return at the agent's own outside option must be positive, that is

$$\Upsilon(\mu) - \Lambda^f(\pi) \geq 0. \qquad \left(IR^f\right)$$

Thus, private investors are essential for financing projects where the information rent π required for public financing exceeds the NPV given by $\Upsilon(\mu)$. On the other hand, subject to the fixed monitoring costs ν, low but positive NPV firms cannot be feasibly privately financed, and so must rely on public financing, provided that the information premium π is not too high.

From equation (2) and Theorem 1, a positive PE premium, $\pi - \nu > 0$, reflects a comparative advantage of the private over the public financier. Thus, absent financing constraints, for any project which is individually rational for both investors to compete on and with a positive comparative advantage, competitive bidding results in the public financier's surplus being taken to zero and the private financier's surplus being equal to the PE premium. Given the limited funds, $B < \infty$, the private investor will only finance projects above some information premium cutoff, $\underline{\pi}$, that is with $\pi \geq \underline{\pi}$. This results in the public investor funding the residual firms at the agents' outside option $\underline{q}$.

Finally, because of the budget constraint, although the private financier faces no competition to fund firms which are not individually rational for the public financier, this investor will restrict the financing of such firms to those with expected cash flow return, μ, above a return cutoff $\underline{\mu}$, that is with $\mu \geq \underline{\mu}$. Hence, conditional on a distribution of firms' cash flow characteristics, θ, and a monitoring cost, ν, both cutoffs, $\underline{\pi}$ and $\underline{\mu}$, are pinned down by the private financier's budget, B. Combining the above, Appendix A.2 establishes that equilibrium firm listing patterns are characterized by the following theorem.

Theorem 2. *A unique public listing equilibrium exists and is characterized by private financier's minimum cutoffs for the expected cash flow return, $\underline{\mu}$, and information premium, $\underline{\pi}$. The equilibrium sorting patterns are depicted in Figure 1a.*

Equilibrium firm listing patterns in the expected cash flow return μ and information rent π space are illustrated in Figure 1a, with equilibrium private investor financing cutoffs given by $(\underline{\mu}, \underline{\pi})$. The vertical dashed line depicts the binding individual rationality constraint of the private investor, so that any project with expected cash flow return above this level, i.e. with $\Upsilon(\mu) > \nu$, will generate strictly positive expected profits when privately financed. The dot-dashed diagonal line depicts the binding individual rationality constraint of the public investor, so that any project below this line, i.e. with $\Upsilon(\mu) > \pi$, will generate strictly positive expected profits when publicly financed. Finally, the horizontal dotted line depicts the set of zero PE premium firms, so that any project above this line, i.e. with $\pi > \nu$, has a comparative advantage in being privately financed.

The graph can be divided into three regions which characterize equilibrium firm financing sources conditional on their cash flow characteristics (μ, π). The unshaded region corresponds to unfinanced firms, the dark shaded region to privately financed firms and the light shaded region to publicly financed firms. The set of unfinanced firms consists of the three regions, (Ia), (Ib) and (Ic). In regions (Ia) and (Ib) it is not individually rational for either investor to finance, independently of the PE premium. This set also includes region (Ic), where it is individually rational solely for the private investor to finance, but, due to financing constraints and preference for higher expected return projects, firms remain unfinanced.

The set of privately financed firms consists of two regions, (IIa) and (IIb). Region (IIa) consists of firms which the public financier cannot finance, like region (Ic), but yielding an expected return weakly greater than the cutoff, i.e. $\mu \geq \underline{\mu}$. In this case, the private financier only pays the agent's outside option. Region (IIb) consists of firms which can be feasibly financed by both investors, but where the private outbids the public due to a sufficiently high PE premium, i.e. $\pi \geq \underline{\pi}$.

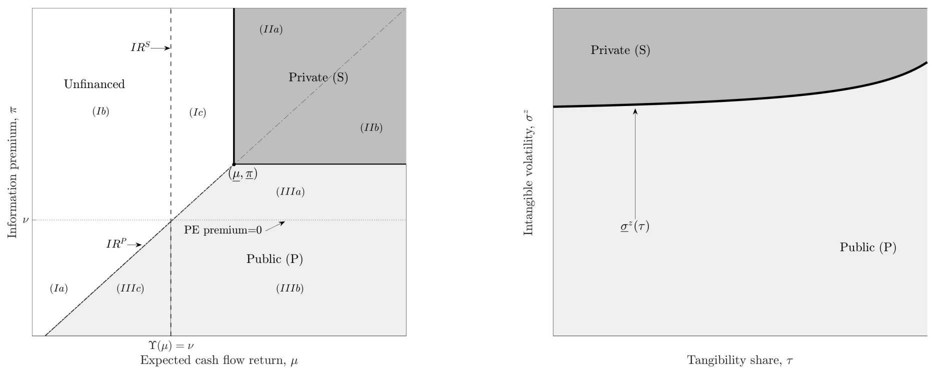
The set of publicly financed firms consists of three regions, (IIIa), (IIIb) and (IIIc). Region (IIIa) consists of firms which both financiers can finance, like region (IIb), but yielding an information premium less than the cutoff, i.e. $\pi < \underline{\pi}$. Similarly, region (IIIb) consists of firms for which the public investor faces no competition, as $\pi < \underline{\pi}$, but would efficiently fund regardless due to a negative PE premium. Finally, region (IIIc) consists of firms which the private financier cannot finance but the public can.

Thus far we have focused on firm sorting across the expected cash flow return and information premium space. Guided by our data consisting of relatively large and established firms, we next move to examining the equilibrium firm sorting patterns holding fixed a given level of expected cash flows, μ, above the cutoff $\underline{\mu}$ such that sorting is described solely based on the information premium level.

Using the expression of the information premium π provided by Theorem 1, and conditional on a given level of private financier's funds, B, and of monitoring cost, ν, we obtain that firms will publicly list when their private cash flow volatility is below a cutoff $\underline{\sigma}^z(\tau)$ which is an increasing function of their tangibility level τ. This sorting prediction over tangibility and private cash flow volatility is summarized in Corollary 1.

Figure 1: Equilibrium Firm Sorting

Figure 1a depicts the equilibrium sorting patterns of Theorem 2 based on firm type θ expected cash flow return μ and information premium π. Figure 1b depicts the equilibrium sorting patterns of Corollary 1 based on firm tangibility τ and private cash flow volatility σ^z, holding fixed a given expected cash flow return μ level above the cutoff $\underline{\mu}$. The unshaded region denotes firms receiving no financing, the dark shaded region denotes firms receiving private financing, and the lightly shaded region denotes firms receiving public financing.



(a) Theorem 2 Sorting

(b) Corollary 1 Sorting

Corollary 1. *For any $\mu \geq \underline{\mu}$ and a given λ^z, firm sorting is driven by their private cash flow volatility σ^z and tangibility τ as described in Figure 1b with cutoff rule*

$$\underline{\sigma}^z(\tau) := \sqrt{\frac{2\pi}{\psi}}\left(\rho + \frac{1}{\lambda^z}\right)\frac{1}{1-\tau}. \tag{7}$$

From the above results we see that the marginally listed firm has a positive association between intangibility and expected cash flow returns for lower levels of intangibility and expected returns, but this relation disappears for sufficiently high expected cash flow return. In the region where both financiers compete, all else equal, firms that are more intangible and have higher intangible volatility give more cover for a CEO to hide misbehaviour and hence it is more costly to design optimal compensation contracts for the public financier. We emphasize that our theory does not imply that highly intangible firms are necessarily privately financed. The bottom left quadrant of Figure 1b indicates highly intangible firms which are nevertheless publicly listed due to their private cash flows being relatively predictable, that is due to a low σ^z. Hence, intangibility or predictability of cash flows alone are insufficient to predict firm sorting.

To conclude, we summarize below the key testable predictions from this section. Reflecting the data we have available on public and private firms, we restrict attention to firms with sufficient long-run expected profitability levels to be financed by either investor, i.e. with $\mu > \underline{\mu}$. While information premia are jointly driven by σ^z and λ^z, to facilitate testing of our core differentiator from the literature of a persistent private cash flow process, we abstract from firm heterogeneity in the persistence parameter.

Firm Listing Testable Prediction. *All else equal, and conditional on having an expected cash flow return above the minimum cutoff, a firm's likelihood to be privately financed is positively associated with (i) higher levels of firm intangibility, and (ii) higher intangible cash flow volatility.*

2.4 Endogenizing PE Funds

Thus far we have taken the amount of funds available to the representative private investor as exogenous. We now endogenize the aggregate supply of PE funds, B, in a general equilibrium, competitive matching model extension.

To do so, we introduce a continuum of households (investors) of fixed mass $M > 1$, each endowed with a unit of funds and a household-specific monitoring technology with a cost $\nu \geq 0$, drawn from some absolutely continuous distribution with CDF $G_\nu(\cdot)$, and paid should they elect to be private financiers. Households compete with each other to finance a unit measure of heterogeneous projects drawn from a distribution with CDF $G_\theta(\cdot)$ via bids of initial promised utility q_0 and committing to a financing type ex-ante. In particular, relative to the partial equilibrium environment, we add a first stage in which each household makes a financier type choice, denoted as $f(\nu) \in \{P, S\}$, and allow bidding strategies to be contingent on the monitoring cost ν in addition to the household's financier type choice f and firm type θ, $q_0^f(\theta, \nu)$. We apply the same notion of equilibrium used in the partial equilibrium setting with these adaptations to construct our general equilibrium.

In lieu of using the monitoring technology at cost ν, a household can invest publicly and pay the expected information rents π. Adapting our partial equilibrium results, a type ν household's return from being a type f financier is $R^f(q_0, \theta, \nu) = \Upsilon(\mu) - \Lambda^f(\pi, \nu) - X(q_0)$ where $\Lambda^S(\pi, \nu) = \nu$ and $\Lambda^P(\pi, \nu) = \pi$.[27] We assume each household is capacity constrained and so may only use the monitoring technology on a single firm.

In equilibrium, since the mass of households is greater than the mass of firms, there is an excess supply of aggregate funds which induces a competitive fringe of unmatched public investors. Since all public investors (and their funds) are perfect substitutes for each other (because they do not elect to use the monitoring technology) and in the absence of risk, this competitive fringe drives the equilibrium expected return of public investing to zero, i.e. $R^P = 0$. In contrast, the equilibrium expected payoff from privately investing in a type θ firm after competing out the public investors is given by

$$R^S(\theta, q_0^*(\theta, \nu), \nu) = R^S(\pi', \nu) = \max\left\{\pi' - \nu, 0\right\},$$

where $q_0^*(\theta, \nu)$ is the winning bid and $\pi' := \min\{\Upsilon(\mu), \pi\}$ is a modified information premium.[28] As a consequence, a cutoff $\bar{\nu}$ arises whereby households with costs exceeding it invest publicly while the rest privately, and the listing patterns are consistent with the partial equilibrium setting. Further, since $R^S(\pi', \nu)$ is submodular in (π', ν), the equilibrium matching exhibits a sorting pattern where the firm with the highest modified information premium π' is matched with the investor with lowest monitoring cost ν, i.e. it exhibits negative assortative matching (NAM).[29] Note that since each investor is constrained to use the monitoring technology on only one firm, private investors returns are heterogeneous and depend on the gap between information premia π and monitoring cost ν. The general equilibrium results are summarized in the next theorem, and established formally in Appendix A.3.

Theorem 3. *For any household mass $M > 1$, a unique general equilibrium exists, and is characterized by firm public and private sorting status as depicted in Figure 1,*

[27]Since households commit ex-ante to use their costly monitoring technology, if unmatched, their payoff is $R^S(\varnothing, \varnothing, \nu) = -\nu$ if they had committed to use the technology, whereas their payoff is $R^P(\varnothing, \varnothing, \nu) = 0$ if they had committed to not use the technology.

[28]Recall from the partial equilibrium setting that if $\Upsilon(\mu) < \pi$, it is not individually rational for public investors to finance a firm, and then a private investor can bid the agent's outside option $\underline{q}$. Instead if $\Upsilon(\mu) \geq \pi$ and $\nu < \pi$, the private financier can outbid.

[29]For details on submodularity and equilibrium matching see, for instance, Becker (1974), Tervio (2008), Gabaix and Landier (2008), and the survey by Chade et al. (2017).

aggregate PE funds B given by

$$B = G_\nu(\bar{\nu})\ M,$$

and NAM between firms indexed by π' and households indexed by monitoring cost ν. In particular, the matching of a type ν household and a firm with modified information premium π', if financing occurs, is given by

$$\pi'_{match}(\nu) := \bar{G}_{\pi'}^{-1}(G_\nu(\nu)\ M),$$

where $G_{\pi'}(\cdot)$ is the CDF of π' and $\bar{G}_{\pi'}(\pi') = 1 - G_{\pi'}(\pi')$ is the survival function.[30]

With the above equilibrium characterization, the average PE premia, Π, public CEO compensation, Ω, and output, O, are given by

$$\Pi := \mathbb{E}[\pi'_{\text{match}}(\nu) - \nu | \nu \leq \bar{\nu}],\ \ \Omega := \frac{\omega_{fix}(\underline{q})}{\rho} + \mathbb{E}[\pi(\theta)|\theta \in \mathbf{P}],\ \ O := \mathbb{E}[\mu|\theta \in \mathbf{P} \cup \mathbf{S}]$$

where $\mathbf{P} = \{\theta : \pi(\theta) \leq \min\{\Upsilon(\mu), \bar{\nu}\}\}$ and $\mathbf{S} = \{\theta : \pi' \geq \bar{\nu}\}$ are the set of publicly and privately financed firms, respectively. Finally, we define L as a firm public listing propensity.

2.5 Comparative Statics and Policy Counterfactuals

In this subsection, we evaluate differential aggregate implications of various policy reforms and technological shifts that have been referenced in the literature. While our testable predictions on firm sorting and CEO compensation up to now have been distribution free, to make predictions on aggregates we must impose some structure on the distributions of firm cash flow characteristics and monitoring costs, $G_\theta(\cdot)$ and $G_\nu(\cdot)$ respectively. We make the following assumptions which facilitate sharp, global monotone comparative static predictions while covering a broad class of distributions.[31]

Assumption 1. The distribution of each component of firm cash flow characteristics is independent, that is $dG_\theta = dG_\mu \cdot dG_\pi$ and $dG_\pi = dG_\tau \cdot dG_\sigma$.

Assumption 2. The survival function $\bar{G}_{\pi'}$ is log convex (concave), and the CDF G_ν is log convex (concave).

[30] Uniqueness holds up to a re-assignment of unmatched and matched public investors who are all indifferent between the two outcomes in equilibrium.

[31] Note that these assumptions mainly facilitate characterization of the average PE premium, Π, dynamics. See Bagnoli and Bergstrom (2005) for a discussion on log concavity and its uses.

In addition to the above assumptions, we also assume that both $G_\theta(\cdot)$ and $G_\nu(\cdot)$ have both positive mass on their whole supports. Equipped with these primitives, we present comparative static predictions consistent with various policy and environmental changes in Theorem 4. In particular, we study changes on firms' propensity to publicly list, average public CEO compensation, PE premium, and economic output from: (i) an increase in the average firm's intangibility $1 - \tau$, like the one occurred over the last fifty years (see Corrado and Hulten (2010)), implemented by an upward scaling of each firm's information premium π; (ii) a relaxation of funding impediments to PE firms, like the one promoted by NSMIA (as studied by Ewens and Farre-Mensa (2020)), implemented by a downward scaling of each investor's monitoring cost ν; (iii) ideas getting harder for new businesses to find (as proposed by Bloom et al. (2020)), implemented by a downward scaling in expected cash flow return μ for new businesses; and (iv) an increase in the public disclosure requirements, like the one caused by SOX (as analyzed by Engel et al. (2007)), implemented by introducing a fixed cost to be public ξ.

Theorem 4. *Suppose that Assumption 1 and 2 hold, and M is sufficiently large. Table 1 presents short-run and long-run impacts on economic aggregates caused by a first-order stochastic shift of (i) G_π to the right, (ii) G_ν to the left, (iii) G_μ to the left, as well as (iv) the introduction of a fixed cost of public listing $\zeta > 0$.*

In Table 1 we consider both short-run (SR) and long-run (LR) consequences. In the SR, the monitoring cost cutoff $\overline{\nu}$ is fixed and so is the supply of PE funds, B, while, in the LR, the cutoff can adjust. In the first row, we consider a rise in firm intangibility implemented through an increase in firm information premia. This has a direct SR effect of raising public CEO pay levels and performance-based shares, and, consequently, magnifying the average PE premia. Furthermore, higher information premia raise the costs of being public above the expected cash flow returns for marginal public firms causing a fall in publicly listed firms and aggregate firm financing, and thus output. In the LR, the supply of PE funds expands as households take advantage of higher average PE premia. This expansion provides financing not only to previously public firms for whom the private advantage became positive, but also to some previously unfinanced firms, increasing output relative to the SR and exacerbating the decline of public listings. However, on net, output falls in LR due to a first-order attrition of public firms to being unfinanced. In contrast, the competing effects of increased information premia of the surviving public firms and selection of high information premium firms switching

Table 1: Comparative Statics

This table presents the theoretical comparative static general equilibrium predictions for various economic aggregates across competing theories of secular trends introduced in Theorem 4. Specifically, we study the impact of increasing (i) intangibility, (ii) PE deregulation, (iii) lack of ideas, and (iv) disclosure costs of being public. The considered economic aggregates are the listing propensity (L), the supply of PE funds (B), the average public CEO compensation (Ω), the average PE premium (Π), and the aggregate output (O). For all the experiments, short-run (SR) and long-run (LR) consequences for each aggregate are considered. We define the SR as the case in which the monitoring cost cutoff $\overline{v}$ is fixed and so is the supply of PE funds, B, while in the LR case we allow the cutoff to adjust. The symbol $+$ $(-)$ denote a rise (fall) relative to the initial level, $\cdot$ indicates not applicable, while ? means that an effect is taken to be ambiguous. See Appendix A.3.3 for details.

	Listing Propensity (L)		PE Funds (B)		Public CEO pay (Ω)		PE Premium (Π)		Aggregate Output (O)	
	SR	LR	SR	LR	SR	LR	SR	LR	SR	LR
(i) Intangibility ($\uparrow G_\pi$)	-	-	·	+	+	?	+	?	-	-
(ii) PE Deregulation ($\downarrow G_v$)	·	-	·	+	·	-	·	?	·	+
(iii) Lack of Ideas ($\downarrow G_\mu$)	?	?	·	-	?	?	?	?	-	-
(iv) Disclosure Costs ($\uparrow \zeta$)	-	?	·	+	?	?	+	?	-	-

to private financing result in ambiguous LR net effects on the average public CEO pay and PE premium.

In the second row, we consider PE deregulation through a decline in monitoring costs. Since we assume that in the SR the household private investing cost cutoff is fixed, our comparative static predictions apply only to the LR. In the the latter case, households switch from public to private financing expanding the supply of PE funds. This expansion of PE funds has two unanmbiguous effects. First, an attrition from public financing of highly intangible firms which results in lower average CEO pay for the remaining public firms. Second, an increase in output due to more high information premium firms that were previously unfinanced receiving private funding.

In the third row, we consider ideas becoming harder to find through a decline in the expected cash flow returns. This reduction makes low productivity firms financed by either investor type no longer profitable to fund. Consequently, privately funded firms and output fall instantaneously to the LR level.[32] The simultaneous fall in both public and private firms results in

[32] Since private investors can choose to not invest when it is not profitable, the supply of private funds provided to firms immediately drops. In the LR the supply of private funds (i.e. the private investor cutoff) also reduces to match the SR invested level, but has no implications

an ambiguous impact on the listing propensity, average public PE premium and CEO pay due to countervailing selection effects.

In the fourth row, we consider additional public disclosure requirements in the case wherein they don't improve transparency and only impose an added fixed cost from being public. This confers an extra advantage to private financing and forces low profitability public firms to exit. Consequently, in the SR unambiguously the average PE premium rises, and both public listing propensity and output fall. Like in the case of increased intangibility, the LR supply of private funds expands to partially offset the attrition of public firms, but doesn't fully counterbalance the SR decline in output. Again, countervailing forces result in an ambiguous LR net effect on average public CEO pay and PE premium and also on firm listing propensity.

While the previous interpretation of additional public disclosure requirements can be thought of as an unproductive reform, this type of policy intervention could have a beneficial effect of improving transparency reducing information asymmetry. The effects of this productive disclosure are equivalent to a reduction in firm intangibility, and hence information premia, with effects corresponding to reversed signs of those in the first row of Table 1.

3 Data

The data for the empirical analysis come from multiple sources. Our main source is S&P Capital IQ which provides financial and accounting data from 1993 to 2016 as well as CEO compensation data from 2001 to 2016 on US firms. Our initial sample consists of corporations that file with the Securities and Exchange Commission (SEC) either a 10-K, a 10-Q or an S-1 Form for the period considered. This includes firms publicly listed on one of the US historically top stock exchanges, i.e. NYSE, Amex and Nasdaq, and non-listed firms with SEC disclosure requirements due to other reasons.[33] In line with the literature, we consider the first category as public and the latter as private.[34]

on the economic aggregates considered. As such, there is no substantive distinction in the SR and LR for this comparative static exercise.

[33]These reasons are having public debt, securities listed on OTC exchanges, or having more than $10 million in assets and a certain number of shareholders. The latter threshold was 500 shareholders before the 2012 JOBS Act, while it increased to 2000 shareholders after. We exclude from our analysis observations of firms listed on minor stock exchanges following previous works studying differences between public and private firms using Capital IQ.

[34]While to some extent the SEC reporting requirements are similar for both publicly listed and non-listed firms, there are two key distinctions between the two categories. First, publicly listed firms have more comprehensive and frequent reporting requirements to the SEC and

We use Compustat Snapshot to obtain information about the listing status of each firm through time as Capital IQ provides only the most recent listing status.[35] For similar reasons, we use Execucomp and Capital IQ corporate events data as in Gao et al. (2017) to detect which executive was the CEO of a firm in a given year. Where ambiguity remains (which occurs in 6.5% of the observations), we consider the highest paid executive in terms of total compensation as the CEO of a firm. Capital IQ corporate events data is used also to exclude observations of firms which underwent LBOs and IPOs. We supplement the IPO information with the data on Jay Ritter's website (for details, see Field and Karpoff (2002) and Loughran and Ritter (2004)).

