Crowdfunding vs. Venture Capital: Complements or Substitutes? A Theoretical Assessment*

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Abstract

Equity crowdfunding (CF) emerged as a new financing source for entrepreneurs and competes with early-stage financing professionals, e.g., venture capital (VC) investors. Entrepreneurs need to decide from who to raise capital and we develop a theory on this financing choice. We model two financing stages where both types of investors are in competition with each of them having competitive advantages. The respective advantages include transaction cost, return requirements, support quality, the efficiency of transaction monitoring and of abandonment decisions. Our model predicts the preferred choice for the entrepreneur contingent on these parameters, the resulting entrepreneurial effort and expected venture value. The model also suggests increased competition in the early-stage financial market and the necessity for VCs to focus on expertise, to specialize, and to move to later financing stages under certain circumstances.

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1 Introduction

The ability to raise capital and from who are important questions for entrepreneurs wishing to found and develop innovative ventures. Access to "traditional" bank financing is limited for innovative startups because they often have little or no collateral while their technology and business models can be too sophisticated and risky to qualify for debt financing. Furthermore, bankers usually neither have the required skills nor the capacity for value adding activities (Ueda, 2002). Venture capital (VC) has therefore developed to an important financing source for innovative projects. It is argued that it has unique features in providing support and creating venture value (Sahlman, 1990, and Kaplan and Stromberg, 2003).

However, crowdfunding (CF) has recently emerged as a web-based financing alternative to VC. The booming market and the number and diversity of CF platforms reveal that almost any project in any economic sector and region of the world can raise capital to a certain level. Policy makers have discovered CF advantages and aim to create attractive and reliable CF investment environments to spur their countries' innovation capacity, creation of startups, and economic growth (Cumming, 2013). The rationale of "the crowd" is either to donate, to pre-pay the develoment of a future product, or to create a financial return (Belleflamme et al., 2014). Pre-payments for a future product on reward-based CF platforms signal customers' demand (Roma et al., 2016) and thus may serve as a proof of concept to entrepeneurs and investors.

Equity CF typically provides early-stage capital and "the crowd" therefore resembles business angels or venture capitalists (VCs) and thus opens the discussion about its selection, monitoring and value adding talent. Surowiecki (2004) argues that the "wisdom of the crowd", i.e., the collective agregation of information, is efficient when selecting investment projects and Mollick and Nanda (2016) find empirical support for "crowd wisdom". However, their paper elaborates on theatre projects, which are simpler

in nature and rather short-term events. This limits the generalizability of their results. Cumming et al. (2021) detect 200 fraudulent CF campaigns and suggest that VCs have superior skills for fraud detection.

When it comes to investee monitoring and value adding, VCs often have specific skills, experience and talent that presumably cannot easily be replicated by "the crowd". Therefore, Hervé and Schwienbacher (2018) raise the question about competition between the two funding sources and entrepreneurs' preferred choices. Both types of investors, "the crowd" and VCs, have similarities but also respective competitive advantages. The literature has not yet elaborated on this competition and on the puzzle if the "crowd's wisdom" can compensate for VCs' experience. It is further interesting to hypothesize how the emergence of CF affects VCs, entrepreneurial wealth, and freedom.

Our paper addresses these questions and proposes a model where a peniless entrepreneur solicits equity financing from either "the crowd" or VCs. We parameterize both types of investors to have a continuum of skills, reflecting their ability and experience in guiding and monitoring startups. Other parameters include transaction cost which we assume to be lower for "the crowd" due to its web-based nature, and to lower due diligence, contracting, and monitoring cost. We calculate the equilibrium of an investee-investor relationship in the absence of equity CF in a first step. In the second, we introduce the CF alternative and discuss the change in the equilibrium outcome, thus allowing us to assess the impact of the emergence of CF on the VC market. Furthermore, entrepreneurs may now chose one of the two financing alternatives, or eventually combine them. We replicate this choice with a two-model setting. In the first one, VCs and "the crowd" compete for one particular financing round. In the second setting, we extend the model to two subsequent rounds and assume that either VCs or "the crowd" finances both, or in combination, where "the crowd" provides seed and the VC expansion capital.

Our model predicts that entrepreneurs weigh lower CF transaction cost against VCs' support quality. VCs' active monitoring yields more efficient decisions to abandon unsuccessful projects. Further, the model suggests that the appearance of CF creates an offer shock in the entrepreneurial finance market in which less talented VCs face a strong competition by "the crowd". This should yield specialization and encourage VCs to strengthen their expertise. The rising competitive pressure is expected to shift VC to later financing stages and to reduce the number of VC players.

Our paper contributes to existing theory on comparisons of entrepreneurial finance investors and also could be applied to different settings, such as business angels financing or ICOs. Business angels could be seen as early VC investors with limited funds. ICOs can also be compared to ECF, even though they are less secured (Andrieu and Sannajust, 2021). Ueda (2004) discusses bank financing vs. venture capital. In her model, the entrepreneur weighs potential higher valuations by VCs against the risk of being expropriated. Hellmann (2002) compares corporate and independant VC. Corporate VC is prefereable with strong synergies or if the young venture poses a threat to the incumbent. Hellmann and Thiele (2015) elaborate on the interrelationship between business angels and VCs and show how the latter can extract rent from the business angels. We also extend Andrieu and Groh (2012) which compares bank-affiliated and independent VC financing. Further our model has links to Chod and Lyandres (2021) who compare initial coin offerings with equity financing.

Recent statistics support the relevance of comparing funding through equity crowdfunding and venture capital. While the average deal size in equity crowdfunding tended te be smaller in comparison to venture capital, the situation has recently changed. In 2018, Seedrs (the main equity crowdfunding platform in UK) declared an average deal size of £728K. From 2018 to 2022, the total amount invested has been multiplied by 4[1].

