A Theory of Income Smoothing When Insiders Know More Than Outsiders*

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Abstract

We develop a theory of income and payout smoothing by firms when insiders know more about income than outside shareholders, but property rights ensure that outsiders can enforce a fair payout. Insiders set payout to meet outsiders' expectations and underproduce to manage future expectations downward. The observed income and payout process are smooth and adjust partially and over time in response to economic shocks. The smaller the inside ownership, the more severe underproduction is, resulting in an "outside equity Laffer curve." (*JEL*: G32, G35, M41, M42, O43, D82, D92)

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The practice of income smoothing has a long tradition in corporate finance. For example, Harold Geneen ran ITT for eighteen years (1959–77), during which the company reported earnings increases for fifty-eight consecutive quarters. It was widely assumed that this streak depended on a certain amount of gray-area fiddling with the numbers, but it was also accepted that investors were not being misled about the big picture. ITT was in fact growing steadily during his tenure and the figures were, on average, a fair reflection of the company's performance. More recently, Microsoft, General Electric, and American Express have all been labeled as "smoothers."

Why do firms smooth income?[1] We argue that a primary reason for income smoothing is the pressure imposed on managers to meet the market's (i.e., analysts') earnings expectations. Although shuffling cash flows backward and forward ("financial smoothing") to level out income fluctuations may be harmless, income smoothing has a darker side.[2] First, managers who are at risk of missing the earnings target may cut investment expenditure and in doing so destroy value. Second, in an attempt to meet market expectations, managers proactively manage expectations by distorting real decisions by doing things like smoothing sales. It is this latter type of real smoothing that is central to this paper.

In a survey by Graham, Harvey, and Rajgopal 2005 among more than 400 executives, 80% of survey participants report that they would decrease discretionary spending on R&D, advertising, and maintenance to meet an earnings target.[3] More than half (55.3%) state that they would delay starting a new project to meet an earnings target, even if such a delay entailed a small sacrifice in value (Graham, Harvey, and Rajgopal 2005, 30–31). Their survey results are supported by a series of empirical studies that show that managers are prepared to destroy value in order to meet the market's expectations.[4] There is also evidence that managers proactively manage expectations. Bouwens and Kroos 2011 examine how retail store managers reduce their sales activity in response to target ratcheting. They find that managers with favorable sales performance in the first three quarters reduce their sales activity in the final quarter in an attempt to mitigate the increase in the next year's sales target (see Indjejikian, Matejka, and Schloetzer [forthcoming] for a review on target ratcheting and incentives). Proactive expectations management also arises endogenously in our model.

Although this interplay between market expectations and managerial incentives is widely acknowledged, it begs the question as to how it is possible that in equilibrium firms can keep managing earnings and expectations, and get away with it –in many cases indefinitely. Why do investors not intervene, or why does the smoothing equilibrium not unravel? If income and expectation management lead to value destruction, why then do insiders and outsiders engage in this game in the first place? Our theory answers these questions by providing a rational expectations equilibrium featuring income smoothing and expectations management that are driven by the pressure imposed on managers to meet income expectations.

We consider a neoclassical firm in which insiders set output on the basis of marginal revenues and marginal costs. Each period, outsiders demand their share of the income that they believe has been generated. Marginal costs are latent and observed by insiders only. Outsiders observe sales, a measure of the firm's output level. Because sales are a monotonically decreasing function of marginal costs, outsiders can perfectly infer the corresponding level of the latent marginal cost variable, and therefore also the level of income. Outsiders' indirect inference of income through sales creates, however, an incentive for insiders to distort production: insiders underproduce in an attempt to downplay the firm's fundamentals and to lower outsiders' income expectations. Outsiders rationally anticipate what insiders are up to, but nevertheless this type of value-destroying manipulation persists in this signal-jamming equilibrium because both parties are "trapped" in a type of prisoners' dilemma. Conditional on outsiders believing that insiders will "behave" it is optimal for insiders to manipulate (i.e., underproduce). As a result, underinvestment and expectations management always prevail in equilibrium. Outsiders infer the correct value of the firm's income, and payout moves in lockstep with realized income (i.e., there is no financial smoothing). Underproduction causes both parties to be worse off than under a first-best policy. Furthermore, the absolute amount of lost output is higher in economic booms than in recessions. This reduces the output variance, a phenomenon we refer to as real smoothing. In our model, underproduction and real smoothing are two sides of the same coin. This direct link between underproduction and real smoothing is not an obvious one. If, for example, insiders were to reduce output at all times by a constant absolute amount, then underproduction would not coincide with real smoothing.

Next, we consider the case where outsiders observe sales with measurement error or "noise." This noise is value-irrelevant, transitory, normally distributed, and independent and identically distributed (i.i.d.) over time. When observing an increase in noisy sales, outsiders cannot distinguish whether the increase is due to a reduction in marginal costs (and therefore represents a real increase in income), or whether the increase is due to value-irrelevant measurement error. Because measurement errors are transitory and shocks to costs persistent, the underlying source of change becomes clear only as time passes by. Therefore, outsiders calculate their best estimate of income on the basis of not only current sales but also past sales, by solving a Kalman filtering problem.

Then, in a rational expectations equilibrium, outsiders form their expectation of actual income on the basis of the complete history of sales and of what they believe insiders' optimal output policy to be. Conversely, in each period, insiders determine their optimal output policy given outsiders' beliefs. We obtain a perfect Bayesian equilibrium in which insiders' actions are consistent with outsiders' beliefs, and outsiders' expectations are unbiased conditional on the information available. Each period, outsiders receive a payout that equals their share of what they expect income to be. Insiders also get a payout, but they have to soak up any under- (over-) payment to outsiders as some kind of discretionary remuneration (charge): if actual income is higher (lower) than outsiders' estimate, then insiders cash in (make up for) the difference in outsiders' payout.

Consequently, income and payout are smooth compared with actual income not because insiders want to smooth income, but because insiders have to meet outsiders' expectations to avoid intervention. With imperfect inference, two types of smoothing take place simultaneously: "payout" (or "financial") smoothing and "real" smoothing.[5] The former is value-neutral and merely alters the time pattern of payout to outsiders without changing the firm's underlying cash flows as determined by insiders' production decision. Insiders also engage in real smoothing by manipulating production in an attempt to manage outsiders' expectations. However, because outsiders' income estimate is now based on the complete history of sales, the instantaneous effect of sales on outsiders' beliefs is weakened. Measurement error and the resulting asymmetry information therefore mitigate the effect of indirect inference on production and real smoothing.

Importantly, real smoothing also has a lagged effect. With imperfect inference, the current output decision affects not only current sales levels but also outsiders' expectations of current and future income. This exacerbates the previously discussed underinvestment problem for insiders because bumping up sales now means the outsiders will expect higher income and payout not only now but also in future. The instantaneous effect of measurement error dominates, however, the lagged effect so that, on balance, measurement error and asymmetric information mitigate underinvestment and reduce real smoothing. However, in the presence of measurement error, outsiders' estimate of the firm's income, although unbiased, is not exact. Payout no longer moves in lockstep with realized income but is smooth relative to income.

In addition to formalizing the type of behavior described in Jensen 2005 (see note 2), our model has implications for a number of areas in corporate finance. First, our model explains key dynamics of corporate payout. We show that in equilibrium, payout follows a distributed lag model and has features as in Lintner 1956. For example, the effect on payout of a positive shock in sales is distributed over time because outsiders do not immediately know whether the increase in sales is due to transitory noise or whether it is caused by a persistent improvement in the firm's fundamental. Importantly, the higher the degree of incomplete information, the more payout is smoothed. Our model provides closed-form expressions for the Lintner constant and speed of adjustment (SOA), allowing these to be linked to economic determinants such as the volatility and growth of income, the persistence of income shocks, the firm's ownership structure, and the variance of income measurement error.

Second, our model has implications for the firm's ownership structure. We show that smoothing and underproduction in particular increase with outside shareholders' ownership stake because it increases insiders' incentives to manage outsiders' expectations. Conversely, a higher level of inside ownership leads to less real smoothing. Indeed, the underinvestment problem disappears as insiders move toward 100% ownership. These effects lead to an "outside equity Laffer curve": the value of the total outside equity is an inverted U-shaped function of outsiders' ownership stake.[6] Morck, Shleifer, and Vishny 1988 document a non-monotonic relation between Tobin's q and managerial stock ownership, and McConnell and Servaes 1990 report an "inverted-U" or "hump-shaped" relation between q and managerial ownership. Our model provides a new theoretical explanation for this empirical phenomenon.

Finally, our model provides new insights as to why firms go public or are taken private. It is well known that firms go public to raise more outside equity capital. However, consistent with empirical evidence by Asker, Farre-Mensa, and Ljungqvist [forthcoming], our model predicts that public firms invest less and are less responsive to changes in investment opportunities compared with private firms. Furthermore, we predict that public firms that have accumulated ample internal sources of funds may be taken private in order to eliminate the investment distortions and costly disclosure requirements public firms are subject to. Our model also implies that public firms smooth payout more than private firms. This implication is consistent with Michaely and Roberts 2012, who show that private firms smooth dividends less than their public counterparts.

Section 1 presents the benchmark case with perfect inference by outsiders. Section 2 analyzes the imperfect inference model and its implications for income and payout smoothing. Section 3 presents novel empirical implications for (1) the time-series and cross-sectional properties of corporate income, (2) real smoothing by firms, (3) corporate ownership structure, and (4) public versus private firms. Section 4 relates our paper to existing literature. Section 5 concludes. Proofs are in the Appendix.[7]

1 Perfect Inference

Consider a firm with an open-ended (infinite) horizon that decides each period on an output level q_t. The firm's income function $\pi(q_t; x_t)$ is strictly concave in q_t and also depends on a strictly positive state variable x_t (i.e., $\pi''(q_t) < 0$, $\pi'(0) > 0$ and $x_t > 0$).

The firm is owned by risk-neutral shareholders who can borrow and save at the risk-free rate, and have a discount factor $\beta \in (0, 1)$. Therefore–unlike Stein 1989–changing the time pattern of cash flows (without changing their present value) through more borrowing or saving is costless. The value of the firm is given by the expected present value of current and future income: $V_t = E_t[\sum_{j=0}^{\infty} \beta^j \pi(q_{t+j}; x_{t+j})]$.

The first-best firm value is given by $V_t^o = \max_{q_{t+j}, j=0\ldots\infty} E_t[\sum_{j=0}^{\infty} \beta^j \pi(q_{t+j}; x_{t+j})]$. Because the production level can be adjusted each period at no cost, the optimal output level q_t^o only depends on the contemporaneous level of x_t, and $q^o(x_t)$ is the solution to the first-order condition $\pi'(q_t^o) = 0$. In other words, the optimal output policy is a myopic one that maximizes at each point in time the firm's current income.

We now introduce inside and outside shareholders who, respectively, own a fraction $(1 - \varphi)$ and φ of the shares, $\varphi \in [0, 1]$. For example, insiders (managers and even board members involved in the firm's operating decisions) typically own the majority of shares of private firms ($\varphi < 0.5$), whereas for public firms it is more common that outsiders own the majority of shares ($\varphi > 0.5$). The risk-neutral insiders set the production (q_t) and payout (d_t) policies. Analogous to Myers 2000, Jin and Myers 2006, Lambrecht and Myers 2007, 2008, and 2012, and Acharya, Myers, and Rajan

2011, we assume that insiders operate subject to a threat of collective action. Outsiders' payoff from collective action is given by $\varphi\alpha V_t$ where α ($\in (0,1)$) reflects the degree of investor protection (or specificity of the firm's technology).[8] Therefore, the value of the outside equity, S_t, must at all times satisfy the following constraint:

$$S_t \;\geq\; \alpha\phi V_t \;\equiv\; \theta V_t \tag{1}$$

Equation (1) is a governance constraint that ensures outside equityholders get a share of the income generated by the firm. How big the share is depends on insiders' effective ownership stake as summarized by the parameter θ with $0 < \theta < 1$.[9] Outsiders can force the firm to pay out by taking collective action. Equation (1) implies that insiders must at all times set the payout d_t high enough so that outsiders are willing to postpone intervention for one more period.[10]

The governance constraint captures parsimoniously a repeated game between insiders and outsiders. At each time t insiders propose to outsiders (e.g., at the annual general meeting) a payout and rent level (d_t, r_t). If outsiders reject this offer, then they get the payoff from intervention, θV_t, insiders get $(1-\theta-c)V_t$, and the game ends. cV_t reflects the cost of intervention to insiders. If outsiders accept, then insiders and outsiders, respectively, get r_t and d_t, and insiders stay in charge for one more period, at which point the game is repeated at $t+1$. In equilibrium, insiders always remain in charge because they propose a pair (d_t , r_t) for which outsiders are indifferent between intervening and leaving insiders in charge–that is,

$$\theta\left[E_{S,t}(\pi_t) + \sum_{j=1}^{\infty}\beta^j E_{S,t}\left[\pi_{t+j}\right]\right] \;=\; d_t + \theta\beta\left[E_{S,t}\left[\pi_{t+1}\right] + \sum_{j=1}^{\infty}\beta^j E_{S,t}\left[\pi_{t+1+j}\right]\right] \tag{2}$$

$$\Longleftrightarrow \quad d_t \;=\; \theta E_{S,t}(\pi_t) \;\;\text{(and therefore } r_t \;=\; \pi_t - \theta E_{S,t}(\pi_t)) \tag{3}$$

where $E_{S,t}[\pi_t] \equiv E[\pi_t|I_t]$ denotes outsiders' conjecture of the firm's income on the basis of all information I_t available to them. The left- (right-) hand side of Equation (2) equals outsiders' expected payoff from intervention (continuation). To avoid collective action, insiders set the payout equal to $d_t = \theta E_{S,t}(\pi_t)$. In other words, outsiders want their share of the income they believe has been realized according to all information available to them.