We consider firm-year observations with positive and non-missing book value of total assets. We exclude financial firms (SIC codes 6000-6999), utilities (SIC codes 4900-4999) and quasi-governmental firms (SIC codes above 9000). All nominal values are adjusted to 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level. Appendix B provides more details on sample construction and data cleaning.

Besides firm listing status and CEO compensation level, the other key outcome variable of interest is CEO pay sensitivity. To measure the latter, we decompose CEO pay following Frydman and Saks (2010) and Edmans et al. (2017) to have consistency across our empirical analyses. We proxy CEO's fixed compensation component as the sum of salary plus fixed bonuses. For the sum of CEO's growth and performance-based pay components we use two measures. The first is the sum of long-term incentive plans and non-equity incentives and any stock and option compensation while the second is the sum of any stock and option compensation. All three proxies are scaled by total CEO compensation.

Having defined our outcome variables, we move to measuring the drivers of firms' private information rents. As before, we decompose these into the

receive much more analysts' attention than non-listed firms. Second, non-listed firms are traded on markets that have less depth (and implicitly fewer shareholders) and where the ease of communicating private information to shareholders while avoiding divulging to a broader public should be greater than in top stock exchanges. Moreover, the lower frequency of trade implies that price adjustments of the non-listed firm value should be lower than those of the publicly listed firms, so that the information sensitivity of stock prices and CEO compensation should lie on a continuum between the totally private firms and those publicly listed on a top stock exchange.

[35] Compustat Snapshot has historical listing information about Compustat firms while standard Compustat has header, that is the latest available, information. We use Compustat Snapshot rather than CRSP due to the fact that it also provides coverage of minor stock exchanges and firms undergoing LBOs, which we exclude from our analysis following the previous literature using Capital IQ.

exposure to private cash flows, $1 - \tau$, and the volatility of private cash flows, σ^z. We proxy the former with a measure of firm intangibility computed as the fraction of intangible capital over total assets where the numerator is obtained following the approach of Peters and Taylor (2017) and the denominator equals the book asset value plus intangible assets off the balance sheet.[36] To proxy the latter for our reduced form empirical analysis, we compute a firm-year proxy using the three year standard deviation of a firm's profitability, measured by EBITDA scaled by total assets. We defer to Section 5 a discussion of our measurement of these different private information components via structural estimation, and to Appendix B a more careful description of how the variables were constructed.

Table 2 reports descriptive statistics for public and private firms. Public firms are larger, both in terms of book assets and sales (both reported in millions of US dollars), and older than private firm. They are also more tangible than their private counterparts and have less volatile earnings, but they are more profitable on average. Public and private firms carry a similar fraction of gross physical property, plants and equipment (PPEGT) on their book assets and have a similar investment level, computed as capital expenditure (CAPEX) divided by PPEGT. Private firms conduct more R&D, have higher SG&A expenditures and hold more goodwill as a fraction of their book assets than public firms. Public CEO pay (reported in millions) is substantially higher than that of private CEOs. The last three rows of the table present the ratio of salary plus fixed bonuses, long-term incentive plans and non-equity incentives plus any stock and option compensation, and stock and option compensation divided by total pay, respectively. For private CEOs the fixed part of their compensation part accounts for a substantially larger portion than for public CEOs. At the same time, approximately two thirds of private CEOs do not report any performance-based pay and, even when they do, the fraction of this compensation is smaller than that of public CEOs.

In addition to our main empirical analysis, we evaluate the empirical content of our theory on historical and international data. For the historical analysis we chiefly use the data about intangible capital stocks of Peters and Taylor (2017), Execucomp, and the historical CEO compensation data of Frydman and Saks (2010). We apply to this data the same filters used to clean our shorter Capital IQ panel data. For the international analysis we use the

[36] In particular, our measure combines Research and Development (R&D) expenses, 30% of Sales, General and Administrative (SG&A) expenses, and changes in other intangible assets on the balance sheet and goodwill as an investment flow into an intangible capital stock.

Table 2: Descriptive Statistics

This table reports descriptive statistics for firm characteristics from 1993 to 2016 as well as CEO compensation characteristics from 2001 to 2016 for companies in our sample. The table is divided between observations related to public and private firms. For each variable we report its mean, median and standard deviation. All nominal values are adjusted to 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level.

	Public				***Private***			
	Mean	Median	Std	N	Mean	Median	Std	N
Firm Characteristics								
Book Assets	3184.06	358.52	15077.42	74409	973.74	34.52	5337.31	45482
Firm Age	34.09	25.00	25.44	74409	20.31	12.00	21.74	45482
Tangibility	0.64	0.64	0.20	74409	0.61	0.63	0.25	45482
Volatility	0.03	0.02	0.04	65523	0.06	0.03	0.08	25689
Profitability	0.07	0.09	0.12	74272	0.01	0.05	0.18	43982
Sales	2837.70	342.40	13386.80	74409	713.05	33.03	3422.82	45482
PPEGT	0.46	0.39	0.31	72805	0.46	0.38	0.34	41608
Capital Expenditure	0.14	0.10	0.14	72614	0.19	0.11	0.22	40117
R&D	0.11	0.08	0.11	28061	0.17	0.11	0.17	12682
SG&A	0.27	0.21	0.25	73386	0.45	0.28	0.53	44729
Goodwill	0.16	0.12	0.15	42369	0.22	0.17	0.19	16007
CEO Pay Characteristics								
CEO Pay	4.18	1.66	9.52	45329	1.20	0.37	4.07	19594
Fixed Share	0.52	0.44	0.34	45329	0.79	0.96	0.29	19594
Incentive Share	0.61	0.67	0.26	32816	0.45	0.43	0.28	6754
Equity Share	0.51	0.53	0.24	30768	0.41	0.37	0.29	5588

World Development Indicators (WDI) data provided by the World Bank, and OECD Structural and Demographic Business Statistics database (see OECD (2019)), and restrict our attention to mainly G7 and OECD member countries.

4 Testing the Theory

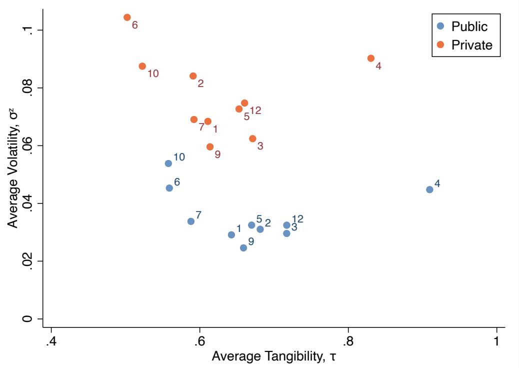
Our theory suggests a common firm-level information premium may drive both firms public listing choices and CEO pay packages. Using the described proxies of firm intangibility and private cash flow volatility we validate the model testable predictions for firm listing choice in Subsection 4.1, and CEO compensation in Subsection 4.2. We then assess the evolution of this information premium and its importance over time and internationally in Subsection 4.3 and Subsection 4.4, respectively.

4.1 Testing Firm Listing Predictions

In Figure 2, we evaluate our cross-sectional sorting predictions by computing the average tangibility and earnings volatility within Fama-French 12 industries separately for public and private firms. The within-industry analysis falls along the patterns predicted by our sorting theory since we see that on average private firms exhibit higher volatility and lower tangibility than public companies. The only departure is the telecommunication industry (Fama-French Industry 7), where, despite the tangibility levels being similar, the joint sorting prediction still holds due to higher volatility of private firms.

Figure 2: Industry Evidence about Firm Listing Patterns

Within-industry average tangibility and 3-year earnings volatility conditional on firm listing status (classified by in-sample median). The 12 Fama-French industries considered are 1 non durables, 2 durables, 3 manufacturing, 4 energy, 5 chemicals, 6 business equipment, 7 telecommunications, 9 shops, 10 health, and 12 other. Given our data cleaning filters, the Fama-French industries 8 utilities and 11 money are excluded.



In Table 3 we present the results from binary regressions of a public listing status indicator on our proxies of intangibility and earnings volatility. We control also for the age of a firm and its size, proxied by the total book asset value, and we include industry and year fixed effects in the first and third columns and industry times year fixed effect in the second one. All the independent variables are in logs and lagged by 1 year. Standard errors are heteroskedasticity-robust and clustered at the firm level. The first two columns display the results from a linear regression model while the third one displays the results from a logistic regression.

The results are consistent with the theoretical listing predictions across the linear specifications. A 10% increase in firm tangibility corresponds to a 0.9 percentage point (pp) increase in the probability of being publicly listed. Similarly, we find a 10% increase in volatility to a 0.1 p.p. reduction in the probability of a firm being listed. All these coefficients are economically and statistically significant at the 1% level and substantive notwithstanding the inclusion of firm age and size (which themselves have the expected effects meaning that older and larger firms are more likely to be public). Similar results are obtained using the logistic regression approach. Untabulated tests show that our results are qualitatively robust to the inclusion of other standard firm controls and firm fixed effects.

Table 3: Firm Listing Regressions

This table shows the results of regressing an indicator variable taking the value 1 if a firm is public and 0 otherwise on one year lagged firm tangibility, profitability volatility, age, and size. All non-indicator variables are in logs. Industry and year fixed effects are included in the regressions of the first and third columns while industry times year fixed effects are included in the regression of the second column. We adopt the Fama-French 48 industry classification in this exercise. The regressions in the first two columns are linear regressions while the regression in the third column is a logistic one. Standard errors are heteroskedasticity-robust, clustered at the firm level and reported in brackets. All nominal values are adjusted to 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level. Superscripts *, **, and *** correspond to statistical significance at the 1%, 5%, and 10% levels, respectively.

	OLS (1)	OLS (2)	Logistic (3)
Tangibility, $\hat{\tau}_{t-1}$	0.089***	0.090***	0.443***
	(0.010)	(0.010)	(0.063)
Volatility, $\hat{\sigma}^z_{t-1}$	-0.011***	-0.011***	-0.048**
	(0.003)	(0.003)	(0.019)
Age	0.049***	0.048***	0.338***
	(0.005)	(0.005)	(0.034)
Size	0.068***	0.069***	0.456***
	(0.002)	(0.002)	(0.017)
Industry and Year FE	Yes	No	Yes
Industry x Year FE	No	Yes	No
Observations	77410	77410	77410
R^2	0.198	0.197	
Pseudo R^2			0.192

4.2 Testing CEO Compensation Predictions

In Table 4 we empirically test our theoretical predictions about the level and composition of CEO compensation. The regression specifications are similar to the listing one with dependent variables the logs of the CEO pay, the fixed share, the incentive share, and the equity share, respectively. Since firm listing status is also a driver of CEO compensation packages based on our theory, we include it as an additional control.

The results are again consistent with our theory and both statistically and economically significant. In the first column, a 10% increase in firm tangibility is associated with a 3% decrease in total CEO pay, while a 10% increase in firm volatility corresponds to a predicted 0.4% increase in CEO total compensation. As has been found previously in the literature, listed, older, and larger firms pay their CEOs more on average. In the second column of Table 4, we find that a 10% increase in firm intangibility is linked to a 1% increase in the fixed share of the CEO pay whereas a 10% increase in firm volatility corresponds to a 0.4% fall of the same fraction. Older firms tend to pay their CEOs with a larger fixed share while listed and larger ones with a smaller one. Finally, in the last two columns we assess the predictions for CEO performance based pay proxied by the incentive and equity shares of CEO pay. A 10% increase in firm tangibility is associated with a roughly 0.6% (0.8%) decrease in the incentive (equity) share of CEO pay while a 10% increase in firm volatility is linked to a slightly milder rise of 0.3% (0.4%) in the incentive (equity) share. Similarly to the listing regressions, untabulated tests show that our results are qualitatively robust to the inclusion of other standard firm controls.

Table 4: CEO Pay Regressions

This table shows the results of regressing the log of the CEO pay, the fixed share, the incentive share, and the equity share on one year lagged firm tangibility, profitability volatility, age, size, and listing status. All non-indicator variables are in logs. Industry and year fixed effects are included in all the regressions, which are all linear. We adopt the Fama-French 48 industry classification in this exercise. Standard errors are heteroskedasticity-robust, clustered at the firm level and reported in brackets. All nominal values are adjusted to 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level. Superscripts *, **, and *** correspond to statistical significance at the 1%, 5%, and 10% levels, respectively.

	CEO Pay (1)	Fixed Share (2)	Incentive Share (3)	Equity Share (4)
Tangibility, $\hat{\tau}_{t-1}$	-0.342***	0.120***	-0.056**	-0.076***
	(0.023)	(0.018)	(0.023)	(0.025)
Volatility, $\hat{\sigma}^z_{t-1}$	0.036***	-0.036***	0.034***	0.044***
	(0.009)	(0.007)	(0.007)	(0.007)
Age	0.092***	0.051***	-0.095***	-0.153***
	(0.014)	(0.010)	(0.011)	(0.012)
Size	0.464***	-0.215***	0.181***	0.176***
	(0.007)	(0.005)	(0.005)	(0.005)
Publicly Listed	0.345***	-0.178***	0.336***	0.296***
	(0.023)	(0.017)	(0.031)	(0.037)
Industry and Year FE	Yes	Yes	Yes	Yes
Observations	43757	43757	30247	28054
R^2	0.555	0.266	0.216	0.190

4.3 Evaluating Time-Series Trends

We now turn to study some of the longer-term historical trends in CEO compensation and how they relate to different policy interventions and other structural changes previously discussed. We view this analysis through the lens of a dynamic version of our model where each year a new measure of projects require financing to replace those that are exogenously discontinued. Thinking of firms as collections of projects as in Klette and Kortum (2004), observed CEO compensation and financing can be seen as the result of the weighted average across these different surviving vintages of projects, so that firms' characteristics change with the composition of their projects.[37]

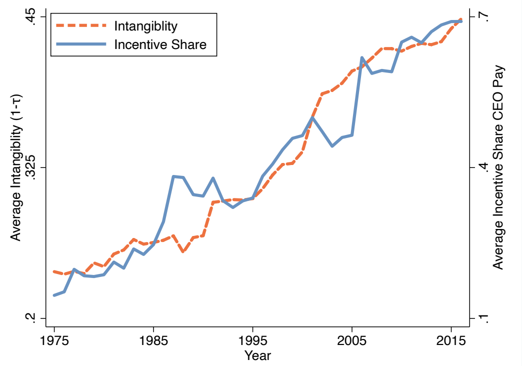
Since our primary data source, has a relatively short time horizon, we use a combination of different data sources extending back to 1975 to evaluate

[37]For instance, Walmart in the past decade has invested heavily into patented IT inventory management, which replaced older less intangible business models.

the longer historical trends. Figure 3 depicts (solid line) the average incentive share from 1975 to 2016 as well as (dashed line) the average intangibility of publicly listed firms.

Figure 3: Historical CEO Pay Performance Sensitivity and Firm Intangibility

This figure depicts the average CEO pay incentive share on the right axes and the average share of intangible assets from 1975 to 2016. The CEO compensation data pre 1991 is taken from Frydman and Saks (2010) while post 1991 is from Execucomp. The average intangibility share is constructed using Compustat and Peters and Taylor (2017) data. The sample begins in 1975 since, as explained in Peters and Taylor (2017), this is the first year that the Federal Accounting Standard Board required firms to report R&D.



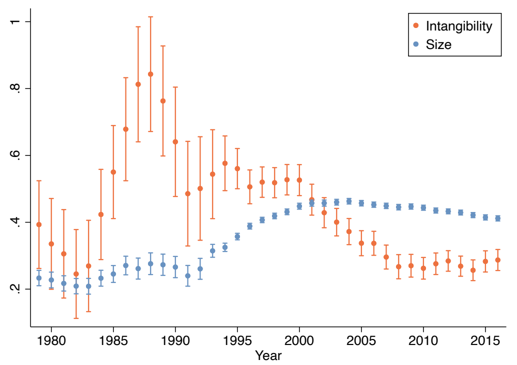
We can see that the two time series move fairly consistently in lockstep upwards. A Dickey-Fuller test statistic of -3.74 (p-value of 0.02%) and a correlation above 95% suggest a co-integrated relationship between the two series. Moreover, since the average annual CEO pay has an 84% correlation with the incentive share, and a Dickey-Fuller statistic of -2.43 (p-value of 0.9%), the data suggests that the level of CEO pay and the incentive share move in virtual lockstep. Thus, the broad compensation trends for CEO compensation from 1975 to 2016 are in line with our theory that the information sensitive component of pay (and, through risk-compensation, the level) is tied to the degree of firm intangibility.

We test how our theory of the influence of firm intangibility on CEO pay holds over the longer historical time-series and cross-section in Figure 4. Leading alternative explanations of the rise in CEO pay seen since the 1970s are based on size as in Gabaix and Landier (2008) and so we examine the relative contributions of size and intangibility over the historical time-series. To do so, we conduct similar exercises as in the earlier subsection, regressing the logarithm of total compensation on the logarithms of firm tangibility, volatility and firm size on a five year rolling window (from year $t-4$ to year t). We plot the estimated coefficients tied to firm intangibility (negative of the tangibility coefficient) and size in Figure 4.

We see substantial time-variation in the magnitudes of association between intangibility and size with CEO pay. In particular, intangibility had a mild impact in the 1970s, which then substantially jumped with the start of the ICT revolution in the 1980s to elasticities about four times higher than those of size, before attenuating in the 21st century. In contrast, the inferred impact of size was relatively flat until the 1990s suggesting intangibility may better explain the inflection point, documented by Frydman and Saks (2010), in the rise in level and equity-based CEO pay which occurred around 1980. The implied information asymmetry stemming from intangible assets declines to the end of the sample consistent with the relative flattening of average public CEO pay over this period.

Figure 4: Estimated Elasticities of Intangibility and Size with CEO Pay

This figure depicts the point estimates and standard error bands of the elasticities of public CEO compensation to intangibility and size over a five year rolling window. The sample begins in 1975 since, as explained in Peters and Taylor (2017), this is the first year that the Federal Accounting Standard Board required firms to report R&D. CEO compensation data prior to 1991 consists of on average 69 firms from Frydman and Saks (2010), while post 1991 consists of on average 1,350 firms from Execucomp.



Finally, we examine the empirical evidence regarding CEO pay, PE premia, and public listing trends guided by the differential predictions of various policy and technological shifts discussed in Theorem 4. Harris et al. (2014) and Harris et al. (2021) document a substantial rise in venture capital (VC) premia relative to public markets from 1984 to 1996. Following the deregulation of PE markets due to NSMIA in 1996, the average VC premia sharply dipped, while the right tail VC premia remained positive throughout the 2000s. These dynamics suggest a rising VC comparative advantage to finance more intangible firms combined with a relatively inelastic PE supply prior to 1996, followed by a substantial decline in both public listings and VC premia as aggregate PE funds expanded. Starting from the early 2000s, the VC premia stayed flat. This fact combined with the moderate drop in listings and CEO pay sensitivity suggests that SOX may have amplified the attrition of public firms but was not a fundamental driver of the exodus.[38] Similarly, the substitution from public to private financing during the aggregate US productivity slowdown from 2004 onwards runs counter to the hypothesis that a primary driver is ideas being harder to find. Thus, overall the dynamics of CEO pay, PE premia, and public listings between the 1980s and the first half of the 2010s are most consistent with a secular rise in intangibility combined with an initially tight constraint on PE funds and a sharp relaxation of them in the late 1990s.

4.4 Evaluating Cross-Country Evidence

Our proposed channel of US public listings decline and rising CEO compensation is a technological one. Consequently, we provide some stylized international evidence about public listing trends and their associations with other economic aggregates in support of this common driver.

Using WDI data in Figure 5 we present the number of domestic publicly listed firms and number of R&D researchers per million of people scaled by their respective 1996 levels. We see that with the exception of Japan and Canada, domestic public listings have declined across all G7 countries.[39] Moreover, the level of country intangibility proxied by their R&D researchers as in Bloom et al. (2020) has increased across all countries but Japan. In addition, when we consider all OECD countries, we find a robust negative correlation between the growth of intangibility and number of listings.[40] Overall, we see that the decline of public listings accompanied by rising intangibility is not only a US phenomenon but it is also exhibited in many other advanced economies. The main difference being that the US decline preceded that in other nations.

Our predictions about the association of public CEO pay with intangibility are also supported by international evidence. While significant differences between US and non-US CEO pay were documented in the 20th century, Fernandes et al. (2013) find that systemic differences in US and non-US CEO pay levels and equity-pay have largely vanished by the early 2000s after adjusting for corporate governance differences. The timing of this convergence

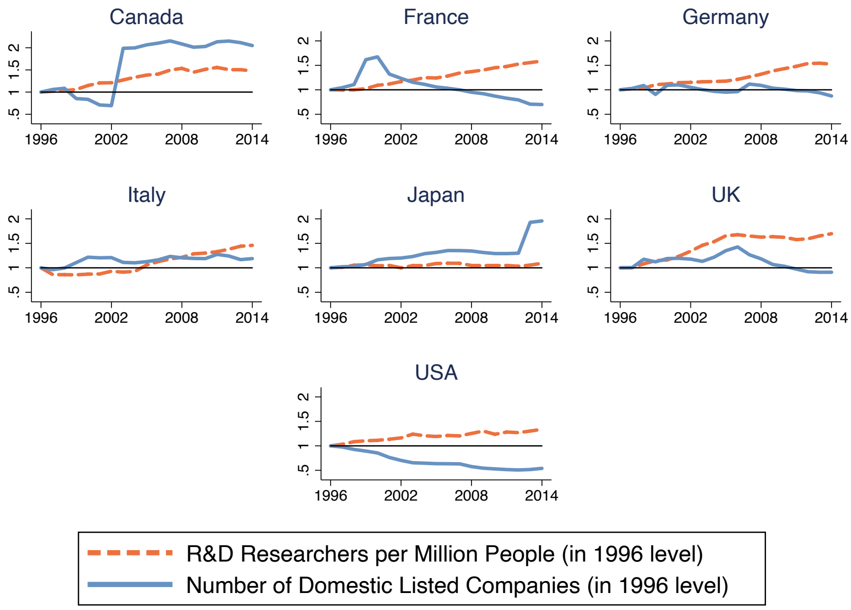
[38] Indeed, analyzing estimated elasticities between intangibility and public listing from 1996 to 2016 in a similar way as we do for CEO pay, we find a reduction of this association around the introduction of SOX, which almost immediately returns to the previous elasticity level.