[1] www.seedrs.com

Large funding rounds are possible on the equity crowdfunding market with millions of pounds (e.g., the recent fundraising of CapitalRise with £1.37 million[2]or the funding of the fintech firm Revolut with £3.9 million). According to Statistica[3], the average deal size for seed-stage in venture capital in Europe was $1.2 million.

The organization of the paper is as follows: We first review adjacent literature and present the model and its assumptions. In the subsequent section we derive the first-best equilibrium and the optimal contracts obtained with and without CF when there is only one investment stage. Next, we extend our analysis to a multiple-stage setting. Finally, we provide empirical predictions and conclude. All proofs are provided in the appendix.

2 Literature review

CF literature is mostly empirical and distinguishes the types of CF. We subsequently discuss the contributions which link VC or other sources of early stage capital and reward-based CF, historically the first type of CF, and then switch to equity CF.

2.1 Reward-based crowdfunding and venture capital or other sources of entrepreneurial finance

Several papers emphasize differences between VC and CF often referring to data from Kickstarter, well known, and one of the first reward-based crowdfunding platforms. Stevenson et al. (2019) focus on the U.S. and show that VCs tend to invest in high tech firms adjacent to wealthy areas such as the Silicon Valley or New York. Compared

[2]https://www.crowdfundinsider.com/2023/05/207727-capitalrise-easily-tops-1-million-target-on-seedrs/

[3]https://www.statista.com/statistics/878807/median-venture-capital-deal-size-in-europe-by-series/

to that, the crowd tends to invest in geographically more dispersed "main street" businessess, and in sectors rather ignored by VCs. This suggests that both financing sources complement. Sorenson et al. (2016) find that CF fund entrepreneurs "in a large number of places that have typically been excluded from VC". Yet, they also note that CF campaigns on Kickstarter are not really focused on the same sectors as VC, e.g., biotechnology versus artisitic projects. However, VCs invest larger amounts. Nevertheless, if "the crowd" backs ventures in "typical" VC sectors, then both funding sources complement each other because such CF investments tend to attract subsequent VC funding. D'Ambrosio and Gianfrate (2016) reveal substitution effects in early financing rounds and complementarity in later rounds. Kaminski et al. (2019) describe reward-based CF as experimental laboratory for investment opportunities in later stages. Thies et al. (2020) investigate the impact of reward-based CF on follow-up investments by VCs and show that CF lowers the likelihood for syndication in subsequent rounds. Colombo and Shafi (2021) shows that the probability to raise VC depends on success characteristics of reward-based CF campaigns. The authors argue that VCs appreciate the CF campaign as a useful quality signal when screening investments. Yu et al. (2017) find that a positive effect of CF on the availability of funding by business angels. Since all these referenced papers refer to reward-based CF, they do not exactly cover the scope of return driven VC investments. VC platforms ex ante select projects and, in particular, want to maintain their long-term stability by trying to detect scams. Cumming et al. (2021) study fraud in the CF sector. Focusing on Kickstarter, they identified 200 fraud cases that finally went through Kickstarter during the period 2010-2015. This tend to suggest that the VCs have specific skills that the crowd may difficultly compete with. Roma et al. (2016) propose a model to link reward-based CF and VC. In their model, entrepreneurs have to chose between pre-payments by "the crowd" or receiving seed money from a VC. The level of informativeness of the CF campaign is then exploited

by VCs in susbsequent rounds. Belleflamme, Lambert and Schwienbacher (2014)model the entrepreneurs' decision between reward-based and equity CF . Their model suggests that with lower financing requirements reward-based CF is preferable while at larger amounts, equity CF becomes the better option.

2.2 Equity crowdfunding and venture capital or other sources of entrepreneurial finance

Another strand of literature compares equity CF with VC or other early-stage financing sources. Walthoff-Borm et al. (2018) find from UK data that equity CF is often used as a "last resort" by ventures which are unable to raise bank financing. Blaseg et al. (2021) support the notion that rather lower quality ventures raise capital on equity CF platforms. Brown et al. (2018) argue that entrepreneurs launching equity CF campaigns are "discouraged borrowers" according to Kon and Storey (2003), unable to get funding from banks,VCs, or business angels. However, they also discuss that entrepreneurs prefer to keep control and autonomy which favors equity CF relative to VC or business angel financing. Löher (2017) focuses on campaign adverstising criteria on German equity CF platforms. He reveals that campaign success is driven by long-term credibility, similar to VC funding criteria. However, the platforms' business model is to attract and inform investors charging transaction fees. Therefore, they have a different incentive compared to VCs who only benefit from the realization of capital gains.

Hornuf et al. (2017) refer to German and UK equity CF data and reveal that successful campaigns also have a higher probability to subsequently raise VC. Buttice et al. (2020) and Dolatabadi et al. (2021) even detect that this VC funding probability can be higher than if young ventures were backed by business angels. However, this effect only emerges in cases where a "direct nominee structure" is chosen, i.e., the crowd

delegates its power to the platform.This is also confirmed in the US by Dolatabadi et al. (2021), but they also find that the effect is weaker in comparison with angel-backed firms.

Kantor (2014) argues that the crowd is unable to perform due diligence and to negotiate investor protection unlike VCs and supports the rationale that CF appears to be a last resort financing alternative for young ventures. Shafi (2021) emphasizes that the crowd has difficulties when evaluating information, which contradicts the "wisdom of the crowd" hypothesis and points to advantages of VCs. Related to that, Hornuf et al. (2019) find that "the crowd" does not revert to their investors' rights if ventures develop below budget and this proves that "the crowd" is a rather passive investor and entrepreneurs keep, in fact, control.

Staging of investments has been a prominent subject in the VC literature, e.g.,Sahlman (1990) or Kaplan and Stromberg (2002). However, only two papers focus on staging in CF. Signori and Vismara (2018) discusses subsequent CF campaigns and follow-up rounds and coinvestments by private equity investors. Correia et al. (2021) find that staged financing is rather infrequent in equity CF.