Insiders' optimization problem can now be formulated as follows:

$$M_t \;=\; \max_{q_{t+j};j=0..\infty} E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\pi(q_{t+j}) \;-\; \theta E_{S,t+j}\left[\pi\left(q_{t+j}\right)\right]\right)\right] \tag{4}$$

Note that E_t and $E_{S,t}$ refer to insiders' and outsiders' expectations, respectively.

Determining the equilibrium policy q_t involves the following key steps: (i) Outsiders make a conjecture $\hat{\pi} \;=\; E_{S,t}(\pi_t)$ regarding the firm's income based on the information I_t available. (ii) Insiders maximize (4) given outsiders' beliefs, and this leads to an optimal policy $q_t^*(x_t)$. (iii) Finally, substituting the optimal policy q_t^* into $\pi(q_t; x_t)$, we determine the conditions that verify outsiders' initial conjecture–that is, the conditions for which $\pi(q^*(x_t); x_t) \;=\; E_{S,t}(\pi_t)$.

A central question of this paper will be to explore how, in equilibrium, the production policy (q_t), realized income $(\pi(q_t; x_t))$, and payout (d_t) vary as a function of outsiders' information set I_t. In this section we consider two scenarios. First, we consider the scenario where the state variable x_t is directly observable to outsiders. Next, we consider the scenario where x_t is unobservable to outsiders, but outsiders instead observe a signal, s_t, of the latent variable (i.e., $I_t \equiv \{s_t, s_{t-1}, s_{t-2}, ...\}$). The signal we use in this paper is the firm's sales, but we show that the results are robust to the use of other observable output or input measures. Importantly, we assume in this section that the relation between the signal and the latent variable is "noise free." Although the link between x_t and s_t is endogenously set by the insiders, it is a deterministic one-to-one mapping that, in equilibrium, allows outsiders indirectly to infer the exact realization of the latent variable. As such, it will turn out that in equilibrium, insiders' private information is fully revealed to outsiders. In Section 2 we consider the scenario where outsiders' observation of the signal is subject to exogenous measurement error ("noise"). As a result, outsiders' inferences will be imperfect, and this will have important implications for the equilibrium. We now consider the various scenarios in turn.

1.1 Observable state variable

Consider first the scenario where outsiders observe the state variable and sales (i.e., $I_t \;=\; \{x_t, x_{t-1}, ...; s_t, s_{t-1}, ...\}$). Hence, (i) outsiders' income estimate always equals the actual income, that is, $\hat{\pi} \;=\; \pi(q_t; x_t)$. (ii) Insiders maximize (4) given outsiders' beliefs. The first-order condition is $(1-\theta)\pi'(q_t) = 0$, which leads to the first-best optimal policy $q^o(x_t)$. (iii) Because outsiders observe sales and the state variable, their beliefs are always true. Note that the first-best outcome results from the fact that (i) outsiders' estimate of the marginal income of production is exact (i.e., $\frac{\partial E_{S,t}(\pi_t)}{\partial q_t} = \pi'(q_t)$), and (ii) insiders get a constant fraction $1-\theta$ of income.

Consider next the scenario in which the outsiders observe the state variable but not output (nor any other measure of output), that is, $I_t \equiv \{x_t, x_{t-1}, ...\}$. In that case, (i) outsiders conjecture that $\hat{\pi} = E_{S,t}(\pi_t) = \pi(q^o(x_t); x_t)$. (ii) insiders maximize (4) given outsiders' beliefs. The first-order condition is $\pi'(q_t) = 0$, which leads to the first-best optimal policy $q^o(x_t)$. (iii) Finally, substituting the optimal policy into $\pi(q_t; x_t)$, it follows immediately that outsiders' initial beliefs are verified since $\pi(q^o(x_t); x_t) = E_{S,t}(\pi_t) = \pi(q^o(x_t); x_t)$. This verifies that the first-best policy is an equilibrium. The first-best outcome results from the fact that a marginal increase in output does not affect outsiders' income estimate (i.e., $\frac{\partial E_{S,t}(\pi_t)}{\partial q_t} = 0$), as no observable measure of output is available to outsiders. Because payout to outsiders is independent of insiders' output policy, insiders have no incentive to distort production policy.

Combining the above two scenarios, it follows that the first-best output policy is achieved irrespective of whether output is observed or not:

Proposition 1. *If x_t is directly observable to outsiders then insiders adopt the first-best production policy, q_t^o, and payout to outsiders (insiders) equals a fraction θ $(1-\theta)$ of realized income $\pi_t(q_t^o; x_t)$.*

Outsiders' and insiders' claim values are, respectively, θV_t^o and $(1-\theta)V_t^o$. Insiders' payoff from intervention, $(1-\theta-c)V_t$, is always less than their payoff from continuation, $(1-\theta)V_t^o$. Therefore, insiders avoid triggering collective action.[11]

1.2 Latent state variable

Consider now the case in which outsiders cannot observe x_t directly but instead observe a signal s_t for x_t. Outsiders' information set consists of the history of signals, that is, $I_t \equiv \{s_t, s_{t-1}, s_{t-2}, ...\}$. Importantly, we assume in this section that the mapping from x_t to s_t is deterministic (i.e., free of noise) allowing x_t to be inferred indirectly. Outsiders use their information to produce a conjecture $\hat{\pi}_t$ of the firm's income, that is, $\hat{\pi}_t = E_{S,t}(\pi_t)$, and require a payout $d_t = \theta E_{S,t}[\pi(q_t)]$ each period,

To solve for the equilibrium, we need to specify the income function $\pi(q_t; x_t)$ explicitly from now onward. For most of this paper, we will assume that $\pi(q_t)$ is quadratic in q_t. Outsiders cannot observe x_t directly but instead observe sales, s_t, defined as output times the price per unit of output. Because sales are a function of the latent variable, outsiders infer x_t indirectly from s_t. We will focus on two cases in the paper: (i) latent marginal costs and (ii) latent marginal revenues. In Section 1.2.3 and online Appendix B we show that the results are valid for alternative income functions (such as an income function based on the Cobb-Douglas production function) and for alternative signals (such as output and input measures).

1.2.1 Case 1: Latent marginal costs. Consider the following income function:

$$\pi_t \;=\; q_t - \frac{q_t^2}{2x_t} \quad \text{with } x_t > 0 \tag{5}$$

The output is sold at a fixed price that is, without loss of generality, normalized to one. Outsiders observe sales, given by: $s_t \;=\; q_t$. Outsiders conjecture that income is a linear function of sales, that is, $\hat{\pi}(s_t) = E_{S,t}(\pi_t) \;=\; a_0 s_t + b_0 \;=\; a_0\, q_t + b_0$. Insiders optimize Equation (4) given outsiders' beliefs. This gives the following first- and second-order conditions:

$$\frac{\partial M_t}{\partial q_t} \;=\; 1 - \frac{q_t}{x_t} - \theta a_0 \;=\; 0 \quad \text{and} \quad \frac{\partial^2 M_t}{\partial {q_t}^2} \;=\; -\frac{1}{x_t} \;<\; 0 \tag{6}$$

Solving gives the following optimal output policy: $q_t^* \;=\; (1-\theta a_0)\, x_t$. Outsiders' conjecture is verified if and only if:

$$\hat{\pi}(s_t = q_t^*; x_t) \;=\; a_0\,(1-\theta a_0)\, x_t + b_0 \;=\; (1-\theta a_0)\, x_t - \frac{(1-\theta a_0)^2\, x_t}{2} \;=\; \pi(q_t^*; x_t) \tag{7}$$

Or, equivalently, if and only if $a_0 \;=\; 1/(2-\theta)$ and $b_0 = 0$. This gives the following proposition:

Proposition 2. *If the firm's income function is given by $\pi(q_t; x_t) \;=\; q_t - \frac{q_t^2}{2x_t}$, then the first-best production policy is $q_t^o \;=\; x_t$ and the realized income under the first-best policy is $\pi^o \;=\; \frac{x_t}{2}$. If outsiders can observe the firm's sales, s_t, but not the latent cost variable x_t, then the production policy adopted by insiders is:*

$$q_t \;=\; \frac{2(1-\theta)x_t}{(2-\theta)} \equiv H x_t \;<\; q_t^o \tag{8}$$

The realized income is given by:

$$\pi(q_t; x_t) \;=\; \frac{2(1-\theta)x_t}{(2-\theta)^2} \;<\; \frac{x_t}{2} \tag{9}$$

The payout to outsiders is given by: $d_t = \theta\hat{\pi}_t \;=\; \frac{2(1-\theta)\theta x_t}{(2-\theta)^2} \;=\; \theta\pi(q_t; x_t)$.

The following properties apply:

Property 1.A. $q_t = Hx_t < x_t = q_t^o$*: Insiders underproduce when* x_t *is not directly observable.*

Insiders underproduce to downplay the firm's fundamentals and to manage downward outsiders' beliefs about income. Of course, outsiders rationally anticipate what insiders are up to, but nevertheless this type of value-destroying manipulation persists in this signal-jamming equilibrium because both parties are "trapped" into a suboptimal equilibrium. Conditional on outsiders believing that insiders will "behave," it is optimal for insiders to manipulate (i.e., underproduce). As a result, underinvestment and expectations management always prevail in equilibrium. The situation is analogous to what happens in a prisoner's dilemma. The preferred cooperative equilibrium would be efficient production by insiders and no conjecture of manipulation by outsiders. This can, however, not be sustained as a Nash equilibrium because insiders have an incentive to underproduce whenever outsiders believe the efficient production policy is being adopted. Note that the degree of underproduction is monotonically increasing in outsiders' ownership stake. As outsiders' stake θ converges to zero (one) production goes to first-best (zero).

Property 2.A. *The firm engages in real smoothing:* $var(q_t) \;=\; H^2 var(x_t) < var(x_t) = var(q_t^o)$.

The property implies that under insiders' policy, the variance of production is lower than in the first-best case, a phenomenon we refer to as real smoothing. This reduction in variance is due to the fact that insiders scale down proportionally the firm's production for all levels of the state variable. As a result, the absolute amount of output lost is higher in booms than in recessions, which reduces the output variance. Therefore, underproduction and real smoothing are two sides of the same coin. This direct link between underproduction and real smoothing is robust to alternative specifications for the income function (see online Appendix B, where we consider a Cobb-Douglas production function rather than a quadratic cost function).

The result that underproduction leads to real smoothing is not obvious. Take the counterexample where insiders reduce output by a constant amount c compared with what is first best, that is, $q_t = x_t - c$. In that case, underproduction does not lead to a reduction in variance, that is, $var(q_t) = var(x_t) = var(q_t^o)$.

Property 3.A. $\hat{\pi}_t = \pi(q_t; x_t)$: *In equilibrium, outsiders infer the correct value of the firm's income and of the latent variable* x_t.

Even though x_t is not observable to outsiders, in equilibrium outsiders can infer the value of the latent variable from the observed sales, s_t. All information asymmetry is therefore resolved and markets are perfectly efficient.

Property 4.A. $\pi(q_t; x_t) < \pi(q_t^o; x_t)$, $d_t < \theta\pi(q_t^o; x_t)$ *and* $r_t < (1-\theta)\pi(q_t^o; x_t)$: *insiders and outsiders are both worse off compared with what they would get under a first-best production policy.*

Underproduction implies that income (and therefore firm value) is lower than what is achieved under a first-best production policy. As in the standard prisoner's dilemma, both parties' values go down relative to what they get under a cooperative, first-best policy.

Property 5.A. $var(d_t) = var(\theta\hat{\pi}_t) = var(\theta\pi(q_t; x_t))$: *Payout moves in lockstep with realized income. Therefore, there is no payout or "financial" smoothing.*

Payout tracks realized income exactly, which is quite different from what is empirically observed. Empirical tests of the Lintner 1956 model show that payout is smooth relative to income. Firms keep payout smooth by borrowing and savings, a process we refer to as "financial smoothing." With financial smoothing, payout is linked not to the actual income realization, but to a smoothed version of income (such as permanent income). Lambrecht and Myers 2012 show that payout smoothing naturally arises when insiders are risk averse or subject to habit formation. We show in the next section that asymmetric information due to imperfect inference can be another explanation for payout smoothing.

Property 6.A. *A myopic policy remains optimal if* x_t *is indirectly but perfectly inferred.*

Outsiders' best estimate of current income is determined by contemporaneous sales only. Similarly, when setting production, insiders consider only the effect of output on current income and insiders' contemporaneous beliefs.

1.2.2 Case 2: Latent marginal revenues.

Consider next the following income function:

$$\pi_t \;=\; q_t\sqrt{x_t} - \frac{q_t^2}{2} \quad \text{with} \quad x_t > 0 \tag{10}$$

On this occasion the latent variable is marginal revenues ($\sqrt{x_t}$). As before, outsiders observe sales, which are now given by $s_t = q_t\sqrt{x_t}$. The observable is now a composite measure of q_t and x_t. However, in equilibrium, q_t is a function of x_t, and provided that the equilibrium value of s_t is monotonic in x_t, outsiders can still unambiguously infer the latent variable x_t from sales. One can prove the following proposition (see Appendix):

Proposition 3. *If the firm's income function is given by* $\pi(q_t; x_t) = q_t\sqrt{x_t} - \frac{q_t^2}{2}$*, then the first-best production policy is* $q_t^o = \sqrt{x_t}$ *and the realized income under the first-best policy is* $\pi^o = \frac{x_t}{2}$*. If outsiders can observe the firm's sales,* s_t*, but not the latent cost variable* x_t*, then the production policy adopted by insiders is:*

$$q_t \;=\; \frac{2(1-\theta)\sqrt{x_t}}{(2-\theta)} \equiv H\sqrt{x_t} \;<\; q_t^o, \tag{11}$$

realized income is given by:

$$\pi(q_t; x_t) \;=\; \frac{2(1-\theta)x_t}{(2-\theta)^2} \;<\; \frac{x_t}{2}, \tag{12}$$

payout to outsiders is given by: $d_t = \theta\hat{\pi}_t = \frac{2(1-\theta)\theta x_t}{(2-\theta)^2} = \theta\pi(q_t; x_t)$.