[39] Note that in contrast to our analysis on US firms, public listing data here include firms in all industries because of data limitations. In addition, while total listings in Canada have increased, the increase is comprised largely of financial vehicles (so called "frankenstocks"), and in fact corporate listings have instead dropped by roughly one third (see Tingle and Pandes (2021) and the Maclean's article here).

[40] We find similar patterns using R&D scaled by GDP rather than the number of R&D researchers as our proxy of country intangibility.

Figure 5: International Evidence about Public Listings and Intangibility

This figure depicts the number of domestic publicly listed firms and the number of R&D researchers per million of people scaled by their respective 1996 levels for the G7 countries from 1996 to 2014 using the WDI data. The sample starts in 1996 since there is no R&D researchers data available before then, and goes until 2014 as there is no listings data for Italy and the United Kingdom after this year.



aligns with the delayed rise of firm intangibility in the EU documented by Corrado et al. (2016) and Corrado et al. (2022). In contrast, Pan and Zhou (2018) document that pay differences between US and Japanese CEOs have not disappeared, consistent with the opposing intangibility trends in Figure 5.

Finally, we examine the international associations of listing patterns, VC funding, GDP growth in relation Theorem 4. To do so, we merge the WDI data with the OECD data on annual total VC investments. Sorting firms by their relative intensity of VC financing to the market capitalization of publicly listed domestic companies in the previous year, we present in Table 5 the correlation between GDP growth and lagged growth in number of public listings. Consistent with the previous literature, we find a positive correlation between public listings and GDP growth on aggregate. However, we document that this positive relationship diminishes and completely reverses for countries with high VC intensity. Countries in the high VC intensity category include US and Israel, the mid categories include France, Germany and UK, and the low category include Poland, and Italy. As such, the sorting is not based solely on size or level of development. The results suggest that for economies more supportive of VC, public markets are no longer the engines of economic growth in line with the PE deregulation predictions of Table 1.

Table 5: Conditional Correlations between Listings and GDP Growth

This table presents the correlations between the time t GDP growth and the time $t-1$ number of public domestic listings growth conditional on time $t-1$ quartiles of VC intensity. The latter is computed as the ratio of VC investments over market capitalization of listed domestic companies. To compute this table we use World Bank WDI data and OECD VC data.

Quartile	1st	2nd	3rd	4th
Correlation	0.102	0.0845	-0.003	-0.251

5 Structural Estimation

In this section we structurally estimate the model and quantify the impact of firms' rising intangibility. Subsection 5.1 describes the estimation method. Subsection 5.2 presents the identification strategy used to separate and measure the intangibility induced information component of the PE premium off of firm listing and performance data as well as the estimation results. Subsection 5.3 presents a separate identification and estimation of the persistence of private information from public CEO pay and firm performance data. These distinct estimations allow us to test for a common underlying driver of both public listing choice and CEO compensation patterns.[41] Subsection 5.4 evaluates a counterfactual where firm intangibility returned to their 1980 levels and measures its impact on public listing propensity and CEO pay packages. Further details on the sample construction for the structural estimation are provided in Appendix B.

5.1 Methodology

We estimate the model using the generalized method of moments (GMM) developed by Hansen and Singleton (1982). Since our model implies a common underlying information premium governing both the firm listing decision and CEO compensation packages, we are interested in estimating the underlying parameters governing the size and distribution of this premium, $\pi(\theta)$. To simplify our exposition, we will drop the z suffixes for all the parameters related to the private cash flows process. To facilitate the testing of our core thesis of a common exposure to persistent private information driving the observed patterns, we ascribe all heterogeneity in the information premium to dispersion in the loading on intangible cash flows and the private cash

[41]We thank Mark Garmaise for the suggestion.

flow volatility.[42] We again treat the loading on private cash flows, $1-\tau$, as observable and given by our proxy of firm intangibility, and take σ^2 to be independently drawn from $\Gamma(\alpha_0, \alpha_1)$.[43] We take the CEO's outside option conditional on the listing status as constant.[44] All other parameters are assumed to be common across firms.

Let δ denote the vector of parameters to be estimated and $h(\delta, v_{it})$ the vector of moment conditions as a function of the parameters δ and the data v_{it}. We estimate the model parameters by minimizing the objective function

$$\hat{\delta} = \arg\min_{\delta} \quad \bar{h}(\delta)' \, W \, \bar{h}(\delta), \tag{8}$$

where $\bar{h}(\delta) = N^{-1}T^{-1}\sum_i \sum_t h(\delta, v_{it})$ is the sample average of the vector of moment conditions for a sample of N firms and T periods.

5.2 Listing Structural Estimation

Guided by our model results on firm listing, we use firm listing and cash flows moments which are informative on the information premium as well as on the productivity parameters. As the listing choice in our model is static, we collapse the panel data into a single cross-section by considering summary firm cash flow statistics and median firm listing status (dropping firms which are public exactly half the time). Since our model abstracts from other considerations dictating a firm's listing choice, we assume that the listing choice is given by $L_i = \mathbb{1}\{\nu - \pi_i + \varepsilon_i \geq 0\}$ where ε_i is an unobserved preference shock which is independently and identically distributed across firms following a logistic distribution. This implies that the log odds ratio of listed versus non-listed is given by $\log\left(\frac{Pr(L_i=1)}{1-Pr(L_i=1)}\right) = \nu - \pi_i$. Taking the expectation of this expression conditional on firms observed tangibility yields

[42] We assume heterogeneity in σ^2 rather than in λ because estimating firm level persistence parameters on short data is more noisy than volatility and consistent with some other works about firms' dynamics.

[43] We assume independence to facilitate closed-form moment expressions. Testing for independence of a copula between the marginal distributions of $1-\tau$ and σ^2 is non-trivial. However, the low Kendall statistic (which is a sufficient statistic for the dependence for some common copulas) of 0.0004 and non statistically significant computed using the earnings volatility and intangibility proxies suggests a tiny distortion.

[44] The moments utilized to estimate the key parameters of interest are independent of the level of the outside option, and hence we do not estimate it.

the logistic regression

$$\mathbb{E}\left[\log\left(\frac{Pr(L_i=1)}{1-Pr(L_i=1)}\right)\middle|\tau_i\right]=\nu+\beta\,(1-\tau_i)^2, \tag{9}$$

where $\beta = \frac{-\mathbb{E}[\sigma^2]\psi}{2(\rho+\lambda^{-1})^2}$ which is independent of firm i's characteristics due to the assumed independence of σ and τ.

Taking the appropriate GMM moments conditions to pin down the logistic regression coefficients, we directly obtain an estimate of ν and we can then decompose the information premium into $\mathbb{E}[\sigma^2]$ and λ by fixing the CEO preference parameters, including setting the discount rate ρ to be 2.5%, which equals the average T-bill rate over our sample period, and the CARA coefficient ψ to 5, as in He (2011).[45]

Identification of the underlying cash flow process parameters then follows from decomposing the average β into its volatility and persistence components. Since the private cash flows are latent, we attempt to back them out by leveraging observed total cash flows and a proxy of tangible cash flows. Following the arguments of Olley and Pakes (1996), we use a version of physical capital investment intensity adapted to our model, that is we scale a firm's CAPEX by its tangible assets, as a measure of firms observable TFP.[46] The volatility and the persistence of the tangible cash flows, σ^x and λ^x, are then identified from the variance and the autocorrelation of this intensity, the latter computed with the methodology of Han and Phillips (2010). The average private cash flow volatility is inferred from partialling out the estimated tangible cash flow volatility from the total earnings volatility. To identify the hyper-parameters associated with the private cash flow volatility Gamma distribution we note that, since $\mathbb{E}[\sigma^2] = \alpha_0\alpha_1$ and $V(\sigma^2) = \alpha_0\alpha_1^2$, the ratio of dispersion and the mean of this inferred private cash flow volatility identifies α_1. Given this, either of the previous moments individually pins down α_0. Finally, identification of λ follows from comparing the level of β with the measurements of $\mathbb{E}[\sigma^2]$. In total we have a just-identified system of 6 moments to identify 6 parameters. Standard errors are bootstrapped using 10,000 resampling draws.

[45]Our specification implicitly normalizes the variance of the idiosyncratic preference shock to a standard logistic distribution through an appropriate scaling of the CARA coefficient. As our quantities of interest from the structural estimation do not depend on the preference parameters (i.e. level of ψ, or the magnitude of ν), their identification is immaterial for our purposes.

[46]Olley and Pakes (1996) reason that the physical investment intensity identifies innovations in productivity on tangible assets.

The results of the listing-based structural estimation are given in column (1) of Table 6. We find that the average size of the private information component of cash flows (σ^2) is about 3.7 times larger than the tangible component of cash flows. Moreover, the annual private information persistence is estimated to be 63% higher than the tangible cash flow persistence.[47] In discrete time, this persistence value corresponds to a relatively high AR(1) coefficient of 0.88. Together these estimates suggest that in the long run firm cash flow volatility is mainly driven by private cash flows rather than their tangible counterparts. The estimated value of ν is positive consistent with our interpretation of it as a monitoring cost. All estimates are statistically significant at the 1% level.

5.3 CEO Pay Structural Estimation

We now test our main theoretical result that the same information premium governs both the cross-sectional listing decisions of firms and the CEO compensation packages. Since in our theory the information premium appears only in the compensation of the public CEOs, the selection of firms into being public makes it difficult to compare our estimates of the distribution of information premia from public CEO compensation data with the estimates obtained before. Instead, we seek to estimate the private information persistence parameter λ, which is assumed to be common to all firms and examine how closely our estimate identified off of CEO compensation data coheres with our results using moments implied by the firm listing decision. To ensure consistency of the sample across estimations we use the Capital IQ compensation data rather than the richer Execucomp for this estimation, although we note that our compensation sample only begins in 2001. To identify the private information persistence parameter λ, we use a measure of pay performance sensitivity of CEO contracts

$$\frac{cov\left(\omega_t, \frac{y_t}{dt}\right)}{Var\left(\frac{y_t}{dt}\right)} = \frac{\mathbb{E}[(1-\tau)^2\sigma^2]}{\mathbb{E}[(1-\tau)^2\sigma^2 + \tau^2(\sigma^x)^2]}\frac{\rho}{\rho+\lambda^{-1}}. \tag{10}$$

Since we want to focus on λ, the other parameters are nuisance parameters. Furthermore, according to our theory CEO pay packages are independent of the observable cash flows, and so we cannot identify σ^x from CEO compensation moments without diluting the exercise. To avoid this issue, we consider

[47]We refer to the persistence here as the discrete time AR(1) coefficient $= e^{-\frac{1}{\lambda^i}}$, $i \in \{x, z\}$.

a biased estimator setting $\sigma^x = 0$ which simplifies the moment condition to

$$\frac{\rho}{\rho + \lambda^{-1}}. \tag{11}$$

This depends only on λ and the fixed parameter ρ which is set to 2.5% as in the previous estimation. Equipped with this biased estimate of λ, we make a correction using our estimates of the parameters governing the cash flow volatility, α_0, α_1 and σ^x, from the firm listing exercise. To obtain the empirical moment, we compute a panel regression of CEO compensation on the yearly change of EBITDA where both variables are scaled by their initial value in the sample in which a firm is publicly listed.

The results from this estimation are given in Column (2) of Table 6. We find from CEO compensation pay sensitivity, our estimate for the persistence λ is 5.38 which falls slightly below the firm listing estimate of 7.67. Using bootstrapped standard errors on the difference between the firm and the bias-adjusted estimate, which equals 8.25, we find that there is no statistically significant difference between our estimates with a z-score of -0.299.

Table 6: Structural Estimation Results

This table presents the estimates from our structural estimation. Column (1) presents the estimates based on firm listing choice and firm performance and our strategy discussed in subsection 5.2. Column (2) presents the estimate for λ based on CEO pay performance sensitivity and our biased moment discussed in subsection 5.3. Column (3) presents the bias-adjusted estimate of λ from Column (2) using volatility estimates from the firm listing based on equation (11). Bootstrapped standard errors are given in parentheses and are based on 10,000 bootstrap samples.

Parameter	Firm (1)	CEO (2)	Bias-adjusted CEO (3)
λ - Persistence of private cash flows	7.673	5.377	8.246
	(1.176)	(1.105)	
ν - Monitoring cost	0.664		
	(0.036)		
α_0 - Shape of distribution of volatility of private cash flows	2.253		
	(0.077)		
α_1 - Scale of distribution of volatility of private cash flows	0.006		
	(0.001)		
$(\sigma^x)^2$ - Volatility of tangible cash flows	0.003		
	(0.000)		
λ^x - Persistence of tangible cash flows	1.623		
	(0.109)		

5.4 Quantifying the Role of Intangibility

To quantify the importance of intangibility-induced persistent private information, we consider a counterfactual experiment where firm intangibility levels remained at the level observed at the start of the ICT revolution in 1980.

We compute that average firm intangibility increased 55% between 1980 and 2016, going from 10.31% to 26.33%. This change, along with our firm-listing based structural estimates, implies that returning intangibility levels to those in 1980 would increase the listing probability from 57.22% to 62.66%, that is a just under 5.5 pp increase. Further, using the implied information premium from public firms only, we find that substituting in the implied tangibility levels of the 1980s leads to a fall in information premium from 0.373 to 0.146. Since this percent fall in information premium is equal to that of the average annual variable pay growth, we conclude that annual public CEO variable pay growth would be 61% lower without the increase in the exposure to persistent private information from intangible assets. The magnitude of these effects suggests that the proliferation of information asymmetries can jointly account for a sizable fraction of public CEO pay and listing trends.

6 Conclusion

The nature and durability of firms' intangible assets are challenging for outside investors to ascertain due to non-separability of intangible cash flows and imitation risks from public disclosure. As private investors have subject matter expertise and are able to communicate with firm insiders behind closed doors, rising firm intangibility may exacerbate persistent private information frictions that differentially impact public investors over private. We build, validate and estimate a market-equilibrium framework in order to quantify the impact of this rising, intangibility-induced persistent private information friction on public listings and CEO compensation. The estimated impact of this channel is large, suggesting significant reforms to public market disclosures, or broadening access to private investments may be important to mitigate distortions from this public listings decline.

Our paper provides one of the first measurements of firm exposure to persistent hidden information. As argued by Gayle and Miller (2015), models which allow misreporting of cash flows are better at rationalizing the observed associations of CEO compensation and firm performance patterns. Persistence in this hidden information magnifies private information distortions beyond the level implied from the level of cash flow volatility. This induces in our optimal contracts more performance based pay and growing expected share of profits independent of firm profitability. In this way we microfound the positive link of high intangible public firms to agency conflicts inferred in

Glover and Levine (2017) and obtain larger information distortions than those suggested in Ai et al. (2022) identified from investment volatility.

Our explanation for the rise in public CEO compensation and listings decline complements productivity based explanations of other related secular trends. Garicano and Rossi-Hansberg (2006), Lustig et al. (2011) and Frydman and Papanikolaou (2018) suggest labour inequality is driven from increased between-firm competition for rising productivity of highly skilled, highly mobile labour. In the case of CEOs this may not be as salient due to their low mobility and tendency to be promoted from within the firm, as documented by Cziraki and Jenter (2021). Karabarbounis and Neiman (2014), Crouzet and Eberly (2018), Ward (2022), Covarrubias et al. (2020), Hartman-Glaser et al. (2019), Autor et al. (2020), and Kehrig and Vincent (2021) propose similar productivity-based explanations for trends in investment, and markups. These studies largely focus on US public firms and typically associate higher intangibility with greater markups, productivity and profitability. In our data encompassing public and large private firms we do not find this overall positive association of intangibility and profitability.

Our framework suggests the decline of US public firms is the efficient market equilibrium response to rising informational asymmetry between firm insiders and the general public. A richer model with welfare costs arising from wealth inequality and advantages to broader access to financial markets may entirely reverse the efficiency of private financing. In our static financing setting, differential returns between private and public investors have no dynamic effects on the future selection of firms. A dynamic extension of the model would imply that the highest ability private investors obtain the highest net return and accumulate ever increasing shares of aggregate wealth over time, crowding out public investors from a widening segment of the economy and leading to a declining correlation between US stock market performance and domestic economic indicators, as found by Greenwald et al. (2022). We leave such extensions and examinations for future work.

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A Appendix Theory

This appendix is divided in four parts. In Section A.1 we formally describe our principal-agent problem and derive the optimal contracts. In Section A.2 we define and characterize the model in partial equilibrium, as well as establish equilibrium existence and uniqueness. In Section A.3 we conduct the same analysis in the general equilibrium setting and present proofs related to the comparative statics exercises.

A.1 Optimal Contracts

We consider a principal-agent contracting problem in continuous time between an uninformed financier (or investor) and an entrepreneur (or CEO) with a project of type $\theta = (\mu^x, \mu^z, \lambda^x, \lambda^z, \sigma^x, \sigma^z, \tau)$. The setting is that of Williams (2011) (hereon W11) with the addition of a mixture of publicly and privately observed cash flows, and hidden savings, as analyzed in Bloedel et al. (2020) (hereon BKS20).[48]

In addition to extending and adapting the arguments of W11 and BKS20 to this mixture of publicly observed and unobserved cash flows, we appeal to a recent stochastic maximum principle (SMP) for infinite horizon discounted stochastic control problems to establish that the set of first-order incentive compatible (FO-IC) contracts contains the set of incentive compatible (IC) contracts in this setting, ensuring the contract is in fact the optimal one.

The remainder of this section is as follows. We first provide a formal description of the contracting environment, the agent's reporting problem with hidden savings given a contract, and the principal's optimal contract design problem. We then use a change of variables to tractably reformulate the agent's reporting problem into a tractable control diffusion problem on the infinite horizon, as in W11 and Cvitanić and Zhang (2013). By doing so, we can solve for the agent's optimal reports given no hidden savings and conditional on a given contract. This allows us to characterize the set of FO-IC contracts. We then use a stochastic variant of the dynamic programming principle and a guess and verify approach to characterize the optimal contract

[48]W11 claims to characterize the optimal (insurance) contract with persistent private information and without hidden savings. BKS20 provide a counterexample to establish the generic sub-optimality of the relevant contract of W11, demonstrating an issue with the reliance of W11 on a numerical observation for his solution. However, BKS20 show that the contract of W11 is indeed optimal for the class of stationary contracts amongst the class of first-order incentive-compatible contracts.

with no hidden savings. This is done in three steps. First, we characterize the requisite aspects of an optimal contract constrained to a fixed initial condition on promised marginal utility and for our guess on the principal's value function. Second, we characterize the restrictions on optimal contracts required to preclude hidden savings. Third, we verify that the combined restrictions on the optimal contract from the first two steps yields indeed an optimal contract which induces truth-telling and no-hidden savings by the agent. We conclude by establishing that the infinite horizon problem conforms to the primitives needed to invoke the results from Haadem et al. (2012), Haadem et al. (2013), Maslowski and Veverka (2014), and Øksendal and Sulem (2019) of an appropriate (necessary) SMP. This assures global optimality of the contract, that is amongst all IC contracts rather than just FO-IC contracts for our setting.

A.1.1 Contracting Environment

Time is continuous and infinite. Let $\mathbf{W} = (W_t)_{[0,\infty)}$ be a bivariate Wiener process on a filtered probability space $(\Omega, \mathscr{F}, \mathbb{F} = \{\mathscr{F}_t\}_{t\geq 0}, \mathbb{P})$ as presented in Øksendal and Sulem (2019) and Pham (2009).

The agent receives a random endowment $\mathbf{y} = (y_t)_{[0,\infty)}$, with $y_t \in \mathbb{R}$, of cash flows adapted to the filtration $\mathbb{F}$ as a mixture of two univariate Ornstein-Uhlenbeck processes, $\mathbf{x} = (x_t)_{[0,\infty)}$ and $\mathbf{z} = (z_t)_{[0,\infty)}$, described by

$$di_t = \mu^i(i_t)dt + \sigma^i dW_t^i,$$

where $\mu^i(i_t) = \mu^i - \frac{i_t}{\lambda^i}$, with $\mu^i, \lambda^i, \sigma^i > 0$ and i_0 fixed with $i \in \{x, z\}$.

The principal designs and commits to a compensation contract $\boldsymbol{\omega} : C[0,\infty)^2 \to C[0,\infty)$, where $C[0,\infty)$ is the set of continuous sample paths (functions). In return the principal receives the reported cash flows $\widehat{y}_t := y_t + m_t^y$, where m_t^y is the $(\mathbf{x}, \mathbf{z})$-adapted misreporting process. Thus, $-m_t^y$ is the amount of the cash flows which the agent diverts for their own consumption. For convenience, define $m_t := \frac{m_t^y}{1-\tau}$ as the misreports in terms of intangible cash flows (rather than total cash flows).

The principal observes jointly the realizations of the tangible cash flow process and the reports about the total cash flow $(\mathbf{x}, \hat{\mathbf{y}})$, but, besides the initial value, z_0, not the realizations of the private cash flow process $\mathbf{z}$, and hence of the total cash flows $\mathbf{y}$. Hybrid moral hazard arises as the principal can fully commit to a contract, while the agent cannot commit to a set of actions post-contracting and has the opportunity to misreport.