3 The model

We begin with a stylized model and consider only one financing round assuming the absence of equity CF but presence of VCs. Hence, we replicate the status before CF existed. Subsequently, we introduce the emergence of CF and discuss staged financing.

An entrepreneur is endowed with an innovative and unique project in a risk-neutral world without discounting. She is penniless and required to solicit outside financing. The project needs to be backed initially with investment I. If financed, it may generate

R or 0 at harvesting. R is normalized such that $R \in [0,1]$[4] and we assume that the entrepreneur's effort is correlated with the probability to reach the harvesting state.

The entrepreneur selects a VC contingent on perceived support quality and fit with the project. Both aspects are modelled by the investor's ability parameter α This ability affects the probability turn the venture successful and determines the investor's capability to value the project (as will be explained later). The probability to reach a goodstate of nature is thus a combination of both, entrepreneurial effort and VC-provided support: $e(1+\alpha) \in [0,1]$. Conversely, with probability $(1 - e(1+\alpha))$, the project leaves 0 value.

The entrepreneur is irreplaceable and needs to spend effort to turn the project successful with $\frac{e^2}{2}$ as a function of her cost of effort. We assume that the contract is negotiated *ex ante*.

3.1 First-best equilibrium

In the first-best equilibrium, the entrepreneur sets her effort in order to maximize the value for all participants:

$$\max_e e(1+\alpha)R - I - \frac{e^2}{2}.$$

The entrepreneurial effort would thus be: $e^* = R(1+\alpha)$.

3.2 Optimal contract in the absence of crowdfunding

With CF being absent, the entrepreneur may only get funding from a VC. The VC is partially competitive, i.e., its competitiveness is unperfect due to a reserve utility. This utility is contingent on ability $\alpha \in [0,1]$ and defined by $r_H * \alpha$. The rationale is

[4]This assumpation, purely formal, is taken to derive results later.

that ability gives the investor negotiation power. In addition, r_H describes the state of competition, including reputation, and alternative deal opportunities.[5]

The VC receives a cash flow right δ against the capital contribution. The remainder $(1-\delta)$ is held by the entrepreneur. The fractions are determined endogeneously in the contract negotiations and the final revenues are obtained at the end of the game.

We assume that both the VC and the entrepreneur are equally and fully able to assess the final project value R. The VC's participation constraint is therefore:

$$e(1+\alpha)\delta R - I \geq r_H * \alpha \; (PC).$$

It means that the VC's rationale is to obtain a share of an expected value minus the invested capital equal or higher to its reserve utility, and depending on ability and market conditions. From the entrepreneur's viewpoint, this reserve value resembles a fee she has to pay to be able to launch her project.

The entrepreneur having imperfect negotiation power will set the contract parameters maximizing her benefit. Her incentive constraint is therefore

$$\underset{e}{max}\; e(1+\alpha)(1-\delta)R - \frac{e^2}{2} \; (IC),$$

and the maximisation program is the following:

$$\underset{e}{max}\; e(1+\alpha)(1-\delta)R - \frac{e^2}{2},$$

$$s.t.\; e \in argmax\; e(1+\alpha)(1-\delta)R - \frac{e^2}{2},$$

[5]The reserve utility represents the cost of VC financing for the entrepreneur.

$$e(1+\alpha)\delta R - I \geq r_H * \alpha.$$

To maximize the incentive constraint, the VC's cash flow claim

$$\delta = \frac{r_H \alpha + I}{e(1+\alpha)R}$$

needs to be as small as possible.

Hence, the entrepreneur's incentive constraint becomes

$$\underset{e}{max}\ e(1+\alpha)\left[1 - \frac{r_H \alpha + I}{e(1+\alpha)R}\right] R - \frac{e^2}{2}.$$

She will set her effort to maximize the incentive constraint such that:

$$e = (1+\alpha)R.$$

This means that her effort corresponds to first-best effort under the above introduced conditions.

3.3 Optimal contract in the presence of crowdfunding

In a second step, we expand the model and include the emergence of equity CF. It may be considered an exogenous shock in the entrepreneurial finance landscape where entrepreneurs may now select between VC or CF.

3.3.1 Optimal contract with the venture capitalist

The VC keeps above developed characteristics with the exception of the reserve utility. We assume that CF has a negative impact, on average, with respect to competition among VCs. The reserve utility therefore becomes $r_L * \alpha$ with $r_H > r_L \geq 0$.

The optimal contract is equivalent to the development in section 3.2, but with r_H being replaced by r_L.

3.3.2 Optimal contract with the crowd

Following Hornuf et al. (2019), we assume that "the crowd" is rather passive and has no ability to support the project. Thus, we consider that with CF, the probability of reaching a good state only depends on the entrepreneurial effort. In addition, "the crowd" has no negotiation power and is therefore perfectly competitive. Yet, there are campaign and portal costs. Following CF platform standards, we assume these costs to be a percentage $k < 1$ of the funds raised $k * I$.

Consequently, the entrepreneur needs to raise $\frac{I}{1-k}$ to start the project and the cost are born by "the crowd". For simplicity and modelling convenience, we assume that the entrepreneur anticipates CF campaign success without error, so that we do not consider campaign failures.

Accordingly, the entrepreneur sets her level of effort such that:

$$\underset{e}{max}\, e(1-\delta_{CF})R - \frac{e^2}{2}\ (IC).$$

Assuming that "the crowd" were able to perfectly estimate its expected utility, its participation constraint is:

$$e\delta_{CF}R - \frac{I}{1-k} \geq 0\ (PC).$$

We require that, unlike the VC, lacking a due diligence ability, "the crowd" is not able to perfectly predict the final project value but only $\widetilde{R}_{CF}$ with an estimation error corresponding with the aggregated value of each crowd individual's etimate. However, "the wiser the crowd", the smaller the estimate's standard deviation, getting closer to

the real value R.