The results for latent marginal revenues are similar to the ones we discussed in previous section. For example, one can easily verify that Properties 1.A to 6.A are valid.

1.2.3 Robustness.

So far we have considered two scenarios in which insiders perfectly but indirectly infer a latent variable and income from observable sales. We have shown that this process of indirect inference gives insiders incentives to manage downward outsiders' beliefs and expectations. We found that in equilibrium (1.A) insiders underproduce, (2.A) firms engage in real smoothing, (3.A) markets are efficient and all information asymmetry is resolved, (4.A) there is no payout or "financial" smoothing, (5.A) insiders and outsiders are both worse off compared with what they would get under a first-best production policy, and (6.A) a myopic policy remains optimal.

In online Appendix B we show that these results are robust. Instead of using sales as the observable, we first consider the case in which output (q_t) is the observable. Next, we depart from the quadratic income specification by introducing a Cobb-Douglas production function with labor as input. We then treat the input measure (labor) as the observable. We show that Properties (1) to (6) remain valid. More generally, changing the income specification or the variable that is being observed does not alter the results in any essential way.

2 Imperfect Inference

In what follows we focus on the latent marginal cost variable and assume that the firm's income function is as in case 1 (we return to the latent marginal revenue case in Section 2.6):

$$\pi(q_t) = q_t - \frac{q_t^2}{2x_t} \tag{13}$$

So far we have considered a model where in equilibrium all information asymmetry gets resolved (because outsiders can perfectly infer the value of the latent variable from the firm's sales s_t). From now on we assume that outsiders observe sales with measurement error. Instead of observing $s_t = q_t$, outsiders observe $s_t = q_t + \epsilon_t$ where ϵ_t is an i.i.d. normally distributed noise term with zero mean and variance R (i.e., $\epsilon_t \sim N(0, R)$). s_t can, for example, be interpreted as analysts' estimate of sales. We assume this estimate to be noisy, but unbiased (i.e., $E(\epsilon_t) = 0$).

Outsiders are aware that observed sales are an imperfect proxy for actual sales and know the distribution from which ϵ_t is drawn. Importantly, insiders implement the production decision (q_t) after the realization of x_t but before the realization of ϵ_t is known. Since ϵ_t is value irrelevant,the firm's actual income is still given by $\pi(q_t) = q_t - \frac{q_t^2}{2x_t}$. However, as q_t and x_t are unobservable, outsiders have to estimate income on the basis of noisy sales figures.

We know from the previous section that there is a mapping from the latent variable x_t to both q_t and π_t. The presence of the noise term ϵ_t obscures, however, this link and makes it impossible for outsiders exactly to infer x_t and π_t from sales. Assuming that insiders cannot trade in the firm's stock and that information asymmetry cannot be mitigated through monitoring or some other mechanism, the best outsiders can do is to calculate a probability distribution of income, π_t, on the basis of all information available to them. On the basis of the initial estimate $\hat{x}_0$ and the sales history, I_t, outsiders can infer a probability distribution for the latent marginal cost variable x_t, which in turn maps into a probability distribution for income π_t. Outsiders then use this distribution to calculate their estimate $\hat{\pi}_t$ of the firm's income, that is, $\hat{\pi}_t = E[\pi_t | I_t] \equiv E_{S,t}(\pi_t)$.

In order to operationalize the inference process, we now need to introduce additional assumptions regarding the latent variable x_t. In what follows we assume that the variable x_t is described by the following process:

$$x_t = A\,x_{t-1} + B^* + w^*_{t-1} \quad \textit{with } 0 \le A < 1 \;\; \textit{and} \;\; B^* > 0 \tag{14}$$

where w^*_{t-1} is an i.i.d. noise term drawn from a truncated normal distribution $N(0, Q^*; -B^*)$, that is, w^*_{t-1} is drawn from a normal distribution with zero mean and variance Q^*, but conditional on $-B^* < w^*_{t-1}$ for all t. Truncating the distribution for w^*_t at $-B^*$ ensures that x_t remains positive at all times.[12] The mean and variance for w^*_{t-1} are, respectively, given by:

$$E[w^*_{t-1}] = \frac{n(\frac{-B^*}{\sqrt{Q^*}})\sqrt{Q^*}}{1 - N(\frac{-B^*}{\sqrt{Q^*}})} \equiv m \text{ for all } t \tag{15}$$

$$var[w^*_{t-1}] = Q^* \left[1 + \frac{\frac{-B^*}{\sqrt{Q^*}} n(\frac{-B^*}{\sqrt{Q^*}})}{1 - N(\frac{-B^*}{\sqrt{Q^*}})} - \left(\frac{n(\frac{-B^*}{\sqrt{Q^*}})}{1 - N(\frac{-B^*}{\sqrt{Q^*}})} \right)^2 \right] \equiv Q \text{ for all } t \tag{16}$$

where $n(.)$ and $N(.)$ denote the standard normal density and cumulative distribution, respectively. We now define the following variable: $w_{t-1} \equiv w^*_{t-1} - m$, for all t. Hence $E[w_{t-1}] = 0$ and $var[w_{t-1}] = Q$, for all t. Note that the truncated distribution for w_{t-1} converges to the (un-truncated) normal distribution $N(0, Q^*)$ as B^* becomes large. Our stochastic process can now be written as:

$$x_t = A\,x_{t-1} + B + w_{t-1} \quad \text{where } B \equiv B^* + m \tag{17}$$

Hence, the (inverse) marginal production cost variable x_t follows an AR(1) process with auto-regressive coefficient $A \in [0, 1)$, a drift B, and an i.i.d. noise term w_{t-1} with zero mean and variance Q.[13] w_{t-1} is drawn from a truncated Gaussian distribution with support $]-B, +\infty[$. We assume for simplicity that the measurement error is uncorrelated with the marginal cost variable x_t (i.e., $E(w_k \epsilon_l) = 0$ for all k and l). As before, the realizations of x_t are observed by insiders only. All model parameters remain common knowledge, however. Outsiders also have an unbiased estimate $\hat{x}_0$ of the initial value x_0.[14]

As before, the capital market constraint requires that d_t satisfies the constraint $d_t = \theta E_{S,t}(\pi_t)$ (and therefore $r_t = \pi_t - \theta E_{S,t}(\pi_t)$). Insiders then solve the optimization problem (4) that was previously described.

The complete derivation of the solution is given in the Appendix. We present a short, heuristic derivation of the rational expectations equilibrium here. Outsiders conjecture that insiders' production policy is given by $q_t = Hx_t$, where H is some constant. Therefore, $E_{S,t+j}\left[\pi(q_{t+j})\right] = \left(H - \frac{H^2}{2}\right) E_{S,t+j}\left[x_{t+j}\right] \equiv hE_{S,t+j}\left[x_{t+j}\right]$. Define $\hat{x}_t \equiv E_{S,t}[x_t]$ as outsiders' estimate of the latent variable x_t conditional on the information available at time t. Because $s_t = q_t + \epsilon_t$ and $q_t = Hx_t$, sales are an imperfect (noisy) measure of the latent variable x_t, as is clear from the following "measurement equation":[15]

$$s_t = H\,x_t + \epsilon_t \quad \text{with} \quad \epsilon_t \sim N(0, R) \tag{18}$$

Outsiders also know the variance R of the noise, ϵ_t, and the parameters A, B, and Q of the "state equation":

$$x_t = A\,x_{t-1} + B + w_{t-1} \quad \text{with} \quad w_t \sim N(0, Q; -B) \text{ for all } t \tag{19}$$

where $E(w_k \epsilon_l) = 0$ for all k and l. Outsiders now solve what is known as a "filtering" problem. Using the Kalman filter (see appendix), the measurement equation can be combined with the state equation to make inferences about x_t on the basis of current and past observations of s_t.[16] This allows outsiders to form an estimate of actual income π_t. While the measurement equation is usually given exogenously, our Kalman filter has the novel feature that the constant slope coefficient H in the measurement equation is set endogenously by insiders.

The solution is formulated in terms of the steady-state or "limiting" Kalman filter, which is the estimator $\hat{x}_t$ for x_t that is obtained after a sufficient number of measurements s_t have taken place over time for the estimator to reach a steady state.[17] The steady-state estimator $\hat{x}_t$ allows us to analyze the long-run behavior of income and payout and is given by (see proof of Proposition 4):

$$\hat{x}_t = (A\hat{x}_{t-1} + B)\,\lambda + K s_t \tag{20}$$

where λ and K are as defined in Proposition 4. K is called the "Kalman gain," and it plays a crucial role in the updating process. Substituting $\hat{x}_{t-1}$ in Equation (20) by its estimate, one obtains after repeated substitution:

$$
\begin{aligned}
\hat{x}_t &= B\lambda\left[1+\lambda A+\lambda^2A^2+\lambda^3A^3+...\right] + K\left[s_t+\lambda A s_{t-1}+\lambda^2A^2s_{t-2}+\lambda^3A^3s_{t-3}+...\right] \\
&= \frac{B\lambda}{1-\lambda A} + K\sum_{j=0}^{\infty}\lambda^j A^j s_{t-j}\ . \qquad (21)
\end{aligned}
$$

Thus, outsiders' income estimate is determined not only by their observation of current sales but also by the whole history of past sales. Due to outsiders' retrospection and long-term memory, insiders' optimization problem is no longer myopic in nature but inter-temporal. Indeed, the current production decision affects outsiders' expectations about not only current but also future income.

Substituting outsiders' beliefs $E_{S,t+j}[\pi(q_{t+j})] = E_{S,t+j}[Hx_t - H^2x_t/2] = h\hat{x}_{t+j}$ into insiders' objective function (4), insiders optimize:

$$
M_t = \max_{q_{t+j};j=0..\infty} E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\pi(q_{t+j}) - \theta h\hat{x}_{t+j}\right)\right] \qquad (22)
$$

Using (21) and the fact that $s_t = q_t + \epsilon_t$ gives the following first-order condition:

$$
\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta hK - \theta hK\beta\lambda A - \theta hK\left(\beta\lambda A\right)^2 - \theta hK\left(\beta\lambda A\right)^3 - ... = 0 \qquad (23)
$$

Or equivalently, since $0 \leq \beta\lambda A < 1$:

$$
q_t = \left[1 - \frac{\theta hK}{1-\beta\lambda A}\right]x_t \qquad (24)
$$

Outsiders' conjectured output policy $q_t = Hx_t$ is a perfect Bayesian equilibrium if and only if:

$$
H = 1 - \frac{\theta hK}{1-\beta\lambda A} \qquad (25)
$$

At the fixed point, we have a perfect Bayesian equilibrium in which outsiders' expectations are rational given insiders' output policy, and insiders' output policy is optimal given outsiders' expectations. Equation (25) has a unique positive root for H, which pins down the equilibrium value for H. This root is less than one (i.e., $H < 1$), and therefore insiders underproduce compared with what is first best (see proof of Proposition 4 and online Appendix C for further details). The results are summarized in the following proposition.

Proposition 4. *Insiders' optimal production plan is given by:*

$$q_t \;=\; H\,x_t \;=\; Hq_t^o \quad \textit{for all } t \tag{26}$$

Payout to outside shareholders equals a fraction θ of expected income: $d_t \;=\; \theta\hat{\pi}_t$ where

$$\begin{aligned} \hat{\pi}_t \;&=\; \left(H - \frac{H^2}{2}\right)\hat{x}_t \;\equiv\; h\hat{x}_t \;, && (27)\\ \textit{and where } \hat{x}_t \;&=\; (A\hat{x}_{t-1} \,+\, B)\,\lambda \,+\, K\,s_t && (28)\\ &=\; \frac{\lambda B}{1-\lambda A} \,+\, K\sum_{j=0}^{\infty}(\lambda A)^j s_{t-j}\;. && (29)\end{aligned}$$

with $K \equiv \frac{HP}{H^2P+R}$, $\lambda \equiv (1-KH)$ and P is the positive root of the equation:

$$P \;=\; A^2P - \frac{A^2H^2P^2}{H^2P+R} \;+\; Q\;. \tag{30}$$

H is the (unique) positive root of Equation (25) and lies in the interval $]0,1[$. The error of outsiders' posterior income estimate $(\pi_t \,-\, \hat{\pi}_t)$ has mean zero (i.e., $E_{S,t}[\pi_t \,-\, \hat{\pi}_t] \,=\, 0$) and variance $\hat{\sigma}^2 \;\equiv\; E_{S,t}[(\pi_t \,-\, \hat{\pi}_t)^2] \;=\; h^2\,P(1-HK)$.

2.1 Production policy

We know from Proposition 4 that insiders' optimal production policy is given by $q_t \,=\, H\,x_t$ where H is a solution to Equation (25). There exists a unique positive root for H that lies in the interval $]0,1[$. We therefore obtain the following property.

Property 1.B. *If outsiders indirectly infer income from noisy sales (s_t) then insiders underproduce (i.e., $q_t \,=\, Hx_t \,=\, Hq_t^o \le q_t^0$).*

As in Property 1.A, insiders underproduce because outsiders do not observe x_t directly but estimate its value indirectly from sales. This gives insiders an incentive to manipulate sales (engage in "signal-jamming") by underproducing.

Property 2.B. *If insiders indirectly and imperfectly infer income, then the variance of output under insiders' policy is lower than in the first-best case:* $var(q_t) = H^2 var(x_t) < var(x_t) = var(q_t^o)$*. Insiders therefore engage in real smoothing.*

Real smoothing implies that output is less sensitive to changes in the state variable than would be the case under the first-best production policy ($H = 1$). This effect can be economically significant if outside ownership or income volatility is high, or if economic shocks are highly persistent (see Figure 1 in Section 2.4.1). Real smoothing is, as before (see Property 2.A), caused by underproduction. However, information asymmetry introduced by measurement error does not increase real smoothing. On the contrary, as we show below (see corollary 1), information asymmetry mitigates underproduction (by raising H) and, as a result, reduces the amount of real smoothing. For extreme levels of measurement error ($R = \infty$), production becomes first-best, and all real smoothing is eliminated. Intuitively, the noisier sales are, the less outsiders learn from sales in forming their beliefs (as reflected by a lower Kalman gain K). This, in turn, reduces insiders' incentive to manipulate production.