In addition to reporting the cash flows, we allow the agent to privately save and borrow at a risk-free rate r. Denote A_t as the agent's assets at time $t \geq 0$, then $\mathbf{A}$ evolves according to

$$dA_t = (rA_t + \omega_t - m_t^y - c_t)dt, \quad \lim_{t\to\infty} e^{-rt}A_t \geq 0 \text{ almost surely}, \quad \text{and } A_0 \text{ fixed.} \tag{A.1}$$

The agent has instantaneous exponential constant absolute risk-aversion (CARA) utility of consumption c given by $u(c) = e^{-\psi c}$, where $\psi > 0$ is the risk-aversion coefficient, while the principal is risk-neutral. Both parties have the same discount rate ρ.[49]

A.1.2 Agent's Problem

Given the contracted compensation process, $\boldsymbol{\omega}$, taking values $\omega(t, x_{[0,t]}, \widehat{y}_{[0,t]}) \in \mathbb{R}$, where $\mathbf{x}_t := x_{[0,t]}$ and $\widehat{\mathbf{y}}_t := \widehat{y}_{[0,t]}$ are sample paths of x_t and $\widehat{y}_t$ over the interval $[0,t]$, and agent's initial assets A_0, the agent's reporting-consumption problem is given by

$$V(\boldsymbol{\omega}) := \sup_{\mathbf{m}\in\mathscr{M}} \widetilde{V}(\boldsymbol{\omega}, \mathbf{m}) := \sup_{\mathbf{c}\in\mathscr{A}(\boldsymbol{\omega},\mathbf{m})} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right] \tag{A.2}$$

where expectations are taken over the sample paths of the processes $\mathbf{x}$ and $\mathbf{z}$ implied by the probability measure $\mathbb{P}$, and the admissible space $\mathscr{A}(\boldsymbol{\omega}, \mathbf{m})$ is given by

$$\mathscr{A}(\boldsymbol{\omega}, \mathbf{m}) = \{\mathbf{c} : \mathbf{v} = (\mathbf{c}, \mathbf{m}) \text{ which is } (\mathbf{x}, \mathbf{z}) \text{ adapted, and implies } \mathbf{A}^{\mathbf{v}} \text{ given by } (A.1)\}.$$

As the uninformed principal knows the fundamental parameters governing the cash flow process $\mathbf{y}$, i.e. θ, and the initial conditions, x_0 and z_0, and observes the realizations of the processes $(\mathbf{x}, \hat{\mathbf{y}})$, the admissible (undetectable) misreporting strategies must generate the same stochastic law as the total cash flows and exhibit the same statistical properties of $(\mathbf{x}, \mathbf{y})$. Specifically, $\mathscr{M}$ is the set of feasible misreports given by $\mathbf{m}$ which

1. is adapted to $(\mathbf{x}, \mathbf{z})$,

2. has continuous sample paths, i.e. $m_t = \int_0^t \Delta_s ds$ for some process $\boldsymbol{\Delta}$,

[49] Since both the agents and principals share the same discount rate, in a closed economy, via standard arguments, the risk-free asset must have a rate of return equal to the discount rate, that is $r = \rho$, and so hereon we will not distinguish between the two.

3. has square integrable stochastic exponential martingale and is bounded, i.e. $\exists K_m > 0 : |m| \leq K_m$,[50]

4. is uncorrelated with $\mathbf{x}$.[51]

A contract $\boldsymbol{\omega}$ is truthful revelation IC if

$$\widetilde{V}(\boldsymbol{\omega}, \mathbf{0}) \geq \widetilde{V}(\boldsymbol{\omega}, \mathbf{m}) \qquad \forall \mathbf{m} \in \mathscr{M}, \qquad (IC)$$

and no-savings compatible, for a given misreporting process $\mathbf{m}$, if

$$\widehat{\mathbf{c}}(\boldsymbol{\omega}, \mathbf{m}) \in \arg \sup_{\mathbf{c} \in \mathscr{A}(\boldsymbol{\omega}, \mathbf{m})} \mathbb{E}\left[\int_0^\infty e^{-\rho t} u(c_t) dt\right] \quad s.t. \quad \widehat{c}_t = \omega_t - m_t^y \ \forall t. \qquad (NS)$$

Given the outside option of the agent $q_0 \in \mathbb{R}_-$, a contract is individually rational (IR) for the agent if $\widetilde{V}(\boldsymbol{\omega}) \geq q_0$.[52]

A.1.3 Principal's Problem

Given outside option q_0, the principal solves

$$J(q_0) := \sup_{\widehat{\mathbf{m}} \in \mathscr{M}} J(q_0, \widehat{\mathbf{m}}) := \sup_{\widehat{\mathbf{m}} \in \mathscr{M}} \sup_{\boldsymbol{\omega} \in \mathbb{S}(q_0)} \mathbb{E}\left[\int_0^\infty e^{-\rho t}(y_t - \omega_t + \widehat{m}_t^y) dt\right] \quad (A.3)$$

where $\widehat{\mathbf{m}}$ is the recommended misreporting strategy and the feasible space of contracts is given by

$$\mathscr{S}(q_0) = \left\{ \begin{array}{c} \omega : C([0,\infty)^2) \to C[0,\infty), \mathscr{F}_t\text{-predictable}, \\ \mathbb{E}[\int_0^\infty e^{-\rho t} u(\omega_t)^2 dt] < \infty \text{ and satisfies (IC), (NS) and (IR)} \end{array} \right\}.$$

[50] See Klebaner (2012) Chapter 8.8 for a discussion of stochastic exponential martingales. This technical restriction ensures that misreports can be equivalently characterized via the Girsanov's transformation. Note that the restriction imposed here is similar but slightly stronger than that of BKS20, but is used to appeal to an appropriate SMP and ensure, in contrast to BKS20, that the first-order approach to IC constraints is globally optimal.

[51] Since x is commonly observed and independent of private cash flows z, a principal can detect existence of misreports if there is a non-zero correlation of the residual cash flow reports and observable cash flows $\mathbf{x}$. Specifically, observing x, the principal can deduce $\widetilde{y}_t := \widehat{y}_t - \tau x_t = (1-\tau)\big(z_t + m_t\big)$, so that the stochastic law (distribution) governing $\widetilde{y}$ must be equal to that of z to avoid detection of misreports.

[52] Recall the agent's instantaneous payoff is $u(c) = -\exp(-\psi c) \in (-\infty, 0)$, so $q_0 = \int_0^\infty e^{-\rho t} u(c_t^0) dt \leq 0$ for any deterministic, real-valued process $\mathbf{c}^{\mathbf{0}}$. Notice that q_0 can correspond either to $\underline{q}$ if there is no financing competition or to the initial promised utility offered by the competing principal.

That is, feasible contracts are any which are incentive and no-savings compatible, individually rational for the agent, and yield a well-defined (discounted) square-integrable expected lifetime under truthful reporting.

A.1.4 Transforming the Agent's Problem

Fix a path of the observable cash flows $\mathbf{x}$ and an admissible reporting strategy $\mathbf{m}^z := \mathbf{m}(\mathbf{x})$, which, conditional on $\mathbf{x}$, is adapted to the filtration generated by the private cash flow process z. That is, $\mathbf{m}$ is adapted to the filtration of x and z, $\mathbb{F} = \mathbb{F}^{x,z}$, while $\mathbf{m}^z$ is adapted to the private cash flow filtration, $\mathbb{F}^z$. With this, the stochastic exponential (likelihood) process for an admissible misreport strategy given x, m^z, is given by

$$\log \Gamma_t := \int_0^t a_s^z dW_s^z - \frac{1}{2}\int_0^t (a_s^z)^2 ds,$$

where

$$a_t^z := \frac{\mu^z(-m_t^z) + \Delta_t}{\sigma^z}.$$

Conditioning on a fixed path x, the evolution of private cash flow reports, $\tilde{y}_t := \widehat{y}_t - \tau x_t$, is described by $\frac{d\widetilde{y}_t}{1-\tau} = dz_t + dm_t$. Hence, by an application of Girsanov's Theorem, the likelihood process of observing private cash flow reports $\widetilde{y}$ under a misreport strategy m is

$$\frac{dP^{z,m}}{dP^{z,*}}(\mathbf{z}) = \Gamma_\infty, \tag{A.4}$$

where $\mathbb{P}^{z,*}$ is the probability measure of the path of $\tilde{y}$ under truthful revelation, and $\mathbb{P}^{z,m}$ is the probability measure of the path of $\tilde{y}$ induced by m.[53]

Then the evolution of private cash flow reports must satisfy

$$\frac{d\widetilde{y}_t}{1-\tau} \overset{d}{=} \frac{d\widetilde{y}_t^m}{1-\tau} = \big(\mu^z(z_t - m_t^z) + \Delta_t\big)dt + \sigma^z dW_t^{z,m}$$

where $\overset{d}{=}$ denotes equality in distribution and $dW_t^{z,m} = dW_t^z - a_t^z dt$ is the Brownian motion implied by the misreports. In this way, $\widetilde{y}^m$ is a weak solution to the (realized) private cash flow report stochastic differential equation (SDE) $\widetilde{y}$. Restricting attention to weak solutions of $\widetilde{y}$, we may then without further loss of generality use a change of variables $\widetilde{y}$ for Γ which, as first illustrated by Bismut (1978), makes the path $\widetilde{\mathbf{y}}$ deterministic (but only known up to time t).

The problem for the agent is then

[53]Both probability measures are conditional on the path of x.

$$V(\boldsymbol{\omega}) = \sup_{\mathbf{m}\in\mathscr{M}} \mathbb{E}^x\left[\mathbb{E}^{m^z}\left[\int_0^\infty e^{-\rho t}u(c_t)dt\big|\mathbf{x}\right]\right] = \sup_{\mathbf{m}\in\mathscr{M}} \mathbb{E}^x\left[\mathbb{E}^z\left[\int_0^\infty e^{-\rho t}\Gamma_t u(c_t)dt\right]\right]$$

where $c_t = \omega_t(\mathbf{x}, \widehat{\mathbf{y}}) - (1-\tau)m_t$, $\mathbb{E}^x[\cdot]$ is the expectation over the distribution of tangible cash flow paths $\mathbf{x}$, $\mathbb{E}^{m^z}[\cdot]$ is the expectation over $\tilde{y}$ under the measure $\mathbb{P}^{m^z}$, and $\mathbb{E}^z[\cdot]$ is the expectation under the (true) "P" measure for z, $\mathbb{P}^z$. The first equality is simply the law of iterated expectations, while the second equality follows from the independence of z and x.[54]

Noting that the evolution of x_t satisfies the Markov property and the agent's choice of controls does not affect its diffusion, we will conjecture that in any optimal contract $\omega_t(\mathbf{x}_t, \widehat{\mathbf{y}}_t) = \omega_t(x_t, \widetilde{\mathbf{y}}_t)$. Moreover, we will require $\omega_t(x_t, \widetilde{\mathbf{y}}_t)$ to be continuous in its first order derivative with respect to x_t and the latter to be bounded.[55] Finally, for the agent's problem we will take $\omega_t = \omega_t(x_t)$ to be a deterministic function of x_t, taking as given a particular path $\widehat{y}_t$ which are impacted by the agent's cash flow reports, as is standard in the contracting literature (see, for instance, Cvitanić and Zhang (2013)). Thus, the agent's problem is transformed to a controlled diffusion problem with random coefficients.[56]

For convenience, we use an additional change of variables of m to $\tilde{m} = \Gamma m$, so that the agent's controlled state processes (using stochastic integration by parts) are

$$d\Gamma_t = \Gamma_t \frac{\mu^z(-\frac{\tilde{m}_t}{\Gamma_t}) + \Delta_t}{\sigma^z} dW_t^{z,m}, \qquad \Gamma_0 = 1,^{57}$$

$$d\tilde{m}_t = \Gamma_t \Delta_t dt + \frac{1}{\Gamma_t}\tilde{m}_t d\Gamma_t, \qquad \tilde{m}_0 = 0,$$

[54] Notice that the admissibility of misreports requires the tangible cash flows to be independent of the private cash flow reports, since under truthtelling x and z are both uncorrelated Gaussian processes, and so x and $\tilde{y}$ must also be uncorrelated to avoid detection, which, given that uncorrelated multivariate Gaussian implies independence, yields the result. Consequently, $m = m^z$. See Lemma 1 of Szydlowski and Yoon (2022) for a formal discussion of the result of independence of separate change of measures of bivariate brownian motions.

[55] Solving the principal's optimal contracting problem, we will verify that indeed ω_t takes this form. In fact, we'll find that it is independent of x consistent with the full information case.

[56] Observe that if we were to restrict our attention to weak solutions of x_t and do a similar change of variables (in the same way as we did for z_t), then taking Γ^x to be the stochastic exponential and given that the principal perfectly observes x_t, we would require $\Gamma^x = 1$ everywhere so that m would be automatically independent of x in this weak formulation.

[57] Again note that here we've dropped the dependence on the state of the sample space $(z_{[0,\infty)} + m_{[0,\infty)})$ which is taken as fixed.

$$dx_t = \mu^x(x_t)dt + \sigma^x dW_t^x, \qquad x_0 = \lambda^x \mu^x,$$

with $c_t = \omega_t(x_t) - (1-\tau)\frac{\tilde{m}_t}{\Gamma_t}$, and $dW_t^m = (dW_t^{z,m}, dW_t^x)$ are such that the private cash flow evolves as a martingale, i.e. $\frac{d\tilde{y}_t}{1-\tau} = \sigma^z dW_t^{z,m}$.

A.1.5 Agent's Optimal Reports Without Hidden Savings

Given our change of variables, we assume that a_t^z is now a function of $\frac{\tilde{m}_t}{\Gamma_t}$ rather than m_t^z. Let $X_t = (\Gamma_t, \tilde{m}_t, x_t)'$ denote the state vector process with evolution summarized by

$$dX_t = b(X_t, \Delta_t)dt + \Sigma(X_t, \Delta_t)dW_t^m$$

where $b(X_t, \Delta_t) = \begin{pmatrix} 0 \\ \Gamma_t \Delta_t \\ \mu^x(x_t) \end{pmatrix}$, $\Sigma(X_t, \Delta_t) = \begin{pmatrix} \Gamma_t a_t^z & 0 \\ \tilde{m}_t a_t^z & 0 \\ 0 & \sigma^x \end{pmatrix}$, and $dW_t^m = \begin{pmatrix} dW_t^{z,m} \\ dW_t^x \end{pmatrix}$. The associated generalized (current value) Hamiltonian to the above control problem is

$$H(X, \Delta, Y, Z) = b(X, \Delta)'Y + Tr(\Sigma(X, \Delta)'Z) + \Gamma u(X, \Delta) - \rho X'Y$$

where Y is a 3×1 vector and Z is a 3×2 matrix. The adjoint process (Y_t, Z_t) of the Hamiltonian then evolves according to the backward SDE

$$-dY_t = \nabla_X H(X_t, \Delta_t, Y_t, Z_t)dt - Z_t dW_t^m.$$

By inspection, the above problem now matches the setting of a discounted infinite horizon optimal control with controlled diffusions. The SMP invoked by W11 is for finite horizon and, is extended to the infinite horizon case by taking the limit $T \to \infty$. However, potential issues with this approach can arise as pointed out by Halkin (1974). Moreover, BKS20 raise concerns about the lack of an appropriate SMP known in the literature for payoff functions which are not bounded below (e.g. exponential utility). We leverage the works of Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) for a necessary SMP applied to Ito-Levy processes in discounted infinite horizon settings based on the satisfaction of a terminal transversality condition and restricted to payoff functions which are (discounted) integrable and have squared discounted integrable growth, i.e.

$$\mathbb{E}\left[\int_0^\infty |f_t(X_t,\Delta_t)| + ||\nabla_X f_t(X_t,\Delta_t)||^2 dt\right] < \infty \tag{A.5}$$

for any admissible controls Δ_t, where $f_t(X_t,\Delta_t) = e^{-\rho t}\Gamma_t u(c_t)$. A difficulty with the application of this SMP however is the verification of their transversality condition. In a restricted setting of controlled diffusions, that is, without jumps, Maslowski and Veverka (2014) provide a sufficient SMP as well as sufficient conditions to ensure the transversality condition holds.[58] In section A.1.9 of this appendix, we formally verify our problem is amenable to an application of the results of Maslowski and Veverka (2014), Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) in order to establish that all IC contracts are necessarily FO-IC.

Having established the necessity of the Hamiltonian optimality conditions for incentive compatibility, we now characterize the IC constraint as done by W11 and BKS20. To keep the notation close to W11, we'll drop the z superscripts and keep just the x ones except when ambiguity may arise. We take $Y = (q, p, p^x)'$, where, akin to W11, q can be interpreted as the promised utility, p as the negative of the promised marginal utility with respect to z, p^x as the negative of the promised marginal utility with respect to x. Moreover, we also take the diffusion of the adjoint, $Z = \begin{pmatrix} \sigma\gamma & \sigma^x\gamma^x \\ \sigma\iota & \sigma^x\iota^x \\ \sigma\zeta & \sigma^x\zeta^x \end{pmatrix}$, where $\sigma := \sigma^z$ (σ^x) is the volatility of the private (public) cash flows, γ (γ^x) is the sensitivity of the promised utility to private (public) cash flows, ι (ι^x) is the sensitivity of the promised marginal utility with respect to z to private (public) cash flows, ζ (ζ^x) is the sensitivity of the promised marginal utility with respect to x to private (public) cash flows. Suppressing the arguments, the generalized Hamiltonian simplifies to

$$H = \Gamma\Delta p + \mu^x(x)p^x + \sum_{i,j}\Sigma_{ij}Z_{ij} + \Gamma u(c) - \rho(\Gamma q + \tilde{m}p + xp^x) \tag{A.6}$$

with $\sum_{i,j}\Sigma_{ij}Z_{ij} = \Gamma a^z\sigma\gamma + \tilde{m}a^z\sigma\iota + \left(\sigma^x\right)^2\zeta^x$.

Recalling $a^z = \frac{\mu^z\left(-\frac{\tilde{m}}{\Gamma}\right)+\Delta}{\sigma}$ the agent's reporting FOC is given by

[58] Unfortunately, we cannot directly appeal to the Maslowski and Veverka (2014) sufficient SMP since, as is common for sufficient SMPs, it requires the Hamiltonian to be concave.

$$\Gamma p + \Gamma \gamma + \tilde{m}\iota = 0. \tag{A.7}$$

Invoking the Revelation Principle, we have under truthful revelation $\tilde{m} = \Delta = 0$, which combined with $\Gamma > 0$, yields the FO-IC condition

$$p + \gamma = 0. \tag{A.8}$$

Finally, the requisite evolution of the adjoint process is by direct calculation (using that under truthtelling $\Gamma = 1$ and $\tilde{m} = \Delta = 0$) is given by

$$dY_t \equiv \begin{pmatrix} dq_t \\ dp_t \\ dp_t^x \end{pmatrix} = \begin{pmatrix} \rho q_t - u(c_t) - \mu^z \gamma_t \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} - \mu^z \iota_t \\ \rho p_t^x - u'(c_t)\partial_x \omega_t + \frac{p_t^x}{\lambda^x} \end{pmatrix} dt + Z_t dW_t^m \tag{A.9}$$

Using the change of measure, $dW_t^z = dW_t^{z,*} = dW_t^{z,m} - a_t^z dt = dW_t^{z,m} - \frac{\mu^z}{\sigma} dt$, $dW_t^{x,m} = dW_t^x$, and recalling that $W_t = (W_t^z, W_t^x)'$, we have the evolutions of the elements of Y under the truthtelling probability measure are given by

$$dY_t = \begin{pmatrix} \rho q_t - u(c_t) \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} \\ \rho p_t^x - u'(c_t)\partial_x \omega_t + \frac{p_t^x}{\lambda^x} + \zeta_t \mu^z \end{pmatrix} dt + Z_t dW_t, \tag{A.10}$$

with random initial conditions $q_0, p_0, p_0^x \in \mathbb{R}_-$, and terminal conditions

$$\lim_{t\to\infty} \mathbb{E}[e^{-\rho t} Y_t] = 0.$$

A.1.6 Solving for Optimal Contracts

First, by the Revelation Principle, we can without loss of generality restrict attention to optimal contracts inducing truthtelling, $\widehat{m}^y = 0_{[0,\infty)}$.

From the necessary optimality conditions solved for the agent's reporting problem, any incentive compatible contract must have promised utility and marginal utility processes (q, p, p^x) as described above and satisfy the FO-IC condition $\gamma_t + p_t = 0$.

The principal's problem can then be reformulated as

$$J(q_0) = \sup_{p_0, p_0^x} J^*(q_0, p_0, p_0^x) = \sup_{p_0, p_0^x} \sup_{\omega, \gamma, \gamma^x, \iota, \iota^x, \zeta, \zeta^x} \mathbb{E}\left[\int_0^\infty e^{-\rho t}(y_t - \omega_t) dt\right] \tag{A.11}$$

subject to the FO-IC constraint $\gamma_t = -p_t$, the NS constraint, the stochastic evolution of y, x, z and adjoint processes dp_t, dq_t, and dp^x_t.

Fixing p_0 and p^x_0 and ignoring the NS constraint for now, we take a dynamic programming approach (as in W11 and BKS20). That is, we assume that the principal's value function $J_t : [0, \infty) \times \mathbb{R}^5 \to \mathbb{R}$ is twice continuously differentiable in its arguments, and we redefine the state X as $(z, x, q, p, p^x)'$, and the controls as $(p_0, p^x_0, \alpha)'$, where $\alpha = (\omega, \iota, \iota^x, \gamma, \gamma^x, \zeta, \zeta^x)$. Then the associated HJB is given by

$$0 = \sup \mathscr{L}_t J_t + y - \omega, \qquad \limsup_{t\to\infty} \mathbb{E}[J_t] = 0,$$

where $\mathscr{L}_t J_t = \frac{\partial}{\partial t} J_t dt + \frac{\partial}{\partial X'} J_t \mathbb{E}_t[dX_t] + \frac{\partial^2}{\partial X' \partial X} J_t \mathbb{E}_t[dX'_t dX_t]$,

$$dX_t = \widetilde{b}(X_t, \alpha_t)dt + \widetilde{\Sigma}(X_t, \alpha_t)dW_t$$

$$\widetilde{b}(X_t, \alpha_t) = \begin{pmatrix} \mu^z(z_t) \\ \mu^x(x_t) \\ \rho q_t - u(c_t) \\ \rho p_t + (1-\tau)u'(c_t) - \frac{\gamma_t}{\lambda} \\ \rho p^x_t - u'(c_t)\partial_x \omega_t + \frac{p^x}{\lambda^x} + \zeta_t \mu^z \end{pmatrix}, \qquad \widetilde{\Sigma} = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma^x \\ Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \\ Z_{31} & Z_{32} \end{pmatrix},$$

where Z_{ij} is (ij)th entry of the Z matrix.