The crowd's participation constraint is then:

$$e\delta_{CF}\widetilde{R}_{CF} - \frac{I}{1-k} \geq 0 \ (PC).$$

The maximisation program is:

$$\underset{e}{max}\ e(1-\delta_{CF})R - \frac{e^2}{2},$$

$$s.t.\ e \in argmax\ e(1-\delta_{CF})R - \frac{e^2}{2},$$

$$e\delta_{CF}\widetilde{R}_{CF} - \frac{I}{1-k} \geq 0.$$

We assume that the crowd is competitive because the entrepreneurial project is unique. The entrepreneur will therefore set δ as low as possible in order to maximize her utility. Therefore, we have:

$$\delta_{CF} = \frac{I}{e(1-k)\widetilde{R}_{CF}}.$$

The entrepreneur's incentive constraint may be transformed to:

$$\underset{e}{max}\ e(1 - \frac{I}{e(1-k)\widetilde{R}_{CF}})R - \frac{e^2}{2}\ (IC).$$

As a result, we achieve $e = R$. In this setting, the crowd is close to a VC without support ability ($\alpha = 0$) and consequently, without negotiation power ($r_L * \alpha = 0$). However, as introduced above, "the crowd" does not have the ability to perfectly value the project. Thus, it may back lemons. In our model, a lemon is defined as a project

with negative expected value, i.e., where $eR - \frac{I}{1-k} - \frac{e^2}{2} < 0$.

3.4 Optimal choice of investor at the social and entrepreneurial level and discussion

Proposition 1. *At first-best equilibirum, VC funding must be chosen. ECF creates an inefficiency compared to first-best equilibirum* $R^2\left[(1+\alpha)^2 - 1\right] + I\frac{k}{1-k}$ *at the social level.*

It is socially optimum to choose a VC firm to finance the project. First, the VC firm has a positive effect due to its support skilss: it improves the probability of success, thus the pribability to obtain a high revenue R. The VC firm's costs can also be seen as income: they are simply transfert of incomes from the entrepreneur to the VC firms, whereas in the context of ECF they are paid to the platform by the crowdinvestors.

Proposition 2. *If ECF is chosen, the crowd looses money if* $R < \widetilde{R}_{CF}$*. At the social level, the NPV is negative if:* $R < \sqrt{\frac{2I}{1-k}}$.

As we assumed that the crowd estimates the final revenue as $\widetilde{R}_{CF}$, it calculates its payoff accordingly in its participation constraint. If the final revenue is overestimated, it results in the crowd loosing money. At the social level, the NPV might be negative, but it would require that the revenue is very low in comparison to the amount invested. This could only happen in massive overestimation of the final revenue by the crowd. One may link this to very opaque investments such as Initial Coin Offerings.

When looking at the optimal choice of investor by the entrepreneur, she decides by comparing her payoffs in the two settings. We first discuss her choice's determinants by assuming that the crowd perfectly evaluate the final revenue (i.e., $R = \widetilde{R}_{CF}$).

At equilibrium, the NPV with the VC firm would be $\frac{1}{2}(1+\alpha)^2R^2 - (r_L\alpha + I)$; with the crowd it would be $\frac{1}{2}R^2 - \frac{I}{1-k}$. The VC is a better option iff:

$$\frac{1}{2}\left[(1+\alpha)^2-1\right]R^2 > (r_L\alpha + \frac{kI}{1-k})$$

We define $\Delta_{efficiency} = \frac{1}{2}\left[(1+\alpha)^2-1\right]R^2$, which measures the comparative efficiency with the VC firm ; and $\Delta_{cost} = (r_L\alpha - \frac{kI}{1-k})$ the comparative costs of the VC firm. Thus, ECF is a better options if there is a higher Δ_{cost} that overcome the positive support gain with the VC firm $\Delta_{efficiency}$.

Let now assume that the crowd overestimates the final revenue: $R < \widetilde{R}_{CF}$. The following proposition details the criteria for entrepreneur to select between equity CF and VC.

Proposition 3. *The entrepreneur prefers VC if the following condition holds:*

$$\Delta_{efficiency} > r_L\alpha + I\left[1 - \frac{R}{(1-k)\widetilde{R}_{CF}}\right].$$

The proposition reveals that a VC's talent in valuing and supporting the project has two opposite effects. First, it strengthens the VC's attractiveness by its contribution to the project success. This effect appears in the member $\Delta_{efficiency}$, with a positive effect by the VC skill α. However, αraises the VC's reserve utility, that in combination with good market conditions r_L, makes him a less attractive option by raising its cost ; on the opposite side, campaign cost k with ECF must be low. The proposition also reveals the impact of the overestimation of the final revenue by the crowd, i.e. $\frac{R}{\widetilde{R}}$. Optimistic valuations by the crowd makes CF more attractive. In contrast, the better the "wisdom of the crowd", which is determined by a low error level $\frac{R}{\widetilde{R}_{CF}}$ close to 1, reinforces VC as a financing option. .

Crowdfunding appears then as a competitor to CF as it may offer high valuation and low costs. However, it does not add the value to project in comparison to VC-backing. There are thus several consequences of the CF emergence. First, it enrich entrepreneur's

choice set, at a potential lower cost and then make possible more financing of projects. However, some lemons may be financed if valuation skills of the crowd are lower.

We have assumed that the emergence of the ECF has impacted the VC market, resulting in the decrease from r_H to r_L to the market conditions of VC firms. If r_L is very close to r_H, it would mean that the global deal flow is sufficiently robust for the VC market to absorb the threat of the ECF competition. It would mean that VC firms maintain their profits and ECF is here to finance entrepreneurs with lower quality projects, or good projects demanding less support and/or at lower cost. If we now assume that r_L is differently lower than r_H, ECF has had an impact on the VC market and thus on the reserve utility of the VC firms. The impact may particularly hit the less talented VC which may reduce their deal flow, and possibibly go out of the market.