2.2 The dynamics of income and payout

Proposition 4 implies the following property:

Property 3.B. $E_{S,t}[\pi_t - \hat{\pi}_t] = 0$ *and* $E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = h^2 P(1 - HK)$*: In the presence of measurement error (*$R > 0$*), outsiders' estimate of the firm's income and of the latent variable* x_t *is unbiased but not exact.*

As insiders' private information is no longer fully revealed to outside investors, it follows that markets are no longer strong-form efficient (unlike Stein 1989, where the stock price always equals its fundamental value). Recall that this result is based on the assumption that insiders cannot trade on their private information.

Property 4.B. *If outsiders infer income from noisy sales, then insiders' and outsiders' claim values are, respectively, given by* $S_t = \theta V(\hat{x}_t; \theta)$ *and* $M_t = V(x_t; \theta) - S_t$*, where* $V(\hat{x}_t; \theta)$ *is defined* *by Equation (38) in Proposition 6. Insiders and outsiders are jointly worse off compared with the first-best income, that is,* $S_t + M_t = V(x_t;\theta) < V(x_t;\theta = 0)$. *Insiders and outsiders are on average also individually worse off. However, it is possible at any given instance that either insiders or outsiders (but not both) are better off than under the first-best case if, respectively,* $M_t > (1-\theta)V(x_t;\theta = 0)$ *or* $S_t > \theta V(x_t;\theta = 0)$.

Because $\pi(q_t) < \pi(q_t^o)$ at all times, insiders and outsiders are jointly always worse off compared with the cooperative, first-best outcome. This does not exclude, however, that at any given instant one party (but not both) is better off than in the first-best case. This happens if either $M_t > (1-\theta)V(x_t;\theta = 0)$ or $S_t > \theta V(x_t;\theta = 0)$. Hence, outsiders (insiders) are better off compared with the first-best scenario if sales figures and therefore perceived income are sufficiently inflated (deflated) by measurement error, that is, if $\hat{x}_t >> x_t$ ($\hat{x}_t << x_t$).

Proposition 5 presents the dynamics of the firm's expected income and its payout:

Proposition 5. *The firm's expected income,* $\hat{\pi}_t (= E_{S,t}[\pi_t] = h\hat{x}_t)$, *is described by the following partial adjustment model:*

$$\hat{\pi}_t \;=\; \lambda A \hat{\pi}_{t-1} \;+\; Khs_t \;+\; h\lambda B \; . \tag{31}$$

The firm's payout to outside shareholders is described by the following target adjustment model:

$$d_t \;=\; d_{t-1} \;+\; (1-\lambda A)\,(d_t^* \;-\; d_{t-1}) \tag{32}$$

$$\;=\; \lambda A d_{t-1} \;+\; \theta Khs_t \;+\; \theta h\lambda B \;\equiv\; \gamma_2\, d_{t-1} \;+\; \gamma_1\, s_t \;+\; \gamma_0 \; . \tag{33}$$

The payout "target" d_t^* *is given by:*

$$d_t^* \;=\; \frac{\theta h\lambda B}{1-\lambda A} \;+\; \left(\frac{\theta Kh}{1-\lambda A}\right) s_t \;\equiv\; \gamma_0^* \;+\; \gamma_1^*\, s_t \; . \tag{34}$$

The speed of adjustment coefficient is given by $SOA \equiv (1-\lambda A)$ *with* $0 < SOA \leq 1$.

Payout (d_t) follows a target (d^*) that is determined by the contemporaneous level of sales. However, as Equation (32) shows, payout only gradually adjusts to changes in sales because the SOA coefficient $(1-\lambda A)$ is less than unity. This leads to payout smoothing in the sense that the effect on payout of a shock to sales is distributed over time. In particular, a dollar increase in sales leads to an immediate increase in payout of only θhK. The lagged incremental effects in subsequent periods are given by $\theta hK\lambda A$, $\theta hK(\lambda A)^2$, $\theta hK(\lambda A)^3$,... The long-run effect of a dollar increase in sales on payout equals $\theta hK \sum_{j=0}^{\infty} (\lambda A)^j = \frac{\theta hK}{1-\lambda A}$, which is the slope coefficient γ_1^* of the payout target d_t^* (see Equation (34)). In contrast, with symmetric information, the impact of a shock to sales is fully impounded into payout immediately.

Intuitively, payout only partially adjusts to a contemporaneous shock in sales because in the short run, outsiders cannot distinguish between a transitory measurement error and a persistent shock to the latent cost variable. However, as subsequent sales are observed, the transitory or persistent nature of the shock is gradually revealed. Payout can therefore also be expressed as a distributed lag model in which it is a function of current and past sales, by repeated backward substitution of Equation (31):

$$d_t = \frac{\theta h \lambda B}{1-\lambda A} + \theta K h \sum_{j=0}^{\infty} (\lambda A)^j \, s_{t-j} \; . \tag{35}$$

Expected income $\hat{\pi}_t$ displays similar dynamics as payout. In fact, one merely needs to set $\theta = 1$ in Equation (35) for payout to obtain the corresponding expressions for expected income.

Given that (i) payout is based on expected income and (ii) expected income is smooth relative to actual income, it follows that payout is smooth relative to actual income and that insiders soak up the variation. The ratio $\Sigma \equiv var(d_t)/var(\theta\pi_t) = var(\hat{\pi}_t)/var(\pi_t)$ measures the fraction of income variation that is transmitted to payout. We know that in the absence of measurement error (see Section 1) this ratio equals one because payout tracks actual income exactly. A ratio less than one indicates that payout is smoothed relative to income. It follows that $1 - \Sigma$ equals the fraction of income variation that is not reflected in payout but absorbed by borrowing and saving. $1 - \Sigma$ is therefore a measure of financial smoothing, as well as a measure of payout smoothing. One can then prove (see Appendix) the following property.

Property 5.B. *If there is information asymmetry ($R > 0$) due to noisy inference, then payout is* *smooth relative to actual income:*

$$var(\hat{\pi}_t) \;=\; \frac{h^2 H^2 P^2 \left(H^2 Q + R\right)}{\left(H^2 P \,+\, R\right)^2} \;<\; h^2 Q \;=\; var(\pi_t) \;<\; \frac{Q}{4} \;=\; var(\pi_t^o) \tag{36}$$

$$\Sigma \;\equiv\; \frac{var(d_t)}{var(\theta\pi_t)} \;=\; \frac{var(\hat{\pi}_t)}{var(\pi_t)} \;<\; 1 \tag{37}$$

In the presence of asymmetric information ($R > 0$), payout is smooth relative to income. The higher the degree of information asymmetry, the more payout smoothing and, therefore, also financial smoothing. In the extreme case where the observable is pure noise ($R \to \infty$), payout becomes perfectly smooth: $\lim_{R\to\infty} var(d_t) = 0$ and $\lim_{R\to\infty} \Sigma = 0$. In the absence of information asymmetry ($R = 0$), payout moves in lockstep with income: $\lim_{R\to 0} var(d_t) = \theta^2 h^2 Q$ and $\lim_{R\to 0} \Sigma = 1$, which confirms Property 5.A for the symmetric information case.

We previously argued that (i) real smoothing is caused by insiders' incentives to underproduce and (ii) asymmetric information due to imperfect inference mitigates underproduction and therefore the degree of real smoothing. Property 5.B shows that financial (or payout) smoothing is caused by asymmetric information. If there is no asymmetric information, then there is no financial smoothing. However, as Equations (36) and (37) make clear, the degree of financial smoothing is influenced (through H) by insiders' incentives to underproduce (as determined, for example, by outsiders' ownership stake θ). Financial smoothing is, therefore, inextricably linked with insiders' production policy, and we examine this link in more detail in Section 2.4.1.

Property 6.B. *With imperfect inference, insiders no longer follow a myopic policy.*

With imperfect inference, outsiders' best income estimate is determined by the entire history of sales. An estimate based on contemporaneous sales only is no longer optimal, as this estimate is unduly influenced by measurement error. Given that current sales affect outsiders' current and future beliefs, it follows that a myopic production strategy by insiders is not optimal. Instead, insiders consider the effect of output on outsiders' current and future payout demands and optimize accordingly. This cumulative lagged effect on output is captured by the multiplier $1/(1 - \beta\lambda A)$ in Equation (24). Under a myopic production policy, outsiders would adopt a higher output level $q_t \;=\; [1 \;-\; \theta h K]\, x_t$ instead of the output described by Equation (24). Outsiders' retrospection in forming beliefs and the resulting use of current and past sales to determine future income targets (similar to the "ratchet effect") therefore, ceteris paribus, aggravates underproduction.

2.3 Payout smoothing and the Lintner model

The payout model (33) is very similar to the well-known Lintner 1956 dividend model.[18] Hundreds of papers have tested the Lintner model and estimated the Lintner constant and SOA. However, in the absence of a formal theoretical underpinning for the Lintner model, little is known a priori regarding the magnitude and behavior of the Lintner constant and SOA. In particular, Lintner 1956 notes, "The constant will be zero for some companies but will generally be positive to reflect the greater reluctance to reduce than to raise dividends ... as well as the influence of the specific desire for a gradual growth in dividend payments found in about a third of the companies visited."

Lintner 1956 also finds an SOA of about 0.3 using aggregate data on corporate earnings and dividends.[19] Recall that SOA=1 implies instantaneous adjustment and therefore no smoothing, whereas SOA=0 means that payout no longer changes from one period to the next. The Lintner SOA implies a half-life for adjustment of payout to changes in income. Half-life is the time needed to close the gap between the actual and target payout by 50%, after a one-unit shock to the error term in the Lintner model equation. When payout follows an AR(1) process, half-life is log(0.5)/ log(1–SOA). If the SOA equals, say 0.3, then the half-life is about two years, and it would take the firm about 6.5 years to close the gap between the actual payout and the target by 90%. Thus, payout is history dependent, and for reasonable parameters the history extends back several years.

Our model provides closed-form structural expressions for the SOA (given by $1 - \lambda A$) and the Lintner constant (given by $\theta h \lambda B$). This allows us to give an economic interpretation to the Lintner model and to explore how the coefficients in the Lintner model depend on key economic variables.

For example, our model confirms that the constant ($\theta h \lambda B$) is positive provided that income has a positive drift, B. This is consistent with empirical evidence and Lintner's observation that dividends tend to grow over time. More importantly, our model explains that this growth in dividends is linked to growth in the firm's income. The Lintner constant is a nonlinear function of the main model parameters. Numerical model simulations (available upon request) reveal, for example, that the constant $\theta h \lambda B$ is an inverted U-shaped function of outsiders' ownership stake θ. We refer the reader to Section 2.4.3 for more details regarding ownership structure and its effect on firm value. Figure 2 in the comparative statics Section 2.4.2 illustrates and summarizes the effect of the main model parameters (θ, A, R, and Q) on the speed of adjustment (SOA) of payout to the payout target.

2.4 Comparative statics and further results

2.4.1 Asymmetric information and the production decision. The following corollary explains the effect of asymmetric information on production.

Corollary 1. *The noisier the link between the latent variable (x_t) and its observable proxy (s_t) (and hence the more information asymmetry), the weaker insiders' incentive to manipulate the proxy by underproducing. In particular, insiders' production decision converges to the first-best one as the variance of measurement errors becomes infinitely large ($R \to \infty$) or as uncertainty with respect to the latent variable x_t decreases ($Q \to 0$), that is, $\lim_{Q\to 0} H = \lim_{R\to\infty} H = 1$. Conversely, the more precise the link between s_t and x_t, the higher the incentive to underproduce. The lower bound for H is achieved for the limiting cases $Q \to \infty$ and $R \to 0$, that is, $\lim_{Q\to\infty} H = \lim_{R\to 0} H = 1 - \frac{\theta}{2-\theta}$.*

When x_t becomes deterministic ($Q = 0$), then the estimation error with respect to x_t goes to zero (i.e., $P \to 0$). This means that the Kalman gain coefficient K becomes zero too (there is no learning). But if there is no learning ($K = 0$ and $\lambda = 1$), then insiders' output decision q_t no longer affects outsiders' estimate of the cost variable, as illustrated by Equation (20). As a result the production policy becomes efficient (i.e., $H = 1$ and $q_t = x_t$).

Similarly, if there are measurement errors, then the link between sales and the latent cost variable becomes noisy. This mitigates the underinvestment problem, because the noise obscures insiders' actions and therefore their incentive to cut production.

In the absence of measurement errors ($R = 0$) the link between sales s_t and the contemporaneous level of the latent variable x_t becomes deterministic.[20] Outsiders know for sure that an increase in sales results from a fall in marginal costs. Therefore, when observing higher sales, outsiders want higher payout. In an attempt to manage outsiders' expectations downward, insiders underproduce. If $R = 0$, then we get the efficient outcome ($H = 1$) only if insiders get all the income ($\theta = 0$); otherwise we get underinvestment ($H < 1$). As the insiders' stake of income goes to zero ($\theta \to 1$) production also goes to zero (that is, $H \to 0$). Both outsiders and insiders get nothing, even though the firm could be highly profitable. This result is in sharp contrast with the symmetric information or direct inference cases where the efficient outcome is obtained no matter how small the insiders' share of the income. Thus, for firms where insiders have a very small ownership stake (e.g., public firms with a highly dispersed ownership structure), the process of indirect inference by outsiders could undermine the firm's very existence.