By inspection, taking $J_t(z, x, q, p, p^x) = e^{-\rho t} J(z, x, q, p, p^x)$, this problem is of controlled diffusion with principal's payoff satisfying a quadratic growth condition.[59] Hence, appealing to Theorem 3.5.3 of Pham (2009), if the principal's value function J is twice continuously differentiable, satisfies a quadratic growth condition and a transversality condition, $\lim_{t\to\infty} e^{-\rho t} \mathbb{E}[J(X^\alpha_t, \alpha_t)] = 0$, solves the HJB above with a measurable and feasible control function α and admits a unique solution to the state SDE, then J is the value function and α the optimal Markovian control.[60] Then the HJB simplifies to

$$\rho J = \max_{\omega, \iota, \iota^x, \gamma^x, \zeta, \zeta^x} y - \omega + \frac{\partial}{\partial X'} J \widetilde{b}(X, \alpha) + \frac{1}{2} Tr\left(\widetilde{\Sigma}\widetilde{\Sigma}' \frac{\partial^2}{\partial X' \partial X} J\right)$$

Using the binding IC constraint $\gamma = -p$, by direct computation we get

[59] In particular, note that the instantaneous payoff of the principal is $y - \omega$, where $y = \tau x + (1-\tau)z$ is linear in x and z, and hence trivially satisfies a quadratic growth condition, $|y| \le C(1 + |y|^2)$ for $C = 1$, and ω is a control.

[60] The growth condition can be generalized to any N degree polynomial growth condition with suitable restriction on the discount rate. See Fabbri et al. (2017).

$$tr\left(\widetilde{\Sigma}\widetilde{\Sigma}'\frac{\partial^2}{\partial X'\partial X}J\right) = J_{zz}\sigma^2 + J_{xx}\left(\sigma^x\right)^2 + J_{qq}[Z_{11}^2 + Z_{12}^2] + J_{pp}[Z_{21}^2 + Z_{22}^2] + J_{p^x p^x}[Z_{31}^2 + Z_{32}^2]$$

$$+2\left[J_{xq}\left(Z_{12}\sigma^x\right) + J_{xp}\left(Z_{22}\sigma^x\right) + J_{xp^x}\left(Z_{32}\sigma^x\right)\right]$$

$$+2\left[J_{zq}\left(Z_{11}\sigma\right) + J_{zp}\left(Z_{21}\sigma\right) + J_{zp^x}\left(Z_{31}\sigma\right)\right]$$

$$+2\left[J_{qp}\left(Z_{11}Z_{21} + Z_{12}Z_{22}\right) + J_{qp^x}\left(Z_{11}Z_{31} + Z_{12}Z_{32}\right)\right]$$

$$+2\left[J_{pp^x}\left(Z_{21}Z_{31} + Z_{22}Z_{32}\right)\right].$$

For any J, the optimization of the HJB is separable with respect to ω and the other controls, so that taking $\partial_x\omega$ as deterministic function of the states and ω, and using $u'(c) = -\psi u(c)$, we get the following optimization problem for ω:

$$\max_{\omega} -\omega - u(c)J_q - \psi u(c)(1-\tau)J_p + \psi u(c)\partial_x\omega J_{p^x}.$$

We now guess that the value function takes the form

$$J = j^x(x) + J^{p^x}(p^x) + J^z(z,q,p) \tag{A.12}$$

where we further conjecture that

$$J^z(z,q,p) = j^0 + j^z(z) - j^q\log(-q) + h\left(\frac{p}{q}\right). \tag{A.13}$$

With this guess and using the agent's consumption c is linear in ω and $u(c)$ is strictly concave, the resulting FOC is

$$u(c)\psi\left[J_q + (1-\tau)\psi J_p - \partial_x\omega J_{p^x}\right] = 1. \tag{A.14}$$

Plugging in the guess of the value function into the HJB results in the remaining optimizations separating into individual problems:

$$\sigma^2\max_{\iota}\left\{\frac{J_{pp}}{2}\iota^2 + J_{qp}\iota\gamma\right\} +$$

$$\max_{\zeta}\left\{\frac{J_{p^x p^x}}{2}\zeta^2\sigma^2 + J_{p^x}\zeta\mu^z(0)\right\},$$

and

$$(\sigma^x)^2 \max_{\zeta^x} \left\{ \frac{J_{p^x p^x}}{2} (\zeta^x)^2 + J_{xp^x} \zeta^x \right\} +$$

$$(\sigma^x)^2 \max_{\iota^x, \gamma^x} \left\{ \frac{J_{qq}}{2} (\gamma^x)^2 + \frac{J_{pp}}{2} (\iota^x)^2 + J_{qp} \iota^x \gamma^x \right\}.$$

Recalling that with the IC constraint $\gamma = -p$ is fixed, the first two problems are (strictly) concave provided that $J_{pp} \leq 0$ ($J_{pp} < 0$) and $J_{p^x p^x} \leq 0$ ($J_{p^x p^x} < 0$), respectively. In this case, defining $k := \frac{p}{q}$, the FOCs yield

$$\iota = p \frac{J_{qp}}{J_{pp}} = -qk \left[k + \frac{h'(k)}{h''(k)} \right],$$

$$\zeta = -\frac{\mu^z(0) J_{p^x}}{\sigma^2 J_{p^x p^x}},$$

and

$$\zeta^x = -\frac{J_{xp^x}}{J_{p^x p^x}}.$$

The last problem of choosing jointly ι^x and γ^x is strictly concave if, in addition to the above, we have that $J_{qq} J_{pp} > J_{qp}^2$, which we will verify ex-post that holds. In this case, the FOCs yield $\gamma_x = \iota_x = 0$.

By direct computation, the derivatives of the guessed value function with respect to q and p are:

$$J_q = -\frac{1}{q} [j^q + h'(k)k], J_p = \frac{1}{q} h'(k) \tag{A.15}$$

$$J_{qq} = \frac{1}{q^2} \Big([j^q + h'(k)k] + [h''(k)k^2 + h'(k)k] \Big) \tag{A.16}$$

$$J_{pp} = \frac{1}{q^2} h''(k) \tag{A.17}$$

$$J_{qp} = -\frac{1}{q^2} \Big(h''(k)k + h'(k) \Big) \tag{A.18}$$

so that $\frac{J_{qp}}{J_{pp}} = -k - \frac{h'(k)}{h''(k)}$.

Hence the sufficient condition for an interior optimal solution of ι is $J_{pp} = \frac{h''(k)}{q^2} < 0$, and the additional sufficient conditions required for $\iota^x = \gamma^x = 0$ are (a) $J_{qq} < 0$, and (b) $J_{pp} J_{qq} - J_{qp}^2 = \frac{1}{q^4} (h''(k) j^q - h'(k)^2) > 0$.

Conjecturing that the principal provides full insurance of the public cash flows x and concentrates all carrots and sticks to those associated with the private cash flows z, we guess that $J_{p^x} = 0$. The the FOC with respect to ω is necessary and sufficient provided $J_q + \psi(1-\tau)J_p < 0$.

So, from the FOC with respect to ω, the contracted utility is given by

$$u(c) = \frac{q}{\psi} U(k)$$

where $U(k) := -\Big(j^q + h'(k)\big[k - (1-\tau)\psi\big]\Big)^{-1}$ and so compensation is given by $\omega = u^{-1}(\frac{q}{\psi}U(k)) = -\frac{1}{\psi}\log\Big(-\frac{q}{\psi}U(k)\Big)$. Observe that this satisfies our assumptions on ω for the agent's reporting problem, namely that ω is Markov. From this, we have that the principal's instantaneous payoff is

$$y - \omega = y + \frac{1}{\psi}\log\Big(-\frac{q}{\psi}U(k)\Big) = y - \frac{1}{\psi}\log(\psi) + \frac{1}{\psi}\log(-q) + \frac{1}{\psi}\log(U(k)).$$

Then using equation (A.14), the fact that from the guesses we obtain that $-qJ_q = j^q + h'(k)k$ and $J_p p = kh'(k)$, the gradient terms of the HJB in (q,p) are given by

$$\begin{aligned} &J_p\left[\rho p + (1-\tau)u'(c) + \frac{p}{\lambda}\right] + J_q[\rho q - u(c)] \\ &= -u(c)[J_q + (1-\tau)\psi J_p] + \rho[J_p p\Big(1 + \frac{1}{\rho}\frac{1}{\lambda}\Big) + J_q q] \\ &= -\frac{1}{\psi} - \rho j^q + \frac{kh'(k)}{\lambda}. \end{aligned}$$

Similarly, using the FOC for ι, the relation of J_{qq} to J_{qp}, the diffusion correction terms can be written as

$$\sigma^2\Big(\frac{1}{2}J_{qq}(p^2) + \frac{1}{2}J_{pp}(\iota^2) + J_{qp}(-p\iota)\Big) = \frac{\sigma^2 k^2}{2}\Big(j^q - \frac{h'(k)^2}{h''(k)}\Big).$$

Thus, plugging in the guess in the HJB on both sides and matching the coefficients we have

$$\rho j^x(x) = \tau x + j^x_x(x)\mu^x(x) + j^x_{xx}(x)\frac{(\sigma^x)^2}{2} \tag{A.19}$$

$$\rho j^z(z) = (1-\tau)z + j^z_z(z)\mu^z(z) + j^z_{zz}(z)\frac{\sigma^2}{2} \tag{A.20}$$

$$-\rho j^q \log(-q) = \frac{1}{\psi}\log(-q) \tag{A.21}$$

$$\rho h(k) = \frac{1}{\psi}\log(U(k)) + \frac{kh'(k)}{\lambda} + \frac{\sigma^2 k^2}{2}\left(j^q - \frac{h'(k)^2}{h''(k)}\right) \tag{A.22}$$

$$\rho j^0 = -\frac{\log(\psi)}{\psi}. \tag{A.23}$$

The first two conditions are second order linear differential equations with initial conditions $x_0 = \lambda^x \mu^x$ and $z_0 = \lambda \mu^z$ (whose existence and uniqueness is guaranteed), and, by guessing $j^z(z) = j_0^z + j_1^z z$ and $j^x(x) = j_0^x + j_1^x x$, we have solutions $j_0^z = \frac{1}{\rho}\mu^z j_1^z$, $j_1^z = \frac{1-\tau}{\rho+\frac{1}{\lambda}}$, $j_0^x = \frac{1}{\rho}\mu^x j_1^x$, and $j_1^x = \frac{\tau}{\rho+\frac{1}{\lambda^x}}$. The third and fifth conditions are solved directly as $j^q = -\frac{1}{\rho\psi}$ and $j^0 = -\frac{\log\psi}{\rho\psi}$. The fourth condition is a second order non-linear homogeneous ordinary differential equation (ODE) for some initial conditions, $h(k_0) = h_0$, $h'(k_0) = h'_0$, and $k_0 = \frac{p_0}{q_0}$. Recall that k_0 is optimally chosen by the principal in a first stage optimization over the initial choice of p_0 given that q_0 is fixed. Using a change of variables $H(k) = h'(k)$, this second order ODE can be reframed as a system of first order ODEs for which, by the Picard-Lindelof Theorem, a solution exists and is unique. Finally, notice that we find that $J^{p^x}(p^x) = 0$ consistent with our guess that $J_{p^x} = 0$.

Given a solution $h(k)$, we must verify that the resulting value function J satisfies the concavity restrictions imposed earlier to obtain interior solutions, and that J satisfies the transversality condition $\lim_{t\to\infty} e^{-\rho t}\mathbb{E}[J(X_t^\alpha, \alpha_t)] = 0$. The solution of $-h(k)$ is plotted in Figure A.1 of W11 for a fixed ψ and λ. Recalling that W11 frames the principal's problem as a problem of minimization of costs, so that his $h(k)$ is the negative of ours, by inspection we have that, in our framing, $h''(k) < 0$ as well as that $h'(k) \geq 0$ for any $k \leq k_0$.

Recalling that $h(k_0)$ and $h'(k_0)$ are free-variables for the principal, the optimal contract (without no hidden savings constraints) is then pinned down by solving for the optimal initial condition k_0 (assuming an interior optimum exists). Rather than attempting to solve for the unconstrained optimal initial conditions, we will now move to the restricted optimal contract in the case where the agent has access to hidden savings.[61]

A.1.7 Optimal Contracts with Hidden Savings

In Proposition 3.1 of BKS20 they characterize the agent's (self-insurance) problem for a single Ornstein-Uhlenbeck endowment process and exponential

[61] W11 found by numerical solutions a local optimum where $h'(k_0) = 0$, so that in our context $k_0 = (1-\tau)\frac{\rho\psi}{\rho+\frac{1}{\lambda}}$. However, given the counterexample of BKS20 we know that at least without the restriction of no hidden savings, this contract is in fact not optimal.

utility and establish that the solution satisfies the standard consumption intertemporal Euler equation as

$$e^{-\rho t}u'(c_t) = \mathbb{E}_t[e^{-\rho s}u'(c_s)],\ \forall s \geq t. \tag{A.24}$$

Consequently, to preclude hidden savings, a contract must ensure that the agent's associated consumption satisfies this condition.

From the previous section, we have that the contracted compensation fully stabilizes the observable cash flow component, x_t, so that the agent's consumption and hidden savings decision under the contract only depends on the unobservable cash flow process, z_t. Thus we can directly apply the results of Theorem 4 of BKS20. Specifically, given the exponential utility, we have $-\psi e^{-\rho t}u(c_t) = -\psi \mathbb{E}_t[e^{-\rho s}u(c_s)]\ \forall s \geq t$, so that by cancelling $-\psi$ and integrating from t to ∞, we have

$$q_t = \int_t^\infty e^{-\rho(s-t)}\mathbb{E}_t[u(c_s)]ds = \frac{u(c_t)}{\rho}.$$

Similarly, by direct computation

$$p_t = \int_t^\infty e^{-(\rho+\frac{1}{\lambda})(s-t)}\mathbb{E}_t[-(1-\tau)u'(c_s)]ds = (1-\tau)\frac{\rho\psi}{\rho+\frac{1}{\lambda}}q_t$$

so that the ratio of (negative) promised marginal utility to the level of promised utility is given by $k_t = \frac{p_t}{q_t} = (1-\tau)\frac{\rho\psi}{\rho+\frac{1}{\lambda}}$ so that $k_t = k_0^*$.

Thus, introducing the no hidden savings restriction, in order to preclude hidden savings, the initial ratio of marginal utility to promised utility must satisfy $h'(k_0) = 0$. Plugging this into the differential equation for $h(\cdot)$ implies that $h(k_0^*) = \frac{1}{\rho\psi}\log(\rho\psi) - \frac{\sigma^2(1-\tau)^2\psi}{2(\rho+\frac{1}{\lambda})^2}$. With this, the W11 contract, adjusted for our mixture of cash flows, is the unique contract which satisfies the required optimality conditions.

Observing that $h''(k_0^*) < 0$ (see figure A.1 in W11 multiplied by -1 to reflect us maximizing rather than minimizing), it is straightforward to verify the necessary and sufficient conditions for interior optimal solutions of ι, ι^x, γ^x, ζ, and ζ^x are satisfied. Moreover, with this guess, J is finite and hence the transversality condition holds, that is $\lim_{t\to\infty} e^{-\rho t}J = 0$. Thus, we have verified all the requisite conditions for our guess to be the solution.

Combining the results of above, we see that the agent's compensation contracted with the public principal evolves according to equation (4). Since the private principal utilizes the monitoring technology, this compensation offered by this investor collapses to the first term of such equation, yielding Theorem 1.

A.1.8 Verifying Agent's Optimal Reports Given the Contract

Since we only utilized a necessary SMP for the agent's problem, we must make a verification argument. That is, we must verify that the agent finds it optimal to truth-tell and to not privately save, given the the offered contract.

First, observe that if $\tau = 0$, then we are in the setting of W11 and BKS20, where, from BKS20 Lemma D.2 and D.3, the value function of the agent under the optimal FO-IC contract with hidden savings and private reports (indirectly implemented by solving the agent's self-insurance problem in Section 3.1 of BKS20) is given by

$$V(A,z) = V_0 \exp\left(-\rho\psi\left(A + \frac{z}{\rho + \frac{1}{\lambda}}\right)\right)$$

where $V_0 = -\exp\left(\rho\psi\left[\frac{\mu^z(0)}{\rho(\frac{1}{\lambda}+\rho)} + \frac{\log(\rho)}{\rho\psi} - \frac{1}{2\psi\rho^2}\left(\frac{\psi\rho}{\rho+\frac{1}{\lambda}}\sigma)^2\right)\right]\right)$ and with optimal assets (given by BKS20 equation (3.7)) equal to

$$A_t^* = A_0 + \frac{1}{2\psi\rho}(\sigma\frac{\psi\rho}{\rho+\frac{1}{\lambda}})^2 t - \int_0^t \frac{\mu^z(z_s)}{\rho+\frac{1}{\lambda}}ds$$

for optimal initial assets $A_0(z_0, q_0) = \frac{\omega_{fix}(q_0)}{\rho} - \frac{\mu^z(0)}{\rho(\rho+\frac{1}{\lambda})} - \frac{z}{\rho+\frac{1}{\lambda}} + \frac{\left(\frac{\psi\rho}{\rho+\frac{1}{\lambda}}\sigma)\right)^2}{2\psi\rho^2}$ (given by BKS20 eq. 3.11), $\omega_{fix}(q_0) = -\frac{\log(-q_0)}{\psi\rho}$.

Second, observe that with $\tau = 1$, the optimal contract is equal to the full information insurance contract (complete stabilization) so that agent utility is $V^x(q_0) = \omega_{fix}(q_0)$ and can be implemented via agent self-insurance contract by simply giving the agent $A_0^x = \frac{\omega_{fix}(q_0)}{\rho}$.

Finally, it is clear that by similar logic, for $\tau \in (0,1)$, the optimal contract indirectly implemented via a self-insurance contract with

$$A_0(q_0;\tau) = \frac{\omega_{fix}(q_0)}{\rho} - \frac{(1-\tau)\mu^z(0)}{\rho(\rho+\frac{1}{\lambda})} - \frac{(1-\tau)z}{\rho+\frac{1}{\lambda}} + \frac{((1-\tau)\sigma\frac{\psi\rho}{\rho+\frac{1}{\lambda}})^2}{2\psi\rho^2}.$$

A.1.9 Establishing a Stochastic Maximum Principle

By inspection, the above problem matches the setting of a discounted infinite horizon optimal control problem. As discussed in Section A.1.5, we combine results from Maslowski and Veverka (2014), Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) to obtain a necessary SMP for our setting. We first establish that all the conditions required, except the concavity one, for Maslowski and Veverka (2014)'s sufficient SMP are satisfied. We then leverage their mapping to the setting of Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) and their result assuring that the transversality condition, given Maslowski and Veverka (2014)'s restrictions on the environment, holds which is needed to yield their necessary SMP.

Maslowski and Veverka (2014)'s sufficient SMP (presented in their Theorem 4) states that if (1) the control space is bounded and convex, (2) the drift and diffusion coefficients of the state process are sufficiently well-behaved and bounded, (3) the instantaneous discounted payoff function is continuously differentiable in the states X and its derivative with respect to the state is square-integrable, and (4) the Hamiltonian is concave in the states and controls (X, α), then the above optimization problem is equivalent to the static maximization of the above Hamiltonian with the associated adjoint process.[62]

We now verify each of the first three conditions in turn. Condition (1) is directly given by the uniform integrability boundedness imposed on the admissible space of misreports $\mathscr{M}$.

There are six sub-conditions to check for condition (2). The first three sub-conditions are given by the first three assumptions of Maslowski and Veverka (2014), which provide sufficient conditions on the state controlled diffusion process to guarantee existence and uniqueness of a strong solution to the forward SDE. The latter three sub-conditions are given by the latter three assumptions, which guarantee the existence and uniqueness of a solution to the forward-backward SDE (X, Y, Z). We verify one by one that the six assumptions of Maslowski and Veverka (2014) hold in our setting:

1. The first assumption requires drift and diffusion coefficients be continuous in their arguments (controls and states), which holds by direct inspection of $b(\cdot,\cdot)$ and $\Sigma(\cdot,\cdot)$.

2. The second assumption requires dissipative states, i.e. $\exists \mu_1 \in \mathbb{R} : (X_1 - X_2)'(b(X_1, \Delta) - b(X_2, \Delta)) \leq \mu_1 |X_1 - X_2|^2$.

[62] There is also a lower bound restriction imposed on the discount rate, but its satisfaction is not a theoretical concern.

Plugging $b(X, \Delta)$ and X in the latter equation, we will consider a bound for each argument individually, and obtain $\mu_1 = \max\{\mu_1^{\Gamma}, \mu_1^{\tilde{m}}, \mu_1^{x}\}$. First, focusing on the last state, x_t, we have that the x terms of this condition correspond to

$$\mu_1^x (x_1 - x_2)^2 \geq -\frac{1}{\lambda^x}(x_1 - x_2)^2$$

so that $\mu_1^x \geq \frac{-1}{\lambda^x}$.

Similarly, focusing on the first two states, taking μ_1 as defined above, the right-hand side for terms Γ and $\tilde{m}$ is

$$\mu_1 \left[((\Gamma_1 - \Gamma_2)^2 + (\tilde{m}_1 - \tilde{m}_2))^2 \right]$$
$$= \mu_1 \left[((\Gamma_1 - \Gamma_2)^2 + (\tilde{m}_1 - \tilde{m}_2))^2 \right] + 2\mu_1(\Gamma_1 - \Gamma_2)(\tilde{m}_1 - \tilde{m}_2)$$

where the equality follows from completing the square. Consequently, the difference between the right-hand side and the left-hand side for these two terms is

$$\mu_1 \left[((\Gamma_1 - \Gamma_2) + (\tilde{m}_1 - \tilde{m}_2))^2 \right] - (\Gamma_1 - \Gamma_2)(\tilde{m}_1 - \tilde{m}_2)(2\mu_1 - \Delta)$$

$$\geq \mu_1 \bigg((\Gamma_1 - \Gamma_2|) + (|\tilde{m}_1 - \tilde{m}_2|) \bigg)^2 - |(\Gamma_1 - \Gamma_2)| \cdot |(\tilde{m}_1 - \tilde{m}_2)| \cdot (2\mu_1 - K_m) \geq 0$$

where the first inequality is due to the restrictions on the admissible controls $\Delta \in \mathscr{M}$, $|\Delta| \leq K_m$ (note by admissibility of $\mathbf{m}$ in Appendix A.1.2, $|m| \leq K_m$, so given $dm_t = \Delta_t dt$ it then follows $|\Delta| \leq K_m$ is necessary for admissibility) and the second inequality is obtained by setting $\frac{K_m}{\mu_1} = 2$, and noting that $\mu_1 = \max\{\frac{K_m}{2}, -\frac{1}{\lambda^x}\} > 0$.