4 Equilibrium with investment staging

Now, we introduce investment staging in our model and require seed and expansion investments I_0 and I_1 at $t = 0, 1$. The total contributed capital is thus, $I_0 + I_1 = I$. Investors receive the cash flow claims δ in exchange for their contributions. The contract is signed at $t = 0$ and harvesting is reached at $t = 2$. The project may generate R or 0, as above. At the interim stage, $t = 1$, the seed investor may decide to abandon the project. However, we assume that liquidation proceeds are 0 and that the entrepreneur has no incentive to abandon the project because she has private benefits $w > 0$ from running it[6]. It implies that, if the entrepreneur holds control rights, she will never liquidate the project and persists mainting the firm to earn w even in bad states of nature providing expected value 0.

[6]Coppens and Knockaert (2022) examine the persistence of entrepreneurs in maintaining distress ventures.

The entrepreneur has different investees choices at the two stages. We study different sequences of investment in the two stages;

Seed stage	Development stage
VC	VC
ECF	ECF
VC	ECF
ECF	VC

At initial stage, the entrepreneur selects a VC or the crowd. At expansion stage, a private signal about the venture quality may be observed either the VC, but not by the crowd and the venture needs to raise expansion capital either from a VC or the crowd. The entrepeur's level of effort determines the probability to reach a good quality state at interim. In this case, the final revenue at $t = 2$ will be R and otherwise 0.

The timeline of the game is the following:

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Insert Figure 1 here

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The entrepreneur selects ex ante her wealth-maximising combination, contingent on the exogeneous characteristics and the contract's optimal parameters.

4.1 First-best equilibrium

As above, in the first-best equilibrium, the entrepreneur sets her effort according to value maximizing solution. In this case, projects should be abandoned at interim period if the venture quality is bad and if the venture is backed by a VC which reduces the cost

of entrepreneurial effort, unlike with CF. The social value of the project is thus

$$e(1+\alpha)(R-I_1) - I_0 - \frac{e^2}{2}$$

and the first-best effort is $e^* = (1+\alpha)(R-I_1)$.

4.2 Exclusive VC financing

If the venture receives VC in the seed round it will be liquidated in bad states of nature. One of our assumption is that the VC abandons unsuccessful ventures, thus avoiding the expansion round and future losses. Therefore, the prospects need to be positive at interim stage.

Hence, the model setting is exactly the same as in a world without CF, with the exception of parameter r_L which replaces r_H. We concentrate on the latter case for simplicity.[7]

The investor's participation constraint is:

$$e(1+\alpha)(\delta R - I_1) - I_0 \geq r_L \alpha \ (PC).$$

The entrepreneur's incentive constraint is then:

$$\max_e \ e(1+\alpha)(1-\delta)R - \frac{e^2}{2} \ (IC).$$

The maximization programme becomes:

$$\max_e \ e(1+\alpha)(1-\delta)R - \frac{e^2}{2},$$

[7]For simplicity, we assume that one VC makes both investments. It would be equivalent to have two VCs (one seed-stage investor, one later-stage investor) doing both investments and the second investor having the same information set as the first investor.

$$s.t.\ e \in argmax\ e(1+\alpha)(1-\delta)R - \frac{e^2}{2},$$

$$e(1+\alpha)(\delta R - I_1) - I_0 \geq r_L \alpha.$$

We assume that the harvesting value increases with effort and that there is no constraint of too much effort. To maximize her profits, the entrepreneur will thus require the investor's cash flow claim δ to be as low as possible. Accordingly, we get

$$\delta^* = \frac{r_L\alpha + I_0 + e(1+\alpha)I_1}{e(1+\alpha)R}.$$

We replace these values in the incentive constraint and obtain

$$\max_e\ e(1+\alpha)R - (r_L\alpha + I_0 + e(1+\alpha)I_1) - \frac{e^2}{2}.$$

The level of effort maximizing the entrepreneur's profits will then be

$$e^* = (1+\alpha)(R - I_1),$$

and we get:

$$\delta^* = \frac{r_L\alpha + I_0 + (1+\alpha)^2(R-I_1)I_1}{(1+\alpha)^2(R-I_1)R}.$$

4.3 Exclusive CF financing

If the venture receives seed capital from the crowd and envisages to proceed with CF financing, then the entrepreneur remains in full control because of the crowd's passiveness (Hornuf et al., 2019). This also allows expansion financing in bad states of nature due to less efficient monitoring by the crowd and window-dressing. As in Kantor (2014),

we assume that the crowd is less able to make due diligence, thus is unable to have a clear signal of the final revenue.

The entrepreneur always prefers to continue the project because of private benefits $w > 0$, whereas she should obtain 0 if she decided to liquidate. Therefore, the equilibrium replicates the one of section 3.3.2, with $I = I_0 + I_1$. One should note that, in this case, campaign costs are paid twice, i.e. $k(I_0 + I_1) = kI$.

4.4 Seed money from the VC and expansion capital from ECF

We show here that this sequence is not possible. Assume the VC finances the project at the seed stage. After, the VC observes a signal on the project's quality at interim period. If the project's quality is good, the VC investor will prefer to reinvest into the project if it has the funds to benefit from the upside potential of the firm. [8] Finally, only lemons would get to ECF. The crowd would infer that the expansion projects are only lemons and would refuse to finance them. Consequently, the sequence VC then ECF is not possible.