Figure 1 illustrates the effect of the key model parameters (R, Q^*, A, and θ) on production efficiency. Efficiency is measured with respect to two different variables: the output level (q_t) and the income (π_t). The degree of efficiency is determined by comparing the actual outcome with the first-best outcome, that is, $q_t/q_t^o = H$ (dashed line), and $\pi_t \ \pi_t^o = 2h$ (solid line). As discussed before, H can also be interpreted as a measure of real smoothing. $H = 1$ indicates no real smoothing, whereas lower values for H indicate a higher degree of real smoothing.

The figure shows that the efficiency loss is larger with respect to output than income because the loss in revenues due to underproduction is to some extent offset by lower costs of production. Panels A and B confirm that full efficiency is achieved as R moves toward ∞ and for $Q^* = 0$. Panel C shows that a higher autocorrelation in marginal costs substantially reduces efficiency because it allows outsiders to infer more information about the latent cost variable from sales and therefore gives insiders stronger incentives to distort production. Finally, panel D shows that production is fully efficient if outsiders have no real stake in the firm's income (i.e., $\theta = 0$). Efficiency severely declines as outsiders' stake increases. For $\theta = 1$, insiders achieve only 28% of the first-best output level. However, one can show that as $Q/R \to 0$, incentives are fully restored, and the first-best outcome can be achieved even for $\theta = 1$. This confirms that the root cause of underproduction is the process of indirect inference and not the outside ownership stake per se. The firm's ownership structure serves, however, as a transmission mechanism through which inefficiencies can be amplified.

2.4.2 Asymmetric information and payout smoothing. The following corollary results from Property 5.B and summarizes how asymmetric information affects contemporaneous payout smoothing:

Corollary 2. *The variance of payout is a fraction of the variance of (outsiders' share of) contemporaneous income, that is,* $0 \leq \Sigma \equiv \frac{var(d_t)}{var(\theta\pi_t)} \leq 1$*. A lower degree of information asymmetry (i.e., lower R/Q) leads to less contemporaneous payout smoothing. In the limiting case where outsiders* *can accurately infer income (i.e., $R/Q = 0$) there is no contemporaneous smoothing ($\Sigma = 1$). In the other extreme where observable sales are infinitely noisy ($R/Q = \infty$), payout is perfectly smooth relative to income ($\Sigma = 0$).*

Payout (d_t) is less volatile than outsiders' share of contemporaneous income ($\theta\pi_t$). The excess variation in income is absorbed by insiders through borrowing and savings. In the extreme case where $R/Q = \infty$, outsiders no longer learn anything from sales ($K = 0$) and payout becomes deterministic (see Equation (33)). The following corollary summarizes how asymmetric information affects intertemporal payout smoothing:

Corollary 3. *Measurement errors create asymmetric information. This, in turn, leads to intertemporal payout smoothing, that is, the effect of shocks in observable sales on payout is distributed over time. A lower degree of information asymmetry (i.e., lower R/Q) leads to less intertemporal smoothing. In the limiting case where outsiders can accurately infer income (i.e., $R/Q = 0$), payout is always on target and coincides at all times with outsiders' share of actual income (i.e., $d_t = d_t^* = \theta\pi_t$ for all t).*

No payout and financial smoothing whatsoever occurs when $R = 0$ because in that case all information asymmetry is eliminated. In the absence of measurement errors, it is possible to infer the marginal cost variable x_t with 100% accuracy from the observed sales figure s_t. The same result obtains when $Q \to \infty$ because in that case measurement errors are negligibly small compared with the variance of the latent cost variable. This important result confirms that asymmetric information and not uncertainty per se is the root cause of payout smoothing. The corollary also confirms that as the degree of information asymmetry goes to zero, our rational expectations equilibrium converges to the simple sharing rule that prevails under symmetric information. Indeed: $\lim_{R\to 0} d_t = \theta \lim_{R\to 0} \hat{\pi}_t = \theta\pi_t$.

Figure 2 illustrates and summarizes the effect of the main model parameters (θ, A, R, and Q) on (i) contemporaneous smoothing as captured by $1-\Sigma$ and (ii) intertemporal smoothing as captured by $1 - SOA$. Recall that no smoothing (i.e., $SOA = 1$ and $\Sigma = 1$) occurs under symmetric information.

Panel A plots Σ (solid line) and SOA (dashed line) as a function of the (real) outside ownership stake θ. $\Sigma \approx SOA \approx 0.86$ for $\theta = 0$. We know that production is first-best if outsiders do not have a stake in the firm. Consequently, $1 - SOA$ and $1 - \Sigma$ (about 14%) captures financial smoothing only as no real smoothing is involved. However, for $\theta = 0$, payout smoothing is purely virtual as no actual payout is made. Still, insiders try to meet outsiders' expectations at all times (no matter how small outsiders' stake in the firm), causing reported income to match expected income.

$\Sigma = 0.86$ means that payout to outsiders absorbs 86% of contemporaneous variation in income. This figure declines to about 0.64 for $\theta = 1$. The SOA of 0.86 (dashed line) for $\theta = 0$ implies a half-life of about 0.35 years for adjustment of reported income to changes in sales. Increasing θ introduces, however, additional real smoothing, and this reduces the SOA from 0.86 (for $\theta = 0$) to 0.49 (for $\theta = 1$) corresponding, respectively, to a half-life of 0.35 years and 1.03 years. The plot confirms our earlier results that reducing inside ownership leads to severe underproduction, which in turn leads to a smoother payout flow because payout becomes less sensitive to sales.

Increasing the degree of autocorrelation in the latent cost variable (A) increases real smoothing (by reducing H) but also increases the variance of outsiders' income estimate (P), both of which affect $var(\hat{\pi}_t)$ in the opposite direction. This explains why contemporaneous smoothing first increases and subsequently decreases with A (see panel B). Intertemporal smoothing monotonically increases with A. No intertemporal smoothing takes place when $A = 0$ because in that case current and past realizations of x_t are irrelevant for the future. As a result, insiders' private information about x_t is also irrelevant for the future. Note that higher autocorrelation raises both real and financial smoothing substantially.

Finally, panels C and D confirm that the contemporaneous and intertemporal payout smoothing both increase with the degree of information asymmetry (as reflected by a higher R or lower Q). Paradoxically, more intertemporal smoothing coincides with higher production efficiency (see Figure 1): when outsiders can infer less from sales (and payout smoothing is more prevalent), there is also less of an incentive to manipulate production.

To summarize, our model captures the history dependence of payout and allows us to link contemporaneous and intertemporal payout smoothing to key economic determinants. Figure 2 shows that contemporaneous smoothing $(1-\Sigma)$ increases with outsiders' ownership stake (θ) and the variance of measurement errors (R), but decreases with the variance of income (Q). Contemporaneous smoothing is non-monotonic in the degree of income persistence (A). Figure 2 also illustrates that the intertemporal smoothing increases with outsiders' ownership stake (θ), the degree of income persistence (A), and the variance of measurement errors (R), but decreases with the variance of the (untruncated) latent variable (Q^*). These are predictions that could be empirically tested. The figure also shows that for reasonable parameters, our model tends to generate SOAs in the 0.5 to 0.8 range, well above the empirically observed estimates. Our model is, however, not capturing features such as risk aversion and habit formation, which induce further smoothing and reduce the SOA (see Lambrecht and Myers 2012).

2.4.3 Ownership structure and firm value. The following proposition states the outside equity value, $\theta V(\hat{x}_t)$.

Proposition 6. *The outside equity value of the firm is given by:*

$$\theta V(\hat{x}_t;\theta) \;=\; \frac{\theta h}{(1-\beta A)}\left(\hat{x}_t + \frac{B\beta}{1-\beta}\right) \tag{38}$$

We know that, for a given value of $\hat{x}_t$, the firm value $V(\hat{x}_t;\theta)$ monotonically declines in the ownership stake θ and that the first-best firm value is achieved when the outside ownership stake is zero (i.e., $\theta = 0$). Numerical simulations (available upon request) show that as much as half of the firm value can be lost as θ varies from zero to one. Numerical simulations also show that the outside equity value $\theta V(\hat{x}_t;\theta)$ is an inverted U-shaped function of θ that reaches a unique maximum, hereby resembling an "outside equity Laffer curve".[21] This result has important empirical implications for the relation between ownership structure and firm value (see Section 3.3) and the behavior of public versus private firms (see Section 3.4).

2.5 Perfect versus imperfect inference

We now summarize our results so far by comparing Properties 1.A to 6.A (for the perfect inference case) with Properties 1.B to 6.B (for the imperfect inference case):

(i) Indirect inference leads to underproduction. Underproduction is less severe with imperfect inference.

(ii) Indirect inference leads to real smoothing. All real smoothing can be traced back to underproduction, and therefore there is less real smoothing with imperfect inference than with perfect inference.

(iii) With perfect inference, markets are strong-form efficient (i.e., all information asymmetry is resolved in equilibrium). With imperfect information, markets are semi-strong form efficient. Insiders' private information is no longer accurately revealed in equilibrium, but outsiders' income estimates are unbiased.

(iv) With perfect inference, there is no payout smoothing: payout moves in lockstep with actual income. Imperfect inference causes payout to be smooth relative to actual income. The noisier the inference, the stronger the degree of payout smoothing.

(v) With perfect (imperfect) inference, insiders and outsiders are always (on average) worse off compared with a first-best policy.

(vi) With perfect inference, a myopic policy remains optimal (as is the case for the first-best policy). With imperfect inference, insiders adopt an optimal policy that explicitly accounts for the effect of output not only on outsiders' current but also future income beliefs and payout demands. The exception to this latter rule is when the latent variable has zero persistence ($A = 0$). From Equation (31), it follows immediately that outsiders' income estimate depends on contemporaneous sales only if $A = 0$ because current and past shocks to the state variable have no bearing for the future. Similarly, if $A = 0$, then the optimal output policy is a myopic one.

2.6 Latent marginal revenues

Our analysis so far assumed marginal costs to be the latent variable. We now reformulate the imperfect inference model in terms of latent marginal revenues. Similar to Section 1.2.2, the income function is given by $\pi_t = q_t\sqrt{x_t} - \frac{q_t^2}{2}$. Outsiders observe noisy sales $s_t = q_t\sqrt{x_t} + \epsilon_t$ with x_t given by the AR(1) process of Equation (17) and where ϵ_t is an i.i.d. noise term with zero mean and variance R (i.e., $\epsilon_t \sim N(0, R)$). Outsiders conjecture that $q_t = H\sqrt{x_t}$. Substituting outsiders' beliefs into insiders' objective function (4), insiders' optimization gives the following first-order condition:

$$\frac{\partial M_t}{\partial q_t} = \sqrt{x_t} - q_t - \theta h K \sqrt{x_t}\left[1 + \beta\lambda A + (\beta\lambda A)^2 + (\beta\lambda A)^3 + ...\right] = 0 \tag{39}$$

The subsequent derivation and its solution are exactly the same as for Proposition 4. We obtain the result that in equilibrium $q_t = H\sqrt{x_t}$ and $\hat{\pi}_t = \left(H - \frac{H^2}{2}\right)\hat{x}_t$, where H and $\hat{x}_t$ are as defined in Proposition 4.

We could do additional robustness checks by adopting alternative income specifications. Although solutions are likely to be numerical, payout (or "financial") smoothing will always arise if there is measurement error because payout is based on outsiders' expectations, which are smooth relative to actual realizations. Similarly, underproduction and real smoothing are generic features of the equilibrium, even in the absence of measurement error (see online Appendix B).

3 Empirical Implications

Our paper provides empirical implications for a variety of literatures in financial economics.

3.1 Time-series and cross-sectional implications

The time-series properties of income and payout were discussed in great detail in Sections 2.2 and 2.3. In terms of cross-sectional analysis, our model predicts that the speed of adjustment toward the payout target should decrease with the degree of information asymmetry between inside and outside investors and with the degree of persistence (autocorrelation) in income. Our predictions are novel and can be easily tested using panel data on income and payout.

3.2 Real smoothing

Our model predicts that if insiders face capital market pressure, then asymmetric information and the resulting inference process lead to underproduction by firms. Furthermore, outsiders' use of current performance (sales) as a basis for determining future income targets shares similarities with what is often referred to as "target ratcheting." The tendency of performance targets to increase following good performance creates incentives for managers to withhold effort, a phenomenon commonly referred to as the "ratchet effect" (see, for example, Weitzman 1980). Similarly, in our model, insiders know that increasing production raises outsiders' future expectations, and this strengthens insiders' incentive to underproduce.

A large stream of analytical research has studied how the efficiency of centrally planned economies is undermined by the ratchet effect (for example, Weitzman 1980, among others). The accounting literature has contributed empirical evidence to this area of research by establishing that target ratcheting can potentially affect market economies as much as centrally planned economies. Several studies (see Indjejikian, Matejka, and Schloetzer 2014 for a review) document that firms engage in target ratcheting. There is also some evidence (see Indjejikian, Matejka, and Schloetzer 2014) that managers who are successful at meeting most of their annual performance target reduce their effort at the end of the year.[22] Greater peer group quality increases sensitivity of target revisions to past peer performance, reduces sensitivity to past own performance, and alleviates the extent to which managers withhold effort at the end of the year. This is consistent with our model, which predicts that incentives to manipulate decrease when the link between insiders' production decision and the income target is weakened.

The accounting literature provides convincing evidence of a related, but somewhat different form of real smoothing –namely the fact that managers underinvest in order to meet (rather than manage) analyst's expectations.[23] There is a subtle difference between this type of real smoothing and the one described in our paper. The former type of real smoothing is driven by managers' need to meet an earnings target, while underproduction and real smoothing in our paper are motivated by managers' incentive to manage outsiders' expectation (similar to a ratchet effect). In some way managers in our model are proactive and forward looking as in Bouwens and Kroos 2011: they know they need to meet expectations, and therefore proactively manage expectations by underinvesting. Managers' behavior as described by Graham, Harvey, and Rajgopal 2005, Roychowdhury 2006, Bhojraj et al. 2009, and Daniel, Denis and Naveen 2012 appears reactive in nature: managers are at risk of missing the earnings target and respond by cutting investment to make up for the shortfall.