3. The third assumption requires a Lipschitz continuous diffusion, i.e. $\exists L > 0$: $||\Sigma(X_1, \Delta) - \Sigma(X_2, \Delta)|| \leq L|X_1 - X_2|$.[63]

   We establish below assumption six a stronger condition of bounded derivatives on the diffusion which yields this assumption by implication. In particular, we show that the euclidean normed gradient of Σ is bounded by a constant M, which then an application of the mean-value theorem establishes the desired result.

[63] $|| \cdot ||$ denotes the L2 or spectral norm on matrices.

To see this, note that using standard relations of the spectral $||\cdot||$ and Frobenius norm $||\cdot||_F$, we get

$$||\Sigma(X_1,\Delta)-\Sigma(X_2,\Delta)|| \leq ||\Sigma(X_1,\Delta)-\Sigma(X_2,\Delta)||_F \equiv \left(\sum_{i,j}|A_{ij}|^2\right)^{\frac{1}{2}},$$

where $A \equiv \Sigma(X_1,\Delta)-\Sigma(X_2,\Delta)$.

Consequently, using the Mean-value theorem and existence of a bound $M>0$ on all elements of the gradient, $|A_{ij}| = |\Sigma_{ij}(X_1,\Delta)-\Sigma_{ij}(X_2,\Delta)| \leq M|X_1-X_2|$. The result immediately follows.

4. The fourth assumption requires continuously differentiable drift, diffusion and instantaneous payoff in the states, which holds by inspection.

5. The fifth assumption requires a drift growth which dissipates, i.e. $\exists \mu_2 \in \mathbb{R} : (d_1-d_2)'(\nabla_1 b(X,\Delta)(d_1-d_2)) < \mu_2|d_1-d_2|^2,\ \forall d_1,d_2 \in \mathbb{R}^3$.

   Taking $D = d_1-d_2 \in \mathbb{R}^3$, $D=(D_1,D_2,D_3)'$, direct computation yields

$$D'(\nabla_1 b(X,\Delta)D) = \Delta D_1 D_2 \leq K_m D_1 D_2 =$$

$$\frac{K_m}{2}(D_1^2+D_2^2-(D_1-D_2)^2) \leq \frac{K_m}{2}|D|^2.$$

   So, $\mu_2 = \frac{K_m}{2}$ yields the result.

6. The sixth assumption requires a bounded diffusion growth, i.e. $\exists M \geq 0 : \sum_{i=1}^{2}||\nabla_1\Sigma^i(X,\Delta)|| \leq M$ where Σ^i is the i-th column of Σ.

   Take $a_0^z = \frac{1}{\sigma}\left(\frac{\mu^z(0)}{\lambda^z}+\Delta\right)$ and using $\tilde{m}_t = \Gamma m_t$, we have $\Sigma^1 = (\Gamma a^z, \tilde{m}a^z, 0)', \Sigma^2 = (0,0,\sigma^x)'$, and so directly

$$\nabla_1\Sigma^1 = \begin{pmatrix} a_0^z - \frac{\tilde{m}}{\Gamma\sigma\lambda} & \frac{1}{\sigma^z\lambda^z} & 0 \\ -\frac{\tilde{m}^2}{\Gamma^2}\frac{1}{\lambda^z\sigma^z} & a_0^z + \tilde{m}\Gamma\frac{1}{\lambda^z\sigma^z} & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } \nabla_1\Sigma^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

   Let $A^1 = \nabla_1\Sigma^1$ then $A_{11}^1 = \frac{\mu^z}{\sigma}$, $|A_{21}^1| = |-\frac{m^2}{\lambda^z\sigma^z}| \leq K_m^2\frac{1}{\lambda^z\sigma^z}$, and $|A_{22}^1| \leq \frac{\mu^z+K_m}{\sigma^z} + 2K_m\frac{1}{\lambda^z\sigma^z}$.

   Again using standard relations of spectral and Frobenius norms, we get

$$||\nabla_1 \Sigma^1(X,\Delta)|| \leq ||\nabla_1 \Sigma^1(X,\Delta)||_F \leq$$

$$\left(\frac{\mu^z + K_m}{\sigma^z}\right)^2 + \left(K_m^2 \frac{1}{\lambda^z \sigma^z}\right) + \left(\frac{\mu^z(z) + K_m}{\sigma^z} + 2K_m \frac{1}{\lambda^z \sigma^z}\right)^2 + \left(\frac{1}{\sigma^z \lambda^z}\right)^2 \equiv M$$

$$\sum_{i=1}^{2} ||\nabla_1 \Sigma^i(X,\Delta)|| \leq M.$$

We next move to verifying Maslowsi and Veverka (2014)'s condition (3).

By inspection $\Gamma u(X,\Delta) = \Gamma(-\exp(-\psi c_t(X,\Delta)))$ is continuously differentiable. Thus, it remains to show that

$$\mathbb{E}^* \left[\int_0^\infty e^{-\rho t} ||\nabla_1 \Gamma u(X,\Delta)||^2 dt\right] < \infty,$$

where

$$\nabla_1 = \begin{pmatrix} \frac{\partial}{\partial \Gamma} \\ \frac{\partial}{\partial \tilde{m}} \\ \frac{\partial}{\partial x} \end{pmatrix},$$

$$\frac{\partial}{\partial \Gamma} \Gamma u(X,\Delta) = u(X,\Delta) + (1-\tau)\psi u(X,\Delta) m = u(X,\Delta)[1 + (1-\tau)\psi m],$$

$$\frac{\partial}{\partial \tilde{m}} \Gamma u(X,\Delta) = -\psi u(X,\Delta), \text{ and}$$

$$\frac{\partial}{\partial x} \Gamma u(X,\Delta) = -\Gamma \psi u(X,\Delta) \partial_x \omega.$$

Thus, $||\nabla_1 \Gamma u(X,\Delta)||^2 \leq (u(X,\Delta))^2 \big((1+\psi\tilde{m})^2 + (-\psi)^2 + (\psi\Gamma)^2(\partial_x \omega)^2\big)$. Then note that $\tilde{m} = m\Gamma \leq K_m \Gamma$. By the restriction imposed on the contracts, we have that $\partial_x \omega_t$ is bounded. and that $u(X,\Delta)$ is discounted square integrable. Noting restrictions on the admissible report strategies require Γ to be square integrable, we thus assure the required well-posedness of the problem.

To obtain a suitable necessary SMP we rely on the corresponding results of Haadem et al. (2012), Haadem et al. (2013), and Øksendal and Sulem (2019) which requires a transversality condition and square integrability of a derivative process to be satisfied. From the above, we have established our setting conforms to Maslowski and Veverka (2014) wherein they establish that the transversality condition is satisfied and assures that the derivative process $\xi(t) := \left.\frac{\partial X^{\Delta+s\beta}(t)}{\partial s}\right|_{s=0}$ is square integrable (see Øksendal and Sulem (2019) for more details). Combining this result with direct inspection that the domain of controls Δ is a convex, open, bounded subset of $\mathbb{R}$, the necessity of the SMP for the problem in our setting follows.

To see the square integrability of the derivative process, note that by direct computation the derivative process is given by

$$d\xi_i(t) = \lambda_i(t) + \sum_{j=1}^{2} \sigma_{ij}^{\xi} dW_j, \qquad \text{with } i = 1, 2, 3\,, \tag{A.25}$$

where $\lambda_i(t) := \Big(\nabla_x b_i' \xi(t) + \nabla_\Delta b_i' \beta(t)\Big)$ and $\sigma_{ij}^{\xi}(t) := \Big(\nabla_x \Sigma_{ij}' \xi(t) + \nabla_\Delta \Sigma_{ij}' \beta(t)\Big)$.

Sufficient conditions for the square integrability of $\xi(t)$ are standard conditions of (i) a bounded growth condition and (ii) a Lipschitz condition on the derivative process. By inspection of the drift and diffusion components of this derivative, the conditions verified above for Maslowski and Veverka (2014) are sufficient to yield the requisite result.

A.2 Firm Listing Equilibrium

This section is divided in two parts. In the first part, we formally state our partial equilibrium definition, while, in the second part, we present a proof of Theorem 2 and Corollary 1.

A.2.1 Partial Equilibrium Definition

For any $\underline{q} < 0$, $B^f > 0$ with $f \in \{S, P\}$, $\nu > 0$ and absolutely continuous CDF $G_\theta(\cdot)$, we define a public listing market equilibrium as a collection of financiers' bidding strategies $q_0^f : \mathcal{Q} \times \Theta \to \mathbb{R}_-$ and financier selection rules of the entrepreneurs $\mathbb{I}^f : \mathcal{Q}^2 \times \Theta \to \{0, 1\}$ such that

1. $q_0^f, q_0^{-f}, \mathbb{I}^f$ is an extensive form trembling hand perfect equilibrium, that is for each $\theta \in \Theta$
   (a) q_0^f is a best-response to (any sequence of trembles of) q^{-f} and $\mathbb{I}^f$, and
   (b) $\mathbb{I}^f \in \{0, 1\}$ is a best-response to (any sequence of trembles of) q_0^{-f} and q_0^f, and
2. firm listing choice is feasible, that is $\mathbb{I}^f + \mathbb{I}^{-f} \leq 1$, as well as equilibrium financing is feasible for both financiers, that is

$$\int_\theta \mathbb{I}^f(q^f(\theta), q^{-f}(\theta)) dG(\theta) \leq B^f$$

where $\mathscr{Q} = \{-\infty\} \cup [\underline{q}, 0]$ and $\Theta := \{\theta : \theta \in \mathbb{R}^7_+\}$.

A.2.2 Proof of Theorem 2

First note that, given the agents' preferences over optimal contracts solved for in Theorem 1, the agent utility varies over contracts solely based on the initial promised utility q_0. Hence, in any equilibrium, a type θ agent's best-response to the financiers' bids of initial promised utilities (q_0^f, q_0^{-f}) is

$$(\mathbb{I}^f, \mathbb{I}^{-f}) = \begin{cases} (1,0) & \text{if } q_0^f > \max\{q_0^{-f}, \underline{q}\}, \\ (0,1) & \text{if } q_0^{-f} > \max\{q_0^f, \underline{q}\}, \\ (0,0) & \text{if } \underline{q} > \max\{q_0^f, q_0^{-f}\}, \\ (a,b), a+b \in [0,1], a,b \in \{0,1\} & \text{else.} \end{cases}$$

Assuming a successful bid, we can rearrange the payoff of a type f financier financing a type θ firm with a bid of initial promised utility q_0 as

$$R^f(q_0, \theta) = \Upsilon(\mu) - \Lambda^f(\theta) - X(q_0).$$

Notice that, as we have seen in Section 2.3, we have that $\Upsilon'(\mu) > 0$, $\Lambda^P(\theta) = \pi$ while $\Lambda^S(\theta) = \nu$, and $X'(q_0) > 0$ with $X(\underline{q}) = 0$. Then, absent competition, so that $q_0 = \underline{q}$, it is individually rational for a type f financier to finance a type θ firm if

$$\Upsilon(\mu) - \Lambda^f(\theta) \geq 0. \qquad \left(IR^f\right)$$

This implies that the two representative financiers have two imperfectly overlapping sets of firms which are individually rational to finance. As a result, three regions arise where it is individually rational for neither financier, it is individually rational for either one or the other financier, and it is individually rational for both financiers. In the first region, trivially no financing occurs in equilibrium as both financiers bid less than $\underline{q}$ (or do not bid at all) and $\mathbb{I}^f = \mathbb{I}^{-f} = 0$ is the unique equilibrium.

In the second region, only one financier can earn a positive payoff from financing a type θ firm. In this situation, a given type f financier can act as a monopolist and bid the agent's outside option $\underline{q}$ so to extract full surplus.

In the third region, both financiers can earn a positive payoff at the agent's outside option, $\underline{q}$. Denote $\bar{q}(\theta) := \min\{\bar{q}^S(\theta), \bar{q}^P(\theta)\}$ where $\bar{q}^f(\theta)$ solves $R^f(\bar{q}^f(\theta), \theta) = 0$ for $f \in \{S, P\}$. Since a type θ agent's best-response is to choose the higher bid of initial promised utility, standard arguments imply that the best response for a type f financier in this region is to weakly outbid the competitor as long as the payoff from financing is strictly positive, that is

$$q_0^f(q_0^{-f}, \theta) = \begin{cases} q_0^{-f} + \epsilon, & \text{if } R^f(q_0^f(q_0^{-f}, \theta), \theta) > 0, \\ [\underline{q}, q_0^{-f}], & \text{else} \end{cases}$$

for $\epsilon \to 0$. This implies that any $q^f < \bar{q}(\theta)$ cannot be an equilibrium. Without loss of generality suppose that $\bar{q}(\theta) = \bar{q}^{-f}(\theta)$. If the firm selection rule is such that $\mathbb{I}^f(\bar{q}(\theta), \bar{q}(\theta)) < 1$, then f can profitably deviate to $\bar{q}(\theta) + \epsilon$, and hence it is not an equilibrium. If instead $\mathbb{I}^f(\bar{q}(\theta), \bar{q}(\theta)) = 1$, then from the best-response functions neither financier has incentive to deviate and the firm is indifferent. For a given θ, by the financier decomposition, and since $X'(q_0) > 0$, this intersection point is unique.

Lemma 1 (No BC version)**.** *In a public listing equilibrium with the private financier's budget constraint is not binding, we have that a type θ firm*

1. *if $\pi \leq \Upsilon < \nu$, receives public financing at bid $\underline{q}$,*
2. *if $\nu \leq \Upsilon < \pi$, receives private financing at bid $\underline{q}$,*
3. *if $\Upsilon \geq \max\{\nu, \pi\}$*
   (a) *if $\pi < \nu$, receives public financing at bid $\underline{q}$,*
   (b) *if $\pi \geq \nu$, receives private financing at bid $\bar{q}_0^P(\theta)$,*
4. *if none of the above, receives no financing.*

Introducing the private financier's limited funds, B, there are two cases to consider. If the private financier is not constrained by its funds, the equilibrium is as specified in the lemma above. Suppose instead that the private financier is constrained, and consider the region of Θ where the specialist has a comparative advantage, that is $\bar{q}^S \geq \bar{q}^P$.

If financing a given type θ firm is individually rational only for the private financier, then this investor's payoff from financing is $\Upsilon(\mu) - \nu - X(\underline{q})$ and zero otherwise. If instead financing a given type θ firm is individually rational for both financiers, then this investor's payoff from financing is $\Upsilon(\mu) - \nu - X(\bar{q}(\theta))$. By the definition of $\bar{q}(\theta)$, we have $X(\bar{q}(\theta)) = \Upsilon(\mu) - \pi(\theta)$ so that $\Upsilon(\theta) - \nu - X(\bar{q}(\theta)) = \pi - \nu$, consistent with Theorem 1. It follows that in the first case the private investor is indifferent over π and has strictly increasing preferences over μ, while in the second case the private investor is indifferent over μ and has strictly increasing preferences over π.

The private financier is indifferent between θ in the first and the second case when $\Upsilon(\mu) - \nu - X(\underline{q}) = \pi - \nu$ which simplifies to

$$\pi = \Upsilon(\mu). \tag{A.26}$$

As an immediate consequence of the above and the fact that the private financier equilibrium payoffs, without financing constraints, are non-decreasing in π and μ, subject to limited funds, the private financier will impose a cutoff rule $(\underline{\mu}, \underline{\pi})$ satisfying equation (A.26) such that

$$\int_{\mu \geq \underline{\mu}} \int_{\pi \geq \underline{\pi}} dG_{\mu} dG_{\pi} = B.$$

For all (μ, π) in the region where it is individually rational for both financiers to finance but where $\mu < \underline{\pi}$ or $\pi < \underline{\pi}$, the public financier faces no competition in financing these firms. Hence, this investor can earn positive returns, and thus in equilibrium bids $\underline{q}$ and finances these firms.

Finally, for all (μ, π) in the region where it is not individually rational for the public financier to finance but where $\mu < \underline{\mu}$ or $\pi < \underline{\pi}$, the public financier faces no competition in financing these firms but cannot profit from financing, thus, this set of firms is unfinanced. Noting that trembling hand perfection rules out other possible equilibria with alternative bidding strategies of the non financing financier yields this equilibrium as unique. With this we have the results presented in Theorem 2. The result of Corollary 1 is an immediate implication, since taking $\underline{\pi}$ as fixed given B and decomposing π using the result of Theorem 1 yields $\underline{\sigma}^z(\tau, B)$.

A.3 General Equilibrium and Comparative Statics

This section is divided in three parts. In the first part, we formally state our general equilibrium definition. In the second and third part, we present a proof of Theorem 3 and 4, respectively.

A.3.1 General Equilibrium Definition

We endogenize the funds to private equity and the distribution of PE premia in general equilibrium by introducing a first stage in which households, endowed with heterogeneous monitoring costs ν, make a financier type choice

$f(\nu) \in \{P, S\}$. The definition of this general equilibrium corresponds to that of partial equilibrium, but with the addition of an equilibrium matching function $m(\nu)$ which maps the household financier of type ν to firm type θ.

A.3.2 Proof of Theorem 3

Since $M > 1$ there is an excess of financial resources over financing needs so that in equilibrium at least some measure of investors will not be matched to a firm. For these unmatched investors $R^S(\varnothing, \varnothing, \nu) = -\nu < 0 = R^P(\varnothing, \varnothing, \nu)$, so that it is not sustainable to have all investors electing to use the monitoring technology. Thus, in any equilibrium there is a positive mass of unmatched public investors.

Since the returns to public investing are not contingent on the investor type, these unmatched public investors can generate the same total surplus as any matched public investor. Therefore, Bertrand competition ensues yielding $q_0^P(\theta)$ such that $R^P(\theta, q_0^P(\theta), \cdot) = 0$ for any θ where it is individually rational for a public investor to invest (i.e. $R^P(\theta, \underline{q}, \cdot) \geq 0$). With this, we have a zero profit condition for public investors, that is the equilibrium public investor return on any financed firm is zero.

Combining this result with the capacity constraint on the monitoring technology usage, which implies that each private investor bids $q_0^S > -\infty$ on at most one firm and that no two private investors bid on the same firm, there are two distinct bidding regions where private investors compete, which in aggregate correspond to the private financing regions found in the partial equilibrium setting.

In one region, for public investors it is not individually rational to bid, whereas for a private investor with $\nu < \pi$ it is. Fixing an investor with monitoring cost ν, these are regions (Ic) and (IIa) of Figure 1a. In this case, as found in the partial equilibrium setting, a private financier is an effective monopolist, and can earn $\Upsilon(\mu) - \nu$.

In the other region where private investors bid, both public and private investors compete to finance firms, and so by outbidding the public investors a given private investor can earn $\pi - \nu$. These are regions (IIb) and (IIIa) of Figure 1a.

Consequently, a private investor is indifferent between a firm of type θ in the first region and a firm of type $\hat{\theta}$ in the second region when $\Upsilon(\mu) = \hat{\pi}$. Re-indexing firms by a change of variables $\pi' := \min\{\Upsilon(\mu), \pi\}$ and noting the free-exit of private investors (by switching to public), the return of a prospective private investor financing a given firm is

$$R^S(\theta, q_0, \nu) = [\pi' - \nu]_+ - [X(q_0) - X(q_0^*(\theta))]_+$$

where $q_0^*(\theta) = \max\{q_0^P(\theta), \underline{q}\}$, and $[a]_+ \equiv \max\{0, a\}$ denotes the positive part of a.

By inspection, the surplus function, which corresponds to the private investor's return absent a competing bid above $\underline{q}$, that is $\mathbb{S}(\pi', \nu) := R^S(\theta, \underline{q}, \nu) = [\pi' - \nu]_+$, is submodular. Thus, NAM is an equilibrium (pairwise stable) sorting. We can redefine $m(\nu)$ as the equilibrium matching function of an investor with cost ν to firm type π' (rather than firm type θ). Then, since an equilibrium firm matching with NAM satisfies $\bar{G}_{\pi'}(\pi') = M \cdot G_\nu(\nu)$, we have that $m(\nu) = \bar{G}_{\pi'}^{-1}(M \cdot G_\nu(\nu))$. Note we use $\pi'_{match}(\nu) = m(\nu)$ in the main body.

With this, equilibrium private investor returns are given by $R^*(\nu) := m(\nu) - \nu$. By direct computation, $m'(\nu) < 0$, $m(0) > 0$ and $\lim_{\nu \to \infty} R^*(\nu) < 0$, and using the Intermediate Value Theorem, there exists a unique fixed point, $\bar{\nu} > 0$, $R^*(\bar{\nu}) = 0$. Noting that $B = M \cdot G_\nu(\bar{\nu})$, and all sorting patterns are the same as in the partial equilibrium setting, we have established that the result is an equilibrium.

Finally, we move to establish equilibrium uniqueness. We will show that without trembles other matching functions of $\nu \le \bar{\nu}$ and $\pi' \ge \bar{\nu}$ can constitute an equilibrium, but with trembling perfection only the NAM function can.