One may note that, in our model, the sequence is impossible as we have taken a standard assumption that investors have no fund restrictions. If we consider that the VC investor has limited funds and if it is perfectly observable by the crowd, then the sequence VC in the seed stage and crowdfunding in the second stage is possible. However, it may adapt to very specific contexts, such as a financing by a business angel or a "small" VC firm. However, in such a setting, the crowd may also compete with other VCs at the second stage. We would ultimately be in the same situation when the crowd and the VC compete in the seed-stage in which the crowd may overvaluate the project but offer lower cost.In such a setting, ECF will complement the business angel

[8] We could also infer another VC firm would be prefered (if we had assumed he could not refinance it) as CF implies campaign costs.

or small VC financing. In practice, VC tend to have large ressources to finance series B stage: according to Statistica[9], the average deal size for series B in Europe for venture capital is $30 million.

4.5 Seed money from the crowd and expansion capital by a VC

If the venture received seed capital from the crowd and solicits expansion capital from a VC, then there was no private signal about the venture's quality. We denote with δ_{CF} the crowd's cash flow claim and with δ_{VC} that of the VC. Their respective estimates of the harvesting value are $\widetilde{R}_{CF}$ and R. One should not that, here, the positive effect of VC is no longer possible as VC comes in at interim stage.

Since the entrepreneur is in control at interim, the project is not abandond in a bad state. Hence, the crowd's participation constraint is

$$e\delta_{CF}\widetilde{R}_{CF} - \frac{I_0}{1-k} \geq 0.$$

At interim, the expansion VC's participation constraint is

$$e\delta_{VC}R - I_1 \geq r_L\alpha \; (PC).$$

The entrepreneur sets her level of effort such that

$$\max_e \; e(1-\delta_{CF}-\delta_{VC})R - \frac{e^2}{2} \; (IC).$$

Maximize her utility, the entrepreneur requires δ_{CF} and δ_{VC} to be as low as possible.

Thus, we get

[9]https://www.statista.com/statistics/878807/median-venture-capital-deal-size-in-europe-by-series/

$$\delta_{CF} = \frac{I_0}{(1-k)e\widetilde{R}_{CF}},$$

and

$$\delta_{VC} = \frac{r_L\alpha + I_1}{eR}.$$

We replace δ_{CF} and δ_{VC} by their respective values in the incentive constraint and obtain

$$\max_e e(1 - \frac{I_0}{(1-k)e\widetilde{R}_{CF}} - \frac{r_L\alpha + I_1}{eR})R - \frac{e^2}{2}.$$

The optimal level of effort is then

$$e = R,$$

and we get

$$\delta_{CF} = \frac{I_0}{(1-k)R * \widetilde{R}_{CF}},$$

and

$$\delta_{VC} = \frac{r_L\alpha + I_1}{R^2}.$$

4.6 The entrepreneur's decision problem

The entrepreneur needs to make a choice among the financing alternative. She decides the one that gives her the highest value.

Proposition 4. *Exclusive CF financing is preferred over CF seed and VC expansion*

capital if the following condition holds:

$$(r_L\alpha + I_1) > \frac{I_1 R}{(1-k)\widetilde{R}_{CF}}$$

With seed capital injected by the crowd, the entrepreneur does not receive VC support in the early stage and bad projects do not get liquidated. Both lowers the project NPV. However, combining ECF then VC could remain a better choice than choosing exclusive CF financing. The costs of both financing particularly matters. When VC comes into the project, it has its costs. They depend on the competition parameter $r_L\alpha$. If crowdfunding increases not enough the competition in the VC market when it emerges, maintaining a high $r_L\alpha$, the combination of CF and VC becomes less relevant and CF becomes competitive. Low campaign costs in the second rounds makes exclusive CF more attractive. In addition, exclusive CF is favored if the crowd's estimated harvesting value is high, in comparison with the real value. If the crowd is able to be close or higher to the real value, CF is attractive to the VC. However, it also enable the financing and refinancing of lemons. VC may be a better choice if the crowd valuation is low. Optimistic crowd and costly VC firms may, on the opposite, create financing opportunities for second rounds for the ECF industry.

Proposition 5. *Exclusive CF financing is preferred to exclusive VC financing if the following condition holds:*

$$\frac{R^2 - (1+\alpha)^2(R-I_1)^2}{2} - \frac{R(I_0+I_1)}{(1-k)\widetilde{R}_{CF}} + (r_L\alpha + I_0) > 0.$$

Backed exclusively by the crowd, the entrepreneur retains a higher cash flow right but the venture does not benefit from the support offered by the VC. Here, once again, the valuation made by the crowd $\widetilde{R}_{CF}$ is a strong determinant. If it is very optimistic, it reinforces CF as a valuable option from entrepreneurial viewpoint ; notably if the

harvesting revenue estimate of the crowd is high enough compared to the two required capital contributions and campaign costs in both rounds. The support quality α has two opposite effects. It reduces the cost of entrepreneurial effort. But it also reinforces the reserve utility of the VC firm, which in combination with a low level of competition r_L may makes CF a more valuable option, taking into account campaign costs k.

Proposition 6. *Seed CF and expansion VC financing is preferred to exclusive VC if the following condition holds:*

$$\frac{R^2}{2} - \frac{I_0 R(1+k)}{\widetilde{R}_{CF}} + (I_0 - I_1) > \frac{(R-I_1)^2}{2(1-\alpha)}$$

$$\frac{R^2}{2} - \frac{I_0 R}{(1-k)\widetilde{R}_{CF}} + (I_0 - I_1) > \frac{(1+\alpha)^2(R-I_1)^2}{2}.$$

A combination of the two financing sources is advantageous to exclusive VC backing if the crowd values the project high in comparison with its real value and the seed capital. Campaign costs should be low. In contrast seed stage VC financing offers support but has some costs. If the support quality is high in comparison with the difference $R - I_1$, exclusive VC is attractive. Indeed, the costs associated with VC (i.e., $r_L\alpha$) are paid under both financings and do not matter in the decision process. One important difference is the continuation decision made. With mixed financing, bad projects are continued and this misallocates resources.