3.3 Corporate ownership structure

First, our model predicts that the degree of income smoothing should increase in the cross-section of firms as outside ownership increases. Kamin and Ronen 1978 and Amihud, Kamin, and Ronen 1983 show that owner-controlled firms do not smooth as much as manager-controlled firms. Prencipe, Bar-Yosef, Mazzola, and Pozza 2011 also provide direct evidence for this. They find that income smoothing is less likely among family-controlled companies than non-family-controlled companies in a set of Italian firms.

Second, in our model underproduction is more severe the smaller is the inside ownership, and this results in an "outside equity Laffer curve." Morck, Shleifer, and Vishny 1988 document a non-monotonic relation between Tobin's q and managerial stock ownership, and McConnell and Servaes 1990 report an "inverted-U" or "hump-shaped" relation between q and managerial ownership. Numerous successors investigate the ownership-performance relation using different data, various measures of performance and ownership structure, and alternative empirical methods. The standard interpretation of the hump-shaped performance-ownership relation is that incentive alignment effects dominate for low inside ownership, but as managerial ownership increases, these incentive benefits eventually are overtaken on the margin by the cost of an increased managerial ability to pursue non-value-maximizing activities without being disciplined by shareholders. Our paper provides a new explanation for the non-monotonic relation between Tobin's q and managerial stock ownership.[24]

3.4 Public versus private firms

Public (private) firms tend to have a high outside (inside) ownership. Our model therefore has a number of implications for the behavior of public versus private firms.

(i) The model's main prediction is that public firms underproduce and that their output is less sensitive to economic shocks. Asker, Farre-Mensa, and Ljungqvist Forthcoming evaluate differences in investment behavior between stock-market-listed and privately held firms in the United States. Listed firms invest less and are less responsive to changes in investment opportunities compared with matched private firms, especially in industries in which stock prices are particularly sensitive to current earnings. Their result is consistent with what is predicted by our model, in that firms with a higher outside ownership produce less and production is less sensitive to changes in the marginal cost variable. This result follows from the fact that insiders become increasingly concerned about "ratcheting up" outsiders' expectations as outsiders' stake in the firm increases.

(ii) Because smoother income leads to smoother payout, one would expect, all else equal, that public firms also smooth payout more than private firms. This implication is consistent with Michaely and Roberts 2012, who show that private firms smooth dividends less than their public counterparts.

(iii) Our model shows that as real ownership of outside shareholders approaches 100%, the existence of the firm is in doubt. How can public firms with a low ownership stake then exist? One solution may be the introduction of audited disclosure, provided that disclosure and accounting statements more generally provide a direct link between the firm's fundamentals and stakeholders' income. For example, income figures that are independently provided by auditors and based on direct inference of fundamentals improve production efficiency because they reduce insiders' incentives to manipulate income through their production policy. Thus, all else equal, higher quality accounting information should increase firm productivity, stock market capitalization, and, more generally, economic growth (as confirmed, for instance, by Rajan and Zingales 1998). On the other hand, if auditing simply focuses on getting more accurate measurements of signals (such as sales) that depend on the firm's fundamentals, then audited disclosure may be counterproductive because it will increase insiders' incentives to manipulate these signals.

4 Related Literature

We now briefly review the related literature. In a seminal paper concerning the firm and capital market interaction, Stein 1989 considers an environment where insiders can pump up current earnings by secretly borrowing at the expense of next period's earnings. Stein 1989 shows that insiders do not engage in value-destroying manipulation if they only care about current and future earnings. Incentives to manipulate arise, however, if insiders also care about the firm's stock price. The market anticipates, however, that insiders engage in this form of "signal jamming" and is not fooled. Despite the fact that stock prices instantaneously reveal all information, insiders are "trapped" into behaving myopically. Thus, stock market pressures can have a dark side, even if markets are fully efficient.

There are several important differences between our model and Stein's. In Stein 1989, myopic managerial behavior takes the form of an attempt to inflate earnings so as to boost stock prices. In contrast, in our model, insiders are not directly concerned about stock prices, but fear intervention by outsiders when their expectations are not met; as a result, insiders manage earnings downward and underproduce so as not to set outsiders' expectations about future income too high. Further, in Stein 1989 the time-series properties of observed earnings and unmanipulated earnings are essentially the same. In contrast, in our model, outsiders' income is smooth compared with actual income and follows a simple partial adjustment model that can be linked to the underlying economic fundamentals in a very transparent and empirically testable fashion.[25] Finally, our model has important implications for payout smoothing and corporate ownership structure, whereas Stein 1989 remains silent on these matters.

In our model, market pressures imply that insiders must meet payout expectations and disgorge cash to outside investors. To this end, we call upon the investor protection framework described in Fluck 1998, 1999, Myers 2000, Jin and Myers 2006, Lambrecht and Myers 2007, 2008, and 2012, Acharya, Myers, and Rajan 2011, among others. With the exception of Jin and Myers 2006, these papers assume symmetric information between insiders and outsiders. While under symmetric information outsiders know exactly what they are due, under asymmetric information outsiders refrain from intervention for as long as the reported income (and corresponding payout) meet their expectations. Therefore, in Jin and Myers 2006 insiders pay out according to outsiders' expectations of cash flows and absorb the residual variation, as is also the case in our model.[26] But Jin and Myers 2006 also differs from our model in a number of fundamental ways. While in their model the actual income process is completely exogenous, in our model income is endogenously determined through insiders' output decision. This allows us to identify the effect of asymmetric information on insiders' production decisions (real smoothing). Also, in Jin and Myers 2006 outsiders base their income estimates at each moment in time on their initial prior information, and they do not learn about the evolution of the latent income component. As a result, there is no intertemporal smoothing in their model. In our model, outsiders observe sales, a noisy proxy for output, which allows them to update their expectations regarding the latent marginal cost variable.

Note that the basic mechanism in our model can be considered similar to that in a strand of signal-jamming equilibrium models in which the indirect inference process distorts corporate choices. This informational effect is similar to the ones discussed (albeit in different economic settings) in Milgrom and Roberts 1982, Riordan 1985, Gal-Or 1987, Stein 1989, Holmström 1999, and more recently Bagnoli and Watts 2010.[27] The learning process (which we model as a filtering problem) and the resulting intertemporal smoothing are, however, quite different from existing papers.

Finally, our paper is also linked to a small but growing literature on payout smoothing. Kumar 1988 derives a coarse signaling equilibrium in which a firm's dividends are more stable than its performance and prospects. Guttman, Kadan, and Kandel 2010 derive an equilibrium in a Miller and Rock 1985 setup in which dividends are constant over a range of earnings. In DeMarzo and Sannikov 2011, the agent and the firm start out with zero cash, but accumulate cash in order to build a buffer stock to absorb cash and avoid inefficient liquidation. Once sufficient cash is accumulated, dividends are paid, and the optimal dividends are smoother than earnings. Lambrecht and Myers 2012 derive a Lintner model of payout based on managerial risk aversion and habit formation. Unlike these papers, our model delivers income and payout smoothing jointly, and these are associated with underinvestment and therefore a real cost for the firm.

5 Conclusion

The theory of income smoothing developed in this paper assumes that (i) insiders have information about income that outside shareholders do not, but (ii) outsiders are endowed with property rights that enable them to take collective action against insiders if they do not receive a fair payout that meets their expectations. We showed that insiders try to manage outsiders' expectations. Furthermore, insiders report income consistent with outsiders' expectations based on available information rather than the true income. This gives rise to a theory of inter-temporal smoothing – both real and financial – in which observed income and payout adjust partially and over time in response to economic shocks, and insiders underinvest in production. The root cause of underproduction and real smoothing is the process of indirect inference through which outsiders make inferences about the firm's fundamentals from observables (such as sales) that can be manipulated by insiders. Insiders underproduce in an attempt to manage downwards outsiders' expectations about future income. The amount of output lost is higher in booms than in recessions, which reduces the output variance (i.e., real smoothing).

Interestingly, the underproduction problem is more severe the smaller the inside ownership is and thus should be a greater hindrance to the functioning of publicly (or dispersedly) owned firms. We show that the firm's outside equity value is an inverted U-shaped function of outsiders' ownership stake. This "outside equity Laffer curve" shows that the underinvestment problem severely limits the firm's capacity to raise outside equity. However, a disclosure environment with independent auditing that focuses on reporting economic fundamentals rather than improving the measurement accuracy of variables that can be manipulated (such as sales) can help mitigate the problem. This leads to the conclusion that accounting quality can enhance investments, size of public stock markets, and economic growth. This theory of inter-temporal smoothing of income and payout conforms not only to several existing findings (such as the Lintner 1956 model of payout policy), but also leads to a range of testable empirical implications in the cross-section of firms as information asymmetry and ownership structure are varied. These implications are worthy of empirical investigation.

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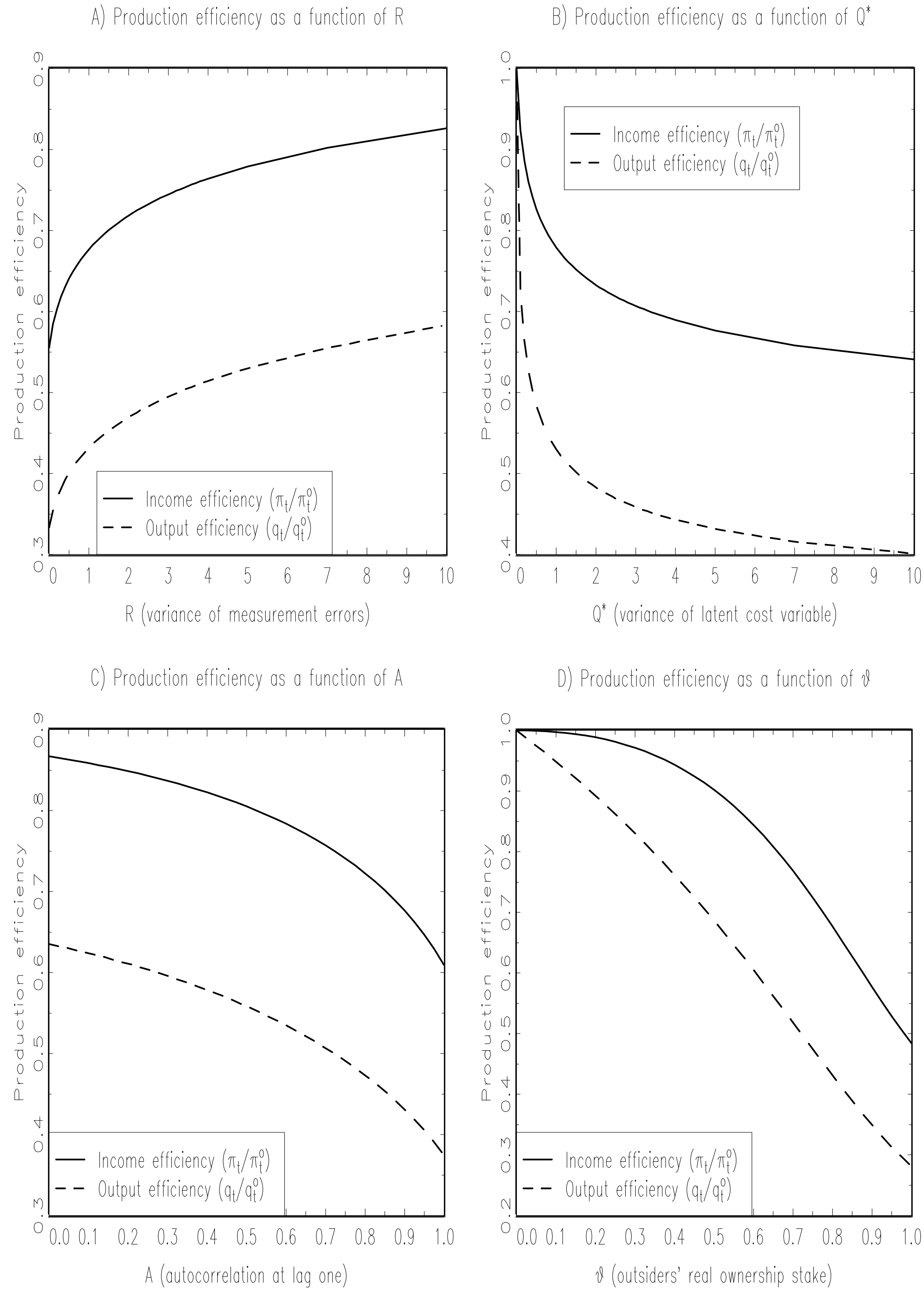

Figure 1.
Production efficiency
The figure examines how production efficiency is affected by the variance of measurement errors (R), the variance of the latent cost variable x_t (Q^*), the autocorrelation at lag one of the latent cost variable (A), and outsiders' real ownership stake (θ). Production efficiency is measured by comparing output (q_t) and income (π_t) relative to their first-best level. The baseline parameter values used to generate the figures in this paper are: $A = 0.9$, $B = 10$, $Q^* = 5$, $R = 1$, $\beta = 0.95$, and $\theta = 0.8$.