To see this, define $q_0^S(\pi', \nu)$ to be the promised utility bid of a financier of type ν to a firm of type π' conferring the full surplus transfer to the firm, so that $X(q_0^S(\pi', \nu)) = \pi' - \nu$. Observe that then for any $\hat{\nu} < \nu$ and $\hat{\pi}' > \pi'$

$$R^S(\hat{\pi}', q_0^S(\hat{\pi}', \nu), \hat{\nu}) - R^S(\pi', q_0^S(\pi', \nu), \hat{\nu})$$

$$= ([\hat{\pi}' - \hat{\nu}]_+ - [\hat{\pi}' - \nu]_+) - ([\pi' - \hat{\nu}]_+) - [\pi' - \nu]_+) \ge 0, \tag{A.27}$$

holding with equality iff $\pi' \ge \nu$ and strict inequality otherwise. As this inequality is the same as condition (7) in Chade et al. (2017), we have generalized decreasing differences in the bidder's surplus globally over the full support of (π', ν). However, restricting to $\nu \le \bar{\nu}$ and $\pi' \ge \bar{\nu}$, (A.27) is identically zero so that any matching between types is an equilibrium. With trembles of investor financing type choice, (A.27) holds with strict inequality for any $\hat{\nu} < \bar{\nu}$ and $\nu = \bar{\nu} + \epsilon$, violating the optimality of any non-NAM function.

A.3.3 Introduction to Comparative Statics

As established in the previous section, the general equilibrium is fully characterized by two equations, the equilibrium matching condition and the equilibrium cutoff condition, which are respectively

$$\pi'_{match}(\nu) : \bar{G}_{\pi'}(\pi'(\nu)) = G_\nu(\nu) \cdot M \tag{A.28}$$

and

$$\bar{\nu} : \pi'(\bar{\nu}) - \bar{\nu} = 0. \tag{A.29}$$

We now move to characterizing comparative statics of various economic aggregates of interest. To do so, we distinguish between the short- and long-run. Denoting ν^{SR} (ν^{LR}) as the short-run (long-run) monitoring cost cutoff. In the first case we hold fixed the set of (potential) private financiers, $\{\nu : \nu \leq \bar{\nu}\}$, but allow adjustment of the matching of this fixed set of financiers to firms, denoted $\tilde{\pi}'_{match}(\nu)$, and for free-exit, so that any (potential) private financier ($\nu \leq \bar{\nu}$) with negative profits may exit and earn zero return instead. We preclude free-entry of new private financiers, so that (A.28) holds in the short-run, but only a weaker condition than equation (A.29) holds of $\tilde{\pi}'(\bar{\nu}^{SR}) - \bar{\nu}^{SR} \geq 0$. In the long-run, free-entry is allowed, imposing in addition $\tilde{\pi}'(\bar{\nu}^{LR}) - \bar{\nu}^{LR} \leq 0$ so that equation (A.29) holds with equality.

In the remainder of this section, we'll drop the *match* subscript to simplify notation. To assist with some of the comparative statics, consider the average PE premium conditional on a $\bar{\nu}$ monitoring cost cutoff, $\Pi(\bar{\nu})$. Observe that $\Pi(\bar{\nu}) = E[\pi'(\nu) - \nu | \nu \leq \bar{\nu}] = E[\pi' | \pi' \geq \pi'(\bar{\nu})] - E[\nu | \nu \leq \bar{\nu}]$ where the second equality follows from the bijective, monotonic matching function and equality of the sets $\{\theta : \pi'(\theta) \geq \pi'(\bar{\nu})\}$ and $\{\theta : \pi'(\nu) = \pi'(\theta), \nu \leq \bar{\nu}\}$.

By an application of the inverse function theorem, the matching function of type π' firms with type ν financier, $\pi'(\nu)$, is strictly decreasing in ν for any $\nu \geq 0$ (since $\partial G_{\pi'}(\pi')/\partial \pi' > 0$ for any π' with positive mass).

Adding and subtracting $\bar{\nu}$ and using the equality (A.29) yields

$$\Pi(\bar{\nu}) = MRL_{\pi'}(\bar{\nu}) + MAI_\nu(\bar{\nu})$$

where $MRL_{\pi'}(\bar{\nu}) := E[\pi' | \pi' \geq \pi'(\bar{\nu})] - \pi'(\bar{\nu})$ is the mean residual lifetime function of π' and $MAI_\nu(\bar{\nu}) := \bar{\nu} - E[\nu | \nu \leq \bar{\nu}]$ is the mean advantage over inferiors function of ν.

Drawing from the above and results from Bagnoli and Bergstrom (2005) we have the next two lemmas.

Lemma 2. *If Assumption 2 (converse) holds, then the mean residual lifetime of π' $MRL_{\pi'}(\bar{v})$ is strictly increasing (decreasing) in $\pi'(\bar{v})$ and decreasing (increasing) in $\bar{v}$.*

Lemma 3. *If Assumption 2 (converse) holds, then the mean advantage over inferiors of v $MAI_v(\bar{v})$ is strictly decreasing (increasing) in $\bar{v}$.*

Proof of Lemma 2. Taking $\pi'(\bar{v}) = \underline{\pi}$ fixed, then if $\bar{G}_{\pi'}$ is log convex (concave), appealing to Theorem 6 of Bagnoli and Bergstrom (2005), $MRL_{\pi'}(\bar{v})$ is strictly increasing (decreasing) in $\underline{\pi}$.[64] Combining this result with $\pi'(v)$ being a strictly decreasing function of v gives the result.

□

Proof of Lemma 3. An application of Theorem 5 of Bagnoli and Bergstrom (2005) directly yields $MAI_v(\bar{v})$ being strictly decreasing (increasing) in $\bar{v}$ if $G_v(v)$ is log convex (concave). □

A.3.4 Proof of Theorem 4 - (i) Intangibility

We consider an increase in firm intangibility culminating in first-order stochastic increase in the information premium, π, so that

$$\tilde{\pi}(\theta) = (1+\epsilon(\theta))\pi \text{ for } \epsilon(\theta) > 0, \forall\theta.^{65} \tag{A.30}$$

For simplicity, let $\epsilon(\theta) = \epsilon > 0$ be constant. Moreover, a first-order stochastic increase in firm intangibility by Assumption 1 corresponds to a first order stochastic increase in π', the modified information premium.[66] Denote $G^{SR}_{\pi'}$ as the short-run CDF of π' after the increase in intangibility. By the definition of first-order stochastic dominance (FOSD), we have

$$G^{SR}_{\pi'}(\pi') \leq G_{\pi'}(\pi') \;\forall\pi' \Rightarrow \bar{G}^{SR}_{\pi'}(\pi') \geq \bar{G}_{\pi'}(\pi').$$

Let $\underline{\pi}' = \pi'(\bar{v})$ prior to the shock and $\underline{\pi}'^{SR}$ denote the interim cutoff prior to extensive margin adjustment of $\bar{v}/B$. Then from (A.28) at the cutoff we

[64] Observe that Bagnoli and Bergstrom (2005) has a typo in Table 3 where the MRL(x) for both the Weibull and Gamma (with $c \in (0,1)$) is proven to be increasing rather than decreasing.

[65] Recall $\pi \propto (1-\tau)^2$, so a transformation of $(1-\tau)^2 \mapsto (1-\tau)^2(1+\tau c)$ for some constant $c > 0$ results in $\tilde{\pi} = (1+\epsilon(\theta))\pi$ with $\epsilon(\theta) = \tau c$. Moreover, $(1-\tau)^2 \in (0,1) \forall\tau \in (0,1)$ and $(1-\tau)^2 > (1-\tau)^2$ for any $\tau \in (0,1)$, thus this transformation maintains the same support for τ as the original.

[66] By definition $\pi' = \min\{\Upsilon(\mu), \pi\}$, so $Pr(\tilde{\pi}' \leq \underline{\pi}) = Pr(\Upsilon(\mu) \leq \underline{\pi}) \cdot Pr(\epsilon\pi \leq \underline{\pi}) = G_\mu(\Upsilon^{-1}(\underline{\pi}))G_\pi(\frac{\underline{\pi}}{1+\epsilon}) < G_\mu(\Upsilon^{-1}(\underline{\pi}))G_\pi(\underline{\pi}) = Pr(\pi' \leq \underline{\pi})$.

have

$$\bar{G}_{\pi'}^{SR}(\underline{\pi}') > \bar{G}_{\pi'}(\underline{\pi}') = G_\nu(\bar{\nu}).$$

Since $\bar{G}_{\pi'}(\cdot)$ is strictly decreasing, we have that $\underline{\pi}'^{SR} > \underline{\pi}'$. This also holds for any equilibrium matching of financiers to firms $\tilde{\pi}'(\bar{\nu}) > \pi'(\bar{\nu})$, where $\tilde{\pi}'(\cdot)$ is the short-run matching function following the increase in intangibility and $\pi'(\cdot)$ the original matching function.

In the long-run, the financier cutoff $\bar{\nu}$ can increase to ensure (A.29) holds. Since $\tilde{\pi}'(\bar{\nu}) > \pi'(\bar{\nu})$ and the matching function $\tilde{\pi}'(\cdot)$ is monotonically decreasing, it follows immediately that $\bar{\nu}^{LR} > \bar{\nu}$ and so $\underline{\pi}'^{LR} > \underline{\pi}'$ by (A.29). Thus, in summary,

$$\underline{\pi}' < \underline{\pi}'^{LR} < \underline{\pi}'^{SR}. \tag{A.31}$$

Since $\pi' = \min\{\pi, \Upsilon(\mu)\}$, $\Upsilon(\underline{\mu}) = \underline{\pi}$ and so

$$\underline{\mu} < \underline{\mu}^{LR} < \underline{\mu}^{SR}. \tag{A.32}$$

Combining these results with the definitions of the equilibrium economic aggregate definitions, we have the following results.

Short-run

1. PE premium increases.

   - Proof

     The mass of privately financed firms $|S|$ remains fixed and the financier cutoff type $\bar{\nu}$ is unaffected (since B is constant), so MAI_ν is unchanged and $MRL_{\pi'}$ is increasing in π' by FOSD, i.e.

$$\Pi^{SR} = E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - E[\nu|\nu \leq \bar{\nu}^{SR}]$$

$$= E[\tilde{\pi}'(\theta)|\{\theta : \pi'(\theta) \geq \underline{\pi}\}] - E[\nu|\nu \leq \bar{\nu}^{SR}]$$

$$\geq E[\pi'(\theta)|\{\theta : \pi'(\theta) \geq \underline{\pi}\}] - E[\nu|\nu \leq \bar{\nu}^{SR}] = \Pi.$$

2. Output decreases.

   - Proof

     Re-expressing O in terms of the total potential output minus unfinanced output, and noting that, by independence, $E[\mu]$ is invariant to transformations of π, we have

$$O^{SR}(\underline{\mu}^{SR}) = E[\mu] - \int_0^{\underline{\mu}^{SR}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi^{SR} dG_\mu = E[\mu] - \int_0^{\underline{\mu}^{SR}} \mu \bar{G}_\pi^{SR}(\Upsilon(\mu)) dG_\mu$$

$$\leq E[\mu] - \int_0^{\underline{\mu}^{SR}} \mu \bar{G}_\pi(\Upsilon(\mu)) dG_\mu$$

$$\leq E[\mu] - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi dG_\mu = O(\underline{\mu}),$$

where the first inequality follows from FOSD of G_π^{SR}, the second inequality from $\underline{\mu} < \underline{\mu}^{SR}$ by (A.32).

3. Mass of publicly financed firms and public listing propensity decrease.

   - Proof
     Observe $|P \cup S| = 1 - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu$ so that

     $$|P| = |P \cup S \setminus S| = 1 - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu - \int_{\underline{\mu}}^{\infty} \int_{\underline{\pi}}^{\infty} dG_\pi dG_\mu.$$

     By definition of short-run $|S|$ (and hence the third term) is constant, while using the same argument as for output O, $|P \cup S|$ falls in short-run, yielding the result. The change in public listing propensity follows immediately.

4. CEO pay increases.

   - Proof
     By definition of Ω, $\Omega^{SR} > \Omega$ iff $\tilde{E}[\pi(\theta)|\theta \in P(\underline{\mu}^{SR})] > E[\pi(\theta)|\theta \in P(\underline{\mu})]$. By direct computation, for the short-run:

     $$\tilde{E}[\pi(\theta)|\theta \in P(\underline{\mu}^{SR})] =$$

     $$\frac{1}{|P^{SR}|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi dG_\pi^{SR} dG_\mu + \int_{\underline{\mu}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} \pi dG_\pi^{SR} dG_\mu + \int_{\underline{\mu}^{SR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{SR})} \pi dG_\pi^{SR} dG_\mu\right]$$
     $$> \frac{1}{|P|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi dG_\pi^{SR} dG_\mu + \int_{\underline{\mu}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} \pi dG_\pi^{SR} dG_\mu + \int_{\underline{\mu}^{SR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{SR})} \pi dG_\pi^{SR} dG_\mu\right] \tag{A.33}$$

     where the inequality follows from $|P^{SR}| < |P|$, as shown above. Similarly, prior to the shift

     $$E[\pi(\theta)|\theta \in P(\underline{\mu})] = \frac{1}{|P|}\left[\int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} \pi dG_\pi dG_\mu + 0 + \int_{\underline{\mu}}^{\infty} \int_0^{\Upsilon(\underline{\mu})} \pi dG_\pi dG_\mu\right]. \tag{A.34}$$

     The first term of (A.33) exceeds that of (A.34) by (A.30) (i.e. $\tilde{\pi} = \pi\varepsilon,\ \varepsilon > 1$).

The second term of (A.33) exceeds that of (A.34) since $\underline{\mu}^{SR} > \underline{\mu}$.
The difference in the third terms of (A.33) -(A.34) is greater than or equal to

$$\bar{G}_{\mu}(\underline{\mu}^{SR})\left[\int_0^{\Upsilon(\underline{\mu}^{SR})} \pi dG_{\pi}^{SR} - \int_0^{\underline{\mu}} \pi dG_{\pi}\right]$$
$$\geq \bar{G}_{\mu}(\underline{\mu}^{SR})\left[\int_0^{\Upsilon(\underline{\mu})} \pi dG_{\pi}^{SR} - \int_0^{\underline{\mu}} \pi dG_{\pi}\right]$$
$$\geq \bar{G}_{\mu}(\underline{\mu}^{SR})\left[\int_0^{\Upsilon(\underline{\mu})} \pi dG_{\pi} - \int_0^{\underline{\mu}} \pi dG_{\pi}\right] = 0$$

where the first inequality follows from $\underline{\mu}^{SR} > \underline{\mu}$, and so $\bar{G}_{\mu}(\underline{\mu}^{SR}) < \bar{G}_{\mu}(\underline{\mu})$, the second inequality follows from $\Upsilon(\underline{\mu}^{SR}) > \Upsilon(\underline{\mu})$, and the third from the transformation of π by (A.30).

Long-run

1. PE funds expands, as shown above, and so the mass of privately financed firms increases relative to the short run and the initial level, that is

$$|S^{LR}| > |S| = |S^{SR}|.$$

2. Mass of publicly financed firms declines relative to the short-run, and it hence declines even more relative to the initial level, that is

$$|P^{LR}| < |P^{SR}| < |P|.$$

- Proof

$$|P^{SR}| - |P^{LR}| =$$
$$\int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} dG_{\pi}^{SR} dG_{\mu} + \left[G_{\pi}^{SR}(\Upsilon(\underline{\mu}^{SR}))\bar{G}_{\mu}(\underline{\mu}^{SR}) - G_{\pi}^{SR}(\Upsilon(\underline{\mu}^{LR}))\bar{G}_{\mu}(\underline{\mu}^{LR})\right]$$

Observe that

$$\int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \int_0^{\Upsilon(\mu)} dG_{\pi}^{SR} dG_{\mu} \geq \int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} G_{\pi}^{SR}(\Upsilon(\underline{\mu}^{LR})) dG_{\mu}$$
$$= G_{\pi}^{SR}(\Upsilon(\underline{\mu}^{LR}))\left(\bar{G}_{\mu}(\underline{\mu}^{LR}) - \bar{G}_{\mu}(\underline{\mu}^{SR})\right)$$

where the inequality follows from G_{π}^{SR} being non-decreasing and $\Upsilon(\mu) \geq \Upsilon(\underline{\mu}^{LR})$ for $\mu \geq \underline{\mu}^{LR}$, and the last equality follows from adding and subtracting $G_\pi^{SR}(\Upsilon(\underline{\mu}^{LR}))$ and using the definition $\bar{G}_\mu(\underline{\mu}^{LR}) = 1 - G_\mu(\underline{\mu}^{LR})$. Using this result, $|P^{SR}| - |P^{LR}| \geq \left[G_\pi^{SR}(\Upsilon(\underline{\mu}^{SR})) - G_\pi^{SR}(\Upsilon(\underline{\mu}^{LR}))\right]\bar{G}_\mu(\underline{\mu}^{SR}) > 0$ where the last inequality follows from $\underline{\mu}^{SR} > \underline{\mu}^{LR}$.

3. Public listing propensity decreases relative to the short-run and decreases even more in the long-run since

$$|S^{LR}| > |S^{SR}| = |S| \text{ and } |P^{LR}| < |P^{SR}| < |P|,$$

imply that

$$L = \frac{|P|}{|P|+|S|} = \frac{1}{1+\frac{|S|}{|P|}} > \frac{1}{1+\frac{|S|}{|P^{SR}|}} = \frac{1}{1+\frac{|S^{SR}|}{|P^{SR}|}} > \frac{1}{1+\frac{|S^{SR}|}{|P^{LR}|}} > \frac{1}{1+\frac{|S^{LR}|}{|P^{LR}|}} = L^{LR}.$$

4. Output increases relative to the short-run but is lower than the initial level, that is

$$O^{SR} < O^{LR} < O.$$

- Proof of $O^{LR} > O^{SR}$

$$O^{LR}(\underline{\mu}) = \int_0^{\underline{\mu}}\int_0^{\Upsilon(\mu)} \mu dG_\pi dG_\mu + \int_{\underline{\mu}}^{\infty}\int_0^{\infty} \mu dG_\pi dG_\mu = E[\mu] - \int_0^{\underline{\mu}} \mu G\bar{S}R_\pi(\Upsilon(\mu))dG_\mu.$$

Thus, $O^{LR} - O^{SR} = \int_{\underline{\mu}^{LR}}^{\underline{\mu}^{SR}} \mu \bar{G}_\pi^{SR}(\Upsilon(\mu))dG_\mu > 0$.

- Proof of $O^{LR} < O$

$$O - O^{LR} = \left[\int_0^{\underline{\mu}} \mu G_\pi(\Upsilon(\mu))dG_\mu - \int_0^{\underline{\mu}^{LR}} \mu G_\pi^{SR}(\Upsilon(\mu))dG_\mu\right] + \int_{\underline{\mu}}^{\underline{\mu}^{LR}} \mu dG_\mu$$

$$= \left[\int_0^{\underline{\mu}} \mu\big(G_\pi(\Upsilon(\mu)) - G_\pi^{SR}(\Upsilon(\mu))\big)dG_\mu\right] + \left[\int_{\underline{\mu}}^{\underline{\mu}^{LR}} \mu\big(1 - G_\pi^{SR}(\Upsilon(\mu))\big)dG_\mu\right] > 0$$

where the inequality follows from $\underline{\mu}^{LR} > \underline{\mu}$, $G_\pi^{SR} \leq 1$ and FOSD of $\tilde{\pi}$.

5. PE premium falls relative to the short-run, and the net long-run effect is ambiguous if Assumption 2 holds.

   - Proof of $\Pi^{LR} < \Pi^{SR}$

$$\Pi^{LR} = E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{LR}] - E[\nu|\nu \leq \bar{\nu}^{LR}]$$

$$= E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{LR}] - \underline{\pi}^{LR} + MAI_\nu(\bar{\nu}^{LR})$$

$$\leq E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - \underline{\pi}^{SR} + MAI_\nu(\bar{\nu}^{LR})$$

$$\leq E[\tilde{\pi}'(\nu)|\tilde{\pi}'(\nu) \geq \underline{\pi}^{SR}] - \underline{\pi}^{SR} + MAI_\nu(\bar{\nu}^{SR}) = \Pi^{SR}$$

   where the second equality uses long-run condition (A.29), the first inequality uses Lemma 2 and $\underline{\pi}^{LR} < \underline{\pi}^{SR}$, and fourth uses Lemma 3 and $\bar{\nu}^{SR} < \bar{\nu}^{LR}$.

   - Proof of $\Pi^{LR} - \Pi$ ambiguous

   By Leibniz rule,

$$\frac{\partial \Pi(\bar{\nu})}{\partial \epsilon} =$$

$$\frac{\partial E[\pi'(\nu) - \nu|\nu \leq \bar{\nu}]}{\partial \epsilon} =$$

$$\int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon} \frac{dG_\nu}{G_\nu(\bar{\nu})} + \int_0^{\bar{\nu}} (-1) G'_\nu(\bar{\nu}) \frac{\bar{\nu}}{\partial \epsilon} \pi'(\nu) \frac{dG_\nu}{G_\nu(\bar{\nu})^2} + \frac{\bar{\nu}}{\partial \epsilon} [\pi'(\bar{\nu}) - \bar{\nu}] \frac{dG_\nu(\bar{\nu})}{G_\nu(\bar{\nu})} =$$

$$\int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon} \frac{dG_\nu}{G_\nu(\bar{\nu})} + \int_0^{\bar{\nu}} (-1) G'_\nu(\bar{\nu}) \frac{\bar{\nu}}{\partial \epsilon} \pi'(\nu) \frac{dG_\nu}{G_\nu(\bar{\nu})^2} =$$

$$\int_0^{\bar{\nu}} \frac{\partial \pi'(\nu)}{\partial \epsilon} [1 - \pi'(\nu) \frac{G'_\nu(\bar{\nu})}{G_\nu(\bar{\nu})}] \frac{dG_\nu(\nu)}{G_\nu(\bar{\nu})}$$

   since $\pi'(0) \to \infty$ for $\bar{\nu} \to 0$ $\frac{\partial \Pi}{\partial \epsilon} < 0$, while for $\bar{\nu} \to \infty$, $\frac{\partial \Pi}{\partial \epsilon} > 0$ (given MAI_ν decreasing $\iff$ $\frac{G_\nu(\nu)}{G'_\nu(\nu))}$ increasing - see proof of Lemma 1 of Bagnoli and Bergstrom (2005)).