4.7 Discussion on the impact of the crowdfunding emergence

First, our results are in line with the findings of Dolatabadi et al. (2021) in the US. They found that ECF investors tend to invest in less sophisticated sectors, perform less than their VC or angel-backed counterparts. However we do not predict that the ECF is only a market for lemons, which is also in line with their paper and Ibrahim

(2015). Our model provides interesting insight in term of investment sequence. Hence, the following venture capitalizations are possible:

Seed stage	Development stage
VC	VC
ECF	VC
ECF	ECF

Either VCs or the crowd are able to provide both, seed and expansion capital. Exclusive VC or ECF financing can be the best options for entrepreneurs, who weight up the costs of each financing, the positive impact of VC to support the project and the valuation offered by the crowd which may exceed the real venture value. Interestingly, VC may come after ECF. Even though the positive impact of VC in supporting the project at early stage is limited, VC remains an attractive option is its costs are reasonnable or its better capacity to value the project makes the difference. However, it is unlikely, that VCs engage in CF campaigns for follow-on financing. VC prefer to refinance good projects to benefit from their inner upside potentiel, and may not ge rid of lemons. However, ECF may finance later-rounds if seed stage investors have limited funds, such as business angels or small VC firms.

This predicted sequence of investments follows the analysis by Signori and Visamara (2018). They study the potential exits for firms financed by equity crowdfunding according to their data in the UK market. First, they link failures to "the crowdfunding community systematically underinvesting in due diligence". This can be linked to the continuation of lemons in the model at interim, as well as a lower capacity to value projects' revenue which makes the funding of negative NPV projects possible. Private SEOs, as they define by refinancing through VC or BA investors happen when there is a first round by ECF and a second round by VC. Aquisition of the crowdfunded firm is also another exit, which corresponds to the successful case in our model.

Our theory is founded on the idea that the emergence of CF increases competition in the entrepreneurial finance landscape. It reduces VCs competitive advantage, unless they are protected by strong expertise. This expertise is required to create sufficient value while backing entrepreneurs. It suggests that less talented VCs exit the market or shift towards later-stage investments where the competition by CF is less severe. CF becomes a stronger threat to VCs if the crowd increases its capability to detect lemons. CF is preferential to entrepreneurs who require less backing. However, CF campaign costs are relevant, and competition among CF platforms needs to maintain the attractiveness of this industry.

The impact of crowdfunding from VC firms' viewpoint has been defined by the shift in the parameter r_H to r_L. We defined a continuum of VC investors of different quality of support α. Investments are done if the VC breakevens from his reserve utility. The impact of the emergence of CF is a threat for the VC, especially if they have lower support capacities. We can predict from our model's results that CF has reduced investment possibilities for VCs. VCs may have to move to lower stages to face competition, or even to disappear if their value-added as investors is not high enough. We should also observe that CF has made VC investors reduce their shares to maintain their competitiveness on the market. Finally, our theory also show that ECF platforms should continue to develop follow-up financing for previous ECF-backed startups.

5 Conclusion

Empirical literature has pointed the drawbacks and advantages of CF and VC but no theory exists yet to explain how entrepreneurs select among the two types of financing. We drive such a theory on contribute to the selection decision of an entrepreneur at the seed and expansion stage of an innovative project. Entrepreneurs can refer to either CF

or VC exclusively or have the choice to combine the two with the crowd providing seed and a VC injecting expansion capital. Entrepreneurs need to weigh campaign cost as well as lower profit requirements of the crowd against the support of VCs. In addition, VCs make efficient abandonment decision and thus improve resource allocation which benefits the relationship. A passive crowd cannot detect lemons and thus creates model frictions.

The model also predicts that the emergence of CF has created a shock for the VC industry. It has increased competition, and thus reduced VCs' deal flow, and their profits to the benefit of entrepreneurs. The model suggests that CF forces VCs to strengthen their own expertise and to specialize. CF may have reduced the number of VC actors, or makes them shift towards later financing stages. Our analysis could be extended also to other settings, such as the ICO markets. ICOs are more sophisticated, less regulated crowdfunding operations. In such a context, platform fees may be almost inexistant, but we should expect more severe overvaluations by the crowd.

Our analysis also has some regulatory implications. Rather than the "wisdom of the crowd", we pointed the problem of the "indifference of the crowd". To develop multiple rounds of financing by platforms, it must be important to find ways to create more implication from the crowd as well as better disclosure process by ventures on the project's advancement after the initial campaign. We also predict that multiple rounds of equity crowdfunding are possible and should be developed by platforms. It is linked to the growth of funds invested on the ECF market, such as the fast development of the seedrs platform in UK.

Our work has some limitations. First, we do not endogeneize the impact of increased competition directly but rather focus on its consequences. Second, we only focus on ECF whereas reward crowdfunding is also widely used to finance innovative projects. Future development could also come from the presence of "sophisticated investors" (Sig-

nori and Vismara, 2018) that have a certification role.

