Discussion of Activism, Strategic Trading and Liquidity
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The Model

• $C(v)$ – the cost of achieving price $v$
  – Remove overpaid managers – then $\Delta v$, not $v$
  – True value – then $v$ is not a choice variable
  – Conduct trade that will bring the price to $v$

• Optimal holdings by the activist

$$G(x) = \max_v \left( vx - C(v) \right)$$

$$V(x) = \arg \max_v \left( vx - C(v) \right)$$

It is not clear how to unwind the position.
Initially the activist has $X_0$ shares, while others know that this amount is distributed $N(\mu_x, \sigma_x)$. With time the number of shares changes as $X_t$.

Denote the cumulative number of shares purchased by time $t$ by noise traders by $Z_t$, distributed $N(0, \sigma)$.

Aggregated purchases are $Y_t = Z_t + X_t - X_0$.

Denote $P(t, Y_t)$ the share price at time $t$.

Not clear why the price $P$ is path independent if we consider liquidity effects.
Activist choses to maximize

\[
E \left[ G(X_T) - \int_0^T P(t, Y_t) \theta_t \, dt \mid X_0 \right]
\]

Applying this dynamically the activist’s value function at time \( t \) is

\[
J(t, x, y) \overset{\text{def}}{=} \sup_{\theta} E \left[ G(X_T) - \int_t^T P(u, Y_u) \theta_u \, du \mid X_t = x, Y_t = y \right]
\]

Eventually this leads to the following equations:

\[-P + J_x + J_y = 0,\]

\[J_t + \frac{1}{2} \sigma^2 J_{yy} = 0.\]
Optimal trading strategy:

\[ \theta_t = \frac{1}{T-t} \left( \frac{X_t - \mu_x - \Lambda Y_t}{\Lambda - 2} \right) \]

If you are close to T and still do not have the required amount, you will have to trade a lot, since (T-t) is close to zero!

A more realistic (and more difficult) task is to chose T dynamically.