6. Public CEO pay has a net increase in the long-run, but the change relative to the short-run is ambiguous.

   - Proof of $\Omega^{LR} > \Omega$

   Observe that since $|P^{LR}| < |P|$ and $\underline{\mu}^{LR} > \underline{\mu}$, the same proof used in the short-run applies here.

A.3.5 Proof of Theorem 4 - (ii) PE deregulation

Define the PE deregulation transformation as $\tilde{\nu} = \frac{\nu}{\epsilon}$, with $\epsilon > 1$, so that each private financier's cost is lower, thereby implying that ν FOSD $\tilde{\nu}$ (i.e. $G_{\tilde{\nu}}(x) = G_{\nu}(\epsilon x) > G_{\nu}(x)$ for any x).

Short-run

As this transformation only affects private financiers' costs, allocations do not change in the short-run and the only impact is a direct increase in the PE premium Π, since $\Pi^{SR} = E[\pi | \pi \geq \underline{\pi}] - E[\tilde{\nu} | \tilde{\nu} \leq \bar{\nu}] > \Pi$.

Long-run

1. PE funds increase while the monitoring cost cutoff decreases, that is

$$\bar{\nu}^{LR} < \bar{\nu} < \epsilon \bar{\nu}^{LR}.$$

- Proof of $B^{LR} > B$

$$B \equiv G_{\nu}(\bar{\nu}) \cdot M = \bar{G}_{\pi}(\bar{\nu})$$

By contradiction, suppose that $B \geq B^{LR}$ then $B \equiv G_{\nu}(\bar{\nu}) \cdot M \geq B^{LR} \equiv G_{\tilde{\nu}}(\bar{\nu}^{LR}) \cdot M = G_{\nu}(\epsilon \bar{\nu}^{LR}) M$, where the last equality uses the definition of the transformation. By monotonicity of G_{ν} it follows that $\bar{\nu} \geq \epsilon \bar{\nu}^{LR}$. From (A.28) and (A.29)) we have

$$B^{LR} \equiv G_{\tilde{\nu}}(\bar{\nu}^{LR}) \cdot M = \bar{G}_{\pi}(\bar{\nu}^{LR})$$

so $B^{LR} = \bar{G}_{\pi}(\bar{\nu}^{LR}) \geq \bar{G}_{\pi}(\epsilon \bar{\nu}^{LR})$ given that $\bar{G}_{\pi}$ is decreasing. Using $\bar{\nu} \geq \epsilon \bar{\nu}^{LR}$, $\bar{G}_{\pi}(\epsilon \bar{\nu}^{LR}) > \bar{G}_{\pi}(\bar{\nu}) = B$ where the last equality follows from (A.28) and (A.29)) at $\bar{\nu}$, which is a contraddiction. Thus, $B^{LR} > B$.

- Proof of $\bar{\nu}^{LR} < \bar{\nu}$

By FOSD of ν and (A.28)

$$\bar{G}_{\pi'}(\pi'(\nu)) = G_{\nu}(\nu) \cdot M < G_{\tilde{\nu}}(\nu) \cdot M = \bar{G}_{\pi'}(\tilde{\pi}'(\nu =)$$

thus, by $\bar{G}$ decreasing, $\tilde{\pi}'(\nu) < \pi'(\nu)$. Substituting this into the equilibrium financier cutoff condition (A.29), we have immediately $\bar{\nu}^{LR} < \bar{\nu}$.

2. Mass of publicly financed firms and public listing propensity decline.

- Proof

  Since $B^{LR} > B$, we have immediately from (A.28) and (A.29) and the definitions of $\underline{\pi}$ and $\underline{\mu}$ that $\underline{\pi}^{LR} < \underline{\pi}$ and $\underline{\mu}^{LR} < \underline{\mu}$.

  Finally,

$$|P^{LR}| = \int_0^{\underline{\mu}^{LR}} \int_0^{\Upsilon(\mu)} dG_\pi dG_\mu + \int_{\underline{\mu}^{LR}}^{\infty} \int_0^{\Upsilon(\underline{\mu}^{LR})} dG_\pi dG_\mu$$

$$< \int_0^{\underline{\mu}} \int_0^{\Upsilon(\mu)} dG_\pi dG_\mu + \int_{\underline{\mu}^{LR}}^{\infty} \int_0^{\Upsilon(\underline{\mu})} dG_\pi dG_\mu = |P|.$$

  As $|S^{LR}| > |S|$ and $|P| > |P^{LR}|$ we have immediately that the listing propensity L drops.

3. Output increases.

- Proof

  Since $\underline{\mu} > \underline{\mu}^{LR}$, we have that s

$$O^{LR} = E[\mu] - \int_0^{\underline{\mu}^{LR}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi dG_\mu > E[\mu] - \int_0^{\underline{\mu}} \int_{\Upsilon(\mu)}^{\infty} \mu dG_\pi dG_\mu = O.$$

4. CEO pay is ambiguous.

   We will prove this for any degenerate distribution of dG_μ, noting that this is sufficient to establish ambiguity for more general dG_μ as well. We establish this by providing conditions resulting in the increase and decrease of Ω below.

- If MAI_π is decreasing[67], then CEO pay decreases in the long-run, that is

$$\Omega^{LR} < \Omega.$$

  - Proof

$$\Omega - \Omega^{LR} = E[\pi|\pi \le \underline{\pi}] - E[\pi|\pi \le \underline{\pi}^{LR}]$$

$$= MAI_\pi(\underline{\pi}^{LR}) - MAI_\pi(\underline{\pi}) + \underline{\pi} - \underline{\pi}^{LR} > 0$$

    since $\underline{\pi}^{LR} < \underline{\pi}$ and by assumption on MAI_π decreasing.

[67] No "named" distribution is known with this property.

- If $\underline{\pi}$ is large then CEO pay increases in the long-run, i.e.

$$\Omega^{LR} > \Omega,$$

while if $dG_{\pi}(\underline{\pi}) > 1$, then CEO pay decreases in the long-run, i.e.

$$\Omega^{LR} < \Omega.$$

  – Proof

    By Leibnitz rule, and integrating by parts, we have

$$\frac{\partial E[\pi|\pi \leq \underline{\pi}]}{\partial \underline{\pi}} = \frac{\underline{\pi} G_{\pi}(\underline{\pi})(g_{\pi}(\underline{\pi}) - 1) + \int_0^{\underline{\pi}} G_{\pi}(\pi) d\pi}{G_{\pi}(\underline{\pi})^2}.$$

    if $g_{\pi}(\underline{\pi}) > 1$, then, given that $\underline{\pi} > \underline{\pi}^{LR}$, the result follows immediately. If $\underline{\pi}$ is large, then, noting that $\int_0^{\underline{\pi}} G_{\pi}(\pi) d\pi < \underline{\pi} G_{\pi}(\underline{\pi})$ and given that $g_{\pi}(\underline{\pi}) \to 0$ is necessary for any well defined distribution, the result of $\Omega^{LR} > \Omega$ follows.

5. PE premium decreases if Assumption 2 holds.

   - Proof

     By definition

$$\Pi = MRL_{\pi}(\underline{\pi}) + MAI_{\nu}(\bar{\nu})$$

     and

$$\Pi^{LR} = MRL_{\pi}(\underline{\pi}^{LR}) + MAI_{\tilde{\nu}}(\bar{\nu}^{LR}).$$

     Given our first result that $\epsilon \bar{\nu}^{LR} > \bar{\nu} > \bar{\nu}^{LR}$, and using the transformation, we have $MAI_{\tilde{\nu}}(\bar{\nu}^{LR}) = MAI_{\nu}(\bar{\nu}^{LR}\epsilon)$, yielding the result.

A.3.6 Proof of Theorem 4 - (iii) Ideas harder to find

We implement ideas becoming harder to find through a first-order stochastic leftward shift in G_{μ}, so that the new distribution $G_{\mu}^{SR} > G_{\mu}$ (i.e. the expected μ is lower). Since $G_{\pi'}(\underline{\pi}) = G_{\mu}(\Upsilon^{-1}(\underline{\pi})) \cdot G_{\pi}(\underline{\pi}))$, this maps to the modified information premium distribution being FOSD after the transformation (so π' is lowered).

Using the equilibrium matching function, the post change financier-firm matching is given by

$$\bar{G}_{\tilde{\pi}'}(\tilde{\pi}'(\nu)) = G_{\nu}(\nu) \cdot M = \bar{G}_{\pi}(\pi'(\nu)).$$

Since $\bar{G}_{\tilde{\pi}}(\tilde{\pi}'(\nu)) = \bar{G}_{\pi}(\epsilon\tilde{\pi}'(\nu)) > \bar{G}_{\pi}(\pi'(\nu))$ we have that $\tilde{\pi}'(\nu) < \pi'(\nu)$. That is, each private financier is matched with a lower information premium firm than prior to the change. Evaluating at the marginal (potentially) private financier profit, we have $\tilde{\pi}'(\bar{\nu}) < \bar{\nu}$ and hence, since $\tilde{\pi}'$ is a decreasing function in ν, we have (by free-exit) that for any ν such that $\tilde{\pi}'(\nu) < \nu$, this financier will switch out from being a private financier. Let $\bar{\nu}^{SR}$ denote the maximal $\nu < \bar{\nu}$ such that the zero profit condition holds with equality, $\tilde{\pi}'(\bar{\nu}^{SR}) = \bar{\nu}^{SR}$, which given above, we know exists. But this satisfies the long-run GE equilibrium definition, so we have immediately that the short-run equals the long-run in this counterfactual. In light of this, $\underline{\pi}^{LR} < \underline{\pi}$, $\underline{\mu}^{LR} < \underline{\mu}$ and $B > B^{LR}$.

Short-run and Long-run

1. PE funds fall, that is

$$B > B^{LR}.$$

   - Proof

     Follows directly from above.

2. Output falls, that is

$$O > O^{LR}.$$

   - Proof

     $|P \cup S| = 1 - \int_0^{\underline{\pi}} \int_0^{\Upsilon^{-1}(\pi)} dG_{\mu} dG_{\pi}$ and

$$|P \cup S| - |P \cup S|^{SR} = -\int_{\underline{\pi}^{LR}}^{\underline{\pi}} [\bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi)) - \bar{G}_{\mu}(\Upsilon^{-1}(\pi))] dG_{\pi} > 0$$

     where the inequality follows from $\bar{G}_{\mu} > G_{\tilde{\mu}}$. Thus, the mass of financed firms falls.

     Observe that by definition aggregate output is $O = \int_{\theta:\theta \in P \cup S} \mu dG_{\mu} dG_{\pi}$. Since $P \cup S \subseteq (P^{SR} \cup S^{SR})$ we have

$$O = \int_{\theta:\theta \in P \cup S \setminus (P^{SR} \cup S^{SR})} \mu dG_{\mu} dG_{\pi} + \int_{\theta:\theta \in P^{SR} \cup S^{SR}} \mu dG_{\mu} dG_{\pi} > O^{SR}.$$

3. Mass of publicly financed firms and public listing propensity are ambiguous.

- Proof

$$|P| - |P^{LR}| = |P \cup S| - |P \cup S|^{SR}| - (|S| - |S^{LR}|) > 0$$
$$\iff \frac{\int_{\underline{\pi}^{LR}}^{\underline{\pi}} [\bar{G}_{\mu}(\Upsilon^{-1}(\pi)) - \bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi))] dG_{\pi}}{B - B^{LR}} > 1$$

where $B - B^{LR} = M \cdot (G_{\nu}(\bar{\nu}) - G_{\nu}(\bar{\nu}^{LR}))$. That is, public listings can increase if the substitution away from private towards public dominates the expansion of unfinanced firms. This depends on the relative curvature of G_{ν} vs $\bar{G}_{\mu}$ around the cutoffs. Since $|S|$ falls and $|P|$ may either (a) increase or (b) fall, the listing propensity will rise in the case of (a) and could rise or fall with (b) depending on the magnitudes.

4. Public CEO pay is ambiguous.

- Proof

Suppose $|P| < |P|^{LR}$, then

$$\Omega - \Omega^{LR} = \frac{1}{|P|} E[\pi | \theta \in P] - \frac{1}{|P^{LR}|} E[\pi | \theta \in P^{LR}]$$

$$\geq \frac{1}{|P^{LR}|} \left(\int_{\underline{\pi}^{LR}}^{\underline{\pi}} \pi [\bar{G}_{\mu}(\Upsilon^{-1}(\pi)) - \bar{G}_{\tilde{\mu}}(\Upsilon^{-1}(\pi))] dG_{\pi} \right) > 0$$

where the inequality follows from $\Upsilon(\cdot)$ monotonic increasing, $\bar{G}_{\mu} > \bar{G}_{\tilde{\mu}}$. On the other hand, if $|P| > |P^{LR}|$ then sign is ambigous

5. PE premium is ambiguous.

- Proof

Note that, under Assumption 2, MRL_{π} decreases since $\underline{\pi} > \underline{\pi}^{LR}$, and MAI_{ν} increases since $\bar{\nu} > \bar{\nu}^{LR}$.

A.3.7 Proof of Theorem 4 - (iv) Costly (unproductive) disclosure

We model an increase in unproductive disclosure cost as a fixed cost $\zeta > 0$ to being publicly listed, so $\tilde{J}^P = J^P - \zeta$. This results in a downward shift in the IR region for public financiers, $\{\Upsilon(\mu) \geq \pi + \zeta\}$ and an upward shift in the premium earned by private financier competing with the public, $\{\pi + \zeta : \Upsilon(\mu) \geq \pi + \zeta\}$. That is, the modified info premium indifference curve for the private financier is now $\tilde{\pi}' = \min\{\Upsilon(\mu) - \zeta, \pi\}$ (i.e. there is a rightward shift in the indifference curves of the private financiers). In the short-run, private financiers' funding remains fixed (as there is no free-entry in short-run binds), so it must be that $\underline{\mu}^{SR} > \underline{\mu}$ while $\underline{\pi}^{SR} < \underline{\pi}$. [68]

Short-run

1. Set of financed firms and output fall.

   - Proof

     By definition $|P \cup S|^{SR} = 1 - \int_0^{\underline{\mu}^{SR}} \int_{\Upsilon(\mu)-\zeta}^{\infty} dG_\pi dG_\mu$, thus,

     $$|P \cup S| - |P \cup S|^{SR} \geq \int_0^{\underline{\mu}} [\bar{G}_\pi(\Upsilon(\mu - \zeta)) - \bar{G}_\pi(\Upsilon(\mu))] dG_\mu > 0$$

     where the first inequality follows from $\underline{\mu}^{SR} > \underline{\mu}$ and the second from $\bar{G}$ decreasing and Υ increasing. By similar logic, $O = E[\mu] - \int_0^{\underline{\mu}} \mu \int_{\Upsilon(\mu)}^{\infty} dG_\pi dG_\mu$ so

     $$O - O^{SR} \geq \int_0^{\underline{\mu}^{SR}} \mu [\bar{G}_\pi(\Upsilon(\mu - \zeta)) - \bar{G}_\pi(\Upsilon(\mu))] dG_\mu > 0.$$

2. Publicly listed firms and listing propensity fall.

   - Proof

     $|P| = |P \cup S| - |S|$, and $|S|$ fixed in short-run, hence by above result follows. Since $|S|$ fixed and $|P|$ falls in short-run, listing propensity falls.

3. Public CEO pay is ambiguous.

   - Proof

     Similar arguments as ideas get harder to find.

4. PE premium rises.

   - Proof

     The mass of privately financed firms doesn't change (and set of private financiers), and hence, the sum of PE premiums is sufficient to characterize the difference. Consider the initial match-

[68] Suppose not, if $\underline{\mu}^{SR} < \underline{\mu}$, then the implied private financier set strictly contains the original, hence $|S^{SR}| > |S|$, which is a contradiction. By definition of $\underline{\pi}^{SR}$, $\underline{\pi}^{SR} = \Upsilon(\underline{\mu}^{SR}) - \zeta$, so $\underline{\pi} - \underline{\pi}^{SR} = \Upsilon(\underline{\mu}) - (\Upsilon(\underline{\mu}^{SR}) - \zeta) = \zeta + (\Upsilon(\underline{\mu}) - \Upsilon(\underline{\mu}^{SR})) > 0$, where the last inequality follows from $\underline{\mu} > \underline{\mu}^{SR}$.

ing, $m(\nu) = \{\theta : \pi'(\theta) = \min\{Y(\mu), \pi\}\}$, $\pi'(\nu)$. After the reform, their payoffs matched to the same firm is $\pi'_{SR}(\nu) - \nu = \min\{Y(\mu), \pi + \zeta\} - \nu > \min\{Y(\mu), \pi\} - \nu$. Since these matchings are feasible, the optimal matching generated in equilibrium (which given the submodularity is efficient) it must be that this total sum exceeds this.

Long-run

In the long-run, PE funds expand so that (A.29) holds with equality. Thus, using analogous arguments as for previous comparative statics, $B^{LR} > B$, $\bar{v}^{LR} > \bar{v}$, $\underline{\mu} < \underline{\mu}^{LR} < \underline{\mu}^{SR}$ and $\underline{\pi} > \underline{\pi}^{LR} > \underline{\pi}^{SR}$. Using similar arguments as above, we have the following results:

1. Public listings fall relative to short-run, but listing propensity change ambiguous;
2. Output rises relative to short-run, but still below initial;
3. Public CEO pay and average PE premium ambiguous.

Proof of Theorem 4 - (v) Productive disclosure

The effects of a productive disclosure are equivalent to a reduction in firm intangibility, and thus a reduction of information premia, with effects corresponding to reversed signs of those explained above.

B Appendix Empirical

B.1 Sample and Variable Construction

We download Capital IQ accounting and financial data about US firms from 1993 to 2016. We apply three filters as a firm-year observation must (1) be US incorporated, (2) have a positive and non-missing book asset value, and (3) not be a financial firm (SIC codes from 6000 to 6999), an utility (SIC codes 4900 to 4999), or a quasi-governamental firm (SIC codes from 9000). We drop firms with missing SIC code or central index key (CIK) for merging reasons.

To find which Compustat firm identifier, GVKEY, is associated with a given Capital IQ firm identifier in a given year, we use the linking tables provided on the Wharton Research Data Services website. To obtain information about firms' listing status we use the Compustat Snapshot data since it has historical (rather than the most recent) information. Furthermore, we assume that if a firm-year observation is not present in Compustat Snapshot, it is not publicly listed. To compute a firm age, we obtain information about foundation years from Capital IQ. We augment this data with the one provided by Jay Ritter on his website. We consider a firm's foundation year as the one provided by Jay Ritter if available, otherwise we consider the one provided by Capital IQ. If none of the two sources has this information, we consider the first year in which a firm appears in our sample as the end of its first year of life. Following Jay Ritter's convention, we cap the age of a firm at 80. We obtain information about LBOs and IPOs from Capital IQ. For the first type of events, we categorize a given firm-year observation as undergoing an LBO if either Capital IQ or Compustat Snapshot reports this type of episode and the listing status changes accordingly (otherwise, we consider the LBO as attempted but not completed). For the second type of events, we categorize a given firm-year observation as undergoing an IPO if either Capital IQ or Jay Ritter's data reports this type of episode. We analyze the Capital IQ text information about IPOs to be sure that IPOs were actually completed and not just initiated, and check that the listing status changes accordingly.

To compute the stock of intangible capital we follow Peters and Taylor (2017). We adapt their methodology to our sample assuming that firms start to accumulate intangible capital since their inception and their IPO does not affect the way in which intangible expenses contribute to a firm intangible capital accumulation. We use the depreciation rate of knowledge capital (the one stemming from R&D expenses) estimated by Li and Hall (2020), and we set those which are not reported to 15% as standard in the literature. Similarly, we set the depreciation of organization capital (the one stemming from SG&A expenses) to 20% following Falato et al. (2020).

We download the Capital IQ CEO data about US firms from 2001 to 2016. We restrict our attention to the set of firms for which we have also accounting and financial data. Given that Capital IQ has header information about which executive was the CEO of a given firm in a given year, we use Execucomp as well as Capital IQ data about corporate events to identify the CEO from the set of executives linked to a given firm in a given year following Gao et al. (2017). Where ambiguity remains in the identify of a CEO for a given firm-year observation, we take the highest paid executive in terms of total pay and exclude all the executives whose salary is either missing or non-positive.

We interpolate EBITDA, PPEGT and CAPEX using non-missing neighbour values. Given the structure of Capital IQ data, we consider 0 EBITDA and CAPEX and non-positive PPEGT as being missing. We drop observations with non-positive assets and total revenues. The tangible assets of a firm are given by the difference between its book asset value and the intangible assets on its balance sheet, that is goodwill and other intangibles on the balance sheet. The intangible assets of a firm are given by the sum of knowledge capital, organizational capital, goodwill and other intangibles on the balance sheet. Our proxy of τ is computed as the ratio of tangible assets over total assets. The profitability measure, y_t is given by the ratio of EBITDA over the denominator of τ. In the reduced form analysis, our proxy of σ_z^2 is the 3-year firm level standard deviation of profitability.

The total CEO compensation and individual parts (meaning, the salary, the fixed bonuses, other incentives, stocks and options) are built following the Execucomp manual using the Capital IQ data. The compensation shares are built so to have consistency between the different types of analysis. This means that the fixed share of CEO compensation is computed as the sum of salary and fixed bonuses divided by total compensation, the incentive shares as the sum of long-term incentive plan and non equity incentive payments and stocks and options divided by total compensation, and the equity share as the sum of stocks and options divided by total compensation.

We exclude observations that underwent an IPO or an LBO, listed on a minor stock exchange, with a ROA (computed as the ratio of EBITDA over book assets) less than -100% (since these firms would never be able to go public conditional on being private and they might be forced to delist otherwise). All nominal values are adjusted to 2016 US dollars. We annually winsorize scaled variables without clear upper or lower bounds at the 1% and 99% level.

Finally, to build the data for the structural estimation analysis, we use the panel built for the cross-sectional reduced form analysis, and we consider a firm as being public if it has appeared more often as such in our sample and vice versa if has appeared less (dropping firm with an equal number of appearances).

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