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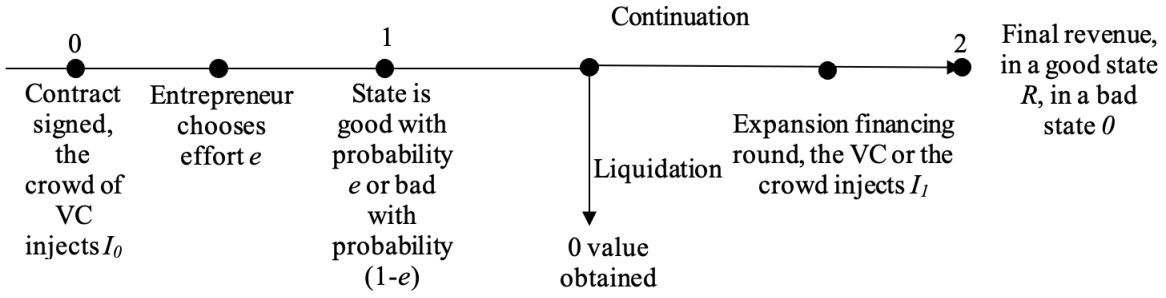
Figure

Figure 1



Exogeneous Model Parameters

Revenue in good states	R
Investment required at seed-stage	I_0
Investment required at development stage	I_1

List of contract's parameters:

Shares held by investors	δ

Others

Entrepreneur's effort	e
VC's level of competence	α
Revenue observed by the crowd at t=0	$\widetilde{R}_{CF}$
Cost of CF campaign (fixed), as a fraction of funds raised	k

Proofs

Proof of proposition 1

With VC financing, to maximize the social output , the level of effort should be such that:

$$\max_e e(1+\alpha)R - I - \frac{e^2}{2}.$$

We then have: $e^* = R(1+\alpha)$.

With ECF, ito maximize the social output , the level of effort should be such that::

$$\max_e R^2 - \frac{I}{1-k} - \frac{e^2}{2}.$$

We then have in this setting $e^* = R$.

The social output with VC financing is higher than with ECF iff:

$$(1+\alpha)^2 R^2 - I > R^2 - \frac{I}{1-k}$$

.

As $0 \leq k \leq 1$, the inequality is always verified.

If we compare the two optima, we may note that ECF creates an inefficiency by reducing the social output. The loss is then:

$$R^2\left[(1+\alpha)^2 - 1\right] + I\frac{k}{1-k}.$$

Proof of proposition 2

The final crowd's payoff is $e\delta R - \frac{I}{1-k}$. If we repace the contract's parameters by its optimal values, we find that it is negative iff:

$$\frac{IR}{(1-k)\widetilde{R}_{CF}} - \frac{I}{1-k} < 0$$

$$\iff \frac{R}{(1-k)\widetilde{R}_{CF}} - \frac{1}{1-k} < 0$$

$$\iff R <) \widetilde{R}$$

At the social level, the NPV is negative iff:

$$eR - \frac{I}{1-k} - \frac{e^2}{2} < 0$$

As $e = R$, it is equivalent to:

$$\frac{R^2}{2} < \frac{I}{1-k}$$

$$R < \sqrt{\frac{2I}{1-k}}$$

Proof of proposition 3

Let us assume that the crowd overestimates the final revenue: $R < \widetilde{R}_{CF}$.

With VC, the entrepreneurial revenue obtained from the optimal contract's parameters is unmodified:

$$\frac{1}{2}(1+\alpha)^2 R^2 - (r_L \alpha + I)$$

With CF, it is:

$$\frac{R^2}{2} - \frac{RI}{(1-k)\widetilde{R}_{CF}}$$

Venture capital is more attractive iff:

$$\frac{R^2}{2}\left[(1+\alpha)^2 - 1\right] - (r_L\alpha + I) + \frac{RI}{(1-k)\widetilde{R}_{CF}} > 0,$$

$$\Longleftrightarrow \Delta_{efficiency} > (r_L\alpha + I) - \frac{RI}{(1-k)\widetilde{R}_{CF}}$$

Proof of proposition 3

With exclusive CF financing, the value obtained by the entrepreneur is:

$$\frac{R^2}{2} - \frac{R(I_0 + I_1)}{(1-k)\widetilde{R}_{CF}}$$

With CF seed and VC expansion capital, it is:

$$\frac{R^2}{2} - \frac{I_0 R}{(1-k)\widetilde{R}_{CF}} - (r_L\alpha + I_1).$$

Exclusive CF is selected iff:

$$\frac{R^2}{2} - \frac{R(I_0 + I_1)}{(1-k)\widetilde{R}_{CF}} - \frac{R^2}{2} + \frac{I_0 R}{(1-k)\widetilde{R}_{CF}} + (r_L\alpha + I_1)$$

$$= -\frac{I_1 R}{(1-k)\widetilde{R}_{CF}} + (r_L\alpha + I_1) > 0.$$

Proof of proposition 3

With exclusive CF financing, the value obtained by the entrepreneur is:

$$\frac{R^2}{2} - \frac{R(I_0 + I_1)}{(1-k)\widetilde{R}_{CF}}$$

With exclusive VC financing, this value obtained is:

$$e(1+\alpha)R - (r_L\alpha + I_0 + e(1+\alpha)I_1) - \frac{e^2}{2}$$

with $e^* = (1+\alpha)(R - I_1)$.

It becomes:

$$(1+\alpha)^2(R-I_1)R - (r_L\alpha + I_0 + (1+\alpha)^2(R-I_1)I_1) - \frac{(1+\alpha)^2(R-I_1)^2}{2} = \frac{(1+\alpha)^2(R-I_1)^2}{2} - (r_L\alpha + I_0)$$

Exclusive CF is preferred to exclusive VC iff:

$$\frac{R^2}{2} - \frac{R(I_0 + I_1)}{(1-k)\widetilde{R}_{CF}} - \frac{(1+\alpha)^2(R-I_1)^2}{2} + (r_L\alpha + I_0) > 0.$$

Proof of proposition 4

With seed CF and expansion VC financing, the value obtained by the entrepreneur is:

$$\frac{R^2}{2} - \frac{I_0 R}{(1-k)\widetilde{R}_{CF}} - (r_L\alpha + I_1).$$

With exclusive VC, it is:

$$\frac{(1+\alpha)^2(R-I_1)^2}{2} - (r_L\alpha + I_0)$$

Seed CF and expansion VC is then preferred to exclusive VC iff:

$$\frac{R^2}{2} - \frac{I_0 R}{(1-k)\widetilde{R}_{CF}} - \frac{(1+\alpha)^2(R-I_1)^2}{2} + (I_0 - I_1).$$