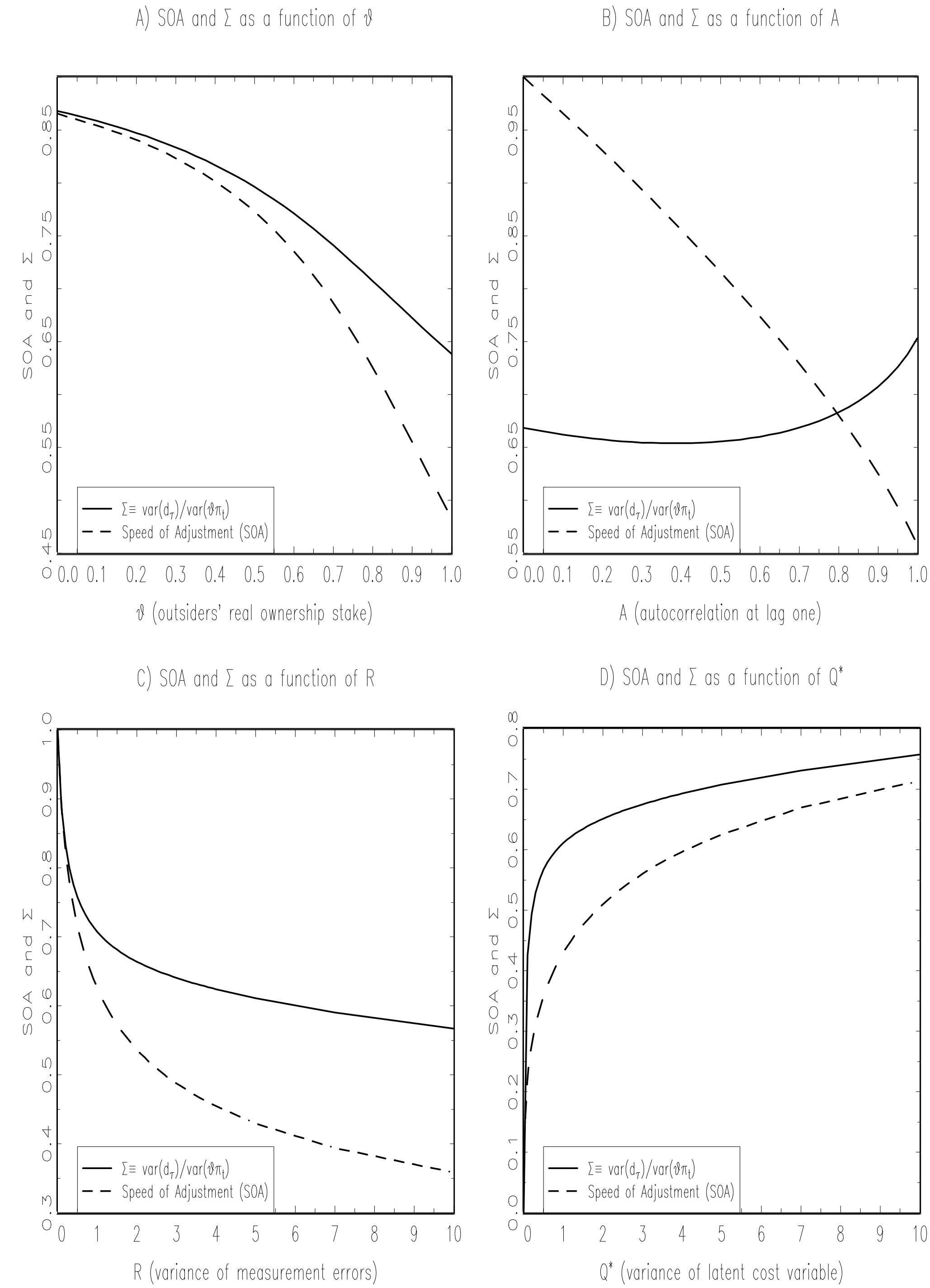

Figure 2.
Payout smoothing

The figure examines how outsiders' real ownership stake (θ), the autocorrelation at lag one of the latent cost variable (A), the variance of measurement errors (R), and the variance of the latent cost variable x_t (Q^*) affect (i) the ratio of the variance of payout to the variance of the corresponding contemporaneous income ($\Sigma \equiv var(d_t)/var(\theta\pi_t)$) and (ii) the speed of adjustment (SOA) of payout to the payout target. The speed of adjustment is given by $SOA = 1 - \lambda A$, with λA capturing the degree of inter-temporal smoothing. The baseline parameter values used to generate the figure are the same as before, that is, $A = 0.9$, $B = 10$, $Q^* = 5$, $R = 1$, $\beta = 0.95$, and $\theta = 0.8$.

Appendix

A.1. Proof of Proposition 3

As before, outsiders conjecture that income is a linear function of sales, that is, $E_{S,t}(\pi_t) \;=\; a_0 s_t + b_0 \;=\; a_0\, q_t \sqrt{x_t} + b_0$. Insiders optimize Equation (4) given outsiders' beliefs. This gives the following first– and second–order conditions:

$$\frac{\partial M_t}{\partial q_t} \;=\; (1-\theta a_0)\sqrt{x_t} \;-\; q_t \;=\; 0 \quad and \quad \frac{\partial^2 M_t}{\partial {q_t}^2} \;=\; -1 \;<\; 0 \tag{40}$$

Solving gives the following optimal output policy: $q_t^* \;=\; (1-\theta a_0)\sqrt{x_t}$. Outsiders' conjecture is verified if and only if:

$$\hat{\pi}(s_t = q_t^* \sqrt{x_t}) \;=\; a_0\,(1-\theta a_0)\,x_t \;+\; b_0 \;=\; (1-\theta a_0)\,x_t \;-\; \frac{(1-\theta a_0)^2\,x_t}{2} \;=\; \pi(q_t^*;x_t) \tag{41}$$

Or, equivalently, if and only if $a_0 \;=\; 1/(2-\theta)$ and $b_0 = 0$. Using the expressions for a_0 and b_0 leads to Proposition 3.

A.2. Proof of Proposition 4

Insiders' optimization problem can be formulated as:

$$M_t \;=\; \max_{\{q_{t+j};j=0..\infty\}} E_t\left[\sum_{j=0}^{\infty}\beta^j\left(\pi(q_{t+j}) \;-\; \theta E_{S,t+j}(\pi(q_{t+j}))\right)\right] \tag{42}$$

We guess the form of the solution and use the method of undetermined coefficients (and subsequently verify our conjecture). The conjectured solution for outsiders' rational expectations based on the information I_t is as follows:

$$E_{S,t}\left[\pi(q_t)\right] \;=\; b \;+\; \sum_{j=0}^{\infty} a_j s_{t-j} \tag{43}$$

where the coefficients b and $a_j (j = 0, 1, ...)$ remain to be determined.

The first-order condition is

$$\frac{\partial M_t}{\partial q_t} = 1 - \frac{q_t}{x_t} - \theta\left(a_0 + \beta a_1 + \beta^2 a_2 + \beta^3 a_3 + ...\right) = 0. \tag{44}$$

$$\Longleftrightarrow \quad q_t = \left[1 - \theta \sum_{j=0}^{\infty} a_j \beta^j\right] x_t \equiv H x_t. \tag{45}$$

Outsiders rationally anticipate this policy and can therefore make inferences about the latent variable x_t on the basis of their observation of current and past sales s_{t-j} $(j = 0, 1, ...)$. We know that $s_t = q_t + \epsilon_t$. This measurement equation can be combined with the state equation (19) to make inferences about x_t on the basis of current and past observations of s_t. This, in turn, allows outsiders to form an estimate of realized income π_t. Because ϵ_t and w_t are zero-mean, uncorrelated white noise, with known variance, the problem corresponds to the standard Kalman Filter (KF) problem as described in textbooks such as Simon (2006, 124–129). It can be shown that the Kalman filter is the optimal linear filter (in terms of minimizing the mean squared error) for the type of problem we are considering (see Simon 2006, 124–129).[28]

Define $\hat{x}_t$ as the KF estimate for x_t based on the measurements s_k up to and including $k = t$. P_t denotes the variance of the estimation error, that is, $P_t = E\left[(x_t - \hat{x}_t)^2\right]$. The solution to the problem is as follows (see, for example, Simon 2006, 128). First, the KF is initialized: $\hat{x}_0 = E(x_0)$ and $P_0 = E\left[(x_0 - \hat{x}_0)^2\right]$, initial values that are assumed to be known and given. The KF is then defined by the following equations (see online Appendix D or Simon 2006, 128–129, for a derivation):

$$\hat{x}_t = \hat{x}_t^- + K_t\left(s_t - H_t \hat{x}_t^-\right) \tag{46}$$

$$P_t = \left(1 - K_t H_t\right) P_t^- \tag{47}$$

where:

$$P_t^- = A^2 P_{t-1} + Q \tag{48}$$

$$K_t = P_t^- H_t \left(H_t^2 P_t^- + R\right)^{-1} \tag{49}$$

$$\hat{x}_t^- = A \hat{x}_{t-1} + B \tag{50}$$

where $\hat{x}_t^-$ and P_t^- denote a priori estimates of $\hat{x}_t$ and P_t, and K_t denotes the Kalman gain. In what follows we focus on the "steady-state" KF, which is the filter for which the Kalman gain K_t becomes time-invariant.[29] To obtain the steady–state filter, we substitute (47) into (48) to obtain:

$$P_{t+1}^- \;=\; A^2 P_t + Q \;=\; A^2 P_t^- \left(1 - H_t K_t\right) + Q \tag{51}$$

Substituting (49) into (51) gives:

$$P_{t+1}^- \;=\; A^2 P_t^- \left[1 - \frac{H_t^2\, P_t^-}{H_t^2 P_t^- + R}\right] + Q \tag{52}$$

The filter has reached a steady state when $P_{t+1}^- = P_t^- = P$ and $H_{t+1} = H_t = H$. Hence the variance of the estimation error for the steady–state filter is given by:

$$P \;=\; A^2 P \left[1 - \frac{H^2\, P}{H^2 P + R}\right] + Q \tag{53}$$

Substituting the expression for P into (49) gives the steady-state Kalman gain K.

It follows (see also Chui and Chen 1991, 78, or Simon 2006) that the error of the steady–state estimator, $x_t - \hat{x}_t$ has zero mean and variance $P(1 - KH)$, that is, $E_{S,t}[x_t - \hat{x}_t] = 0$ and $E_{S,t}[(x_t - \hat{x}_t)^2] = P(1 - KH)$, where $\hat{x}_t$ is given by:

$$\hat{x}_t \;=\; A\hat{x}_{t-1} + B + K\left[s_t - H\left(A\hat{x}_{t-1} + B\right)\right] \;=\; \left(A\hat{x}_{t-1} + B\right)\lambda + K s_t \tag{54}$$

$$\;=\; \frac{B\lambda}{1 - \lambda A} + K\sum_{j=0}^{\infty} \lambda^j A^j s_{t-j}\;, \text{ where} \tag{55}$$

$$\lambda \;\equiv\; (1 - KH) \qquad \textit{and} \qquad K \equiv \frac{H\,P}{H^2 P + R} \tag{56}$$

and where P is the positive root of Equation (30) (online Appendix C proves that (30) has one positive and one negative root).

K is called the "Kalman gain," and it plays a crucial role in the updating process.[30] Using the conjectured solution for q_t, it follows that outsiders' estimate of income at time t is given by:

$$
\begin{aligned}
E_{S,t}[\pi_t] &= E_{St}\left[Hx_t - \frac{H^2 x_t}{2}\right] = \left(H - \frac{H^2}{2}\right)\hat{x}_t && (57)\\
&= \left(H - \frac{H^2}{2}\right)\left[\frac{\lambda B}{1-\lambda A} + K\sum_{j=0}^{\infty}(\lambda A)^j\, s_{t-j}\right] && (58)\\
&= b + \sum_{j=0}^{\infty} a_j s_{t-j} && (59)
\end{aligned}
$$

where the last step follows from our original conjecture given by Equation (43). This allows us to identify the coefficients b and a_j:

$$
\begin{aligned}
b &= \left(H - \frac{H^2}{2}\right)\left[\frac{\lambda B}{1-\lambda A}\right] && (60)\\
a_j &= \left(H - \frac{H^2}{2}\right) K\,(\lambda A)^j && (61)
\end{aligned}
$$

For this to be a rational expectations equilibrium, it has to be that (see Equation (45)):

$$
H = 1 - \theta\sum_{j=0}^{\infty} a_j \beta^j = 1 - \frac{\theta\left(H - \frac{H^2}{2}\right)K}{1-\beta\lambda A} \tag{62}
$$

Simplifying gives the condition for H in the proposition. Fixing outsiders' beliefs (i.e., $E_{S,t}[\pi(q_{t+j})] = \left(H - \frac{H^2}{2}\right)\hat{x}_{t+j} \equiv h\hat{x}_{t+j}$) and solving for insiders' optimal production, it follows from (22)–(24) that insiders' output strategy is a perfect Bayesian equilibrium. One can also immediately verify that the second–order condition for a maximum is satisfied (assuming x_t is positive).

Next, we prove that there exists a unique positive value for H that satisfies (62). Substituting for λ and K, Equation (62) becomes:

$$
f(H) \equiv 1 - H - \frac{\theta H^2 P\left(1 - \frac{H}{2}\right)}{H^2 P + R(1-\beta A)} \equiv 1 - H - g(H) = 0 \tag{63}
$$

Noting that $f(0) = 1 > 0$ and $f(1) = -\frac{\theta P}{2(P+R(1-\beta A))} < 0$, it follows that there exists an $H \in]0,1[$ for which $f(H) = 0$. In online Appendix C, we prove that $f(H)$ is a decreasing function, and therefore the root is unique.

Finally, we calculate the expected value and variance of the estimate's error: $\pi_t - \hat{\pi}_t$. We make use of the result that the error with respect to the posterior steady-state estimator for x_t has zero mean and variance $P(1-KH)$. Hence,

$$E_{S,t}[\pi_t - \hat{\pi}_t] = E_{S,t}\left[h(x_t - \hat{x}_t)\right] = 0 \tag{64}$$

$$E_{S,t}[(\pi_t - \hat{\pi}_t)^2] = E_{S,t}\left[h^2(x_t - \hat{x}_t)^2\right] = h^2\,P(1-KH) \tag{65}$$

A.3. Proof of Proposition 5

Actual income under insiders' production policy is given by:

$$\pi_t = q_t - \frac{q_t^2}{2x_t} = hx_t \tag{66}$$

We know from the proof of Proposition 4 that $\hat{\pi}_t = E_{S,t}[\pi_t] = b + \sum_{j=0}^{\infty} a_j s_{t-j}$ (where the values for b and a_j are defined there). Lagging this expression by one period, it follows that $\hat{\pi}_t - \lambda A \hat{\pi}_{t-1} = hKs_t + h\lambda B$.

Because $d_t = \theta\hat{\pi}_t$, it follows immediately that $d_t = \lambda A d_{t-1} + \theta K h s_t + \theta h \lambda B$. Substituting this expression into the target adjustment model (32) gives:

$$\lambda A\, d_{t-1} + \theta K h s_t + \theta h \lambda B = d_{t-1} + (1-\lambda A)d_t^* - d_{t-1} + \lambda A d_{t-1} \tag{67}$$

Simplifying and solving for d_t^* gives Equation (34).

A.4. Proof of Property 5.B

$$\tfrac{var(d_t)}{var(\theta\pi_t)} = \tfrac{var(\hat{\pi}_t)}{var(\pi_t)} < 1 \tag{68}$$

$$\iff \quad H^4P^2Q + H^2P^2R < H^4P^2Q + 2H^2PRQ + R^2Q \tag{69}$$

$$\iff \quad H^2P^2 < 2H^2PQ + RQ \tag{70}$$

where we assumed that $R > 0$. From Proposition 4, it follows that P is the solution to the following quadratic equation:

$$H^2P^2 = Q\,R - P\left[R(1-A^2) - QH^2\right] \tag{71}$$

Substituting (71) into (70), and simplifying, condition (70) becomes:

$$H^2 P\,Q \,+\, P\,R(1 - A^2) \;>\; 0 \tag{72}$$

which is always satisfied. Hence, $\frac{var(d_t)}{var(\theta\pi_t)} \,<\, 1$. The inequality $var(\pi_t) \,=\, h^2 Q \,<\, \frac{Q}{4} \,=\, var(\pi_t^o)$ follows directly from the fact that $H < 1$.

A.5. Proof of Proposition 6

We know that $E_{S,t}[x_{t+1}] \,=\, A\hat{x}_0 + B$; $E_{S,t}[x_{t+2}] \,=\, A^2\hat{x}_t + AB + B$; $E_{S,t}[x_{t+3}] \,=\, ...$

Therefore, the firm's outside equity value is:

$$\begin{aligned}
\theta V(\hat{x}_t) &= \theta E_t[\sum_{j=0}^{\infty}\beta^j \pi_{t+j}] \,=\, \theta\left[h\,\hat{x}_t \,+\, \beta\,(hA\hat{x}_t \,+\, hB) \,+\, \beta^2\left(hA^2\hat{x}_t \,+\, hAB \,+\, hB\right) \,+\, ...\right] \\
&= \theta\left[h\hat{x}_t\left(1 \,+\, \beta A \,+\, \beta^2 A^2 \,+\, \beta^3 A^3 \,+ ...\right) \,+\, hB\beta\left(1 \,+\, \beta A \,+\, \beta^2 A^2 \,+\, \beta^3 A^3 \,+ ...\right)\right] \\
&\quad +\theta\left[hB\beta^2\left(1 \,+\, \beta A \,+\, \beta^2 A^2 \,+\, ...\right) \,+\, \frac{hB\beta^3}{1 - \beta A} \,+\, \frac{hB\beta^4}{1 - \beta A} \,+\, ...\right] \\
&= \frac{\theta h}{(1 - \beta A)}\left(\hat{x}_t \,+\, \frac{B\beta}{1 - \beta}\right) \qquad (73)
\end{aligned}$$

Notes

[1]According to Investopedia, "Companies indulge in this practice because investors are generally willing to pay a premium for stocks with steady and predictable earnings streams, compared with stocks whose earnings are subject to wild fluctuations." Related reasons often cited for income smoothing are: risk-averse insiders with limited access to external markets trying to insure themselves (Lambert 1984, Dye 1988), managers aiming to maximize their tenure (Fudenberg and Tirole 1995) or to minimize taxes (Graham 2003). Income smoothing can signal good prospects (Ronen and Sadan 1981) or low volatility to reduce the cost of debt (Trueman and Titman 1988). Income smoothing can also encourage liquidity trading by uninformed investors (Goel and Thakor 2003). We refer to Section 4 for a detailed literature review.

[2]Jensen (2005, 8) notes:"Indeed, earnings management has been considered an integral part of every top managers job for at least the last two decades. But when managers smooth earnings to meet market projections, they are not creating value for the firm; they are both lying and making poor decisions that destroy value...when numbers are manipulated to tell the markets what they want to hear (or what managers want them to hear) rather than the true status of the firm it is lying, and when real operating decisions that would maximize value are compromised to meet market expectations real long-term value is being destroyed."

[3]Related theories that explain income manipulation (but not smoothing) are linked to insiders' myopia (Stein 1989, Bebchuk and Stole 1993) or career concerns (Gibbons and Murphy 1992, Holmström 1999).

[4]See, e.g., Baber et al. 1991, Perry and Grinaker 1994, Bange and DeBondt 1998, Bushee 1998, Cheng 2004 and Gunny 2010, among others.

[5]We do not model how real and financial smoothing are implemented in practice. In Ronen and Sadan 1981, various smoothing mechanisms are discussed and illustrated in great detail. For empirical evidence regarding real smoothing, refer to Section 3.2.

[6]The analogy with the taxation literature is straightforward: outsiders' ownership stake acts ex post like a proportional tax on distributable income and undermines insiders' incentives to produce. Note that our underinvestment result does not require the presence of costly effort by insiders.

[7]The paper has complementary online appendices. Online Appendix A discusses extensions,

in particular, the effect of stock-based and sales-based insider compensation. Online Appendix B examines the robustness of our results by considering different income specifications (e.g., a Cobb-Douglas production function, rather than a quadratic cost function), different observables (such as output or input, rather than sales), and different latent variables (such as marginal revenues, rather than marginal costs). Online Appendix C provides elements of the proofs that have been omitted and a brief discussion of insiders' participation constraint. Online Appendix D presents a technical note on the optimality and accuracy of the Kalman filter adopted in the paper.

[8]When we have a public corporation with a large outside ownership stake, then collective action is as described in the Myers 2000 "corporation model." Outsiders take over the firm and displace insiders. The cost of collective action reflects the loss in managerial human capital, deadweight costs of getting organized, and so on. If we have a private company with a small outside ownership stake, then outsiders are minority stakeholders and the inside majority rules. Minority shareholders are, however, not entirely impotent as company law or commercial code grants minority shareholders either a judicial venue to challenge the decisions of management or the right to step out of the company by requiring the company to purchase their shares. The payoff from collective action to outside minority shareholders under this "oppressed minorities mechanism" (see La Porta et al. 1998) is therefore the fair value of their stake, net of any costs of intervention (such as a possible minority discount or legal costs).

[9]For $\theta = 0$, shareholders have no stake in the firm and the capital market constraint disappears. For $\theta = 1$, managers can no longer capture rents, and their objective function is no longer defined. Therefore $\theta \in (0, 1)$.

[10]Graham, Harvey, and Ragjgopal 2005 provide convincing evidence of how capital market pressures induce managers to meet earnings targets at all costs. As one surveyed manager put it: "I miss the target, I'm out of a job." Mergenthaler et al. 2012 find that CEOs are penalized via bonus cuts, fewer equity grants, and forced turnover when they just miss the latest consensus analyst forecast.

[11]It is possible for insiders' participation constraint to be violated under imperfect inference (see online Appendix C for further details).

[12]It is easy to see that $x_t \geq (\leq) x_{t-1} \iff B^* + w^*_{t-1} \geq (\leq)(1 - A)x_{t-1}$. Because $A < 1$, there is a force that pulls x down. On the other hand, there is an upward force because $B^* + w^*_{t-1}$ is

always positive. Depending on which force is the stronger, the process either moves up or down. As $x_{t-1} \to 0$, the probability of an upward move goes to 1.

[13]Mean reversion (i.e., $A < 1$) is a realistic assumption for production costs. For example, commodity prices (which constitute a large component of production costs in some industries) are often mean reverting due to the negative relation between interest rates and prices.

[14]For example, $\hat{x}_0$ is revealed to outside investors when the firm is set up at time zero.

[15]We could truncate the distribution of ϵ_t to ensure that outsiders' sales measurements, s_t, remain positive. Doing so would not change the results in any fundamental way. Durbin and Koopman 2012 show (see Section 2.2.4 and lemma 4 in Section 4.2) that the formulas for the Kalman filter estimate and its variance are valid, irrespective of whether or not the shocks ϵ_t and w_t are normally distributed.

[16]In the absence of normality for the disturbances w_t, our Kalman filter estimates remain minimum–variance linear unbiased estimates (MVLUE). There is, however, a nonlinear filter with a smaller mean square error, which is derived in online Appendix D. We show that for all economically relevant parameter combinations both filters are indistinguishable, making the Kalman filter (near) optimal. Online Appendix D also performs a series of diagnostic tests for the optimality of our Kalman filter.

[17]Under mild conditions (see note 30 in the Appendix) the Kalman filter converges to its steady state. Convergence is of geometric order and therefore fast (typically within four time steps for the simulations we ran; see Figure 3 in online Appendix D).

[18]The only difference is that in Lintner 1956 the payout target is determined by the firm's net income, whereas in our model the target is a function of sales because net income is not directly observed by outsiders.

[19]Fama and Babiak 1968 test Lintner's model for individual firms over a twenty year period and report a mean SOA of 0.32. Skinner 2008 finds an SOA for total payout of 0.4 and 0.55 for the periods 1980 to 1994 and 1995 to 2005, respectively.

[20]For $R = 0$ we get $P = Q$, $K = 1/H$ and $\lambda = 0$. Therefore, from Proposition 4 it follows that $\hat{x}_t = s_t/H$ and $s_t = Hx_t$. Consequently, $\hat{x}_t = x_t$.

[21]The traditional Laffer curve is a graphical representation of the relation between government revenue raised by taxation and all possible rates of taxation. The curve resembles an inverted

U-shaped function that reaches a maximum at an interior rate of taxation.

[22]For example, Bouwens and Kroos 2011 examine how retail store managers reduce their sales activity in response to target ratcheting. They find that managers with favorable sales performance in the first three quarters reduce their sales activity in the final quarter in an attempt to mitigate the increase in the next year's sales target. Holzhacker, Mahlendorf, and Matejka 2013 use data from 354 service units of a governmental agency and show that service unit targets are revised upward following good "own" performance but also following good peer performance.

[23]Survey-based evidence by Graham, Harvey, and Rajgopal 2005 indicates that: (i) insiders (managers) always try to meet outsiders' earnings per share (EPS) expectations at all costs to avoid serious repercussions; and, (ii) many managers underinvest by postponing or forgoing positive net present value (NPV) projects to smooth earnings and therefore engage in real smoothing. Roychowdhury 2006 finds that firms discount product prices to boost sales and thereby meet analyst earnings forecasts. Bhoraj et al. 2009 find evidence suggesting that firms that cut discretionary expenditures and/or manage accruals to achieve the latest analyst forecast benchmark achieve a short-run stock price benefit, but destroy long-run firm value. Finally, Daniel, Denis, and Naveen 2012 analyze situations in which the firm's cash flow from operations is insufficient to meet its expected levels of dividends and investment. They find that among dividend-paying firms with a cash flow shortfall, over two-thirds reduce investment (relative to median industry levels).

[24]Note that the firm's replacement value is a constant in our model. Therefore, Tobin's q is the outside equity value scaled down by a constant.

[25]Another difference is that in Stein 1989 stock prices are strong-form efficient at all times because outsiders can reconstruct the original earnings stream from the observed earnings. In contrast, stock prices are unbiased but only semi-strong efficient in our model because outsiders constantly learn and update their expectations on the basis of observable signals that act as a noisy proxy for the unobserved output variables seen only by the insiders.

[26]Other important but less closely related papers on smoothing include Ronen and Sadan 1981, Lambert 1984, Trueman and Titman 1988, Dye 1988, Fudenberg and Tirole 1995, Kanodia and Mukherji 1996 and Tucker and Zarowin 2006, among others.

[27]In our model insiders have an incentive not to raise outsiders' expectations regarding income. Opposite incentives arise in Bagnoli and Watts 2010, who examine the interaction between product

market competition and financial reporting. They show that Cournot competitors bias their financial reports so as to create the impression that their production costs are lower than they actually are.

[28]If $\{w_t\}$, $\{\epsilon_t\}$ are zero-mean, uncorrelated, and white (as is the case for our model) then the KF is the best linear solution to our problem, that is, the KF is the best filter that is a linear combination of the measurements. If $\{w_t\}$, $\{\epsilon_t\}$ are also Gaussian, then the linear KF is also optimal among all possible filters. In the absence of normality for the disturbances w_t, our steady-state Kalman filter estimates remain minimum-variance linear unbiased estimates (MVLUE). There is, however, a nonlinear filter with a smaller mean square error, which is derived in online Appendix D. We show that for all economically relevant parameter combinations both filters are indistinguishable, making the Kalman filter (near) optimal.

[29]Simon (2006, 194) shows that the Kalman gain converges to a steady state often after a few time steps, and he argues that "for many problems of practical interest the performance of the steady-state filter is nearly indistinguishable from that of the time-varying filter." In our simulations (see, e.g., Figure 3 in online Appendix D), convergence is obtained within four time steps.

[30]If there is little prior history regarding sales s_t, then K_t itself will vary over time because P_t, the variance of the estimation error, initially fluctuates over time. Once a sufficient number of observations have occurred P_t, and therefore K_t, converge to their stationary level P and K. A sufficient condition for the filter to converge is that $\lambda\ A < 1$. The order of convergence is geometric (see Chui and Chen, 1991, 88, Theorem 6.1).